On the Cost of Misperception: General Results and Behavioral Applications

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Two Ideas Combined

Ecological Rationality

- Simon, Gigerenzer
- objective: performance across all problems in the DM's environment
- optimization over rules

Error Management

- Haselton, Buss, Nettle
- objective: expected loss in a given decision problem
- optimization over distributions of errors

we:

- characterization: loss across the DM's environment
- optimization: management of perception error

In This Paper

loss characterization:

distr. of decision problems $\times$ misperception $\longrightarrow$ expected loss

second-best perception:

- errors are not completely avoidable
- biases help to avoid costly types of errors

applications:

overprecision, anchoring, illusion of control, prob. weighting

Literature

hedonic beliefs
Brunnermeier & Parker (2005), Caplin & Leahy (2001), Köszegi (2006)

evolution of preferences
Robson (2001), Netzer (2009)

rational inattention
Sims (1998, 2001), Woodford (2012a,b)

second-best perception
Steiner & Stewart (2016)

Outline

Loss from Misperception

Loss Characterization

Second-Best Perception

Applications

Bayesian Sophistication

A Decision Problem

$a \in \{1, \ldots, n\}$

$u(a, \theta)$, $\theta \in \{0, 1\}$

p objective probability of the state 1

q subjective probability of the state 1

$u(a, p) = (1 - p)u(a, 0) + pu(a, 1)$

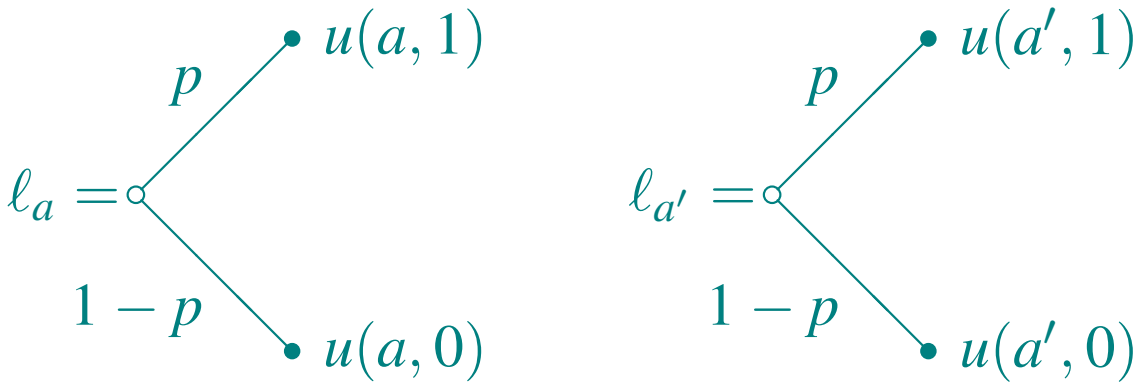
$a_q^* \in \arg\max_a u(a, q)$

$\ell(p, q; u) = \max_a u(a, p) - u\left(a_q^*, p\right)$

Example

the DM chooses one from n binary lotteries

e.g. financial bets on a binary event



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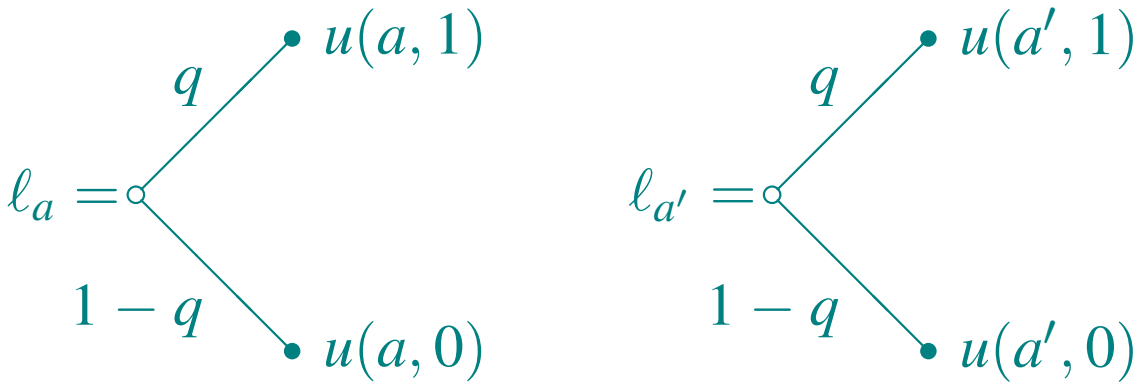
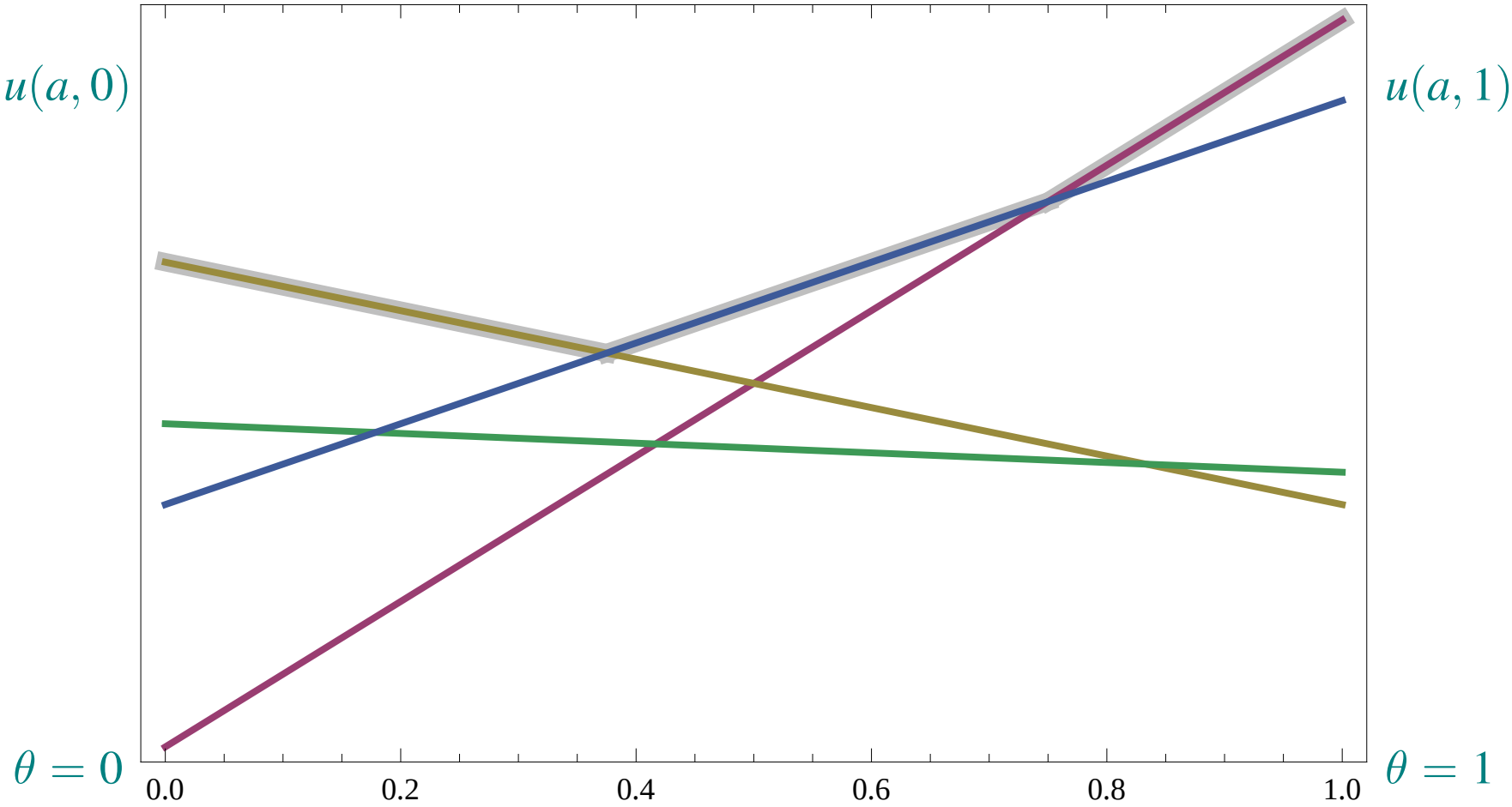
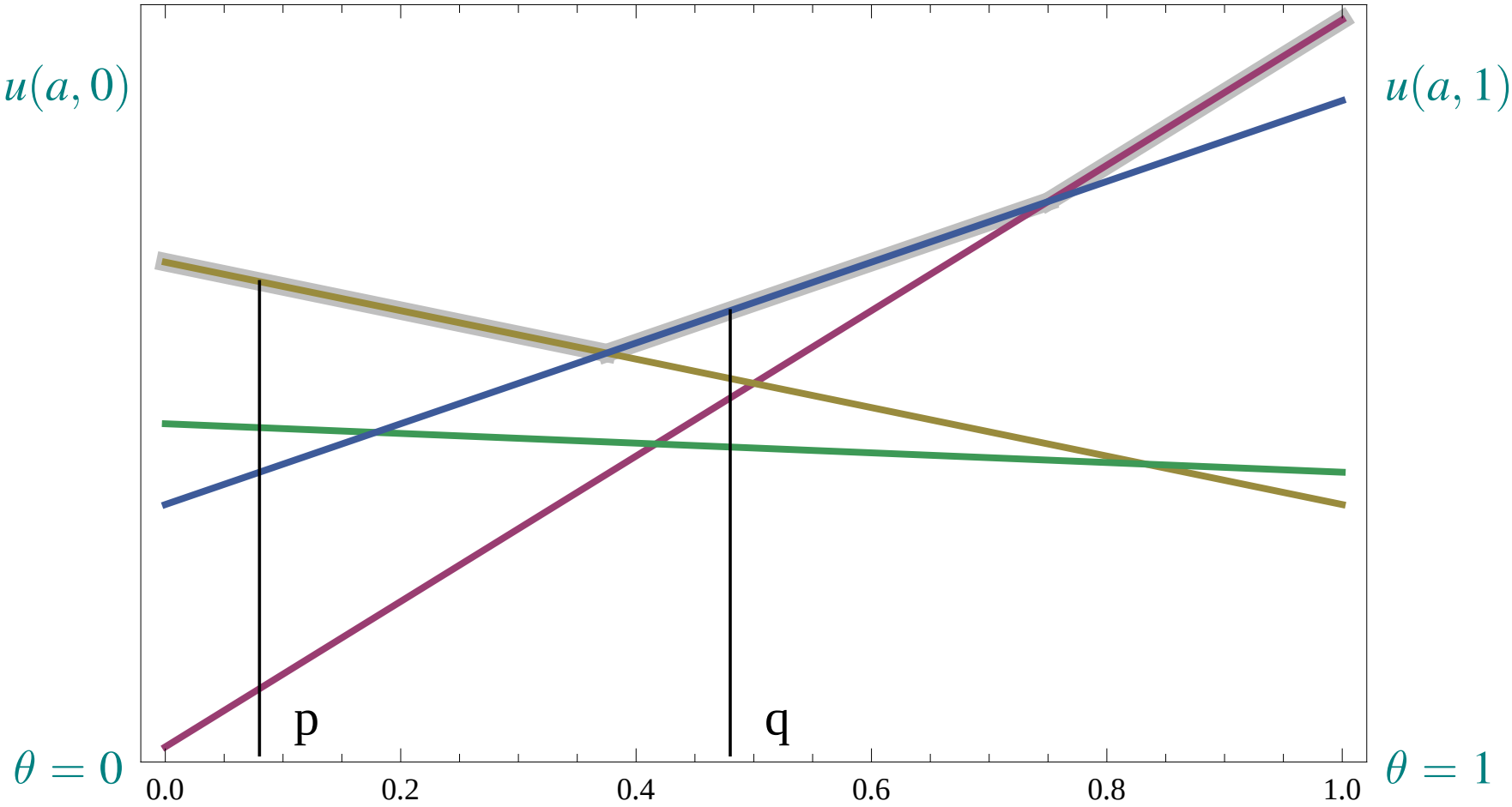
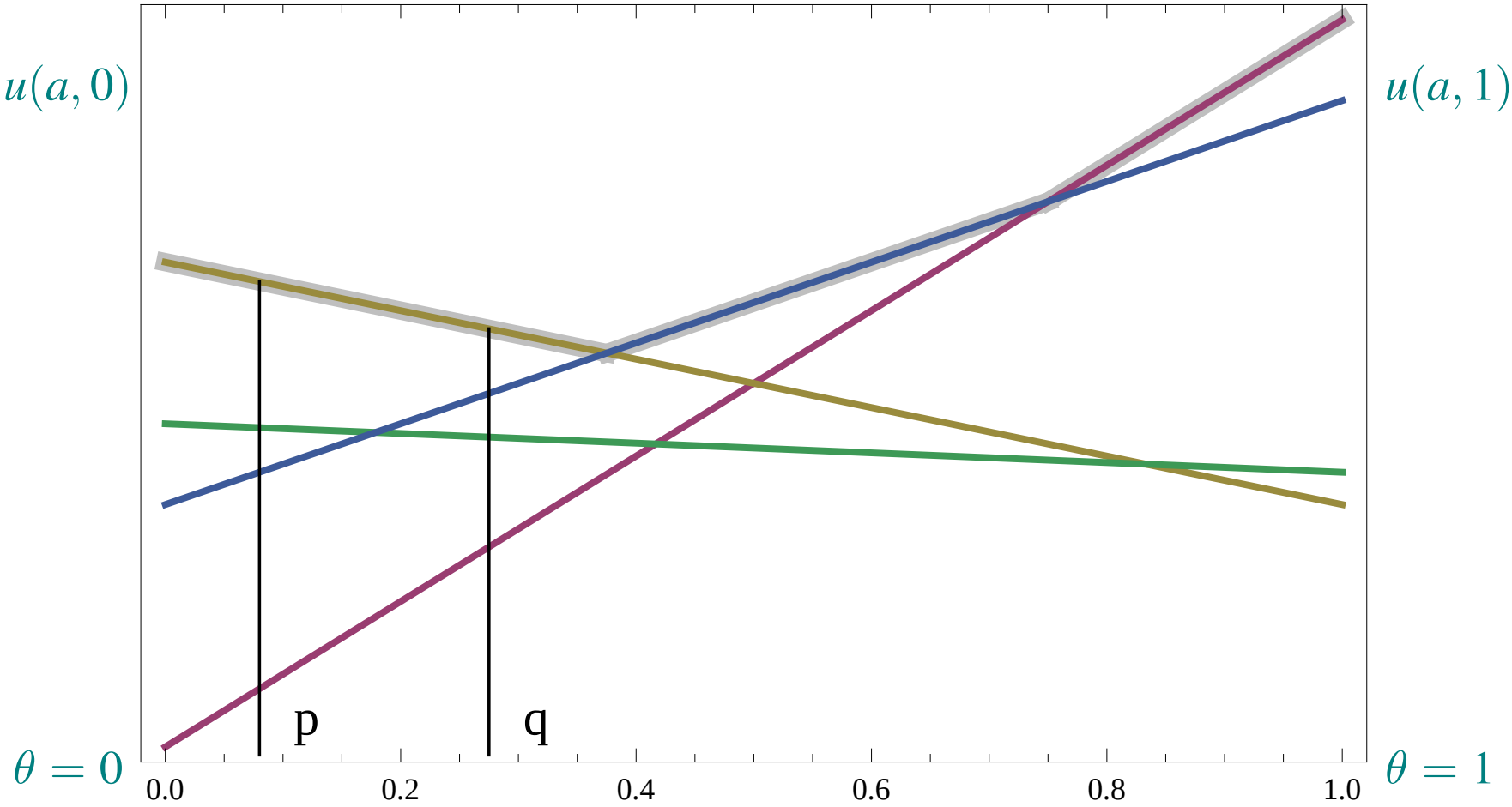



$u(a,0)$
$u(a,1)$
$\theta = 0$
$\theta = 1$
0.0
0.2
0.4
0.6
0.8
1.0



loss



no loss

Environment

the DM encounters many decision problems in her environment

$u = (u(a, \theta))_{a \in A, \theta \in \Theta}$ is a random variable with support $\mathbb{R}^{2n}$

each draw of u is a decision problem in the DM's environment

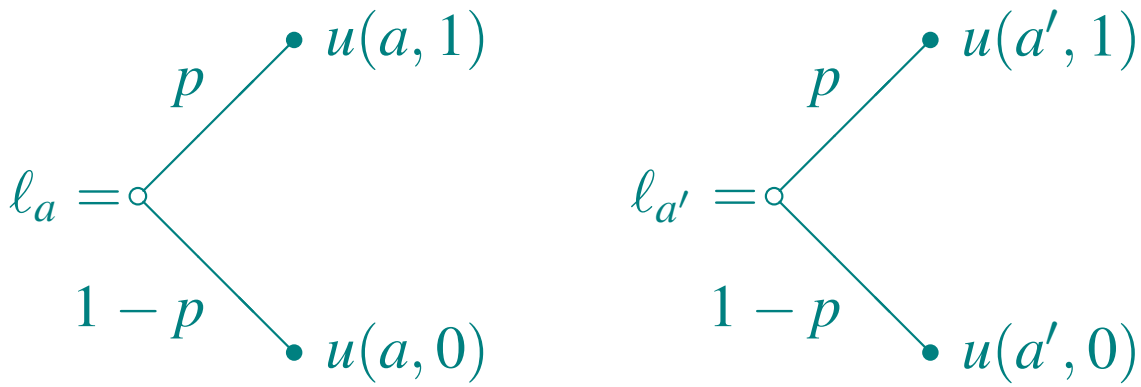
loss from misperception in the DM's environment:
ex ante expected loss from misperceiving true probability p as q

$$L(p, q) = \mathrm{E}_u[\ell(p, q; u)]$$

Example

precision

the DM chooses one from n binary lotteries



$u(a,\theta)$ are iid $\sim \mathcal{N}(0,1)$, observed at the choice stage

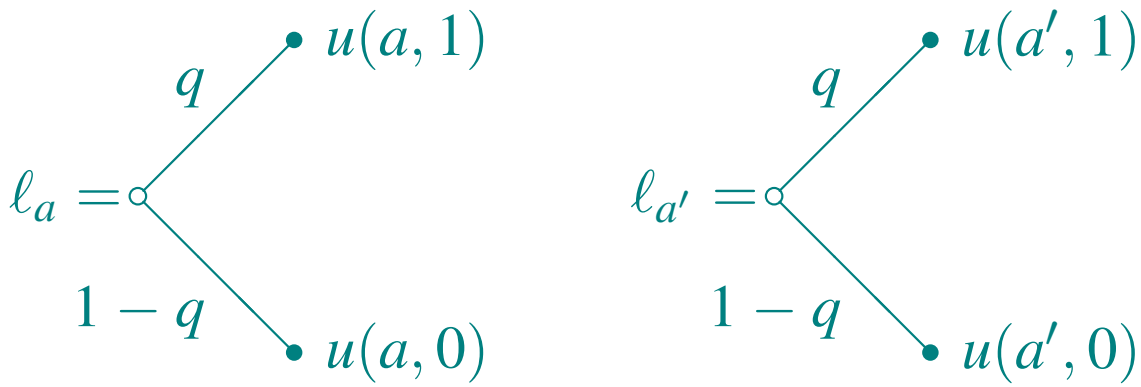
further away q is from $1/2$, the more knowledgeable DM feels

loss from misperception of the precision?

Example

precision

the DM chooses one from n binary lotteries



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Loss Characterization

Second-Best Perception

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Main Result

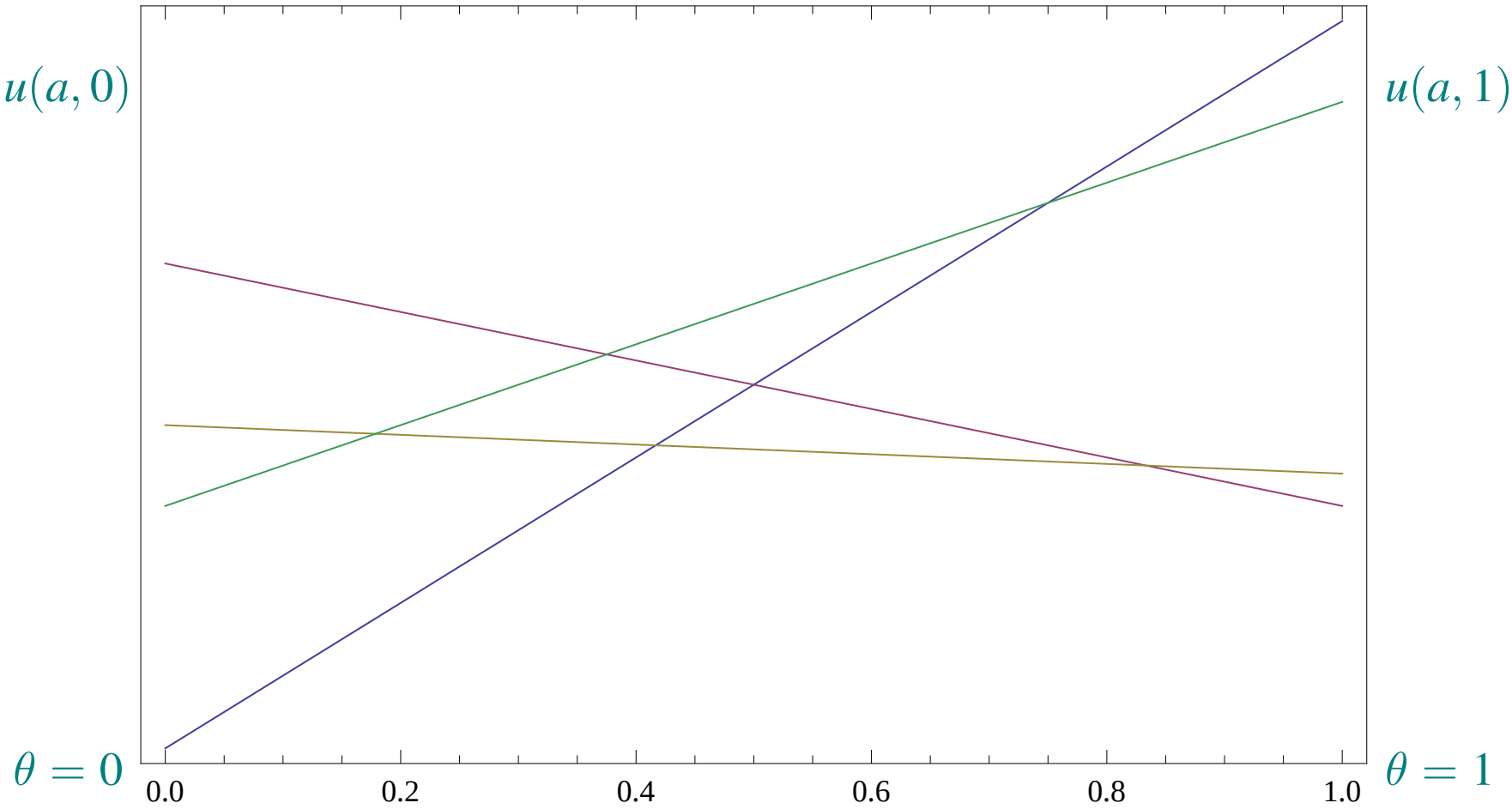
how much does the DM lose if she misperceives p for q?

$$L(p,q) = \int_p^q (s-p)v''(s)ds$$

how well-off is the realistic DM who optimizes?

$$v(p) = \mathrm{E}_u[\max_a u(a,p)]$$

$\ell(0, 1; u)$



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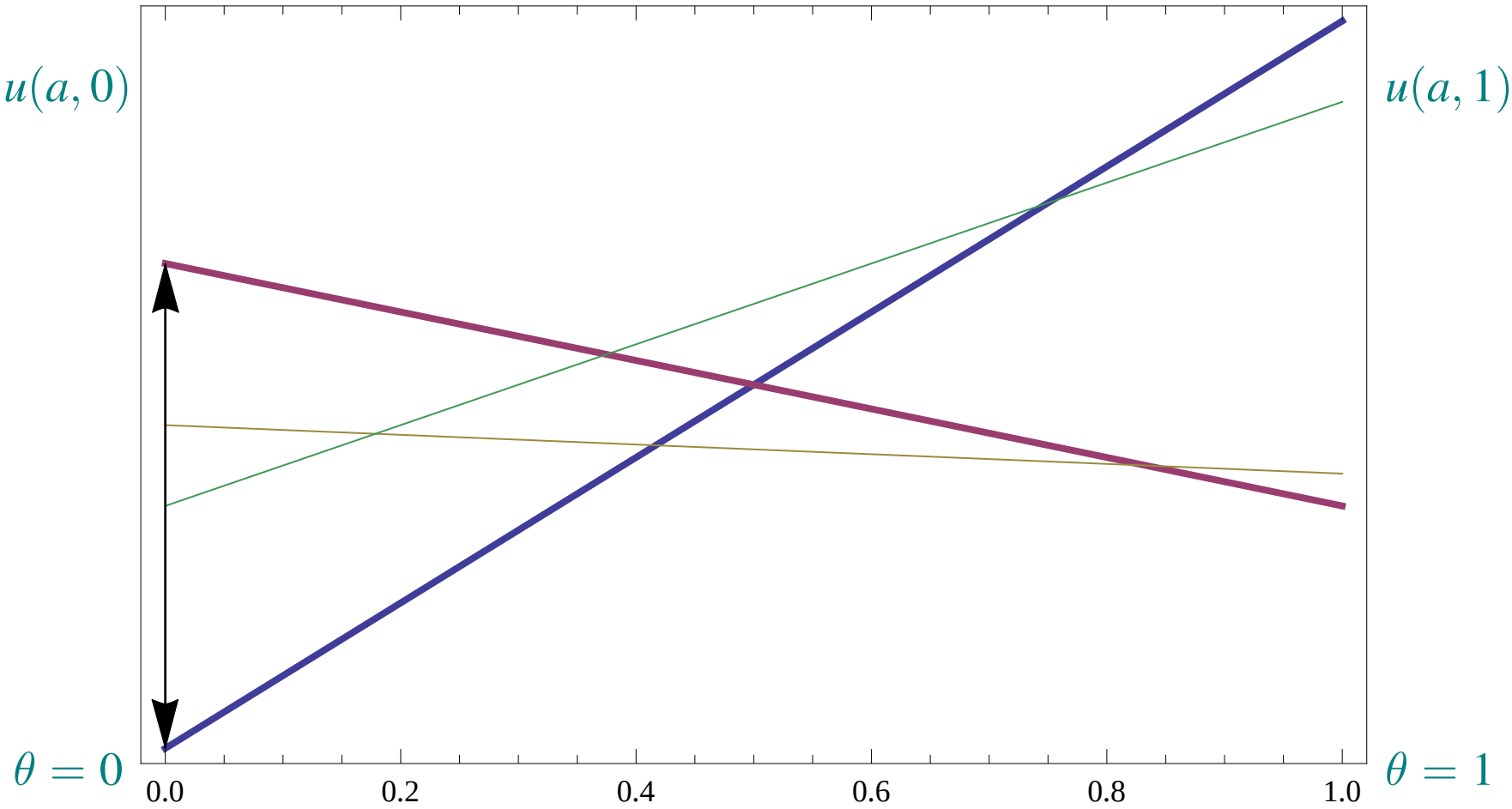

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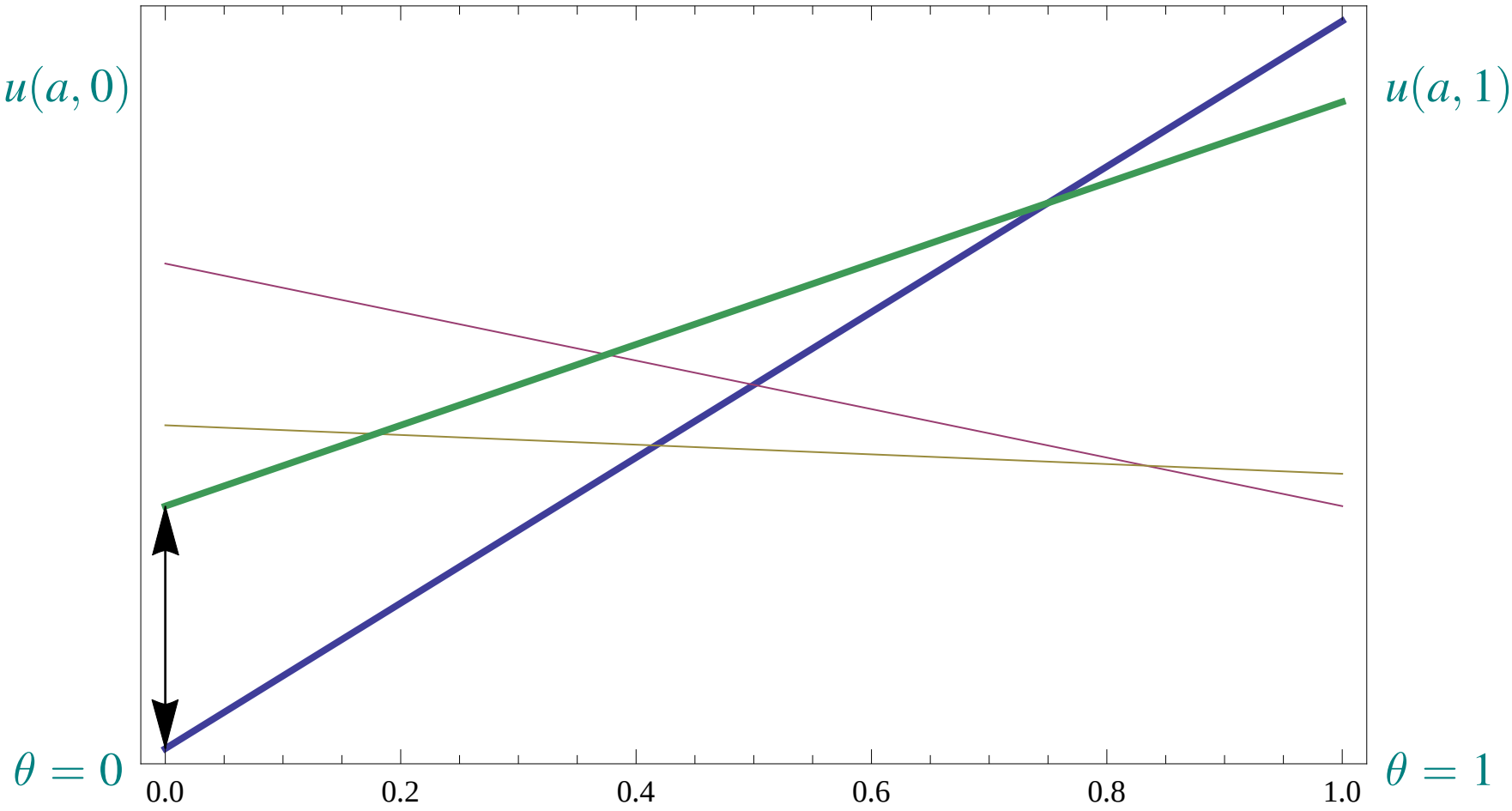

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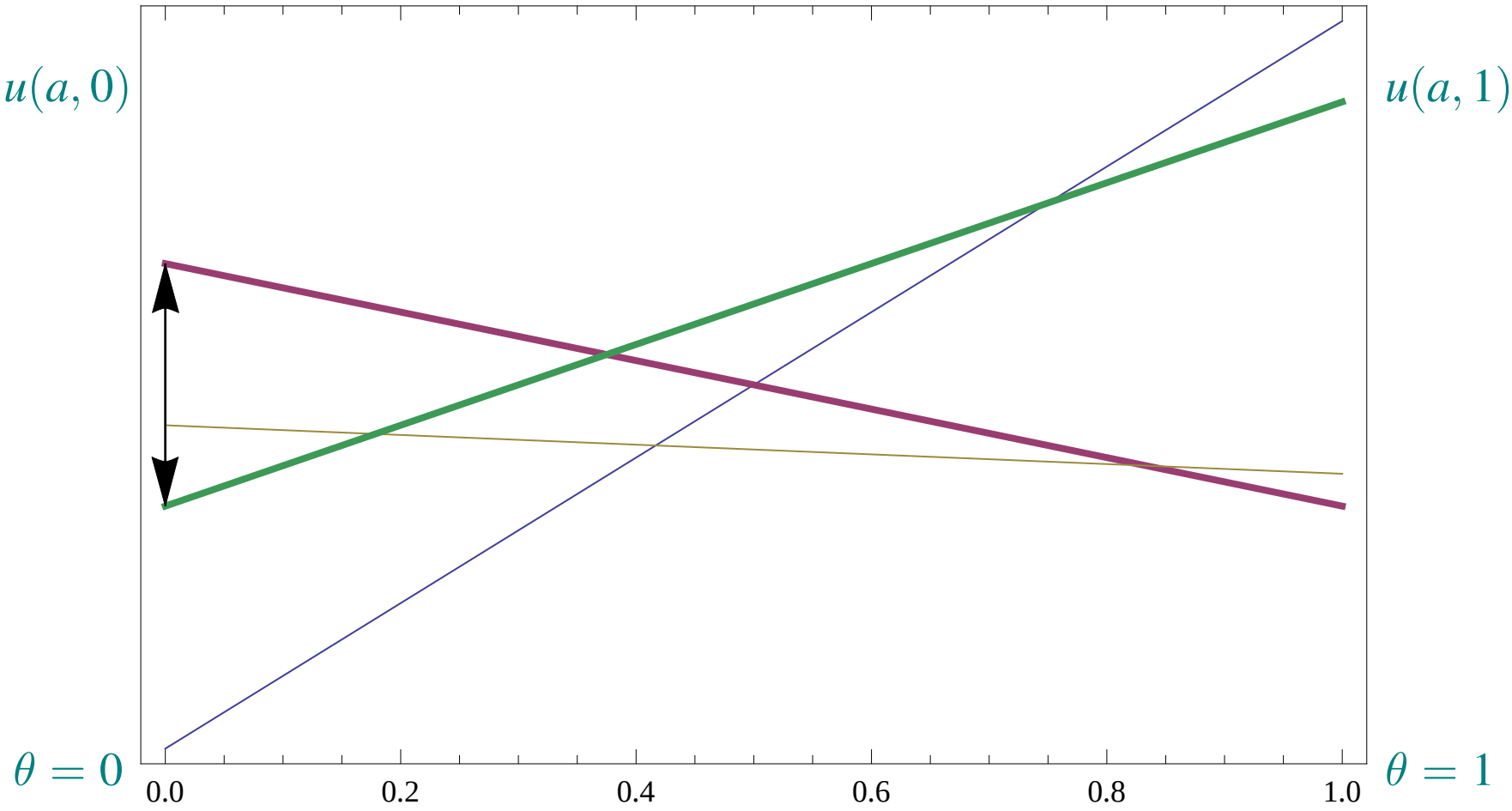


Towards the Ex Ante Loss

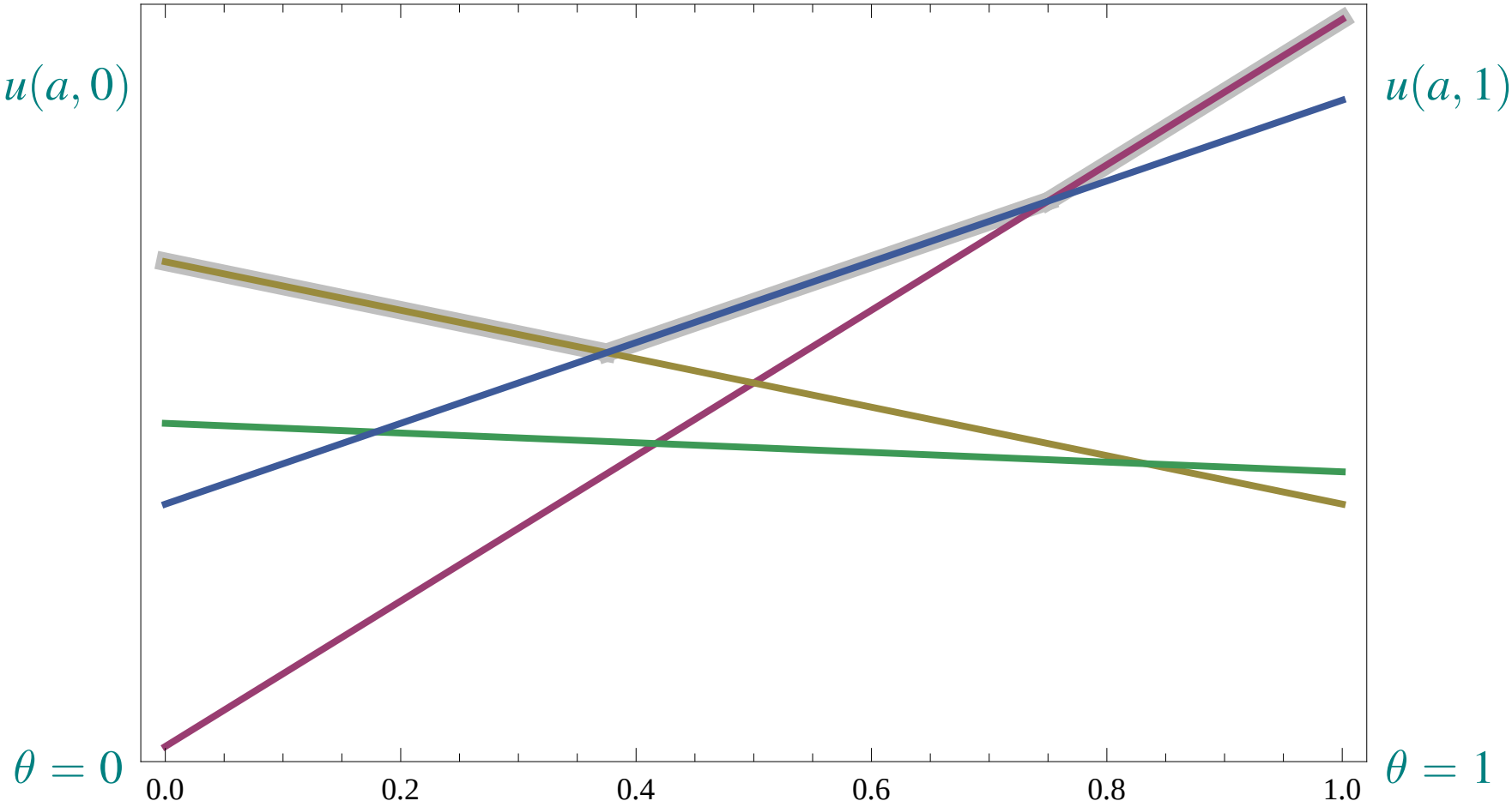
let us build a function $w(s)$ that captures:

- likelihood of a tie at s between the best and the second-best action
- angle of the intersection

then:

$$L(p,q) = \int_p^q (s-p)w(s)ds$$

Value Function



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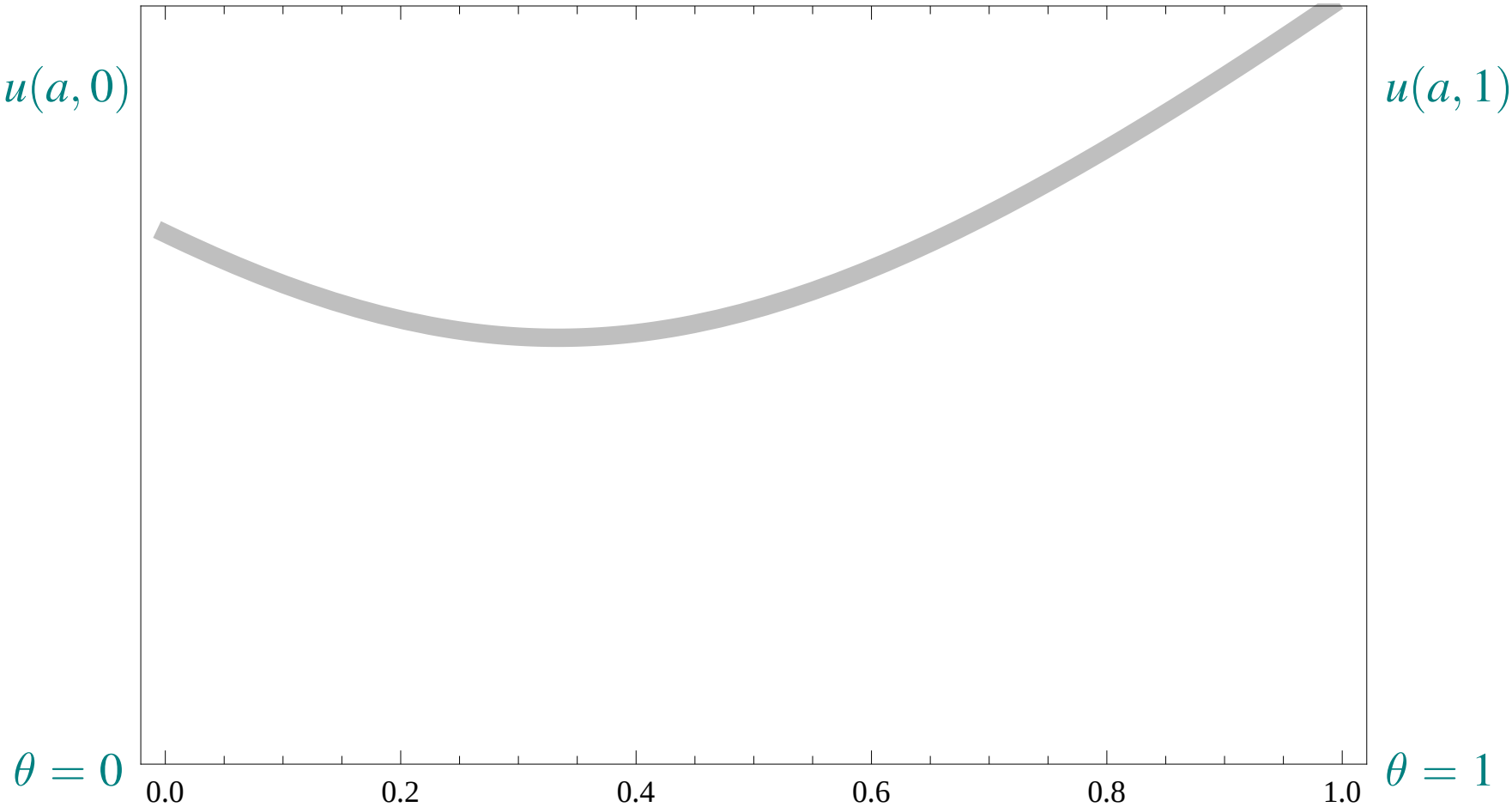
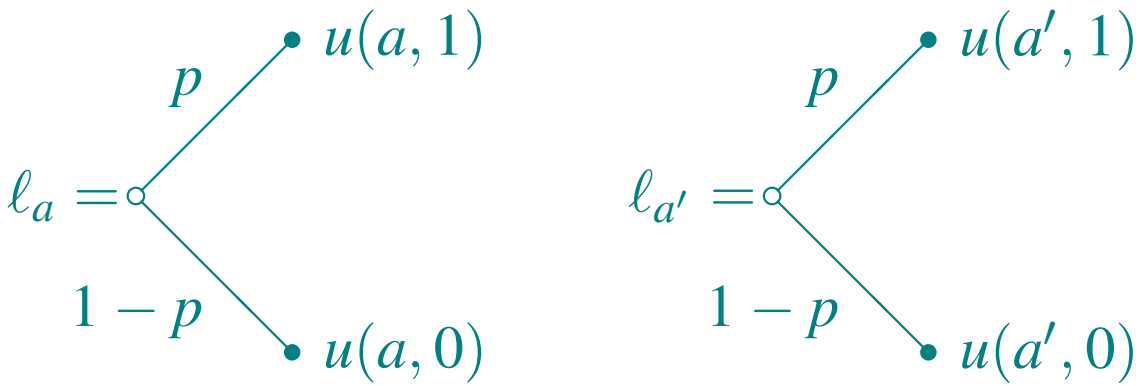


$v''(p)$ is the expected intensity of the kink at p

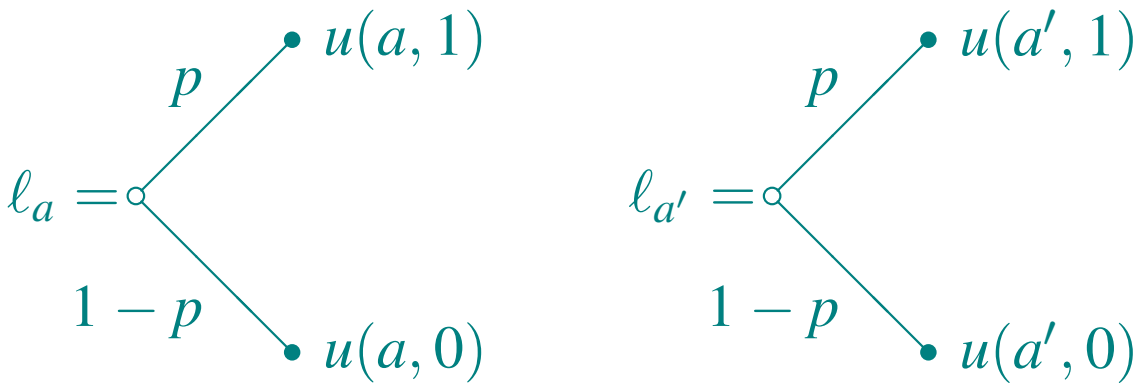
Back to the Example



$v(p)$ is the expectation of the maximum over n draws of

$$u(a,p) \text{ iid} \sim \mathcal{N}\left(0, p^2 + (1-p)^2\right)$$

Back to the Example



$$v(p) = \text{const}_n \times \sqrt{p^2 + (1-p)^2}$$

$$u(a,p) \text{ iid} \sim \mathcal{N}\left(0, p^2 + (1-p)^2\right)$$

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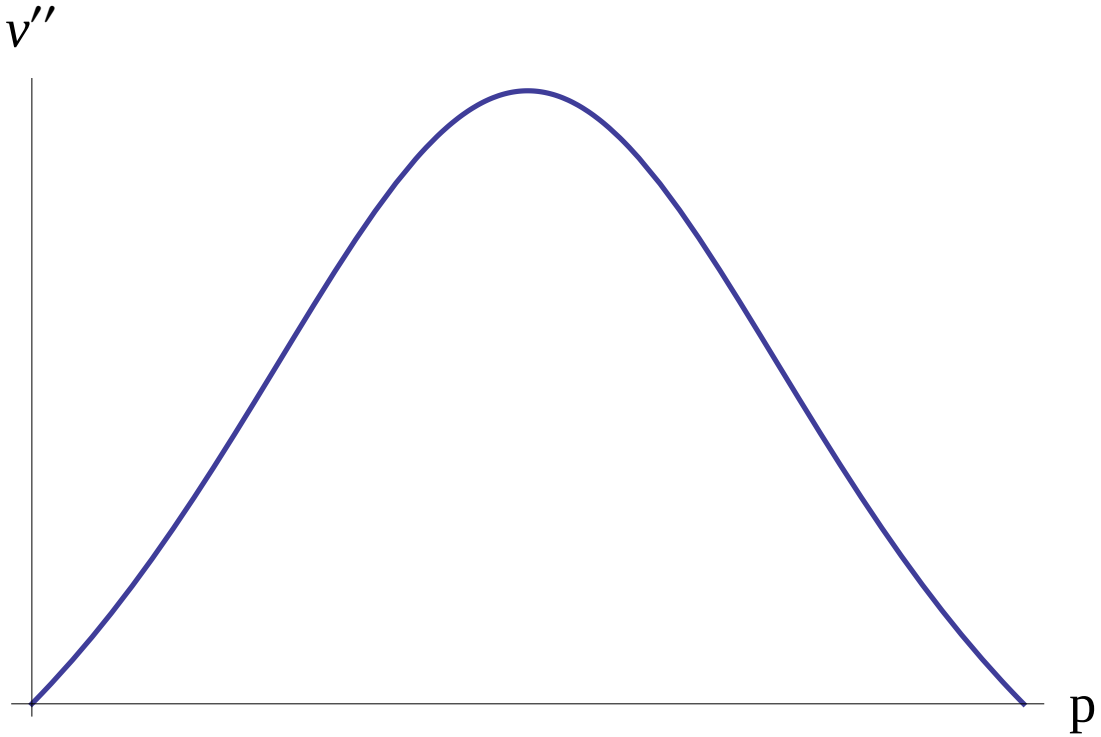

intuition:

- misperception of p for q is costly iff tie arises inbetween
- ties are concentrated near $1/2$
- under-precision is costlier than over-precision

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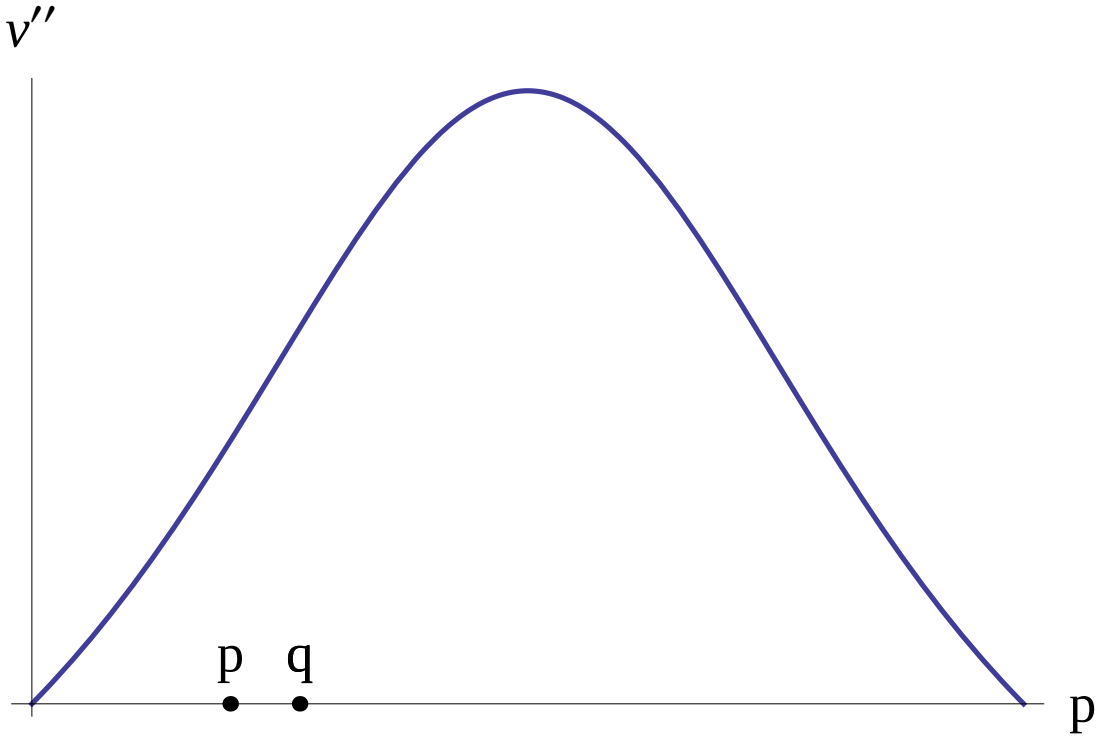

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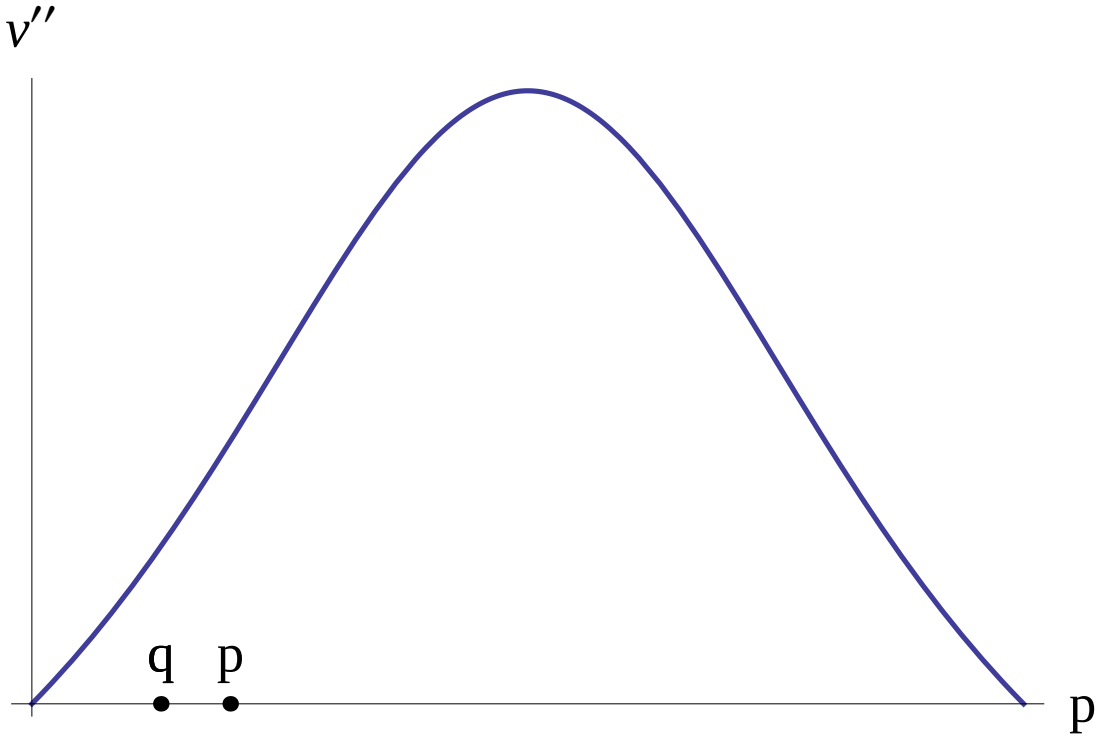

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Loss Characterization

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Perception Problem

the first-best perception, $p = q$, is infeasible

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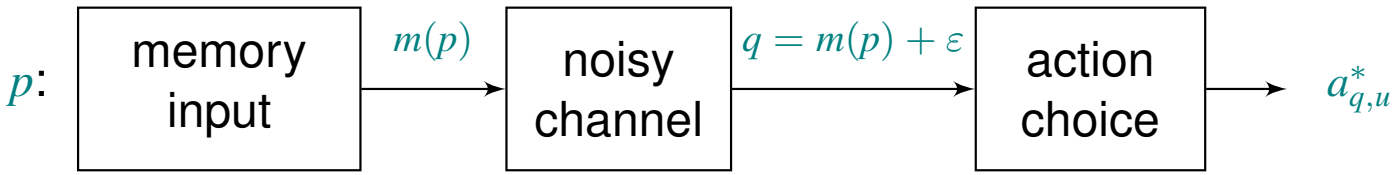

ε symmetrically distributed on $[-\sigma, \sigma]$

Perception Problem

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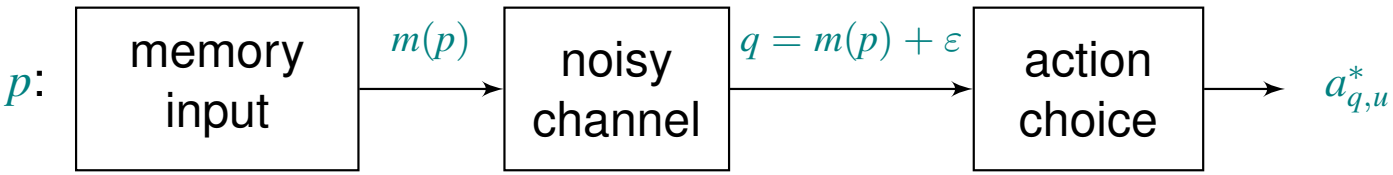

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unbiased perception is feasible: $m(p) = p$ leads to $\mathrm{E}_\varepsilon[q \mid p] = p$

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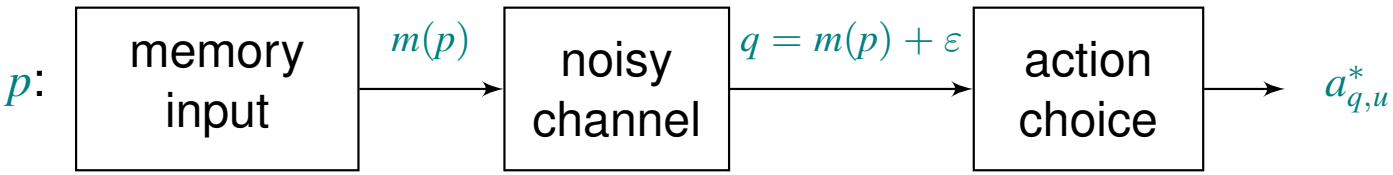

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Definition (Naive Perception problem)

For each p,

$$\min_{m(p)\in[\sigma,1-\sigma]} \mathrm{E}_\varepsilon \left[L\big(p, m(p) + \varepsilon\big)\right]$$

Optimality Condition

mean-square-error intuition corrected

Proposition

Let $q = m^*(p) + \varepsilon$ be the stochastic second-best perception when the true probability is p. Then,

$$\mathrm{E}_q \left[v'' (q) (q - p) \right] = 0.$$

(except of the corners)

Direction of the Bias

Proposition

Let $p \in (\sigma, 1-\sigma)$.

1. *If, for all* $\delta \in (0, \sigma)$,

$$v''(p+\delta) < v''(p-\delta),$$

then an upward bias occurs: $m^*(p) > p$.

2. *If, for all* $\delta \in (0, \sigma)$,

$$v''(p+\delta) > v''(p-\delta),$$

then a downward bias occurs: $m^*(p) < p$.

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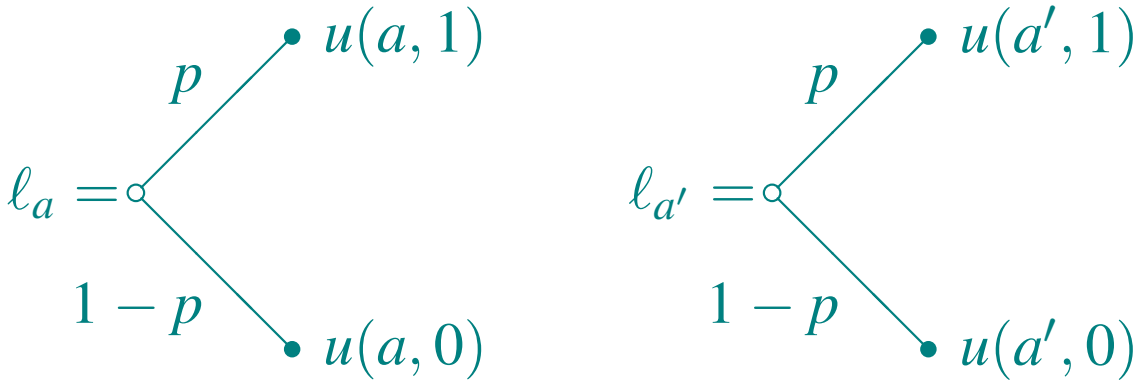
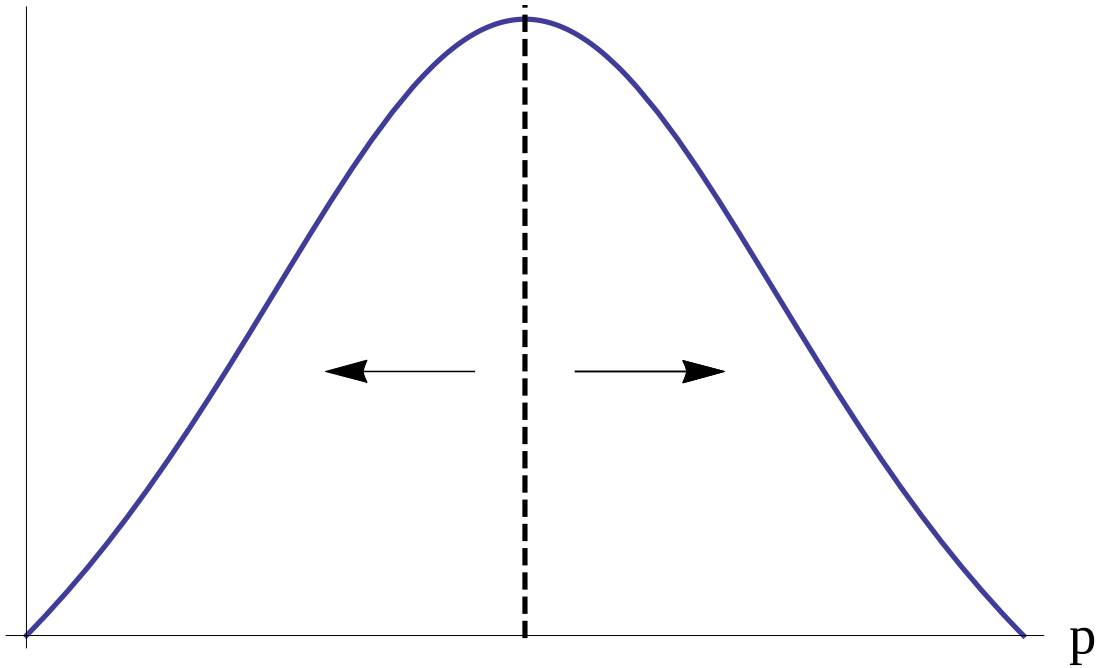
the DM chooses one from n binary lotteries



$u(a,\theta)$ are iid $\sim \mathcal{N}(0,1)$, observed at the choice stage

Overprecision



if $p < 1/2$, then $m^*(p) < p$
if $p > 1/2$, then $m^*(p) > p$

Anchoring

the DM chooses an object $a \in \{1, 2\}$

quality of each object a is $x_1^a + x_2^a$

the DM observes:

- the first components of both objects in round 1
- the second components in round 2

$\Delta_t = x_t^2 - x_t^1$

Δ_1 is the first impression

Anchoring

the DM in round 1:

- observes the true Δ_1
- memorizes a value $m(\Delta_1)$

the DM in round 2:

- recalls subjective $\hat{\Delta}_1 = m(\Delta_1) + \varepsilon$
- observes the true Δ_2
- chooses object 2 iff $\hat{\Delta}_1 + \Delta_2 > 0$

expected loss from misperception:

$$L(\Delta_1, \hat{\Delta}_1) = \mathrm{E}_{\Delta_2}\left[(\Delta_1 + \Delta_2)\mathbf{1}_{\Delta_1+\Delta_2>0} - (\Delta_1 + \Delta_2)\mathbf{1}_{\hat{\Delta}_1+\Delta_2>0}\right]$$

what should the DM memorize to minimize the loss?

Anchoring

assumption: density of Δ_2 is single-peaked, symmetric

result: optimal exaggeration of the first impression:

$$\text{if } \Delta_1 > 0, \text{ then } m^*(\Delta_1) > \Delta_1$$
$$\text{if } \Delta_1 < 0, \text{ then } m^*(\Delta_1) < \Delta_1$$

intuition:

- misperception of Δ_1 for $\hat{\Delta}_1$ causes loss iff it reverts choice
- reversal arises iff tie happens at $\tilde{\Delta}_1$ between Δ_1 and $\hat{\Delta}_1$
- ties are more likely when the first impression is near 0
- $\Rightarrow$ asymmetry in the cost of the errors

Anchoring

$v(\Delta_1)$ — value of the realistic DM across all draws of Δ_2

loss from misperception:

$$L(\Delta_1, \hat{\Delta}_1) = \int_{\Delta_1}^{\hat{\Delta}_1} v''(s)(s - \Delta_1)d\Delta_1$$

v'' is single-peaked, symmetric around 0:

$$v''(\Delta_1) = \psi(-\Delta_1) = \psi(\Delta_1)$$

Illusion of Control

the DM is of either:

- low type—choice has a low impact, or
- high type—choice has a high impact

on her own well-being in most (but not all) decision problems

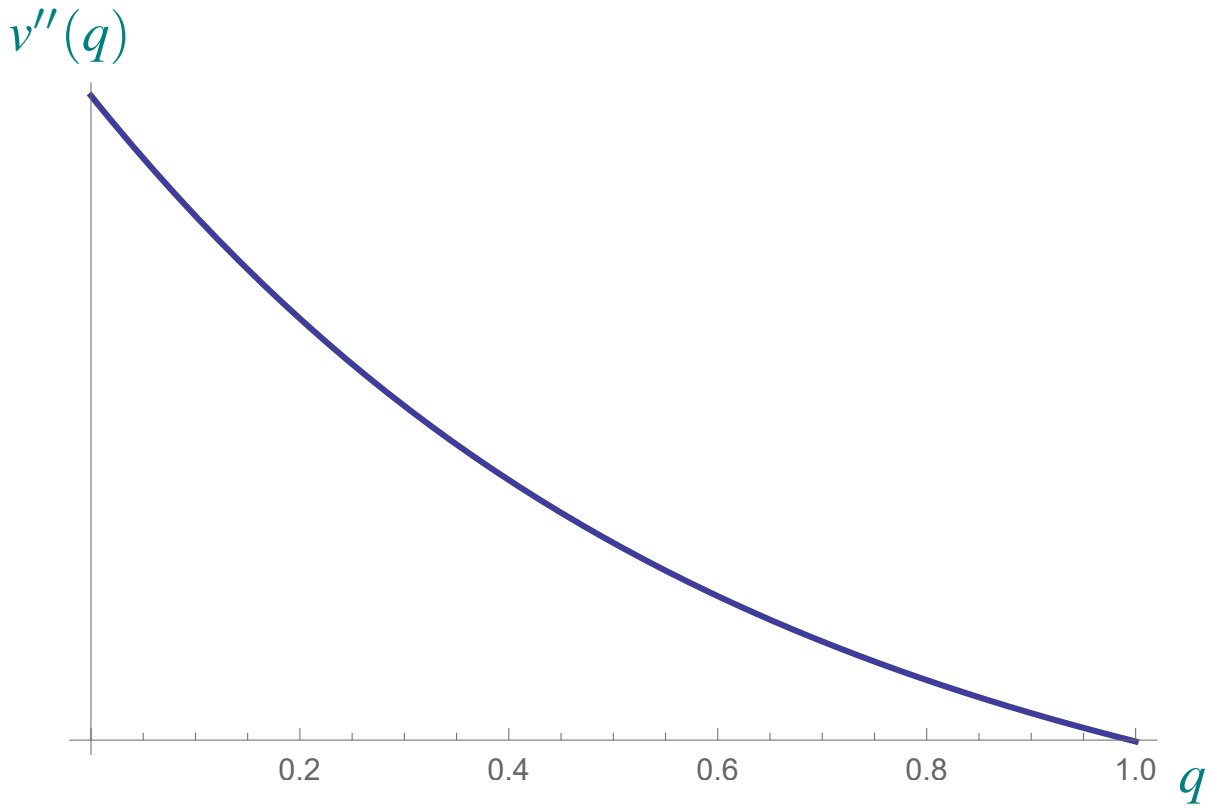
the high type may stand for:

- high talent
- more informed type
- less randomness in the impact of choice

formalization:

- $u(a, \theta) = u_a + \tau_\theta \eta_a$
- u_a, η_a are normal r.v.s, iid across actions, $\tau_1 > \tau_0 > 0$

Illusion of Control



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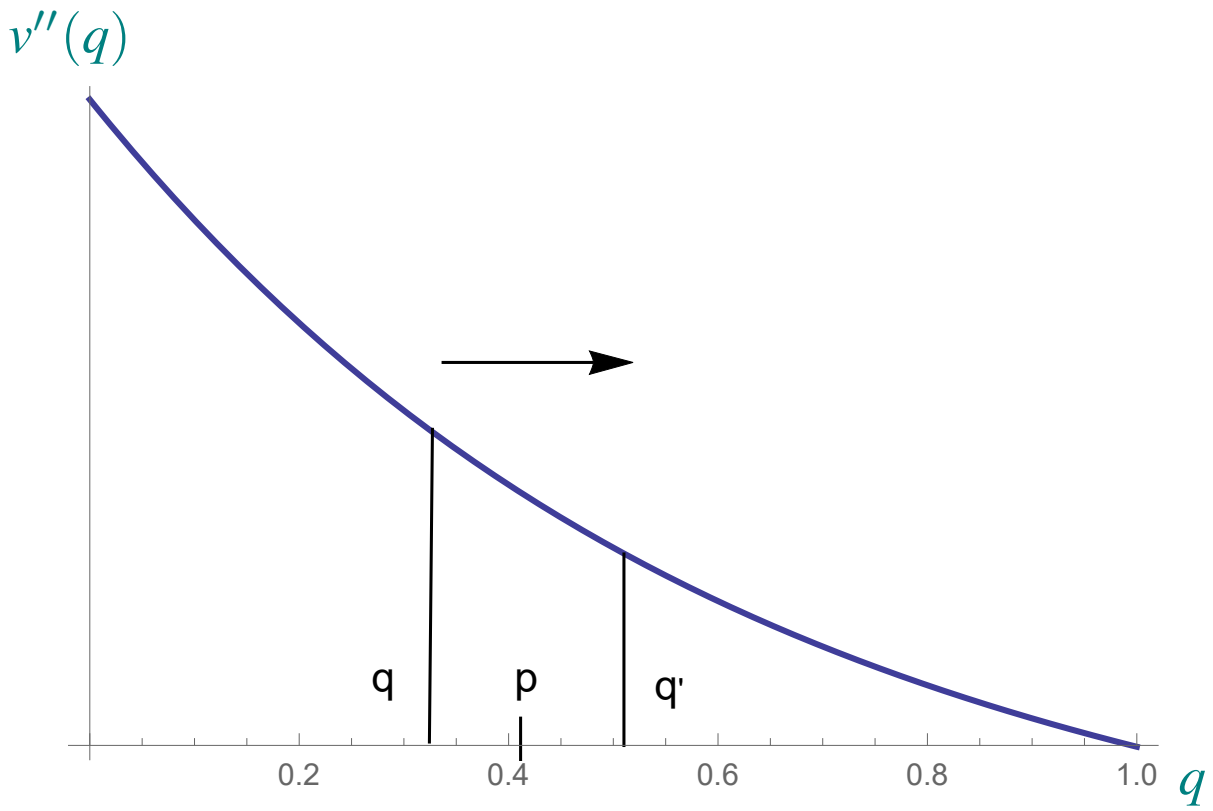

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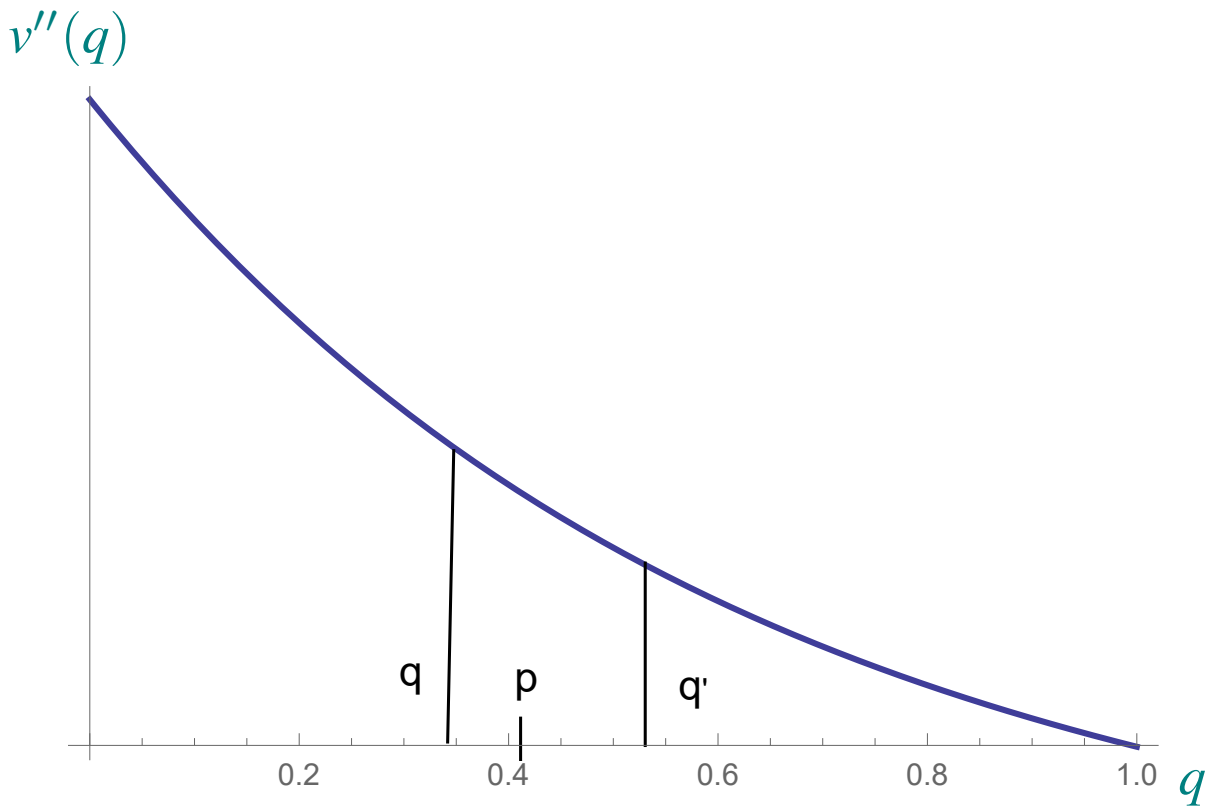


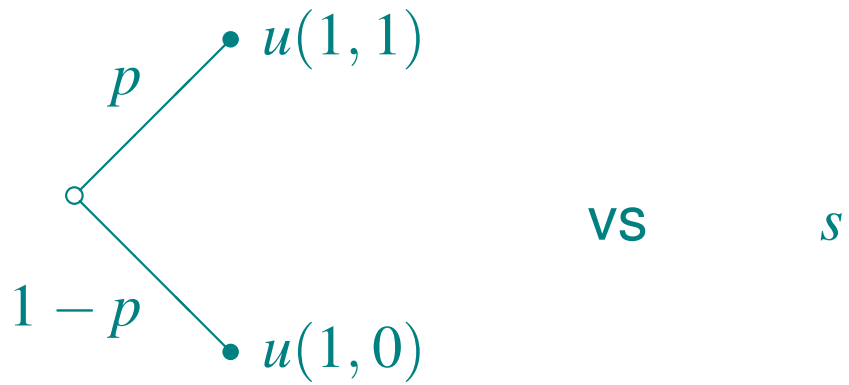
Illusion of Control

intuition:

- variance of $u(a, p)$ increases in p
- $\Rightarrow$ likelihood of tie decreases in p
- $\Rightarrow$ upward error is more likely consequential than the downward one of a same size

Probability Weighting

Steiner and Stewart (2016)



$u(1,\theta)$ are iid $\sim \mathcal{N}(0,1)$

if $s > 3^{1/4}$, then $v''(p)$ is U-shaped

Probability Weighting

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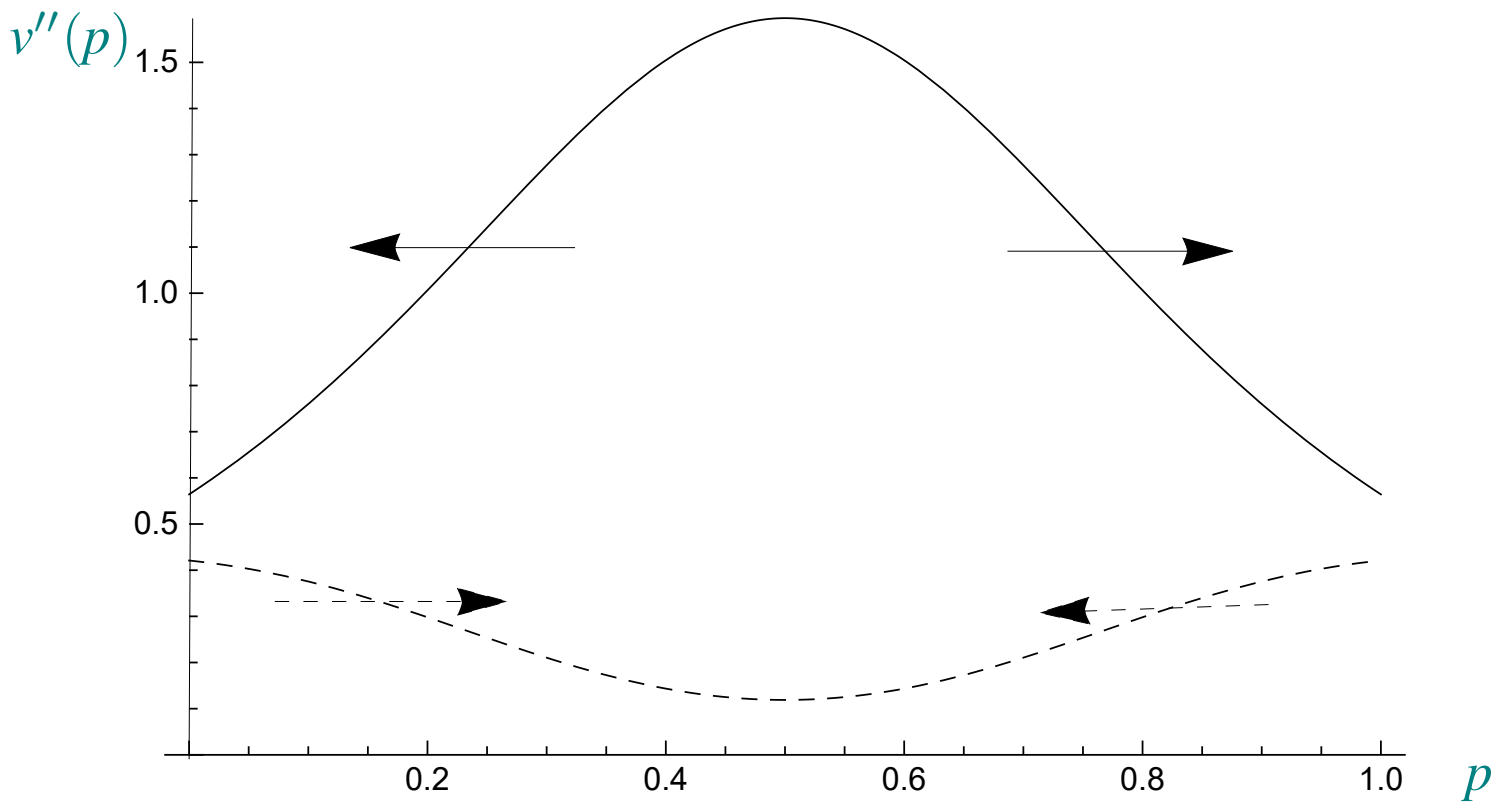

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From Naivety to Sophistication

our naive DM

- hasn't used all the retained information at the choice stage
- has used a rigid decision rule

we now examine a sophisticated DM who

- forgets some information during the decision process
- forms bayesian posteriors using all the retained information
- can employ any decision rule

we'll find

- optimal bias in the same direction
- caused by the same forces

Sophisticated Perception Technology

$\theta \in \{0, 1\}$, uniform prior

the DM:

- observes signal about θ and updates her belief to p
- chooses to memorize whether p is "low" or "high"
- forgets p
- observes $u(.,.)$
- chooses a to maximize $\mathrm{E}[u(a, \theta)|$her information set$]$

Sophisticated Perception Technology

binary message space $M = \{0, 1\}$

perception function $m(p)$, $m : [0, 1] \longrightarrow M$

choice rule $c(u, m)$, $c : \mathbb{R}^{2n} \times M \longrightarrow A$

Definition (sophisticated perception problem)

$$\max_{m(\cdot), c(\cdot,\cdot)} \mathrm{E}_{u,p} \left[u \left(c\big(u, m(p)\big), p \right) \right].$$

Optimal Partition

assumptions: density of p is

- symmetric around $1/2$
- independent from payoffs

Proposition

The DM partitions the probability interval into $\{[0, p^*), [p^*, 1]\}$.
If $v''(p)$ *is decreasing, then* $p^* < 1/2$.

$\Rightarrow$ the high type is more prevalent than the low one

Intuition

$v''(p)$ measures the local value of information:

- consider a DM whose current belief is p
- her value in absence of further information is $v(p)$
- how helpful is a weakly informative signal?

$$\mathrm{E}_{\text{posterior}}\left[v(\text{posterior})\right] - v(p)$$

if v'' decreases, then

- distinguishing among high probabilities is less important than among the low ones
- the optimal partition reflects that

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- consider a DM whose current belief is p
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$$v''(p)\text{Var}(\text{posterior})/2$$

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Discussion

are the perception biases mistakes?

the biases are optimal given the designer's information about the decision problem

what if the outside observer has finer information about the decision problem than the perception designer?