Attention, please!

Olivier Gossner Jakub Steiner Colin Stewart

CNRS/LSE, Zurich/Cerge, Toronto

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"The successful advertisement . . . forces itself upon the attention. . . . Everyone reads it involuntarily,. . . "

Fogg-Meade, Journal of Political Economy, 1901

What do we do

- study dynamic information acquisition and choice
- flexible allocation of attention
- attention need not be optimized
- study attention-grabbing manipulation
  i.e. increase of attention to a target item

What do we do

- study dynamic information acquisition and choice
- flexible allocation of attention
- attention need not be optimized
- study attention-grabbing manipulation
  i.e. increase of attention to a target item

Impact on choice

- under general conditions:

  forcing attention to an item increases demand for it

- even if the item has a low value
- sensitive to the stopping rule, may depend on the outside option

Literature

Empirical

- Psychology: Armel Beaumel Rangel '08: attention in choice
- Marketing: Chandon et al '09: store shelves
- Finance: Seasholes Wu '07: attention-grabbing predicts demand
- Zoology: Yorzinski et al' 13: attention in peahens during courtship

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Sequential Learning

- Econ: Wald '45, Fudenberg Strack Strzalecki '18, Morris Strack '17, Liang Mu Syrgkanis '19, Ke Villas-Boas '19, Austen-Smith Martinelli '18
- Psychology: Ratcliff '78, Krajbich, Armel, Rangel '10: aDDM, Bogacz et al '06: race accumulators

Table of Contents

1 Example

2 Attention and its manipulation

3 Behavioral Consequences

Example

Choice set

- item $i \in \{1, 2\}$ or outside option
- $v^i \in \{-1, 1\}$ unknown value of item i, known $v^{oo} \in (-1, 1)$

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Sequential learning

- ι_t item inspected in t
- x_t signal on ι_t: $\text{pr}(x_t = v^{\iota_t}) = \lambda > \frac{1}{2}$
- log-likelihood ratios

$$p_t^i = \sum_{s<t:\iota_s=i} \log \frac{\Pr\left(x_s \mid v^i = 1\right)}{\Pr\left(x_s \mid v^i = -1\right)} = \text{const.} \sum_{s<t:\iota_s=i} x_s$$

- $\iota_t = \alpha(\mathbf{p}_t)$

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Exogenous stopping rule

- thresholds $\overline{p}^i > 0 > \underline{p}^i$
- stop exploring i if $p_t^i \leq \underline{p}^i$
- stop and choose i if $p_t^i \geq \overline{p}^i$
- stop and choose the outside option if $p_t^i \leq \underline{p}^i$ for both i

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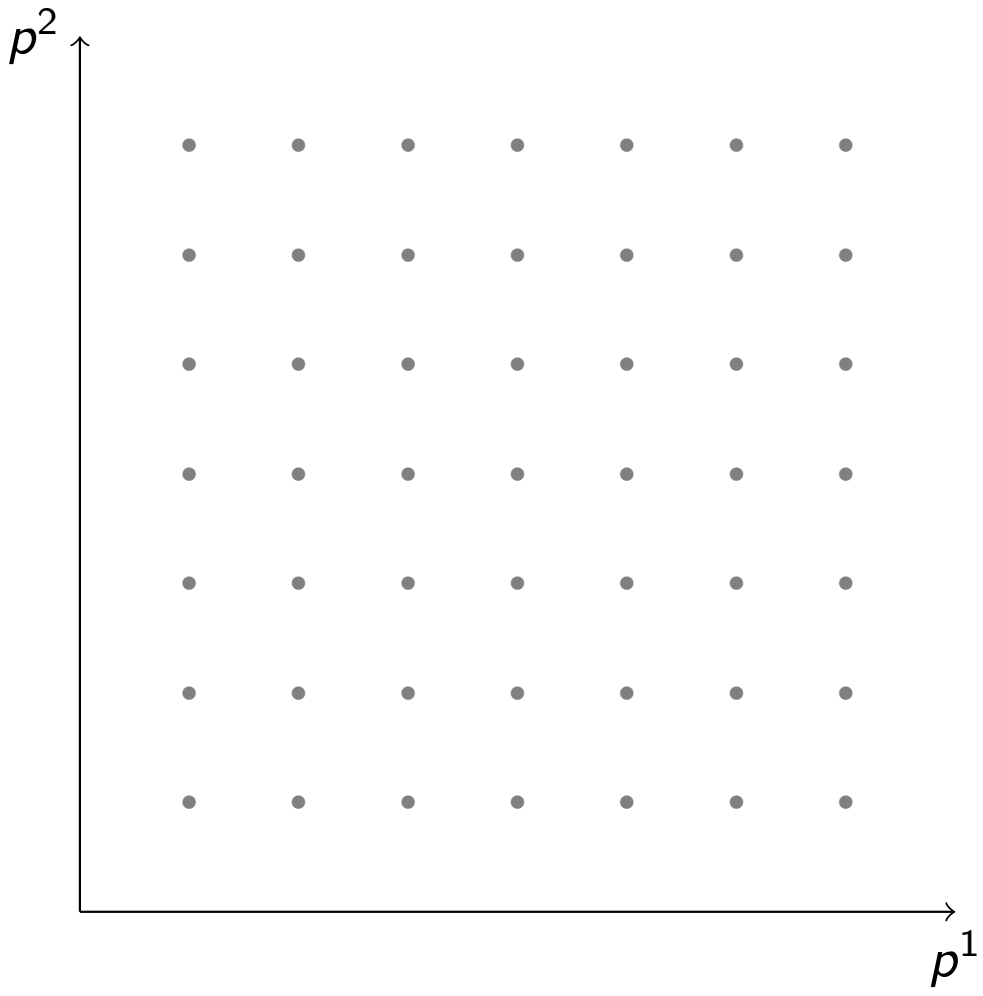

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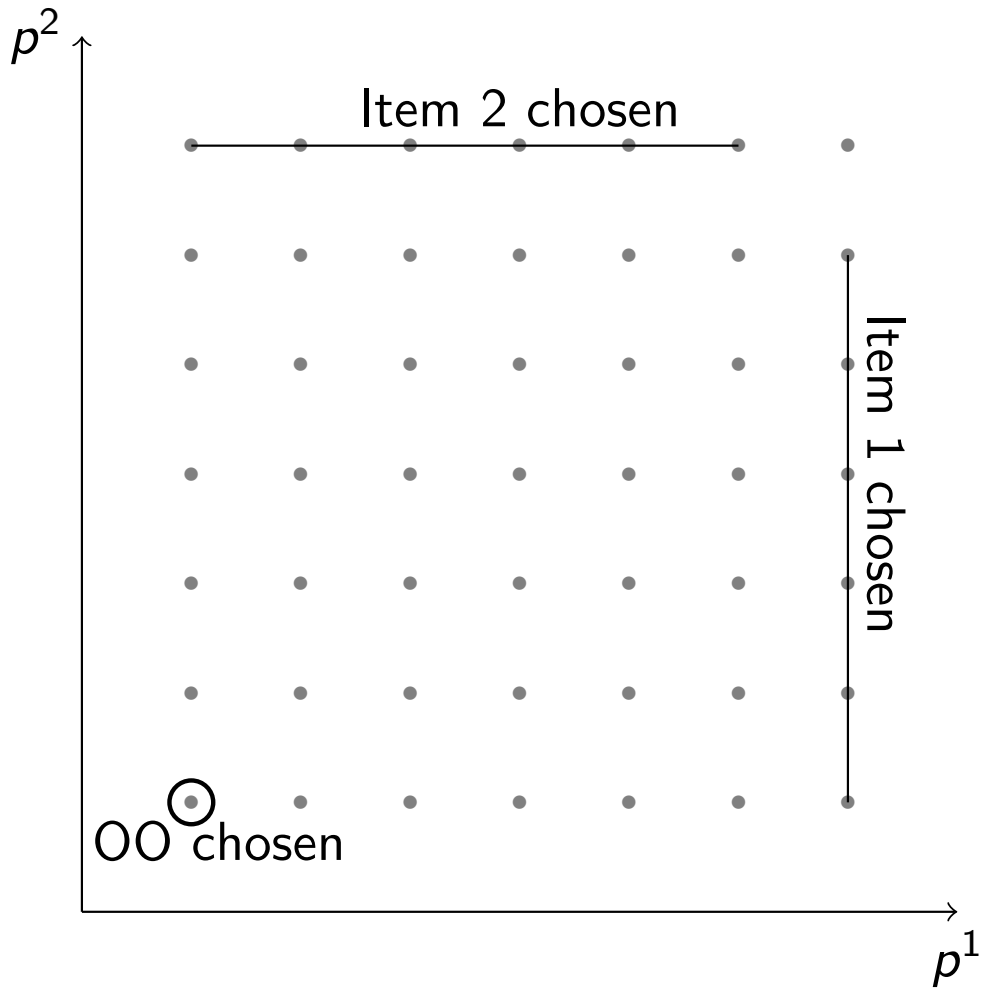

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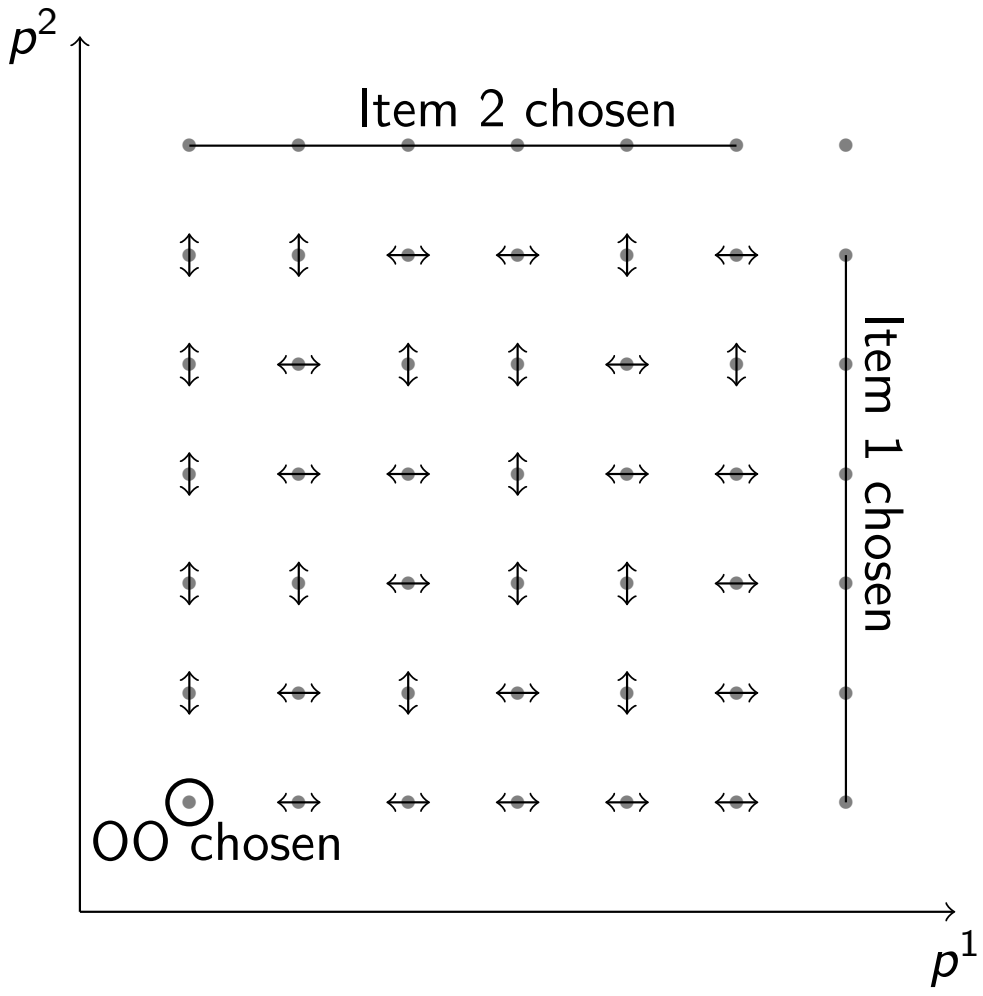

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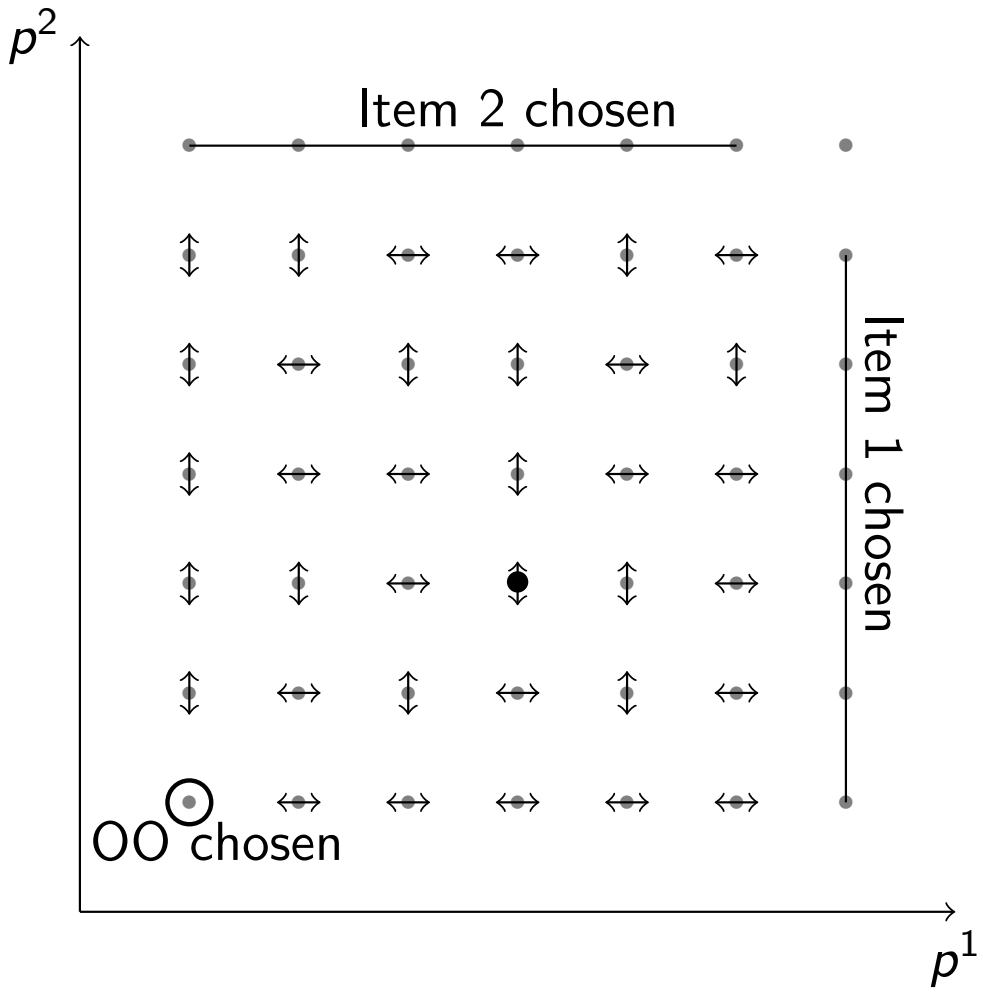

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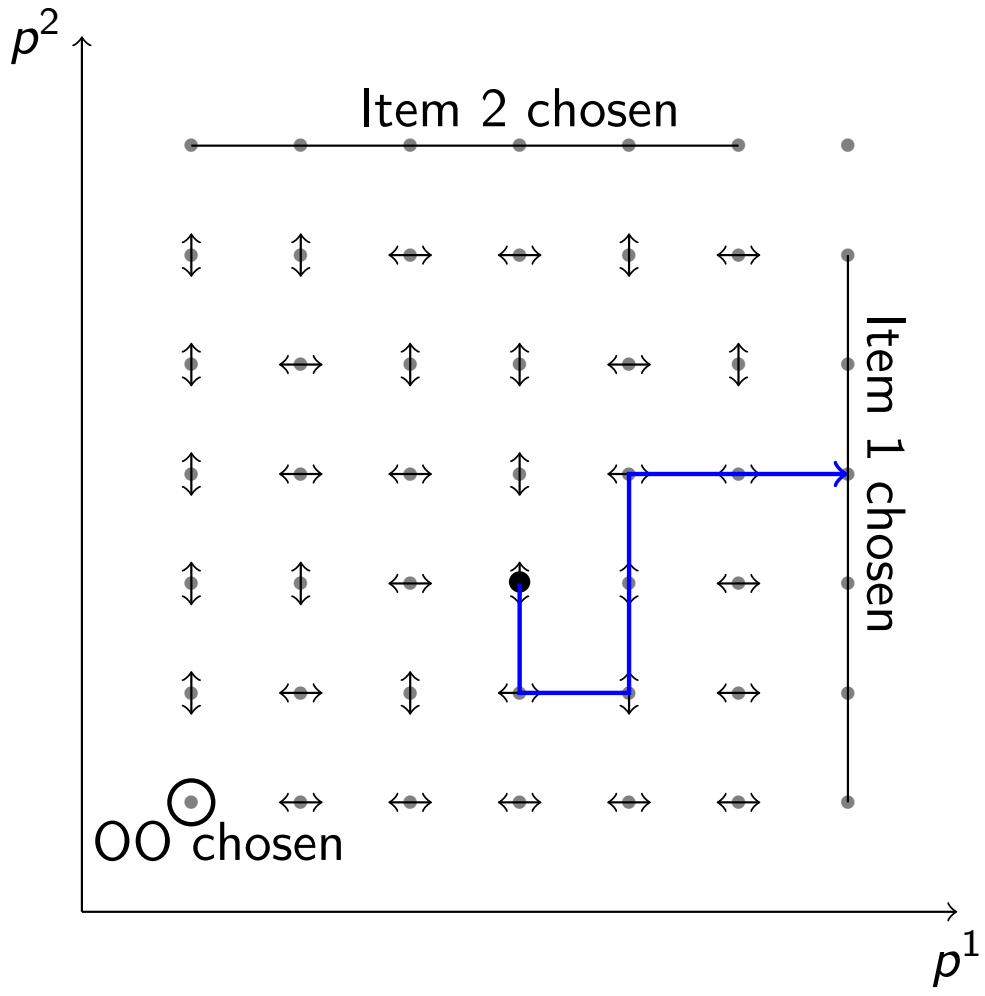

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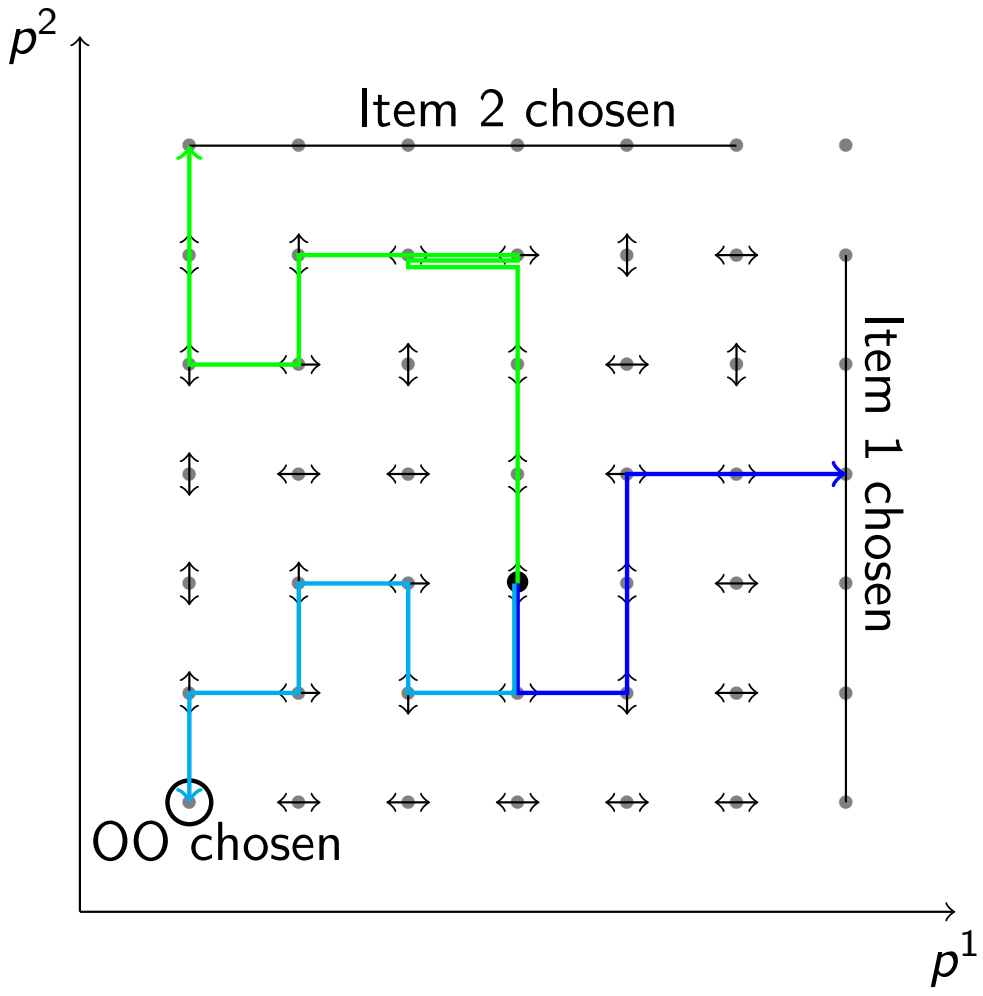

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Forced attention on item 1 for one period

1. increases and "speeds up" demand for item 1
2. decreases and "slows down" demand for item 2

conditional on any realization $\mathbf{v}$ of items' values.

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Forced focus on an item increases demand for it, even if it is bad

Formalization

Baseline strategy α and values $\mathbf{v}$ induce process $\mathbf{p}_t$ with law $P^{\mathbf{v}}_{\alpha}$

Manipulated strategy β and values $\mathbf{v}$ induce process $\hat{\mathbf{p}}_t$ with law $P^{\mathbf{v}}_{\beta}$

Stopping regions: $F^j = \{\mathbf{p} : p^j \geq \overline{p}^j\}$, $F^{oo} = \{\mathbf{p} : p^j \leq \underline{p}^j \text{ for } \forall j\}$

Stopping times: $\tau = \min\{t : \mathbf{p}_t \in F^1 \cup F^2 \cup F^{oo}\}$, $\hat{\tau}$ similarly

$\tau^j = \tau$ if $\mathbf{p}_\tau \in F^j$ and ∞ otherwise, $\hat{\tau}^j$ similarly

Proposition

1. $P^{\mathbf{v}}_{\alpha}(\mathbf{p}_\tau \in F^1) \leq P^{\mathbf{v}}_{\beta}(\hat{\mathbf{p}}_{\hat{\tau}} \in F^1)$
2. $P^{\mathbf{v}}_{\alpha}(\mathbf{p}_\tau \in F^2) \geq P^{\mathbf{v}}_{\beta}(\hat{\mathbf{p}}_{\hat{\tau}} \in F^2)$
3. τ_1 FOSD $\hat{\tau}_1$
4. $\hat{\tau}^2$ FOSD τ^2

for any realization $\mathbf{v}$ of items' values.

Proof
construction of the probability space

Each deck specifies the signal realizations for one of the items

$$\omega = \begin{cases} 1: & + + - + + - \dots \\ 2: & + + + - + + \dots \end{cases}$$

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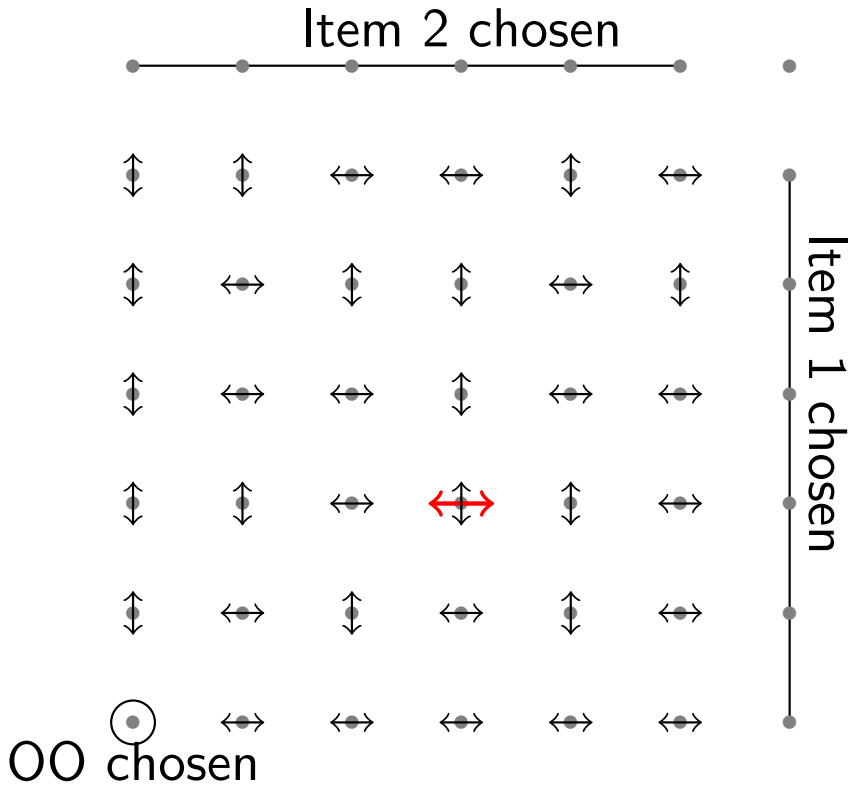
$$\text{draw} = \begin{cases} 1: & + + - + + - \ldots \\ 2: & + + + - + + \ldots \end{cases}$$

Baseline vs Manipulated Process



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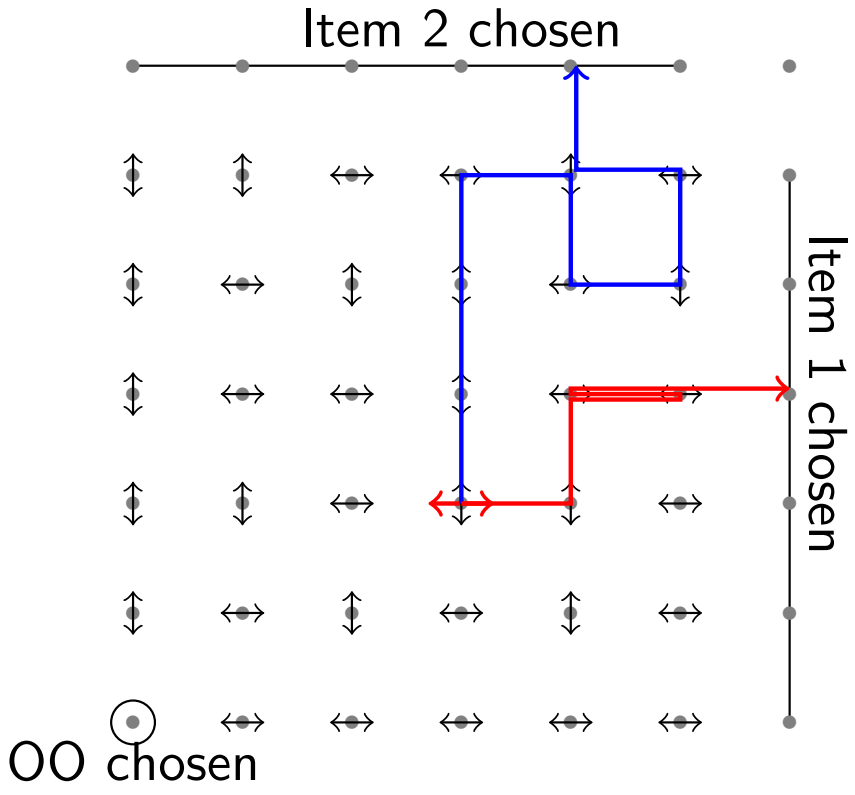

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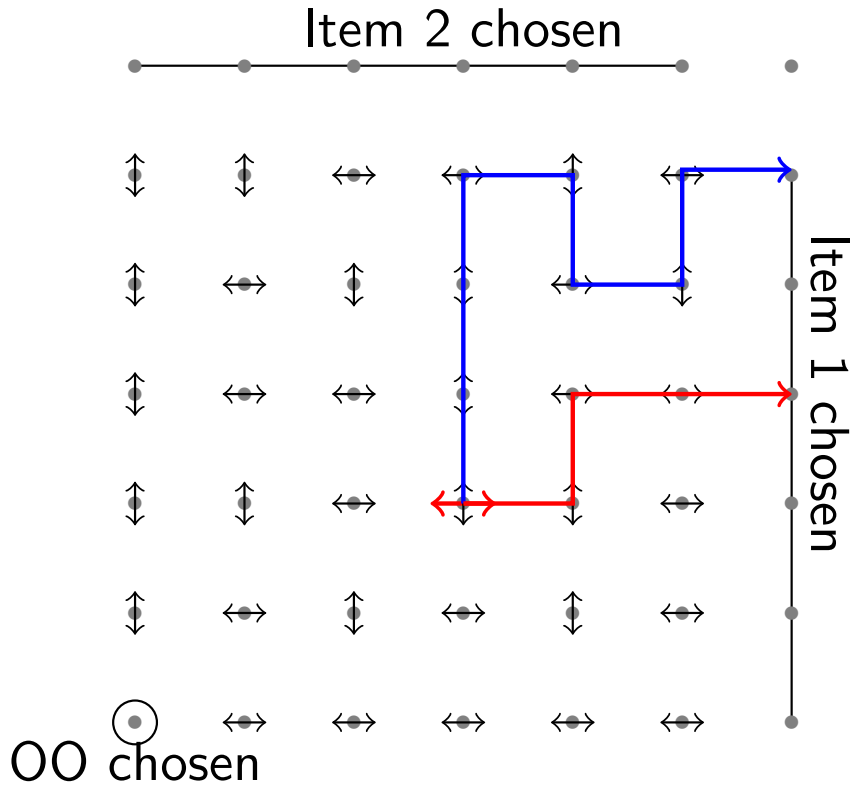

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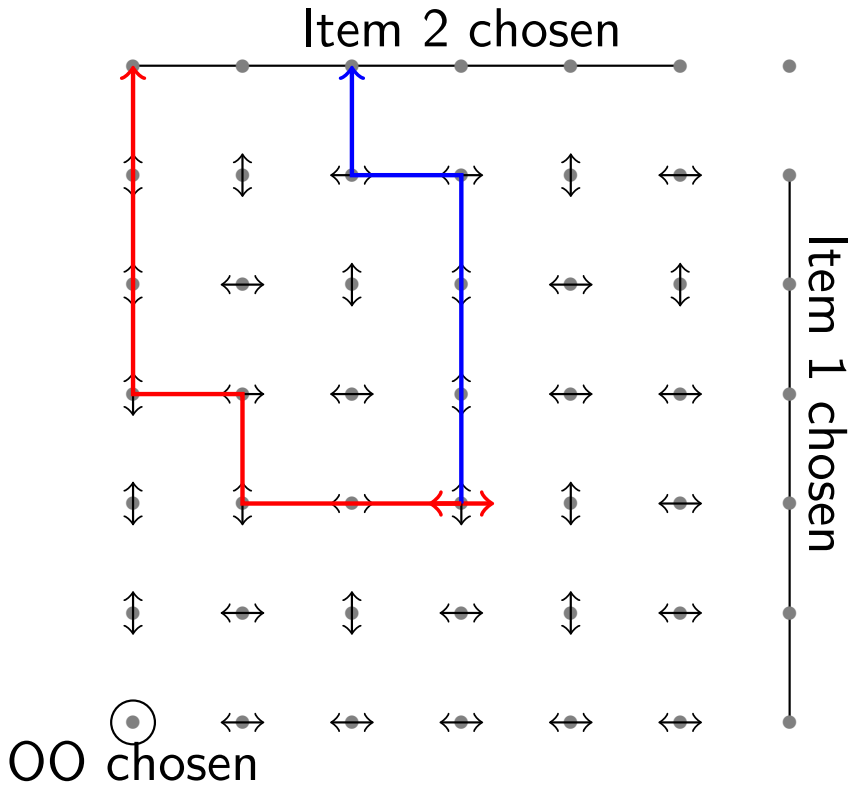

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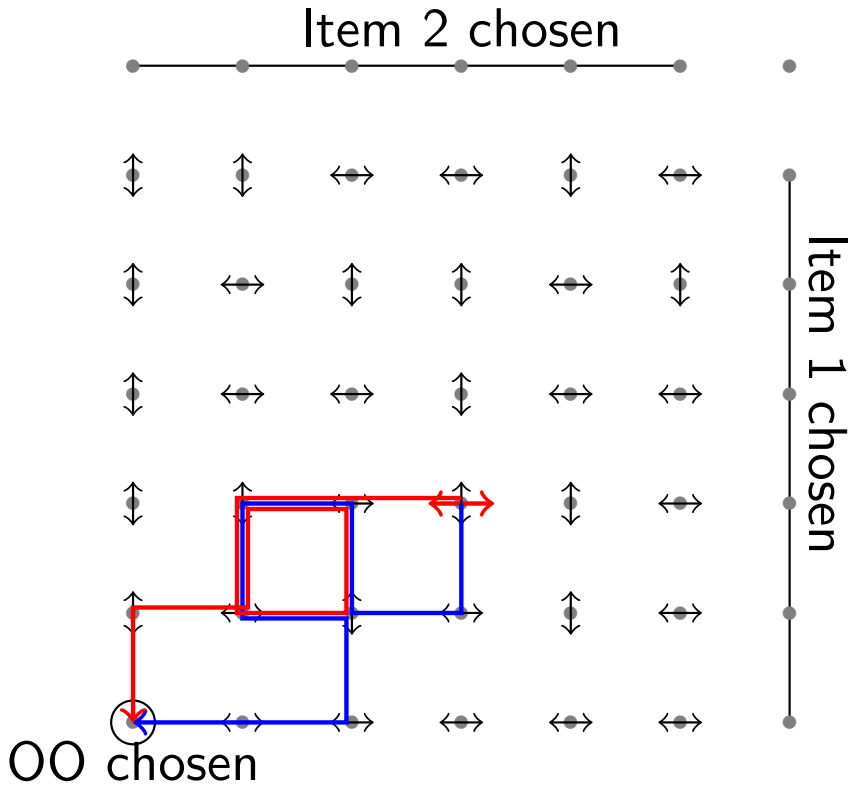

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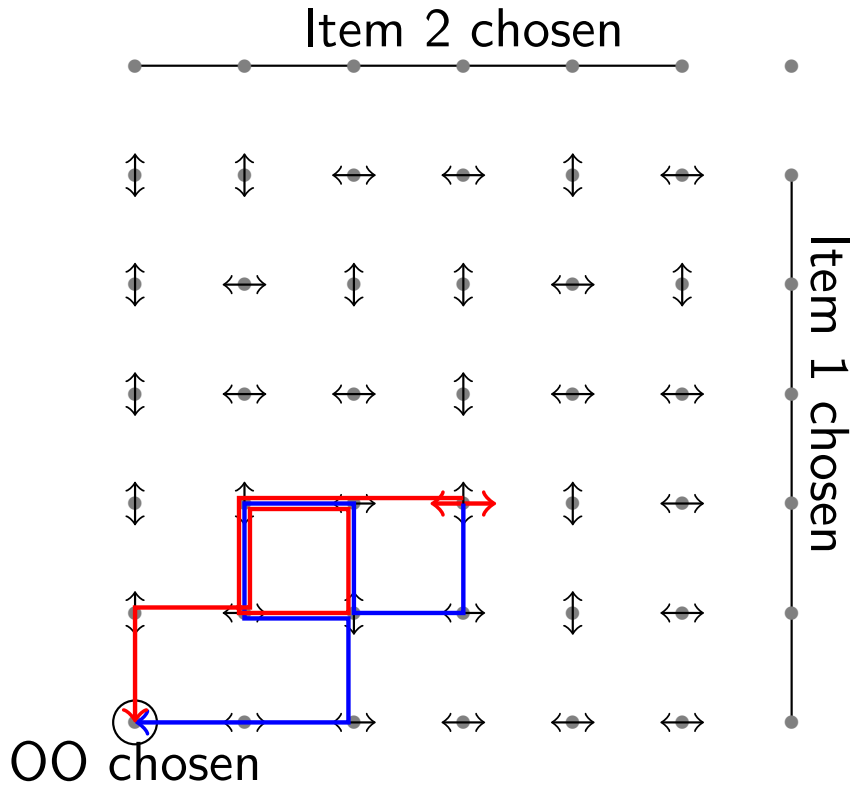


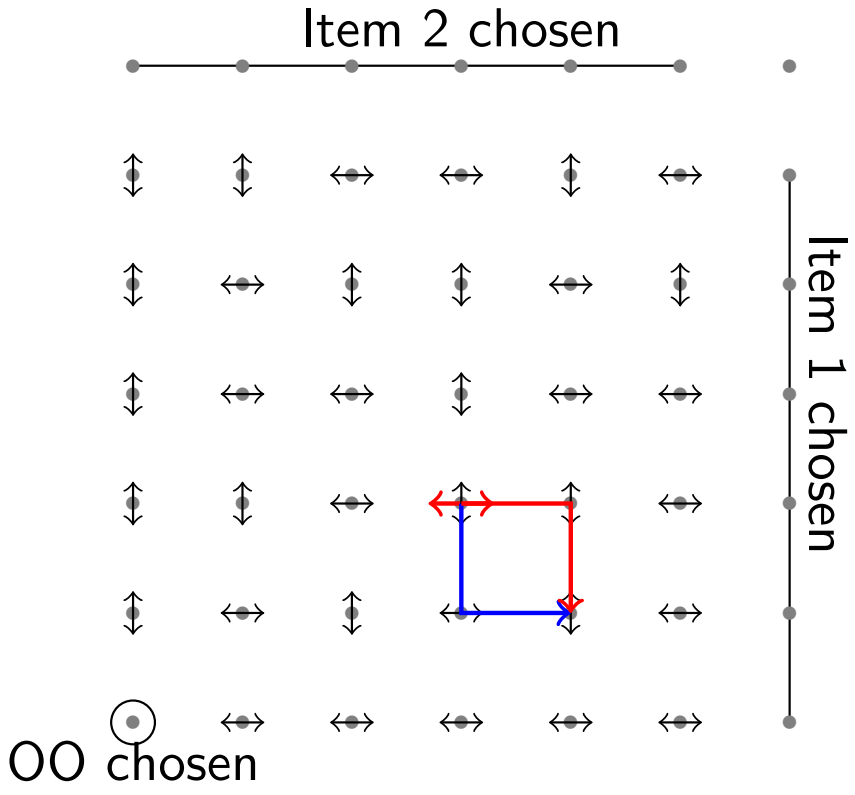
Let's measure attention to item i up to t as:

periods of focus on i up to t: $\begin{cases} a(i,t) & \text{under baseline} \\ \hat{a}(i,t) & \text{under manipulated} \end{cases}$

Baseline vs Manipulated Process

draw

$$\begin{cases} 1: & + \ldots \\ 2: & - \ldots \end{cases}$$



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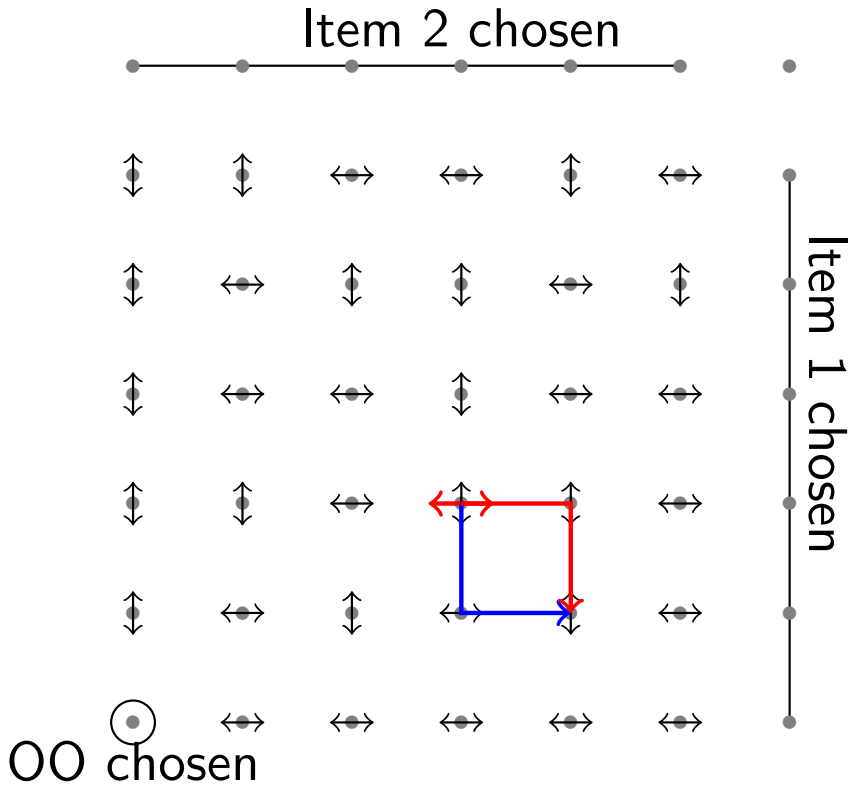
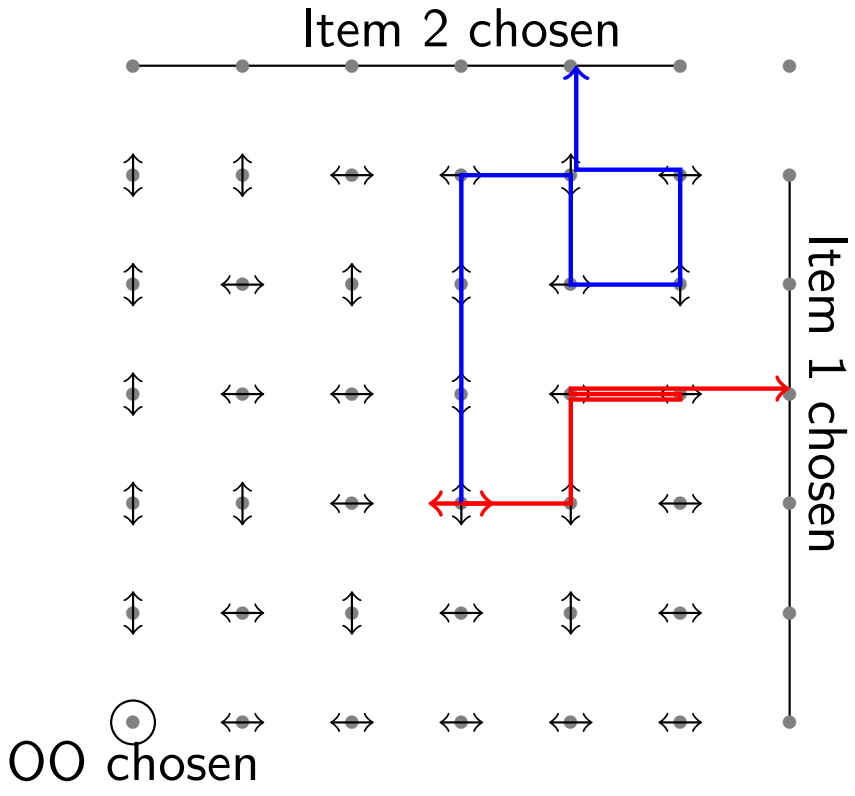


We say that the two processes meet in t if $a(1,t) = \hat{a}(1,t)$

Meeting lemma: If the processes meet then they coincide thereafter

(Property of the coupling construction, not of the model)

Baseline vs Manipulated Process



Attention lemma: For every t and draw $\begin{cases} a(1,t) \leq \hat{a}(1,t) \\ a(2,t) \geq \hat{a}(2,t) \end{cases}$

(The initial attention wedge never reverts)

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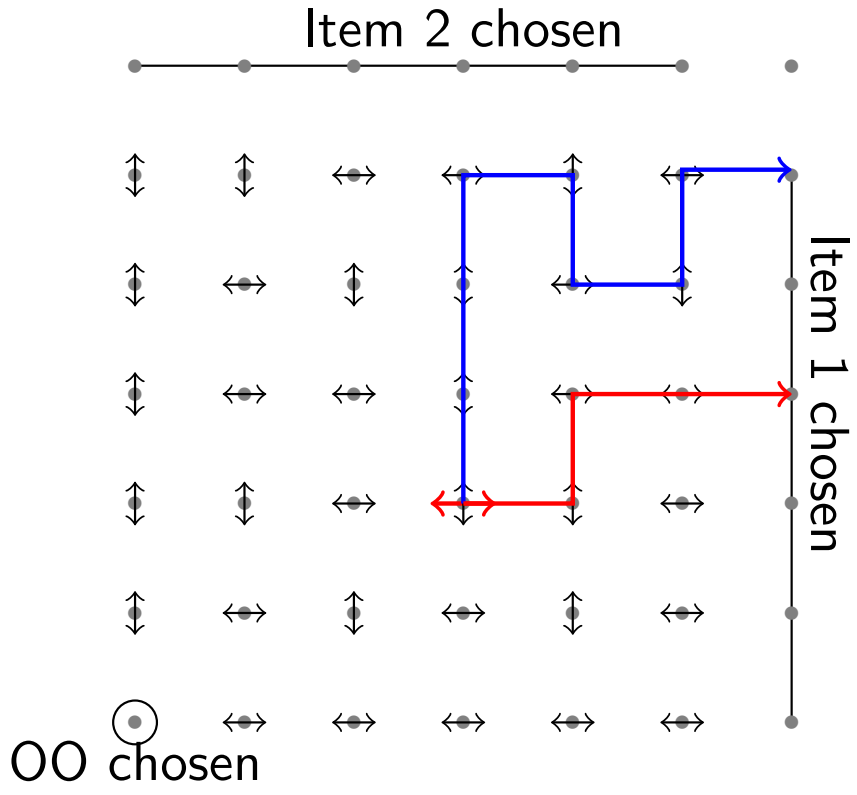

Suppose baseline chooses 1 in τ

Manipulated hasn't reach $\overline{p}^2$ by τ since it is "behind" with item 2 and baseline hasn't reached $\overline{p}^2$ by τ

Manipulated is "ahead" with item 1 and hence it reaches $\overline{p}^1$ no later than baseline

Baseline vs Manipulated Process

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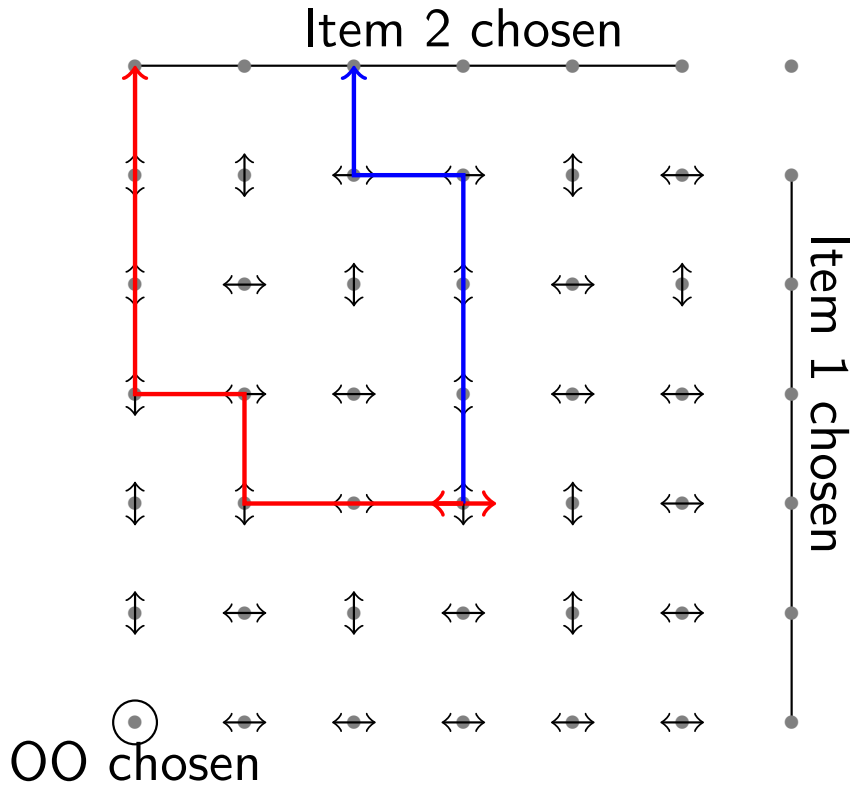

Suppose manipulated chooses 2 in $\hat{\tau}$

Baseline hasn't reach $\overline{p}^1$ by $\hat{\tau}$ since it is "behind" with item 1 and manipulated hasn't reached $\overline{p}^1$ by $\hat{\tau}$

Baseline is "ahead" with item 2 and hence it reaches $\overline{p}^2$ no later than manipulated

Exogenous vs Endogenous Attention

Psychologists distinguish

- exogenous attention: triggered by color contrasts, movements,. . .
- endogenous attention: controlled by DM

Debriefing

Insight based on the specific stopping rule:

- DM is manipulable only if both processes would reach $\overline{p}$:
  - both processes $\rightsquigarrow \underline{p} \Rightarrow$ all strategies choose OO
  - one process $\rightsquigarrow \underline{p}$, the other $\rightsquigarrow \overline{p} \Rightarrow$ all strategies choose the latter
  - both processes $\rightsquigarrow \overline{p} \Rightarrow$ approval race
- Forced focus on an item gives it an edge in the race

General insight on attention manipulation:

- Could the target item be relatively ignored on a continuation path?
- No for 2 items (attention lemma)

Table of Contents

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Counterexample

$\alpha(\mathbf{p}_0) = 2, \qquad \alpha(\mathbf{p}) = 1$ if $p^2 \neq p_0^2, \qquad \alpha(p^1, p_0^2, p^3) = 3$ if $p^1 \neq p_0^1$

manipulation targets 1 for one period

Baseline: focuses on 2, then on 1 until it reaches a threshold

Manipulated: focuses on 1, then on 3, chooses 1 only if 3 is declined

Manipulation reduces demand for its target

What went wrong?

The two processes "meet" after two periods: they both have focused:

- on the target item 1 for one period
- But the inspection within $\{2, 3\}$ differed

Continuation processes need not coincide

Baseline attention on the target may "overtake" the manipulated one

Main Result

General updating and value space, stochastic strategies, many items

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Manipulated strategy β targets item i relative to baseline strategy α if

- β is more likely to focus on i than α for any $\mathbf{p}$ and t
- conditional on not focusing on i, the two strategies coincide

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Attention Theorem

Let α be stationary and satisfy IIA. Then, for any t,

1. $a(i,t;\beta) \geq a(i,t;\alpha)$ for the target i,
2. $a(j,t;\beta) \leq a(j,t;\alpha)$ for any $j \neq i$,

in any probability state.

Argument:

- Non-target items inspected in a same order across the two processes
- $\Rightarrow$ all p_t^j coincide upon meeting

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Corollary

Let α be stationary and satisfy IIA. Then, for any t,

1. $a(i, t; \beta)$ FOSD $a(i, t; \alpha)$ for the target i,
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conditional on any values $\mathbf{v}$.

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Multi-armed Bandits

DM chooses an arm in each period

Arms have fixed types

Stochastic outcome in each period depends on the type of the chosen arm

p_t^j index for j as a function of past outcomes for j

Stationary baseline strategy $\alpha(\mathbf{p}_t)$ chooses arm with the highest index

$d(j, t; \gamma)$ cumulative demand for j up to t under $\gamma = \alpha, \beta$

Proposition

- $d(i, t, \beta)$ FOSD $d(i, t, \alpha)$ for the target i
- $d(j, t, \alpha)$ FOSD $d(j, t, \beta)$ for all $j \neq i$

for any $\mathbf{v}$ and t.

One-shot Choice

Threshold stopping rule

- stop exploring j if $p_t^j \leq \underline{p}^j$
- stop and choose j if $p_t^j \geq \overline{p}^j$
- stop and choose the outside option if $p_t^j \leq \underline{p}^j$ for all j

Proposition

Let the baseline strategy be stationary and satisfy IIA. Then, the attention manipulation

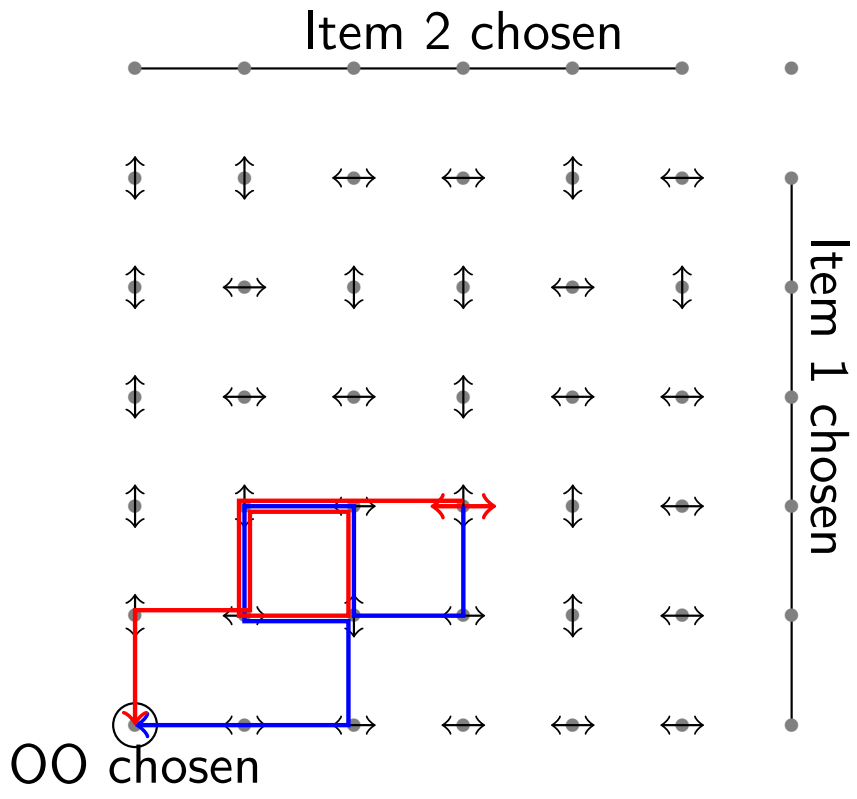
1. increases demand for target item, decreases the time to choose it
2. decreases demand for each other item, increases time to choose it
3. doesn't affect the demand for the outside option

conditional on any realization of items' values.

Whether OO is Chosen and When Is Strategy-independent

draw

$$\begin{cases} 1: & - - + - - \ldots \\ 2: & - + - - \ldots \end{cases}$$



Optimality Argument for IIA

Assume:

- fixed thresholds
- flow cost $c\left(p_t^{\iota}, \iota_t\right)$ of updating
- bayesian DM

Proposition

An attention strategy that maximizes expected discounted value less of the cost is stationary and satisfies IIA.

Map the problem to the multi-armed bandit problem of Gittins Jones '74

Optimal Gittins-index strategy:

- compute an index for each item
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Caveat:

- payoff interdependence since stopping with OO depends on all beliefs

Forced Choice (no outside option)

Armel et al '08, Krajbich et al '10, Milosavljevic et al '12:

Short forced focus on good item increases the probability of its choice

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Short forced focus on bad item decreases the probability of its choice

Let the DM stop as soon she finds a good or bad item:

- p_t^j log-likelihood ratios
- stop and choose j if $p_t^j \geq \overline{p}$
- stop and choose $-j$ if $p_t^j \leq \underline{p}$

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Proposition

Consider any stationary, IIA attention strategy. Send the thresholds to $-\infty$ and ∞. Forced focus on good item weakly increases its demand.

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For a good item, lower thresholds are never reached

Only positive manipulation effect

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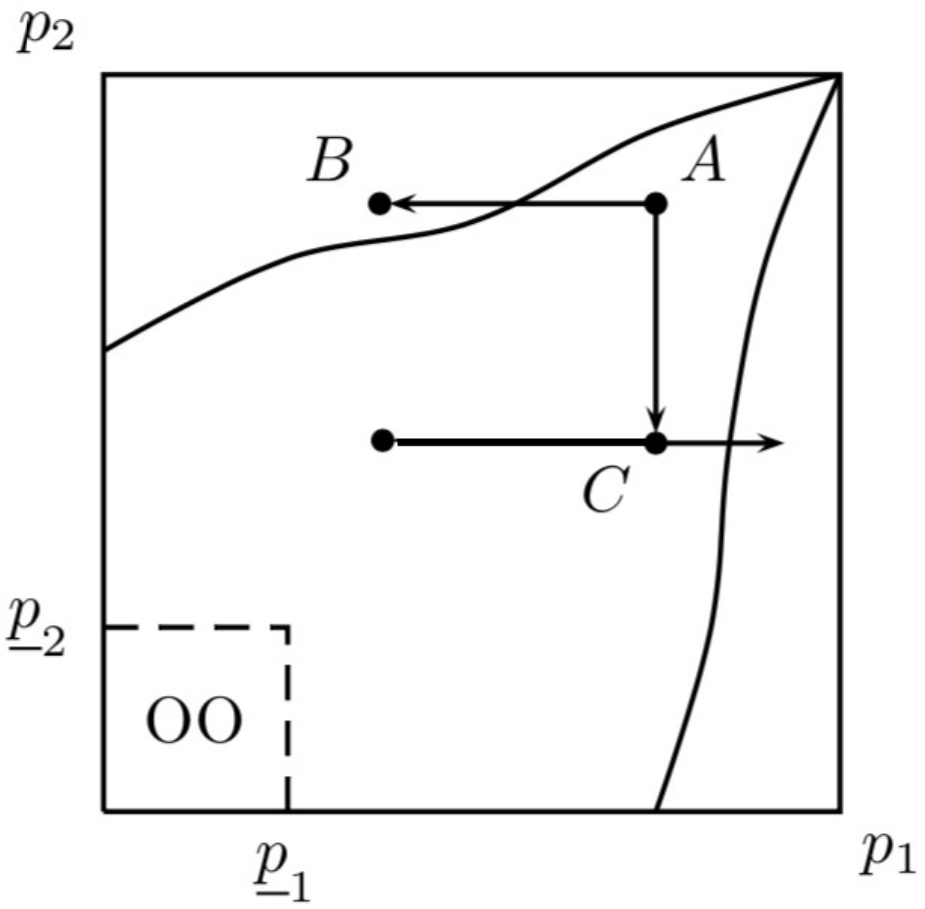
Approval and disapproval races

For a bad item, upper thresholds are never reached

Only negative manipulation effect

Curved Boundaries

When the boundaries are curved, we can find a learning process and an attention strategy for which the result fails



Ke&Villas-Boas '19: optimal boundaries are curved in a Bayesian setting

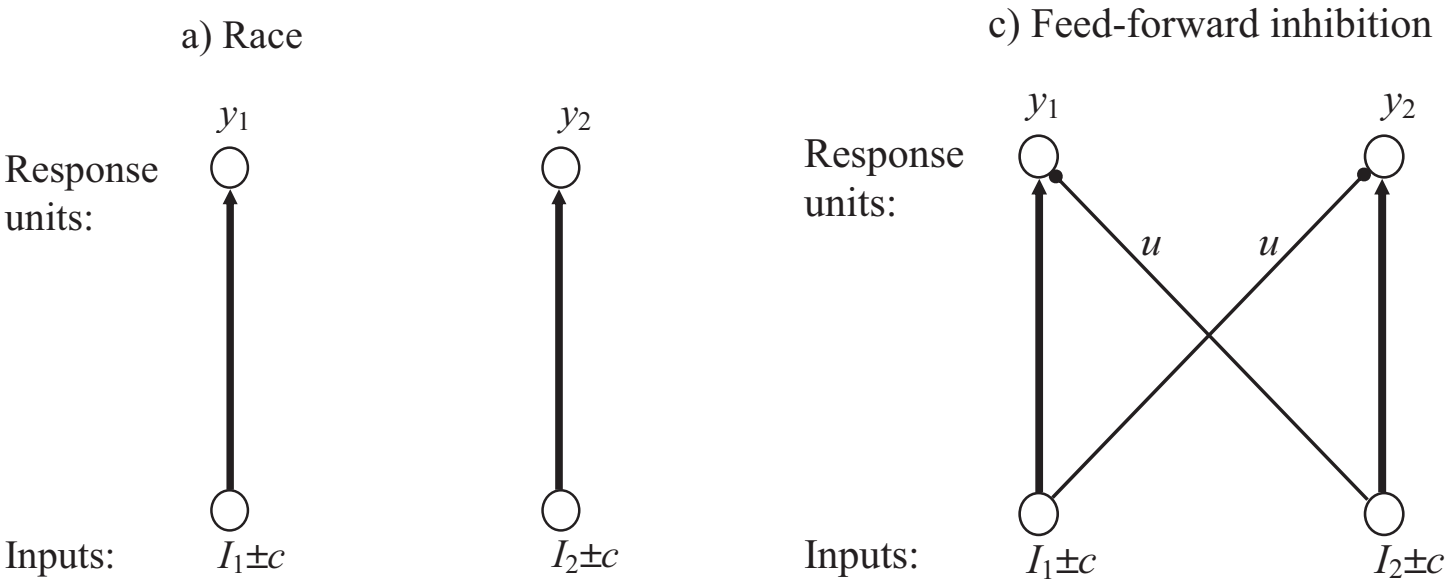
Inhibition

Intuition for the increasing stopping boundaries

- optimistic signal on i makes termination with $j \neq i$ less attractive

Neural Implementation

- constant thresholds and inhibition links, Shadlen & Newsome '01



figures from Bogacz et al. '06

Stimulus increases its accumulator, inhibits the alternative accumulator

An accumulator triggers choice once it reaches a threshold

Adaptive Clinical Trials

Increasingly popular in practice, Berry (2006)

Sequential clinical trial of drugs $i \in \mathcal{I}$ on at most T patients

Patient t is administered a drug $\iota_t = \alpha(\mathbf{p}_t)$

A drug j is dropped if $p_t^j \leq \underline{p}$, approved if $p_t^j \geq \overline{p}$

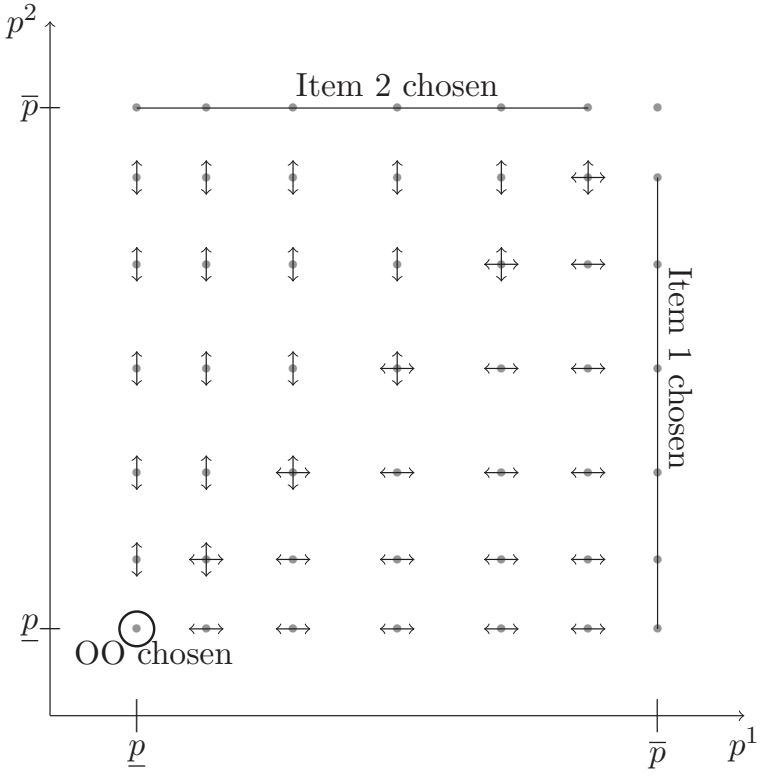
Thresholds determined by regulation

Several drugs can be approved

Manipulation of the trial towards testing drug i on initial patients increases the probability that i is approved.

The Fastest Strategy

Bayesian DM, fixed threshold stopping rule



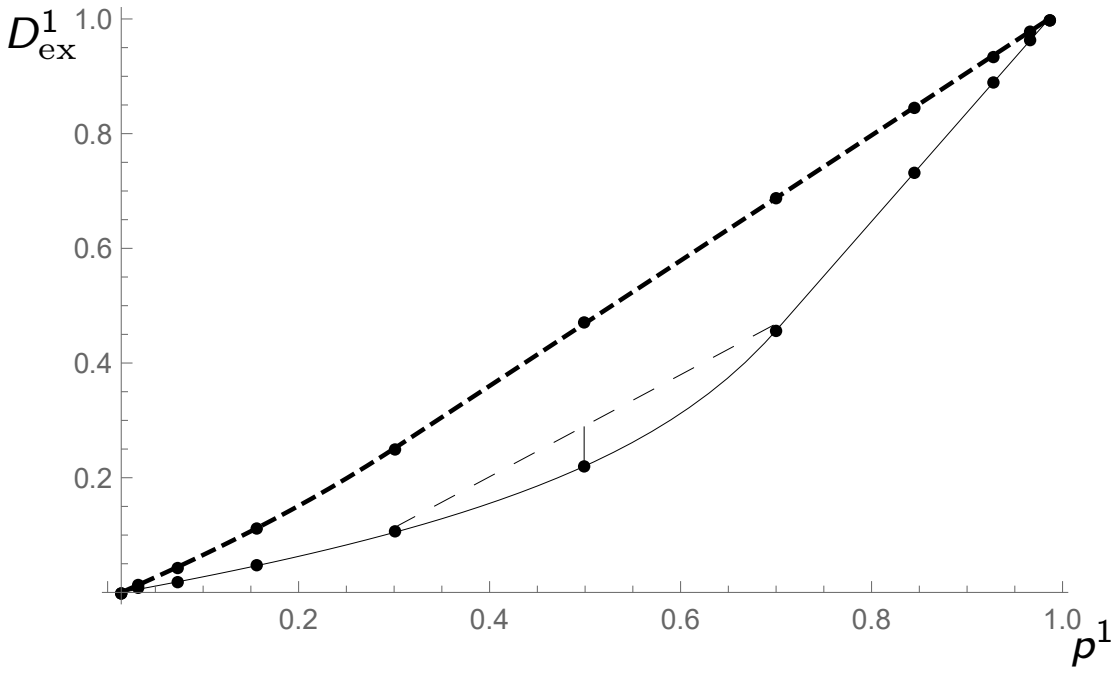
Stopping time of any other strategy FOSD stopping time of this strategy

Proof based on a distinct coupling construction:

- ω specifies how belief p is updated when DM encounters p k'th time

Strength of the Manipulation Effect

for the fastest baseline strategy



Manipulation effect

- is positive if manipulation affects focus
- increases in p^i
- decreases in p^{-i}

Conclusion

That attention-grabbing boost choice seems natural

But attention may lead to both positive and negative news. Why the asymmetric impact on choice?

1. Under natural assumptions, choice process is an approval race

2. Attention-grabbing cannot decrease cumulative focus on the continuation path

Thank you for your attention!