1 Static (one period) model

The problem: $\max U(C, L, X)$, s.t. $C = Y + w(T - L)$ and $L \leq T$.

The Lagrangian: $L = U(C, L, X) - \lambda(C + wL - M) - \mu(L - T)$, where $M = Y + wT$

The FOCs: $U_C(C, L, X) = \lambda$ and $U_L(C, L, X) \geq \lambda w + \mu$

The interior solution: $U_L/U_C = MRS_L(C, L, X) = w$

Solve FOCs for $L^* \equiv L^*(w, M, X)$ and $C^* \equiv C^*(w, M, X)$ or $H^* = T - L^*$.

The reservation wage $= MRS_L(Y, T, X)$.

Response of leisure (time worked) to changes in wages:

$$\frac{\partial L^*(w, M, X)}{\partial w} = \left.\frac{\partial L^*}{\partial w}\right|_{M=current\_income} + \frac{\partial L^*}{\partial M}\frac{\partial M}{\partial w}, \tag{1}$$

where $\frac{\partial M}{\partial w} = T$.

Holding X constant, compute $\left.\frac{\partial L^*}{\partial w}\right|_{M=current\_income}$, the Marshallian demand for leisure, from the Slutsky equation:

$$\left.\frac{\partial L^*}{\partial w}\right|_{M=current\_income} = \left.\frac{\partial L^*}{\partial w}\right|_{U=U^*} - \frac{\partial L^*(w, C + wL^*)}{\partial M}L^*,$$

where $\left.\frac{\partial L^*}{\partial w}\right|_{U=U^*}$ is the Hicksian demand for leisure.

Substitution effect is negative (if the price of leisure goes up, holding utility constant, people buy less leisure) + leisure is a normal good.

So, the Marshallian elasticity is larger (in absolute value) than the Hicksian. Now substitute into equation (1):

$$\frac{\partial L^*}{\partial w} = \left.\frac{\partial L^*}{\partial w}\right|_{U=U^*} + \frac{\partial L^*(w, C + wL^*)}{\partial M}(T - L^*). \tag{2}$$

Substitution effect, income effect and endowment effect. The endowment effect increases consumption of leisure since a rise in the wage rate makes them richer overall (the income effect is smaller than the endowment effect).

The response from a wage change is ambiguous for employed people (but positive for not working).

Individuals that work for only few hours a week are likely to have the substitution effect dominate the income effect.

1.1 Empirics

Empirical studies suggest that changes in labor supply are mainly due to changing participation. The elasticity of the supply of female labor, especially that of married women, is greater than that of men.

For prime age men, evidence indicates that the income effect dominates.

How are the empirical results estimated? Use cross-sectional data to run:

$$\ln H_i = \beta_0 + \beta_1 \ln w_i + \beta_2 X_i + v_i \tag{3}$$

But there is endogeneity (omitted variables).

How to deal with those with $H_i = 0$?

1.2 Extensions

taxes: $wH(1 - \tau)$

transfers: get welfare if $H \leq 0$

fixed costs when $H > 0$ (transportation, eating out, day care).

Rationing, home production, transfers within families, etc.

The effect of a tax is ambiguous (recall opposing subs. and income/endowment effects). Of course, unconditional welfare reduces the incentive to work.

Consider a negative income tax program (NIT): guaranteed income G and a subsidy to work S, such that $S = G - \tau w H$ if $G > \tau w H$, and $S = 0$ otherwise. S stops at $wH = G/\tau$. Analyze the effect graphically and sign it using the Slutsky equation.

1.3 Life Cycle Labor Supply

Understand intertemporal decisions: (a) retirement decisions, (b) reaction to shocks: trading work today if wages are unusually high for leisure tomorrow.

$\max \sum_{t=1}^{T} U(C_t, L_t, t)$.

The budget constraint allows for savings and follows assets over time.

The model suggests: $\triangle \ln H_{it} = \beta_0 + \triangle\beta_1 \ln w_{it} + \triangle\beta_2 X_{it} + \triangle v_{it}$

λ_t can be broken down to an individual fixed effect and a common age effect (interest rates). If the interest rate is constant then β_0 is the result of first differencing this age effect $\beta_0 t$.

β_1 is the elasticity of labor supply w.r.t. a transitory change in w.

1.4 Empirical Analysis of Labor Supply

OLS is inconsistent no matter whether we include or exclude the zero h observations.

Assume unobservable *latent* variable h_i^* (desired hours):

$$\begin{aligned} h_i^* &= \beta^{'} x_i + \delta w_i + u_i \text{ with } u_i \sim N(\mu, \sigma^2) \\ h_i &= h_i^* \text{ iff } h_i^* > 0 \\ h_i &= c \text{ iff } h_i^* \leq 0, \end{aligned}$$

and use Tobit MLE based on the normality assumption:

$$L = \prod_{h_i^* > 0} \frac{1}{\sigma} \varphi \left(\frac{h_i - x_i^{'} \beta - \delta w_i}{\sigma} \right) \prod_{h_i^* \leq 0} \Phi \left(\frac{0 - x_i^{'} \beta - 0}{\sigma} \right).$$

Where does the participation decision come from?

Heckman (1974): $w_i = \theta' z_i + v_i > w_i^R$. Get w_i^R from the $h = 0$ condition. Do *not* work if

$$w_i < w_i^R \Leftrightarrow \theta' z_i + v_i < -\frac{\beta' x_i}{\delta} - \frac{u_i}{\delta} \Leftrightarrow -\frac{u_i}{\delta} - v_i > \theta' z_i + \frac{\beta' x_i}{\delta}.$$

Estimate the model using MLE:

$$L = \prod_{\text{work}} |\mathsf{J}|\, f(\underbrace{h_i - \beta' x_i - \delta w_i}_{u_i}, \underbrace{w_i - \theta' z_i}_{v_i}) \prod_{\text{not}} \Pr\left(-\frac{u_i}{\delta} - v_i > \theta' z_i + \frac{\beta' x_i}{\delta}\right)$$

Because the model delivers w^R from the $h = 0$ assumption, Tobit leads to significant overestimation of some elasticities. In other words, the hours of work decision made when the woman is in the labor force appears distinct from her labor market participation decision.

1.4.1 Sample Selection: Heckman's λ

Heckman (1979) considers a two-equation behavioral model:

$$\begin{aligned} y_{i1} &= x_{i1}^{'}\beta_1 + u_{i1} \\ y_{i2} &= x_{i2}^{'}\beta_2 + u_{i2}, \end{aligned}$$

where wages y_{i1} are observed only for women who work ($y_{i2} > 0$).

$E[y_{i1}|x_i, y_{i2} > 0] = x_{i1}^{'}\beta_1 + E[u_{i1}|selection \quad rule] =$
$= x_{i1}^{'}\beta_1 + E[u_{i1}|y_{i2} > 0] = x_{i1}^{'}\beta_1 + E[u_{i1}|u_{i2} > -x_{i2}^{'}\beta_2]$.

If u_{i1} and u_{i2} are jointly normal with covariance σ_{12}, then $E[y_{i1}|x_i, y_{i2} > 0] = x_{i1}^{'}\beta_1 + \frac{\sigma_{12}}{\sigma_2}\frac{\phi(x_{i2}^{'}\beta_2/\sigma_2)}{\Phi(x_{i2}^{'}\beta_2/\sigma_2)} = x_{i1}^{'}\beta_1 + \sigma\lambda(x_{i2}^{'}\beta_2)$

We can numerically identify σ from β_1 even when $x_{i2} = x_{i1}$ because λ is a non-linear function, but this would be wrong. Need exclusion restrictions (IVs, variables in x_{i2} not included in x_{i1}).

Now, return to Heckman (1974) and consider 3 equations: w_i, w_i^R, and h:

$$\begin{aligned} h_i &= \delta w_i + x_i^{'}\beta + \sigma\lambda(z_i^{'}\widehat{\gamma}) + u_i \\ w_i^R &= x_i^{'}\phi + e_i \\ w_i &= \theta' z_i + v_i. \end{aligned}$$

Correct the hours equation for sample selection: do not participate if:

$$\theta' z_i + v_i - x_i^{'}\phi - e_i < 0 \Leftrightarrow \underbrace{\theta' z_i - x_i^{'}\phi}_{r_i^{'}\gamma} + \underbrace{v_i - e_i}_{\varepsilon_i} < 0$$

Running a Probit on this delivers $\widehat{\gamma}$. Second, IV for w_i:

$$\widehat{w_i} = \theta' z_i + \sigma^* \lambda^*(\cdot)$$

Finally

$$h_i = \delta \widehat{w_i} + x_i^{'} \beta + \sigma \lambda (z_i^{'} \widehat{\gamma}) + \varepsilon_i.$$

You need a variable predicting wages but not hours and another variable affecting participation, but not wages.

1.4.2 Natural Experiments (DDs)

Eissa and Liebman (1996): participation rates of single women with children who became entitled to earned income tax credits (EITC).

A family is eligible for EITC if (a) earned income $<$ \$28,000 in 1996 and (b) there is a child.

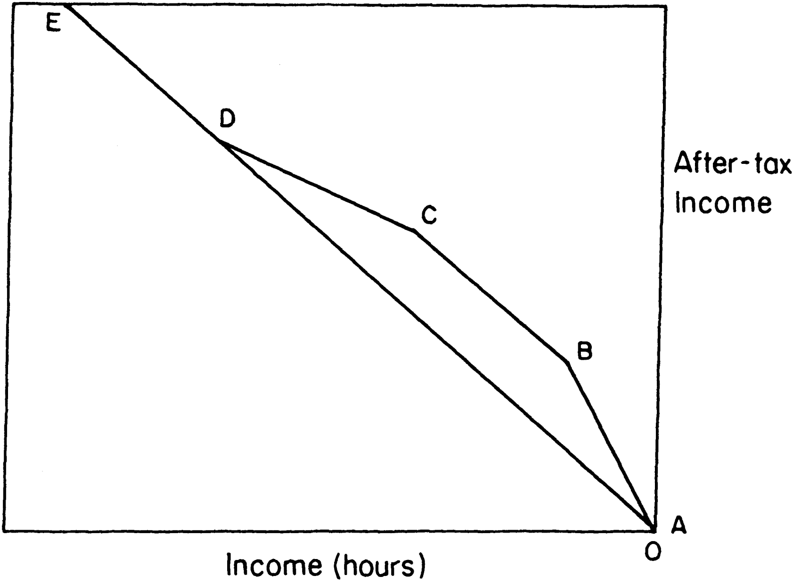

FIGURE I
EITC Budget Constraint

In 1987, the subsidy rate (and maximum) was increased from 11% to 14% (from $550 to $851).

A natural control group for a DDs design are single women without children.

Table 1: Xs differ so focus only on single women with less than high school.

The main result (in Table 2): LFP rises by 4 p.p.

They also find that hours worked did not decrease for those already working.

Of course, this says nothing about the overall costs of the program (to the taxpayer).

Brewer et al. (2006) suggest that a similar UK program (WFTC) increases the lone-parent employment rate by 10 p.p. copmare to "no program" scenario.

TABLE I
SUMMARY STATISTICS

	Group				
	Without children	With children			
			Education		
Variable		All	Less than high school	High school	Beyond high school
Age	26.78	31.17	28.67	30.88	33.97
	(7.02)	(7.07)	(7.39)	(6.79)	(6.21)
Education	13.44	12.05	9.33	12.00	14.63
	(2.33)	(2.28)	(1.81)	(0.00)	(1.54)
Nonwhite	0.15	0.37	0.43	0.37	0.33
	(0.36)	(0.48)	(0.49)	(0.48)	(0.47)
Preschool children	0.00	0.48	0.61	0.48	0.36
	(0.00)	(0.50)	(0.49)	(0.50)	(0.48)

TABLE II

LABOR FORCE PARTICIPATION RATES OF UNMARRIED WOMEN

	Pre-TRA86 (1)	Post-TRA86 (2)	Difference (3)	Difference-in-differences (4)
A. *Treatment group:*				
With children [20,810]	0.729 (0.004)	0.753 (0.004)	0.024 (0.006)	
Control group:				
Without children [46,287]	0.952 (0.001)	0.952 (0.001)	0.000 (0.002)	*0.024 (0.006)*
B. *Treatment group:*				
Less than high school, with children [5396]	0.479 (0.010)	0.497 (0.010)	0.018 (0.014)	
Control group 1:				
Less than high school, without children [3958]	0.784 (0.010)	0.761 (0.009)	−0.023 (0.013)	*0.041 (0.019)*
Control group 2:				
Beyond high school, with children [5712]	0.911 (0.005)	0.920 (0.005)	0.009 (0.007)	*0.009 (0.015)*

1.4.3 Recent Extensions

Income Elasticity, not Hours Instead of analyzing the impact on hours of work and labor force participation, study the impact on taxable income (summary measure capturing also work effort and job mobility).

Household Production and Leisure Use time-diary data.

Burda et al. (2007): female-male difference in total work (for pay + at home) lower when GDP p.c. higher (no gap in rich northern countries). Women's total work is further below men's where their relative wages are lower.

Connolly (2008): on rainy days, US men shift 30 minutes from leisure to work (intertemporal elasticity of labor supply of around 0.01).