i) Household utility maximization problem

\[
\max_{c_t} \left( \frac{(c_t + q_t)^{1-\theta}}{1-\theta} e^{-\theta (c_t - \bar{c})} \right) A_t
\]

Subject to: budget constraint

\[
a_t = r a_{t+1} + \frac{w_t - n a_t}{(1 + \tau_c)} c_t
\]

and No Ponzi game constraint

\[
\lim_{t \to \infty} a_t e^{-\int_{t}^{\infty} (r(s) - \bar{c}) ds} \geq 0
\]

\[
a_0 > 0
\]

\[
a_t = k_t + b_t
\]

where \(k_t\) is a labor income.

b) Set up Hamiltonian

\[
H = \left( \frac{(c_t + q_t)^{1-\theta}}{1-\theta} e^{-\theta (c_t - \bar{c})} \right) A_t + \lambda_t \left[ w_t + (r - \bar{c}) q_t - (1 + \tau_c) c_t \right]
\]

where \(\lambda_t\) is the state variable and \(c_t\) is the control variable.

\[
\frac{\partial H}{\partial c_t} = 0 \quad \Rightarrow \quad (c_t + q_t)^{1-\theta} e^{-\theta (c_t - \bar{c})} = \lambda_t (1 + \tau_c)
\]

\[
\frac{\partial H}{\partial q_t} = 0 \quad \Rightarrow \quad (w_t - n) \lambda_t = -\frac{\partial H}{\partial \lambda_t}
\]

TNC:

\[
\lim_{t \to \infty} \lambda_t a_t = 0
\]
c) The government's flow budget constraint

\[ \mathbf{f}^t + \mathbf{b} = \mathbf{f}^t + \mathbf{b} \]

There is no need for specification of NP6 condition for government because government needs to maintain balanced budget \( y_t \) therefore it follows that \( y_t = \mathbf{c}_t y_t \) and government's not using borrowings to finance its consumption.

\[ y_t = \mathbf{c}_t y_t \]

a) because \( y_t = \mathbf{c}_t y_t \Rightarrow y_t - \mathbf{c}_t y_t \)

becomes

\[ \left( \left( 1 + \mathbf{r}_t \right) \mathbf{c}_t \right)^{-\theta} \mathbf{c}^\theta - (\mathbf{y}_t - \mathbf{y}_t) = \lambda_t \left( 1 + \mathbf{r}_t \right) \]

and differentiates w.r.t. time

\[ \frac{\mathbf{d}^t}{\mathbf{d}t} = \left( 1 + \mathbf{r}_t \right)^{-\theta} \left[ \mathbf{c}^\theta - (\mathbf{y}_t - \mathbf{y}_t) \right] \]

from P.O.C 2) \( \Rightarrow \frac{\mathbf{d}^t}{\mathbf{d}t} = \lambda_t - \mathbf{n}_t \)

Plug-in for \( \mathbf{d}^t \)

and \( \lambda_t \) (from P.O.C 3)

\[ \left( 1 + \mathbf{r}_t \right)^{-\theta} \mathbf{c}_t^\theta - (\mathbf{y}_t - \mathbf{y}_t) \]

\[ \lambda_t - \mathbf{n}_t \]

\[ \left( 1 + \mathbf{r}_t \right)^{-\theta} \mathbf{c}_t^\theta - (\mathbf{y}_t - \mathbf{y}_t) \]

Euler equation

\[ \frac{\mathbf{c}^t}{\mathbf{c}_t} = \frac{1}{\theta} \left[ \mathbf{r}_t - \mathbf{g}_t \right] \]

Explanation:

Households choose consumption to equalise Households.
of return $r_t$ to the rate of time preference $\delta$ and the rate of decrease of marginal utility of consumption $\frac{\delta}{\bar{c}}$ due to change in per capita consumption.

In optimizing environment, Euler equation says that household equates rates of return of consumption (return on assets, return on shifting future consumption to present period) so households are in different between consuming and saving.

e) Firm's profit maximization problem

$$\max_{k_t} AK_t^\delta - (r+\delta) k_t - w_t$$

representative firm omit index $i$.

Use FOC to get

$$\dot{k}_t = -\lambda A k_t^{\delta-1}$$

$$w_t = (1-\delta) A k_t^\delta$$

competitive market equilibrium

$$\dot{A} = -\delta A$$

assuming homogeneity of HH

plug in to HH's budget constraint

to get

$$\dot{c}_t = \delta A k^\delta_t - (r+\delta) k_t - (1+\delta) c_t$$

Euler equation

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\delta} \left[ A k_t^{\delta-1} - \delta - 5 \right]$$

competitive market equilibrium is characterized by (A), (C1) +

The TNC: $\lim_{t \to \infty} k_t \cdot e^{-\frac{1}{\delta}(r+\delta)n} = 0$
i) At steady state: \( c_x = 0 \), \( q_x = 0 \)

Hence:
\[
A_k^{k+1} - 3 - 0 = 0 \quad \Rightarrow \quad k_x = \left( \frac{A_k A_i}{A_i^2} \right)^{1/2}
\]

\[
A_k^{k+1} - (n + 5)k_x - 1 + 2c = 0 \quad \Rightarrow \quad c_x = \frac{A_k^{k+1} - (n + 5)k_x}{1 + 2c}
\]

ii) Because \( c_x(\tau) = \frac{1}{1 + 2c} \left( A_k^{k+1} - (n + 5)k_x \right) \) the shifts in \( c_x \)

just affect the "height" of \( c_x = 0 \) locus.

Unanticipated change.

\[
\begin{array}{c}
\tau_x \\
\tau_i \\
0
\end{array}
\]

See next page.
unanticipated change - temporary

\[ c(t) = \begin{cases} c_1 & t < T \\ c_2 & t \geq T \end{cases} \]

\[ \dot{c}(t) = 0 \]

\[ c(T) = c_2 \]

1) c jumps down at time 0 to \( c_1 \)
2) Paths under taxation, \( C_t \), are now deriving behaviour of \( C_{t+1} \) during this period \( C_T \), \( e_T \)
3) at Time \( T \), \( C_t \) and \( C_{t+1} \) are on path under \( e_1 \) in this period \( C_{t+1}, e_T \)
Plan for consumer solves
\[
\max_{c_t, g_t} \int \frac{(c_t + g_t)^{-1 - \sigma}}{1 - \sigma} \, c_t \, dt
\]

S.t. \quad \tau_t = c_t + I_t + g_t

Now write budget constraint
\[
\tau_t = \beta c_{t+1} + (1 + \delta) g_t - c_t - g_t
\]

Hamiltonian
\[
H = \frac{(c_t + g_t)^{1 - \sigma}}{1 - \sigma} e^{-(g_t - c_t - g_t)} + \lambda_t c_t - \beta (1 + \delta) e^{-(g_t - c_t - g_t)}
\]

F.O.C.
\[
\begin{align*}
\frac{\partial H}{\partial c_t} &= (c_t + g_t)^{1 - \sigma} e^{-(g_t - c_t - g_t)} = \lambda_t - \beta (1 + \delta) e^{-(g_t - c_t - g_t)} \\
\frac{\partial H}{\partial g_t} &= (c_t + g_t)^{1 - \sigma} e^{-(g_t - c_t - g_t)} = \lambda_t \\
\frac{\partial H}{\partial \lambda_t} &= -\beta (1 + \delta) e^{-(g_t - c_t - g_t)} = -\lambda_t
\end{align*}
\]

\[\lim_{t \to \infty} \lambda_t = 0\]

⇒ Indeterminacy between choice of \( c_t \) and \( g_t \). (3 unknowns, only 2 eqns.)

Perfect substitutability + \((c_{t+1} + g_t)\beta < 1\) satisfy \((\sigma)\) ⇒ that any choice

of \( g_t \) such that \( g_t = c_t \) delivers the same consumption as in case without taxes.
when considering path with \( q_t = 0 \) \( t \) this problem collapses to standard problem without distortions.

Therefore this result support our view that any equilibrium with this path \( q_t \) (\( q_0 > 0 \) \( t \)) is socially optimal.

From this follows that for any plausible level of taxation the competitive maintained equilibrium is socially optimal (decentralized).

Change to lump sum taxation will not lead to change in social optimality of solution, because the consumption taxation already delivers socially optimal solution.

Perfect substitutability of private consumption and government purchase plays crucial role, because it makes utility from households indifferent between private and private consumption and utility from government good.
1) \[ u(x, y, t) = \frac{(c(x, y, t))^{1-\theta}}{1-\theta} \]

Hence \[ u(x, y, t) = c(x, y, t)^{1-\theta} \]

Therefore \[ u_c e^{-\theta (n-t)} = \alpha_1 \] changes to \[ \frac{\partial c}{\partial t} (1-\theta)^{-1} \cdot c_t + (1-\theta) c_{x} e^{-\theta (n-t)} = \lambda_1 (1+\pi) \]

Using \[ q_t = c \cdot c_t \rightarrow \text{we get} \]

\[
\begin{align*}
\frac{\partial c}{\partial t} (1-\theta) c_t + (1-\theta) c_{x} e^{-\theta (n-t)} &= \lambda_1 (1+\pi) \\
\frac{\partial c}{\partial t} (1-\theta) c_t - \theta c_{x} e^{-\theta (n-t)} &= \lambda_1 (1+\pi) \\
\end{align*}
\]

Alternatively with time

\[
\frac{\partial c}{\partial t} (1-\theta) c_t - \theta e^{-\theta (n-t)} = \lambda_1 (1+\pi) 
\]

We plug in 
\[
(n_r - n) = \frac{\alpha_1}{\lambda_1} 
\]

\[
\frac{\partial c}{\partial t} (1-\theta) c_t - \theta e^{-\theta (n-t)} = \lambda_1 (1+\pi) 
\]

\[
\frac{\partial c}{\partial t} (1-\theta) c_t - \theta e^{-\theta (n-t)} = \lambda_1 (1+\pi) 
\]

New Euler Equation

\[
\frac{c_x}{c_t} = \frac{1}{\theta} (n_r - n) 
\]
Consider 3 saving rates: low \( s_1 \), golden rule \( s_{cr} \) (consumption's maximized), and high \( s_2 \).

When household are over-saving at rate \( s_2 \) (\( s_2 > s_{cr} \)), then planner can decide to decrease saving rate to \( s_{cr} \) and \( s_{cr} \). Along the transition path to lower saving rate \( s_{cr} \), the consumption is higher at all points of time (per capita).

Then on the path with saving rate \( s_2 \) therefore the economy with too high saving rate is dynamically inefficient.

When thresholds are same at rate \( s_1 \) \(< s_{cr} \), the per capita consumption can be increase increased, but it requires lowering consumption (temporary) and the overall outcome...
has to take into account the discounting of future consumption.

3) Models of endogenous growth usually use capital accumulation, set. I also include human capital into productivity or innovation.

In the view of models of endogenous technological change, capital accumulation and innovation are two aspects of the same process: Because introduction of new forms of capital leads to accumulation of the new forms of capital.

Therefore the models of endogenous technological change induce expansion of the number of varieties of goods.

New technology.

These models, as describe the production of final goods that produced from a large variety of intermediate goods. An expansion of number of varieties requires technological advance-innovation, and the assumption is that in vendor maintains monopoly right on the production of its goods for some period.

For more details, see Chapter 6 or page 15 by Barro and Sala-i-Martin in Economic Growth.