1. **An "AK" Model with Physical and Human Capital.** Consider the following aggregate production function:

\[ Y = K^\alpha (hL)^{1-\alpha}, \]

where \( h = H/L \) is human capital per person, \( Y \) is output, and \( K \) is physical capital. Accumulation of the capital inputs is given by

\[
\begin{align*}
\dot{K} &= s_K Y - \delta K \\
\dot{H} &= s_H Y - \delta H
\end{align*}
\]

where \( s_K \) and \( s_H \) are constant and exogenously given. The labor force, \( L \), grows at the exogenous growth rate \( n \).

(a) What is the per capita growth rate of the economy along the balanced growth path?

(b) Show that along the balanced growth path the production function can be written as \( Y = AK \) where \( A \) is constant. What is the value of \( A \)?

(c) Discuss briefly the "story" one would tell to construct a decentralized, competitive equilibrium for this economy. Are there any externalities needed? Will the decentralized equilibrium be socially optimal if we endogenize the determination of \( s_K \) and \( s_H \)?

(d) Discuss the effect of imposing the inequality restrictions \( I_K \geq 0 \) and \( I_H \geq 0 \).

2. **Capital Externalities in a Neoclassical Growth Model.** Consider the usual Ramsey-Cass-Koopmans model as discussed in class (i.e. with population growth at rate \( n \) and exogenous technological progress at rate \( g \)). Assume instantaneous utility is CRRA. Let final output be produced by a continuum of (identical) firms of mass
one (firms can be continuously indexed on the interval \( i \in [0, 1] \)), with each firm using the same production function

\[
Y_i = \kappa^\beta K_i^\alpha (AL_i)^{1-\alpha},
\]

where \( \kappa = \int_0^1 K_i di \). Assume that there are positive externalities to capital accumulation. That is, firms take the aggregate capital stock, \( \kappa \), as given at each point in time. Notice that because the measure of firms is one, \( \kappa = K = K_i \). The rest of the model is identical to the Ramsey framework.

(a) What is the per capita growth rate of the economy along the balanced growth path? If \( g = 0 \) is there per capita growth? Why?

(b) Solve for the investment rate \( s = I/Y \) along the balanced growth path for the decentralized economy.

(c) What is the socially optimal investment rate along the balanced growth path, and why are these rates different?