Homework 2

1. *Prob. 2.9 from Romer, 2nd ed.: Capital taxation in the Ramsey–Cass–Koopmans model.* Consider a Ramsey–Cass–Koopmans economy that is on its balanced growth path. Suppose that at some time, which we will call time 0, the government switches to a policy of taxing investment income at rate $\tau$. Thus, the real interest rate that households face is now given by $r(t) = (1 - \tau)f'(k(t))$. Assume that the government returns the revenue it collects from this tax through lump-sum transfers. Finally, assume that this change in tax policy is unanticipated.

(a) How does the tax affect the $\dot{c} = 0$ locus? The $\dot{k} = 0$ locus?

(b) How does the economy respond to the adoption of the tax at time 0? What are the dynamics after time 0?

(c) How do the values of $c$ and $k$ on the new balanced growth path compare with their values on the old balanced growth path?

(d) (this is based on Barro, Mankiw and Sala–i–Martin, 1995.) Suppose there are many economies like this one. Workers’ preferences are the same in each country, but the tax rates on investment income may vary across countries. Assume that each country is on its balanced growth path.

(i) Show that the saving rate on the balanced growth path, $(y^* - c^*)/y^*$, is decreasing in $\tau$.

(ii) Do citizens in low–$\tau$, high–$k^*$, high–saving countries have any incentives to invest in low–saving countries? Why or why not?
(iii) Does your answer to part (c) imply that a policy of *subsidizing* investment (that is, making $\tau < 0$), and rising the revenue for this subsidy through lump-sum taxes, increases welfare? Why or why not?

(iv) How, if at all, do the answers to parts (a) and (b) change if the government does not rebate the revenue from the tax but instead uses it to make government purchases?

2. **Prob. 2.10 from Romer, 2nd ed.** *Using the phase diagram to analyze the impact of an anticipated change.* Consider the policy described in Problem 2.9, but suppose that instead of announcing and implementing the tax at time 0, the government announces at time 0 that at some later time, time $t_1$, investment income will begin to be taxed at rate $\tau$.

   (a) Draw the phase diagram showing the dynamics of $c$ and $k$ after time $t_1$.
   
   (b) Can $c$ change discontinuously at time $t_1$? Why or why not?
   
   (c) Draw the phase diagram showing the dynamics of $c$ and $k$ before $\tau_1$.
   
   (d) In light of your answers to parts (a), (b) and (c) do at time 0?
   
   (e) Summarize your results by sketching the paths of $c$ and $k$ as functions of time.

3. **Prob. 2.11 from Romer, 2nd ed.** *Using the phase diagram to analyze the impact of an anticipated change and anticipated temporary changes.* Analyze following two variations on Problem 2.10:

   (a) At time 0, the government announces that it will tax investment income at rate $\tau$ from time 0 until some later date $t_1$; thereafter investment income will again be untaxed.
   
   (b) At time 0, the government announces that from time $t_1$ to some later time $t_2$, it will tax investment income at rate $\tau$; before $t_1$ and after $t_2$, investment income will not be taxed.