Perceiving Prospects Properly

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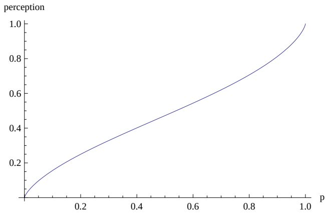
Puzzle



What are perception biases good for?

Terminology

observation: agent observes payoff-relevant parameter x

$\downarrow \varepsilon$

decision: based on a perturbed observation, y

- error in perception: $y \neq x$ (assumed)
- bias in perception: $E[y \mid x] \neq x$ (derived)

First example

tasks:

- n options with values $x_1, \ldots, x_n$
- choose one
- x_k are i.i.d. from $N(0, 1)$

technology:

- memorize $(m_1, \ldots, m_n) \in \mathbb{R}^n$
- recall $(y_1, \ldots, y_n) = (m_1 + \varepsilon_1, \ldots, m_n + \varepsilon_n)$
- ε_k are i.i.d., zero mean
- choose

unbiased perception strategy:

- $m_k = x_k$ for all k
- choose the highest y_k

Unbiased perception is suboptimal

- $m_k = x_k$, for $k = 1, \ldots, n-1$; general $m_n(x_n)$
- choose the highest y_k
- optimize $m_n(x_n)$

Desideratum (from f.o.c.):

- let $s = \max\{y_1, \ldots, y_{n-1}\}$
- unbiased perception at ties:

$$E[x_n \mid y_n = s] = s$$

Unbiased perception $m_n = x_n$:

- s is high compared to typical draws of x_n
- optimistic ε_n increases likelihood of tie with s
- conditional on the tie, the expected error is optimistic
- unbiased perception ex ante
- biased perception at ties

Outline

Model and Result

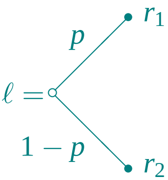
Intuition

Full Optimization

Literature

Is Perception Bias a Mistake?

Individual's problem

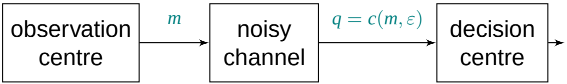


versus alternative s.

The lottery is the optimal choice $\Leftrightarrow pr_1 + (1-p)r_2 \geq s$.

Technology

p:



$$\begin{cases} \text{lottery} & \text{if } qr_1 + (1-q)r_2 > s, \\ \text{alternative} & \text{otherwise.} \end{cases}$$

Evolution's problem

Define performance:

$$f(\ell, q) = \begin{cases} pr_1 + (1-p)r_2 & \text{if } qr_1 + (1-q)r_2 > s, \\ s & \text{otherwise.} \end{cases}$$

Evolution's problem

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Evolutionary problem:

$$\max_{m(\cdot)} E\left[f\Big(\ell, c(m(p), \varepsilon)\Big) \right].$$

Special case

$q = m + \varepsilon$, $\varepsilon = \pm\sigma$ equally likely
$p \in [\sigma, 1 - \sigma]$ symmetric about $1/2$
$r_1, r_2 \sim N(0, 1)$
r_1, r_2, p, and ε independent

Proposition

If $s > 3^{1/4}$ then the individual

1. *overweights small probabilities: $m(p) > p$ for all $p < 1/2$,*
2. *underweights large probabilities: $m(p) < p$ for all $p > 1/2$.*

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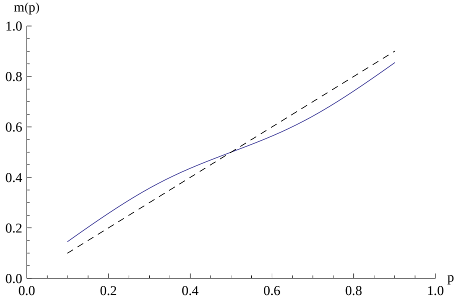

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Pivotal argument

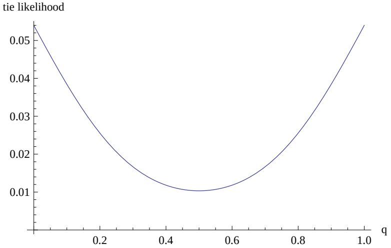
A small error in perception is consequential only if

$$pr_1 + (1-p)r_2 \approx s.$$

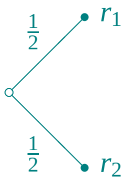
The optimal strategy must condition on a (perceived) tie:

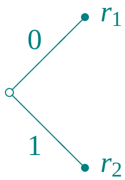
$$qr_1 + (1-q)r_2 = s.$$

Which probabilities are likely to lead to a tie?



Intuition for the U-shape

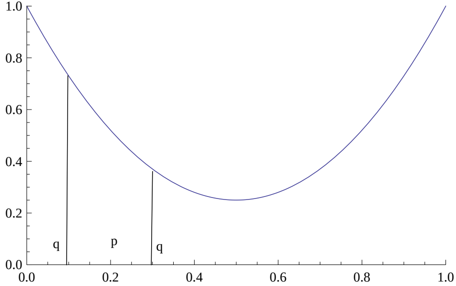




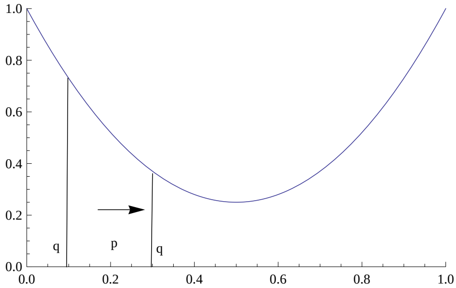
distribution of the lottery expectation:

mean $= 0$	mean $= 0$
variance $= \frac{1}{2}$	variance $= 1$
unlikely tie with s	likely tie with s

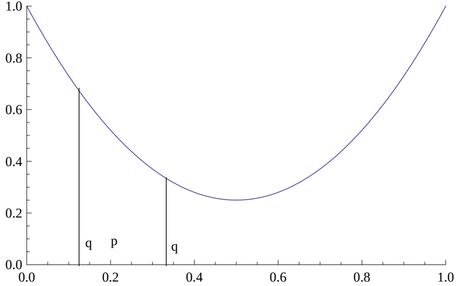
Optimal perception



Optimal perception



Optimal perception



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Motivation

What does the alternative stand for? Why is it high?

What is the role of naivety?

Changes in the setting:

- choice over many lotteries
- sophisticated decision centre
- finite message set instead of the noisy channel

Setting

Individual chooses from a set of the lotteries with measure $\phi(\ell)$.

She selects a subset L of measure $\kappa \in (0,1)$ and receives

$$\int_L \Big(pr_1 + (1-p)r_2\Big)\phi(\ell)d\ell.$$

Individual consists of:

OC: for each lottery, sends message $m(p) \in M$; M is finite.

DC: applies a selection function $c(m, r_1, r_2) \in \{0, 1\}$.

Evolutionary problem:

$$\max_{m,c} \quad E\left[\Big(pr_1 + (1-p)r_2\Big)c(m(p), r_1, r_2)\right],$$

$$\text{s.t.:} \qquad E\left[c\left(m(p), r_1, r_2\right)\right] = \kappa.$$

Solution

Optimal strategy is Bayesian:
$q(m) = E[p \mid m]$,

$$c(m, r_1, r_2) = \begin{cases} 1 & \text{if } q(m)r_1 + (1 - q(m))r_2 \geq s, \\ 0 & \text{otherwise,} \end{cases}$$

where s is the value of the marginal lottery,

$$\Pr(q(m)r_1 + (1 - q(m))r_2 \geq s) = \kappa.$$

When κ is sufficiently small then agent overweights small probabilities:
for all $p < 1/2$,

$$q(m^*(p)) \geq \arg\min_{m \in M} (q(m) - p)^2.$$

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Principal agent approach to evolution

Robson (2001a), Samuelson & Swinkels (2006), Robson & Samuelson (2007)

Ex ante attention allocation

Robson (2001b), Rayo & Becker (2007), Netzer (2009)

Rational inattention

Woodford (2012a,b)

Other foundations of prospect theory

Herold & Netzer (2010), Frenkel, Heller & Teper (2013)

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Is perception bias a mistake?

Removing biases across all decision problems would harm the decision-maker.

Overweighting of danger — proper cautiousness

Examples from the field:

- excessive insurance deductibles (Barseghyan et al 2013),
- fear of the rare market crashes (De Giorgi & Legg 2012).

But, decision-making can be improved upon.

Thaler & Ziemba (1988) document harmful overweighting of small probabilities in gambling.

Why does evolution put up with this?

Because Evolution optimizes perception ex ante and the distributions of marginal lotteries in and out of the casino differ.

We can classify gambling as an error.