Perceiving Prospects Properly

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What are perception biases good for?
Observation: agent observes payoff-relevant parameter $x$

Decision: based on a perturbed observation, $y$

- **Error** in perception: $y \neq x$ (assumed)
- **Bias** in perception: $E[y \mid x] \neq x$ (derived)
First example

tasks:

- $n$ options with values $x_1, \ldots, x_n$
- choose one
- $x_k$ are i.i.d. from $N(0, 1)$

technology:

- memorize $(m_1, \ldots, m_n) \in \mathbb{R}^n$
- recall $(y_1, \ldots, y_n) = (m_1 + \varepsilon_1, \ldots, m_n + \varepsilon_n)$
- $\varepsilon_k$ are i.i.d., zero mean
- choose

unbiased perception strategy:

- $m_k = x_k$ for all $k$
- choose the highest $y_k$
Unbiased perception is suboptimal

- \( m_k = x_k \), for \( k = 1, \ldots, n - 1 \); general \( m_n(x_n) \)
- choose the highest \( y_k \)
- optimize \( m_n(x_n) \)

Desideratum (from f.o.c.):
- let \( s = \max \{y_1, \ldots, y_{n-1}\} \)
- unbiased perception at ties:
  \[
  E [x_n \mid y_n = s] = s
  \]

Unbiased perception \( m_n = x_n \):
- \( s \) is high compared to typical draws of \( x_n \)
- optimistic \( \varepsilon_n \) increases likelihood of tie with \( s \)
- conditional on the tie, the expected error is optimistic
- unbiased perception ex ante
- biased perception at ties
Outline

Model and Result

Intuition

Full Optimization

Literature

Is Perception Bias a Mistake?
Individual’s problem

\[ \ell = \frac{p}{1 - p} \]

versus alternatives.

The lottery is the optimal choice \( \iff pr_1 + (1 - p)r_2 \geq s \).
Technology

\[ p: \quad \text{observation centre} \xrightarrow{m} \text{noisy channel} \xrightarrow{q = c(m, \varepsilon)} \text{decision centre} \]

\[ \begin{aligned}
\{ \text{lottery} \quad \text{if} \quad qr_1 + (1 - q)r_2 > s, \\
\text{alternative} \quad \text{otherwise.} \end{aligned} \]
Evolution’s problem

Define performance:

\[ f(\ell, q) = \begin{cases} 
  pr_1 + (1 - p)r_2 & \text{if } qr_1 + (1 - q)r_2 > s, \\
  s & \text{otherwise.}
\end{cases} \]
Evolution’s problem

Define performance:

\[
f(\ell, q) = \begin{cases} 
pr_1 + (1 - p)r_2 & \text{if } qr_1 + (1 - q)r_2 > s, \\
s & \text{otherwise.}
\end{cases}
\]

Evolutionary problem:

\[
\max_{m(\cdot)} E \left[ f(\ell, c(m(p), \varepsilon)) \right].
\]
**Special case**

\[ q = m + \varepsilon, \quad \varepsilon = \pm \sigma \text{ equally likely} \]
\[ p \in [\sigma, 1 - \sigma] \text{ symmetric about } 1/2 \]
\[ r_1, r_2 \sim N(0, 1) \]
\[ r_1, r_2, p, \text{ and } \varepsilon \text{ independent} \]

**Proposition**

*If \( s > 3^{1/4} \) then the individual*

1. *overweights small probabilities*: \( m(p) > p \) for all \( p < 1/2 \),
2. *underweights large probabilities*: \( m(p) < p \) for all \( p > 1/2 \).
Special case

$q = m + \varepsilon, \; \varepsilon = \pm \sigma$ equally likely
$p \in [\sigma, 1 - \sigma]$ symmetric about $1/2$
$r_1, r_2 \sim N(0, 1)$
$r_1, r_2, p, \text{ and } \varepsilon$ independent

![Graph showing the relationship between $p$ and $m(p)$]
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Is Perception Bias a Mistake?
A small error in perception is consequential only if

\[ pr_1 + (1 - p)r_2 \approx s. \]

The optimal strategy must condition on a (perceived) tie:

\[ qr_1 + (1 - q)r_2 = s. \]
Which probabilities are likely to lead to a tie?
Intuition for the U-shape

distribution of the lottery expectation:

\[
\begin{align*}
\text{mean} &= 0 & \text{mean} &= 0 \\
\text{variance} &= \frac{1}{2} & \text{variance} &= 1 \\
\text{unlikely tie with} \ s & & \text{likely tie with} \ s
\end{align*}
\]
Optimal perception
Optimal perception
Optimal perception
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Is Perception Bias a Mistake?
Motivation

What does the alternative stand for? Why is it high?
What is the role of naivety?

Changes in the setting:
- choice over many lotteries
- sophisticated decision centre
- finite message set instead of the noisy channel
Setting

Individual chooses from a set of the lotteries with measure $\phi(\ell)$. She selects a subset $L$ of measure $\kappa \in (0, 1)$ and receives

$$\int_L \left( pr_1 + (1 - p)r_2 \right) \phi(\ell) d\ell.$$

Individual consists of:

**OC:** for each lottery, sends message $m(p) \in M$; $M$ is finite.

**DC:** applies a selection function $c(m, r_1, r_2) \in \{0, 1\}$.

Evolutionary problem:

$$\max_{m, c} \quad E \left[ \left( pr_1 + (1 - p)r_2 \right) c(m(p), r_1, r_2) \right],$$

s.t.: \quad $E \left[ c \left( m(p), r_1, r_2 \right) \right] = \kappa.$
Solution

Optimal strategy is Bayesian:

\[ q(m) = E[p \mid m], \]

\[ c(m, r_1, r_2) = \begin{cases} 1 & \text{if } q(m)r_1 + (1 - q(m))r_2 \geq s, \\ 0 & \text{otherwise}, \end{cases} \]

where \( s \) is the value of the marginal lottery,

\[ \Pr(q(m)r_1 + (1 - q(m))r_2 \geq s) = \kappa. \]

When \( \kappa \) is sufficiently small then agent overweights small probabilities:

for all \( p < 1/2 \),

\[ q(m^*(p)) \geq \arg \min_{m \in M} (q(m) - p)^2. \]
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Is Perception Bias a Mistake?
Literature

Principal agent approach to evolution

Ex ante attention allocation

Rational inattention
Woodford (2012a,b)

Other foundations of prospect theory
Herold & Netzer (2010), Frenkel, Heller & Teper (2013)
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Is Perception Bias a Mistake?
Is perception bias a mistake?

Removing biases across all decision problems would harm the decision-maker.

Overweighting of danger — proper cautiousness

Examples from the field:

- excessive insurance deductibles (Barseghyan et al 2013),
- fear of the rare market crashes (De Giorgi & Legg 2012).
But, decision-making can be improved upon.


Why does evolution put up with this?

Because Evolution optimizes perception ex ante and the distributions of marginal lotteries in and out of the casino differ.

We can classify gambling as an error.