

Endogenous Risk Attitudes

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JEEA Teaching Materials

Nonlinear Measurement Scales

physical instruments often use nonlinear measurement scales

- this improves precision at some range of inputs
- at the expense of precision at other values

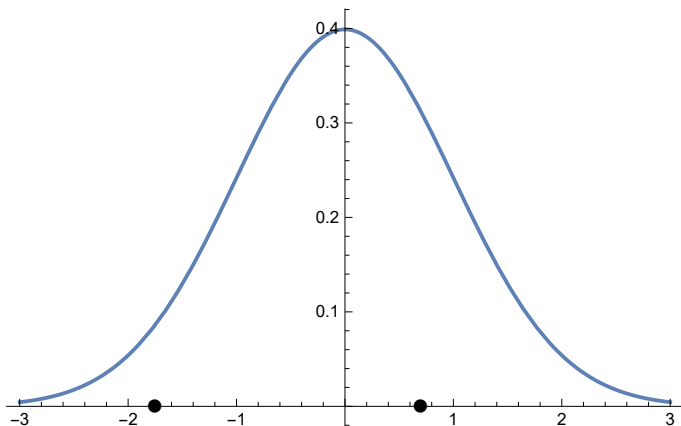
psychophysics literature extends this to human perception

- Kahneman & Tversky '79 use this to justify S-shaped utility

Formalization

Robson '01, Netzer '09

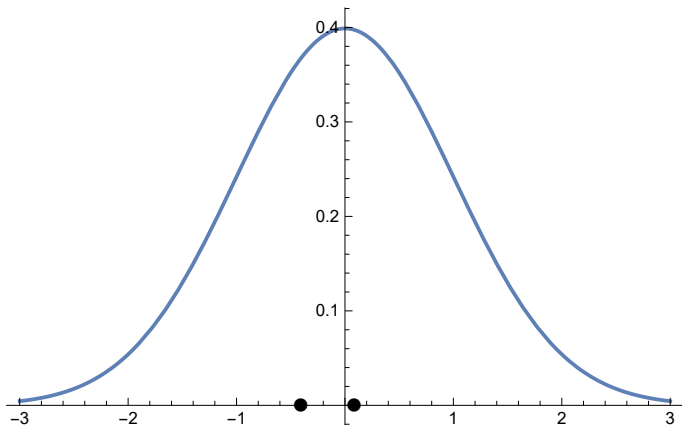
pick one of the two draws:



Formalization

Robson '01, Netzer '09

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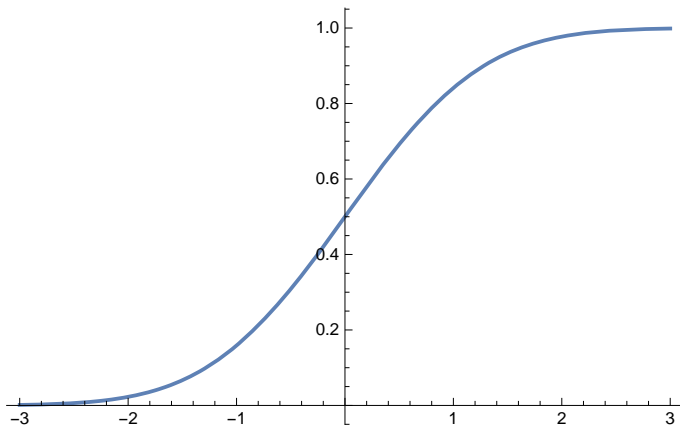
Formalization

Robson '01, Netzer '09

encode reward r_i as $m(r_i) + \varepsilon_i$

choose your encoding function m

optimal encoding function as noise vanishes



Our Contribution

Robson '01, Netzer '09:

- perception of one-dimensional inputs
- encoding function \sim hedonic as opposed to Bernoulli utility
- vanishing implications for choice

this paper:

- 1 exogenous perception \Rightarrow behavior
 - coarse model \Rightarrow perception-driven risk attitudes
 - well-specified model \Rightarrow risk-neutrality
- 2 optimal perception of lotteries
 - microfounded objective
 - s-shaped encoding function
 - over-sampling of low-probability states

psychophysics:

Weber's law, Fechner 1860, Thurstone '27...

encoding of stimuli:

Attneave '54, Barlow et al. '61, Laughlin '81...

econ [riskless]:

Robson '01, Netzer '09, Rayo&Becker '07...

econ [risky, large noise]:

Friedman '89, Khaw&Li&Woodford '20, Frydman&Jin '19...

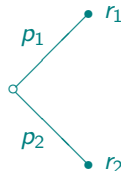
misspecification:

Berk '66, White '82, Esponda Pouzo '16, Heidhues et al. '18...

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Decision Problem



versus

alternative s

risk-neutrality: lottery optimal $\Leftrightarrow r := \sum_i p_i r_i > s$

set of states and probabilities fixed, and DM observes s frictionlessly

the DM:

- measures each reward many times
- estimates the lottery value given the collected data
- controls the encoding function and sampling frequencies

Perception

perception strategy:

- encoding function $m : [\underline{r}, \bar{r}] \longrightarrow [\underline{m}, \bar{m}]$; exogenous span
- sampling frequencies $(\pi_i)_i \in \Delta$ (set of states)

DM samples signals (i_k, \hat{m}_k) , $k = 1, \dots, n$:

- i_k specifies the state; sampling frequencies $\pi_i \neq p_i$
- $\hat{m}_k = m(r_{i_k}) + \varepsilon_k$; iid standard normal noise

DM is sophisticated: knows conditional signal distributions

decoding: a map from perception data to the estimate of the lottery

nearly complete information: $n \rightarrow \infty$

a posteriori optimal choice

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Simple Decoding

fix perception strategy $m(\cdot)$ and $(\pi_i)_i$

def simple decoding: DM's estimate of lottery value $= m^{-1}(\sum_{k=1}^n \hat{m}_k)$

Observation

The probability that the DM chooses the lottery in problem (\mathbf{r}, s) converges a.s. to 1 (0) as $n \rightarrow \infty$ if

$$\sum_i \pi_i m(r_i) > (<) m(s).$$

EU maximizer with Bernoulli utility $m(\cdot)$ and subjective probabilities π_i

two treatments:

- | | | |
|--|----|-------------|
| ① genuine lottery $(p_i, r_i)_i$ | vs | safe option |
| ② certainty equivalent of $(p_i, r_i)_i$ | vs | safe option |

nearly identical choices across the treatments

aggregation friction rather than risk aversion

our simple procedure fits Oprea's subjects

Maximum Likelihood Estimate

the DM is endowed with a compact set $\mathcal{A} \subseteq [\underline{r}, \bar{r}]^I$ of anticipated lotteries

forms ML estimate of the lottery

$$\mathbf{q}_n \in \arg \max_{\mathbf{r}' \in \mathcal{A}} \prod_{k=1}^n \varphi(\hat{m}_k - m(r'_{i_k}))$$

Proposition

Suppose that the DM anticipates that the lottery involves no risk:

$$\mathcal{A} = \{\mathbf{r} \in [\underline{r}, \bar{r}]^I : r_i = r_j \text{ for all states } i, j\}.$$

Then, she follows the simple decoding procedure.

White '82: asymptotic MLE minimizes KL-divergence from the true data-generating process, among all anticipated processes

$$\text{MLE} \xrightarrow{\text{a.s.}} \arg \min_{r' \in \mathcal{A}} D_{KL}(f_r \parallel f_{r'})$$

with Gaussian errors & no anticipated risk

$$D_{KL}(f_r \parallel f_{r'}) = \sum_{i=1}^l \pi_i (m(r_i) - m(r'_i))^2$$

hence MLE of $m(r) \rightarrow \sum_{i=1}^l \pi_i m(r_i)$

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Coarse Anticipation of Risk

DM anticipates lotteries to be measurable w.r.t. a partition of arms \mathcal{K}

Proposition

Prob that DM chooses the lottery in problem (\mathbf{r}, s) converges to 1 [0] if

$$\sum_{J \in \mathcal{K}} p_J r_J^* > [<] s,$$

where, for each $J \in \mathcal{K}$,

- r_J^* is the certainty equivalent $m(r_J^*) = \sum_{i \in J} \frac{\pi_i}{\sum_{j \in J} \pi_j} m(r_i)$
- $p_J = \sum_{i \in J} p_i$ is the true probability of J

- anticipated risk: risk neutrality
- unanticipated risk: risk attitudes

Impact of Prior Information

let's bridge the gap between anticipated and unanticipated lotteries

joint limit of

- number of signals
- precision of prior density of Bayesian DM

effects of

- time pressure
- level of anticipated risk

Prior Belief and Sampling

prior $\propto \exp\left(-\frac{n}{\Delta}\sigma^2(\mathbf{r})\right)$ on $[\underline{r}, \bar{r}]^I$, where $\sigma^2(\mathbf{r}) = \sum_i p_i (r_i - r)^2$

DM samples $a \times n$ perturbed messages

- Δ – degree of the a priori anticipated risk
- a – attention span, sample size increases with a
- as n grows
 - sample size grows
 - risk becomes a priori unlikely

Proposition

The Bayesian estimate of lottery \mathbf{r} converges to

$$\mathbf{q}^*(\mathbf{r}) = \arg \min_{\mathbf{r}' \in [\underline{L}, \bar{r}]^I} \left\{ \frac{1}{a\Delta} \sigma^2(\mathbf{r}') + \sum_i \pi_i (m(r_i) - m(r'_i))^2 \right\}.$$

limiting cases

- $a\Delta$ large: close to risk-neutrality
- $a\Delta$ small: close to the simple procedure

unstable risk attitudes

- $a \rightarrow 0$ vs. $a \rightarrow \infty$: Kahneman's thinking fast/slow
- $\Delta \rightarrow 0$ vs. $\Delta \rightarrow \infty$: Rabin's paradox

Proposition

Consider a lottery with small risk σ^2 . The Bayesian estimate of the lottery value converges a.s. to

$$r + \frac{1}{2} \frac{m''(r)}{m'(r)} \cdot \sigma^2 \cdot \frac{1 + 4a\Delta m'^2(r)}{[1 + a\Delta m'^2(r)]^2} + o(\sigma^2).$$

- $a\Delta \rightarrow 0$: the usual Arrow-Pratt measure for $u(\cdot) = m(\cdot)$
- $a\Delta \rightarrow \infty$: risk-neutrality

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Objective

ex ante distribution of the decision problems (\mathbf{r}, s)

- all r_i iid from continuously differentiable density h
- s independently from continuously differentiable density h_s

ex ante minimization of

$$L(n) = \mathbb{E} [\max \{r, s\} - \mathbb{1}_{q_n > s} r - \mathbb{1}_{q_n \leq s} s]$$

- equivalent to maximization of the expected chosen reward

loss becomes tractable as n diverges

Proposition

If the encoding function m is continuously differentiable, then

$$L(n) = \text{const.} \cdot \mathbb{E} \left[\sum_i \frac{p_i^2}{\pi_i m'^2(r_i)} \mid r = s \right] \frac{1}{n} + O\left(\frac{1}{n^2}\right).$$

Proposition

If the encoding function m is continuously differentiable, then

$$L(n) \propto \mathbb{E}[\text{MSE conditional on tie}].$$

choice is distorted if s falls between r and value estimate q_n

condition on ties: small perception error distorts choice only if $r \approx s$

loss \propto MSE

Proposition

If the encoding function m is continuously differentiable, then

$$L(n) \propto \mathbb{E} \left[\sum_i p_i^2 \text{MSE}(r_i) \text{ conditional on tie} \right].$$

MSE is a weighted sum of MSEs for each r_i

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$\text{MSE}(r_i)$ is mitigated by high π_i and $m'(r_i)$

Information-Processing Problem

$$\min_{m'(\cdot), (\pi_i)_{i>0}} \mathbb{E} \left[\sum_i \frac{p_i^2}{\pi_i m'^2(r_i)} \mid r = s \right]$$

$$\text{s.t.: } \int_{\underline{r}}^{\bar{r}} m'(r) dr \leq \bar{m} - \underline{m}$$

$$\sum_i \pi_i = 1$$

attention allocation:

- high $m'(\tilde{r})$ focuses on the neighborhood of \tilde{r}
- high π_i focuses on the state i

constraints:

- $m(\cdot)$ is bounded – your scale can't be fine everywhere
- $\sum_i \pi_i = 1$ – you can't sample all the states frequently

Optimal Perception

suppose h and h_s are unimodal with a same mode and symmetric

Proposition

- 1 Optimal encoding function is **s-shaped**:
 $m(\cdot)$ is convex below and concave above the modal reward
- 2 Over-sampling of low-probability states:
 $\frac{\pi_J}{\pi_{J'}} > \frac{p_J}{p_{J'}}$ when $p_J < p_{J'}$

intuition:

- 1 focus on reward values that you're likely to encounter at ties
- 2
 - over-sample states that you expect to be poorly informed on
 - you measure tail rewards poorly
 - conditional on tie, low-probability state has more spread-out rewards
since $\sum_{J'} p_{J'} r_{J'} = s$ isn't too informative about r_J when p_J is small

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link between perception and risk attitudes arises when decoding is coarse

- informed comparative statics on perception predicts choice

optimality arguments get some stylized facts about perception right

- we introduce marginal reasoning to psychophysics