Coordination of Mobile Labor*

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December 11, 2006

Abstract

We study coordination failures in many simultaneously occurring coordination problems. Players encounter one of the problems but have the outside option of migrating to one of the remaining ones. Drawing on the global games approach, we show that such a *mobile* game has a unique equilibrium that allows us to examine comparative statics. The endogeneity of the outside option value and of the migration activity leads to non-monotonicity of welfare with respect to mobility friction; high mobility may hurt players. We apply these "general equilibrium" findings to the problem of the labor market during industrialization as described by Matsuyama [11].

JEL classification: C72, D82, D83.

Keywords: Coordination, General Equilibrium, Global Games, Globalization, Industrialization, Mobility.

*I am grateful to Amil Dasgupta, Bernardo Guimaraes, Fuhito Kojima, Peter Kondor, Eugen Kováč, Andreas Ortmann, Avner Shaked, and Hyun Shin for their comments on the earlier version of the paper. Discussions with Jozsef Sakovics, John Moore and other colleagues in the University of Edinburgh helped me to revise the paper. An anonymous referee suggested the application of the game to the problem of industrialization. The usual disclaimer applies.

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1 Introduction

Coordination problems[1] are usually analyzed in isolation. A typical model highlights a single coordination problem characterized by a payoff function with strategic complementarities and all other economic opportunities of players are summarized by an exogenous outside option. Such models are analogous to the partial equilibrium approach and are useful for highlighting a particular economic feature. In this paper we study the mutual interactions of several coordination problems in a setup analogous to the general equilibrium approach. As is often the case, the subtle "general equilibrium" effects lead to surprising comparative statics.

We set up the model within the framework of Matsuyama [11], who studies the coordination problem of workers during industrialization.[2] Matsuyama considers a single emerging industry sector with increasing returns to scale and workers deciding between joining the industry or joining agriculture which serves as a safe outside option. To move to the "general equilibrium", we consider several industries instead of one and let players migrate among them, which allows examining the effects of mobility on the extent of industrialization and, consequently, on welfare. Players are uniformly matched to the industry sectors at the beginning of the game. The outside option of a player born into sector (coordination problem) j, who considers staying in j, consists of retraining/emigration which then allows her to join one of the other sectors. Thus the outside option value in any coordination problem j is endogenously determined by players' behavior in all the other coordination problems. Another channel through which the coordination problems mutually influence each other is that the mass of immigrants to j depends on the coordination outcomes of all the other coordination problems.

Coordinating on an efficient but risky action is difficult to achieve and an increase in the value of the outside option value further undermines the successful coordination. Conversely, an increase in the mass of immigrants enhances coordination: first, the immigrants to sector j directly increase the productivity in j because of the increasing returns to scale; second, the players native to j are more motivated to stay in j which further increases the participation in j.

A player leaving j imposes a direct *negative externality* on the players left behind in j as she lowers productivity in j, and a direct *positive externality* on the players in her new sector. But her migration also influences all the players also indirectly. Her

[1] Coordination problems are common in economics; for a review, see Cooper [3].

[2] We are grateful to the referee who pointed out this application.

emigration from j decreases the outside option values in all the other sectors $j' \neq j$, hence players observing j' coordinate on staying in j' more often. However, the incentive to leave j increases, and thus the coordination on staying in j becomes more difficult.

These causal links are difficult to analyze because of equilibrium multiplicity in coordination games. We therefore use the global games approach, which allows us to predict the coordination outcome in each sector j for given strategy profiles in all other sectors. Global game approach is a reasonable selection tool for our purposes because its comparative statics is indeed in line with the causal links described above.[3]

The uniqueness of equilibrium allows us to study comparative statics. We find that the externalities discussed above lead to counterintuitive effects. Welfare is non-monotonic in mobility: the direct non-strategic effect of an increase in mobility is positive, as, ignoring strategic considerations, moving to a successful sector is cheaper. However, the strategic effect is negative: lower mobility cost increases the outside option value associated with the emigration, which undermines successful coordination. Thus some sectors that would have succeeded had the mobility cost been high, fail when the mobility cost is low. This negative strategic effect may outweigh the positive direct effect and welfare may decrease with mobility.

The described game has a natural self-regulatory property. Consider, for instance, a shift in the distribution of economic fundamentals towards poorer states of the world. This decreases the outside option value as migrants end up in poorer sectors. The lower value of the outside option enhances successful coordination and this positive strategic effect partially counteracts the negative direct effect. Another channel through which the self-regulatory mechanism operates is the increased mass of immigrants: the more sectors that have poor fundamentals, the more players that migrate. This makes coordination attempts more likely to succeed and thus helps to partially counteract the direct effect of the distribution's shift.

The benchmark result of two independent broad strands of literature, global games (Carlsson and van Damme [2]) and the stochastic stability concept (Kandori, Mailath and Rob [8]), is that risk dominance rather than Pareto dominance selects the equilibrium in coordination games. Given this benchmark result, the influence of mobility on the coordination outcomes has been examined in various papers belonging to the latter stream with the main conclusion that, if players are allowed to move and/or choose

[3] Global games were introduced by Carlsson and van Damme [2] and further developed by Morris and Shin [14]. Heinemann, Nagel and Ockenfels [4] tested the theory experimentally and although they rejected the quantitative predictions, they confirmed the qualitative features of the predicted comparative statics.

with whom they interact, then the Pareto efficient equilibrium may prevail.[4] Goyal and Vega-Redondo [6] vary the cost of link formation and find an effect similar to the one we find: welfare is non-monotonic with respect to mobility — the efficient equilibrium prevails only at a high cost — while if the cost of the link formation is low the risk dominant equilibrium prevails.

To our knowledge, mobility has not been studied within the global games literature. However, the outside option value is often varied exogenously in global games applications, which leads, as in our model, to the tension between the positive direct and the negative strategic effects; see the discussion of the strategic effect of a collateral on creditors in Morris and Shin [15] and section 2.3.1 in [14] or the study of the influence of demand-deposit contracts on bank run probability in Goldstein and Pauzner [5].

The model in Steiner [17] shares the non-trivial effects caused by the endogeneity of the outside option with the model at hand but they differ in timing and interpretation. We study many coordination problems simultaneously and emphasize the welfare effects of mobility in the present model while Steiner [17] studies a time sequence of coordination problems and emphasizes cycles endogenously arising in the equilibrium if players fear bankruptcy and the associated loss of future profits.

Jeong [7] and Burdett, Imai and Wright [1] study "break-up" externalities which occur when matched players search for new partners while not taking into account the welfare loss of the abandoned partner. Jeong stresses the possible welfare-improving consequences of mobility restrictions in environments with break-up externalities, which is in line with our main finding. Burdett et al. focus on the multiplicity of equilibria; if matched players search intensively, the partnerships become unstable and the intensive search is the best response. We find a similar multiplicity in the case of a finite number of sectors.

We describe the game formally in Section 2. We informally analyze the limit case of a large number of sectors and precise signals in Section 3.2. In Sections 3.3 and 3.4 we justify the informal shortcuts to the limit cases used in Section 3.2. Based on that we return to the informal solution from Section 3.2 and analyze its comparative statics in Section 4. Section 5 concludes. All proofs are relegated to Appendix.

[4] E.g. Oechssler [16]; Mailath, Samuelson and Shaked [10].

2 The Model

We start by describing a standard *static* coordination game in Section 2.1. Then, building on this, we describe a *mobile* game in which players are able to migrate among the coordination problems in Section 2.2. We link the abstract setup to the economic problem of industrialization in Section 2.3.

2.1 The Static Game

There is a continuum of homogeneous, risk-neutral players of measure 1, one industry sector and the players simultaneously decide whether to *leave* or *stay* in the sector; the actions are denoted by 0 and 1 respectively. The payoff to those who have stayed is

$$\pi(\theta, l) = \begin{cases} 1 & \text{if } l \geq 1-\theta, \\ 0 & \text{if } l < 1-\theta, \end{cases} \tag{1}$$

where l is the measure of players who have stayed and $1-\theta$ is the critical participation needed for the sector's success. The payoff for leaving the industry sector is a constant $V \in (0,1)$. The payoff function (1) exhibits strategic complementarity; incentive to stay in the industry increases in the measure of other players who stay, which typically leads to equilibrium multiplicity. Clearly, for non-extreme values of θ, the game has two pure strategy equilibria in which nobody, respectively everybody, stays.

Building on Carlsson and van Damme [2], Morris and Shin [14] show that the equilibrium multiplicity disappears if a noise in observations of the sector's fundamental θ is assumed. We introduce this standard global game structure in the rest of this paragraph: θ is a random variable distributed according to c.d.f. $\Phi(\cdot)$. The players observe an imprecise signal $x^i = \theta + \sigma\epsilon^i$ of the state θ, which itself is unobserved. The parameter σ describes the size of the noise. The errors ϵ^i are i.i.d. with c.d.f. $F(\cdot)$ and expectation of the error is assumed to be well defined. Pure strategy is a function $s^i : \mathbf{R} \to \{0,1\}$. The static game is denoted by $\Gamma_\sigma(V)$.

2.2 The Mobile Game

In the static game, the productivity of the sector described by the function $\pi(\theta, l)$ and the outside option payoff V were postulated exogenously. Next we consider several sectors simultaneously and assume that players who emigrated from their native sectors immigrate to another of the remaining sectors. In such a mobile world, the payoff for staying in or leaving a particular sector depends on players' behavior in the native sector and all the other sectors as well.

There are J sectors indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$; each sector j is characterized by fundamental θ_j independently drawn from c.d.f. $\Phi(\cdot)$.[5] The corresponding p.d.f. is bounded from above by some $\overline{\phi}$. Players are randomly and uniformly matched to the sectors at the beginning of the game. The measure of players observing each sector is normalized to 1. An individual player observing sector j is denoted by (j, i), $j \in \mathcal{J}$, $i \in [0, 1]$; we will sometimes refer to the observers of sector j as j-players.

Each player (j, i) observes a private signal $x^{(j,i)} = \theta_j + \sigma\epsilon^i$ about the fundamental of sector j and chooses staying in j or leaving it. Players who have left j are randomly matched to one of the remaining sectors $k \in \mathcal{J}/\{j\}$ and each sector k receives $\frac{1}{J-1}$ of the emigrants from j.

We say that sector j is *established early* if the measure l_j of j-players who stay in j exceeds the critical measure $1-\theta_j$, and in such a case those who have stayed in j receive payoff 1. Sector j will be *established late* if $l_j < 1 - \theta_j$ but $l_j + n_j > 1 - \theta_j$, where n_j is the measure of immigrants to j from all the other sectors and in such a case j-players who have stayed in j receive payoff $1 - c$, where the "penalty" $c \in (0, 1)$. Sector j will be *not established* if $l_j + n_j < 1 - \theta_j$, and in such a case j-players who have stayed in j receive 0. The payoff of j-players staying in j is summarized by

$$c\pi(\theta_j, l_j) + (1 - c)\pi(\theta_j, l_j + n_j),$$

with the function $\pi(\cdot, \cdot)$ specified in (1).

The payoffs of emigrants are defined as follows: j-players who have left j receive $1 - c$ if they are matched to a sector which is established early or late and receive 0 if they are matched to a sector which has not established.

Pure strategy is, as in the static game, a function $s^{(j,i)} : \mathbf{R} \to \{0, 1\}$. A *threshold* strategy is a particularly simple pure strategy characterized by a threshold x^* such that a player observing j stays if and only if $x^{(j,i)} > x^*$ and leaves otherwise. s_j is a collection of strategies of all players observing sector j; formally $s_j : [0, 1] \times \mathbf{R} \to \{0, 1\}$, $s^{(j,i)}(x) \equiv s_j(i, x)$. Similarly, s_{-j} is a collection of strategies of all players observing sectors other than j; formally $s_{-j} : \mathcal{J}/\{j\} \times [0, 1] \times \mathbf{R} \to \{0, 1\}$, $s^{(k,i)}(x) \equiv s_{-j}(k, i, x)$, $k \neq j$. We call the whole game a *mobile* game and denote it by Γ^M_σ.

[5]The setup can be generalized to allow independent fundamentals drawn from different distributions.

2.3 Labor Market Interpretation

The mobile game has a intuitive interpretation within the framework of Matsuyama [11]: the migration between the sectors takes place in real time and the penalty c is the cost of delay. Players are interpreted as workers who can either directly join the sector they are born into, or to retrain for another sector; retraining lasts $T > 0$. We interpret the length T of the retraining as a degree of (im)mobility.

Worker receives an income stream $\underline{w}$ while working in a (yet) unestablished sector or during the retraining. A worker working in an established sector receives an income stream $\overline{w} > \underline{w}$. Sector j is established at time 0, T or never, depending on whether $1 - \theta_j < l_j$, $l_j < 1 - \theta_j < l_j + n_j$ or $l_j + n_j < 1 - \theta_j$. The discount factor is $0 < \delta < 1$. The lifetime payoff of a worker who stays in an early established sector is

$$U = \int_0^{+\infty} \delta^t \overline{w} dt,$$

a worker who stays in a late established sector or retrains to an established sector receives

$$M = \int_0^{T} \delta^t \underline{w} dt + \int_T^{+\infty} \delta^t \overline{w} dt,$$

and a worker who stays in or immigrates to a sector which never gets established receives

$$D = \int_0^{+\infty} \delta^t \underline{w} dt.$$

There exists an affine transformation of payoffs which maps U to 1, M to $1-c$ and D to 0 so the labor market setup corresponds to the above abstract setup, with $c = \frac{U-M}{U-D} = 1 - \delta^T$. Note that c does not depend on $\underline{w}$ or $\overline{w}$.

Steiner [18] examines other closely related setups: an immigrant is allowed to opt for an outside option after she observes a signal about the fundamental of her new sector, players are allowed to migrate repeatedly, migration is biased towards sectors with better fundamentals, and general payoffs with strategic complementarities are examined. The insights from the current model are robust to each of these modifications. Steiner [18] assumes a continuum of sectors, which simplifies the analysis. In this paper we emphasize that equilibrium in a game with a continuum of sectors is a good approximation for equilibria under a large but finite number of sectors.

3 Solution

3.1 Solution of the Static Game

The static game can be solved by applying Proposition 2.2 in Morris and Shin [14] to the particular payoff function (1): The proposition states that the equilibrium threshold solves the Laplacian equation

$$\int_0^1 \pi(x^*, l)dl = V. \tag{2}$$

The left hand side of (2) simplifies to

$$L(x^*) = \begin{cases} 0 & \text{if } x^* < 0, \\ x^* & \text{if } 0 \leq x^* \leq 1, \\ 1 & \text{if } 1 < x^*, \end{cases}$$

and because $V \in (0, 1)$, the unique root of (2) is $x^* = V$, which we formally summarize in:

Proposition 1. (Morris and Shin [14]) *For each $\delta > 0$ there exists such $\overline{\sigma} > 0$ that for all $0 < \sigma < \overline{\sigma}$, if strategy s survives iterated elimination of dominated strategies in the game $\Gamma_\sigma(V)$, then $s(x) = 0$ for $x < V - \delta$ and $s(x) = 1$ for $x > V + \delta$.*

The equilibrium threshold x^* increases in V, and thus industrialization is more likely for low V. This confirms the insight of Matsuyama [11] who, using foresight dynamics techniques, finds that productive agriculture has adverse effects on industrialization. An increase of V has two distinct welfare effects. The direct effect is positive, but the negative strategic effect consisting of the decrease in the probability of successful coordination may prevail — welfare is non-monotonic in V.

3.2 Solution of the Mobile Game – Informal Approach

A player leaving sector j receives an expected payoff V, which depends on the equilibrium strategies of all the other players. V is approximately a common value across all sectors if the number of sectors J is large because then the impact of emigrants from any particular sector j on success of any other particular sector $k \neq j$ is negligible, as only $\frac{1}{J-1}$ of them immigrate to k. In the limit $J \to \infty$, each sector receives the same measure of immigrants and we denote this common value by n. Further in this section we informally solve the mobile game under the limit $J \to \infty$ and we postpone the formal interpretation of this limit solution to Sections 3.3 and 3.4.

Values V and n are defined for any equilibrium strategy profile and are bounded: $n \in [0,1]$, and $V \in [\underline{V}, \overline{V}]$, $\underline{V} = (1-c)(1-\Phi(1))$, $\overline{V} = (1-c)(1-\Phi(-1))$ because an emigrant may be matched to a sector with $\theta > 1$, which surely succeeds, or to a sector with $\theta < -1$, which surely fails. Given any pair V and n, the observers of any particular sector j face a global game with the Laplacian threshold condition

$$\int_0^1 \left[c\pi(x^*, l) + (1-c)\pi(x^*, l+n)\right] dl = V,$$

which simplifies to

$$cL(x^*) + (1-c)L(x^*+n) = V. \tag{3}$$

The threshold player with Laplacian beliefs cannot be sure of success or failure of her native sector in the second round, because V is bounded between $\underline{V} > 0$ and $\overline{V} < 1-c$; if she were sure, she would not be indifferent between staying and leaving. Hence $0 < x^* + n < 1$, which allows rewriting (3) as

$$(1-c)(x^*+n) + \begin{cases} cx^* & \text{if } x^* \geq 0, \\ 0 & \text{if } x^* < 0 \end{cases} = V. \tag{4}$$

A sector succeeds (at least) in the second round if and only if its fundamental exceeds x^*. Otherwise the threshold player observing signal $x = x^*$ would be, in the limit of precise signals, sure of the sector's success. Hence

$$V = (1-c)(1-\Phi(x^*)). \tag{5}$$

The measure of immigrants n to any sector is equal to the share of sectors with a fundamental below x^* because the matching for migrants to sectors is uniform:

$$n = \Phi(x^*). \tag{6}$$

Using (5) and (6), (4) can be written as

$$(1-c)(2\Phi(x^*)-1) + \begin{cases} x^* & \text{if } x^* \geq 0, \\ (1-c)x^* & \text{if } x^* < 0 \end{cases} = 0, \tag{7}$$

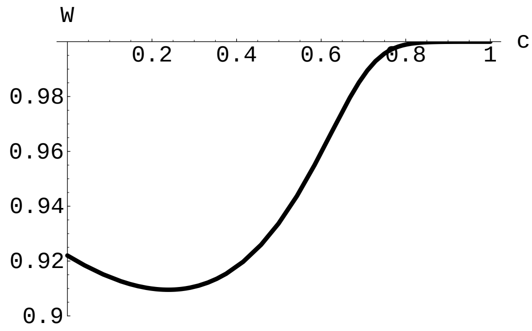

Figure 1: The welfare is non-monotonic in c due to strategic effects.

and the left hand side is strictly increasing in x^*, and thus (7) has a unique root. Below we will refer to (7) as the *informal* equation.

The solution of (7) leads to a complex comparative statics; for instance welfare is non-monotonic in c, see Figure 1. However, before we analyze the comparative statics in Section 4, we need to clarify the formal meaning of this informal solution. Namely, we formally analyze the limit $\sigma \to 0$ for any finite number J of sectors in Section 3.3, and then examine the limit $J \to \infty$ in Section 3.4. Readers uninterested in these technical issues may wish to skip to Section 4.

3.3 Two Sectors

We formulate the results and the proofs in this section for the case of two sectors, ie. $J = 2$. The results are generalized to any $J \geq 2$ in Section 3.4.

In the first step, we fix a strategy profile s_{-j} in sector $-j$ and examine the induced game $\Gamma_\sigma(s_{-j})$; the set of players in $\Gamma_\sigma(s_{-j})$ is the set of all j-players, they decide between staying in j or leaving it and the payoff difference is denoted by $\tilde{\pi}(\theta_j, l_j; s_{-j})$. We find that:

Lemma 1. *The game $\Gamma_\sigma(s_{-j})$ satisfies all the six conditions in Proposition 2.2 in Morris and Shin [14] for any fixed s_{-j}.*

We verify the six conditions in Appendix. In particular, the strategic complementarity holds because an increase of emigration from j increases the incentive to emigrate for two reasons: the measure of participants in sector j decreases, and the measure of participants in the other sector increases.

The game $\Gamma_\sigma(s_{-j})$ induced by any fixed s_{-j} can be solved as a global game:

Lemma 2. *For each $\delta > 0$ there exists, uniformly for all s_{-j}, such $\overline{\sigma} > 0$ that for all $0 < \sigma < \overline{\sigma}$, if strategy s survives iterated elimination of dominated strategies in the game $\Gamma_\sigma(s_{-j})$ then $s(x) = 0$ for $x < \theta_j^*(s_{-j}) - \delta$ and $s(x) = 1$ for $x > \theta_j^*(s_{-j}) + \delta$, where $\theta_j^*(s_{-j})$ is the unique root of the Laplacian equation*

$$\int_0^1 \tilde{\pi}(\theta_j^*, l_j; s_{-j}) dl_j = 0.$$

Lemma 2 is an application of Proposition 2.2 in Morris and Shin [14] to the particular payoff function $\tilde{\pi}(\cdot, \cdot; s_{-j})$ but with making the statement uniform over all s_{-j}. We check in Appendix that the proof in [14] allows such strengthening.

Now we are prepared to formulate the main result of this section:

Proposition 2. *For any $\delta > 0$ there exists $\overline{\sigma} > 0$ such that for any $\sigma \leq \overline{\sigma}$ the mobile game Γ_σ^M has only Bayes-Nash equilibria in which observers of sector j stay for $x^{(j,i)} > x_j^* + \delta$ and leave for $x^{(j,i)} < x_j^* - \delta$, where (x_1^*, x_2^*) solves the system of two equations, $j = 1, 2$:*

$$cL(x_j^*)+(1-c)\left(\int_{-\infty}^{x_{-j}^*} \left[L(x_j^*+1) - L(\theta_{-j})\right] d\Phi(\theta_{-j}) + \int_{x_{-j}^*}^{+\infty} \left[L(x_j^*) - L(\theta_{-j}+1)\right] d\Phi(\theta_{-j})\right) = 0. \tag{8}$$

We denote the left hand side of (8) by $g(x_j^*, x_{-j}^*)$.

Proposition 2 is a fixed point statement. Any strategy profile s_{-j} in sector $-j$ induces a global game in sector j with a unique equilibrium profile s_j and vice versa. We are looking for such a pair of strategy profiles s_1 and s_2 that are mutually a "coordination response" to each other. This "coordination response" is not chosen by players individually, rather the j-players as a group are driven by their individual optimizations to the strategy profile that is a "coordination response" to the strategy profile of $-j$-players.

Lemma 1 and 2 state that, for small σ, the "coordination response" to *any* strategy profile s_{-j} (possibly asymmetric across players and with non-threshold strategies) is a symmetric profile of threshold strategies, up to a small set of signals around the threshold. Thus when looking for the fixed point of the "coordination responses", we can focus on the symmetric profiles of threshold strategies. The proof of Proposition 2 consists of showing that though there exists, for small but positive σ, a set of signals around the threshold x_j^* on which the "coordination response" may differ from the selected action, the set is too small to alter the result.

Further we utilize the equilibrium from Proposition 2 in the following way. We assume that the errors in the observations of fundamentals are small, and therefore the equilibrium selected in the limit $\sigma \to 0$ is a good approximation of the actual equilibrium. Formally we return to the complete information setup with $\sigma \equiv 0$ and say that an equilibrium of the complete information game is *robust to the global game perturbation* if it is summarized by two thresholds satisfying (8). Below we analyze the properties of the robust equilibria.

Unlike the static global game, the mobile game may have multiple robust equilibria. Figure 2 depicts the "coordination responses" according to (8) and each of the three intersections constitutes a robust equilibrium. In addition to the symmetric equilibrium which always exists, there may exist asymmetric equilibria in which one sector has a low and the other sector a high threshold. E.g., if 1-players rarely emigrate then 2-players are highly tempted to emigrate because the sector 1 often succeeds, hence 2-players emigrate often, the sector 2 often fails, and 1-players are not too tempted to emigrate, which confirms that they emigrate only rarely. The existence of asymmetric equilibria allows one of the two ex ante identical sectors to succeed more often than the other.

The equilibrium multiplicity arises because the rate of emigration from any particular sector has a non-negligible influence on the success of the other sector. In the next section, we will consider J sectors, and state that for large J all robust equilibrium thresholds lie in a small neighborhood of the informal solution (7). The intuition is that, for large J, the immigration from any particular sector is a negligible part of the total immigration mass, and thus all sectors receive approximately the same measure of immigrants.

We examine the comparative statics of the informal solution in Section 4, because the informal solution is a good approximation of the robust equilibria if the number of sectors is finite but large. Alternatively, we could have continued the analysis with the two sectors setup, and examine the comparative statics of the symmetric equilibrium. The results are qualitatively the same, however the first approach bypasses the equilibrium selection and a finite but large number of sectors seems to be a reasonable assumption from the applied point of view.

3.4 Many Sectors

Next, we generalize the system of equations (8) from Proposition 2 to the case of $J > 2$ sectors. Let $X_{-j}^J = (x_1^J, \ldots, x_{j-1}^J, x_{j+1}^J, \ldots, x_J^J)$. Robust thresholds $x_1^{*J}, \ldots, x_J^{*J}$ satisfy

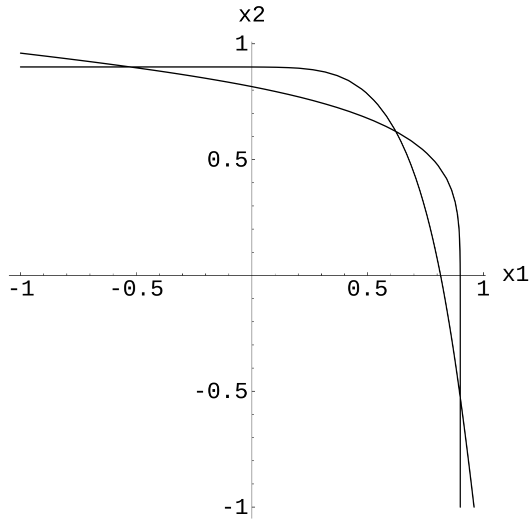

Figure 2: The threshold in sector j as a "coordination response" to the threshold in $-j$. Each of the three intersections constitutes an equilibrium.

$g^J(x_j^{*J}, X_{-j}^{*J}) = 0$ where $g^J(x_j^{*J}, X_{-j}^{*J})$ equals

$$\underbrace{cL(x_j^{*J})}_{I} + (1-c)E\left[\underbrace{L\left(x_j^{*J} + \frac{\sum_{k\neq j} e_k}{J-1}\right)}_{II} - \underbrace{\frac{1}{J-1}\sum_{k\neq j}\int_{l_j=0}^{1} \pi\left(\theta_k, 1-e_k + \frac{\sum_{m\neq j,k} e_m + 1 - l_j}{J-1}\right) dl_j}_{III}\right]. \tag{9}$$

The expectation is taken with respect to the random realization of fundamentals in all $k \neq j$ sectors, and e_k denotes the emigration from sector k, hence e_k is a random variable equal to 1 if $\theta_k > x_k^{*J}$ and 0 if $\theta_k < x_k^{*J}$.

The term I is the expected payoff under the Laplacian belief for staying in j during the first period before the immigrants arrive. The term II is the expected payoff for staying in j in the second period after the immigrants arrive. The term III is the expected payoff for leaving sector j, that is, player is allocated to one of the remaining sectors $k \neq j$, whose success depends on θ_k, and on the measure of players participating in k: the measure $1 - e_k$ of k-players who stay in k are joined by the measure of immigrants from $m \neq k, j$ and by the measure of immigrants $1 - l_j$ from j.[6] Generalization of the proof of Proposition 2 to $J > 2$ is straightforward but notationally cumbersome,

[6]The threshold j-player's beliefs over the immigration from j and $k \neq j$ differ because she has the Laplacian belief about the measure of players staying in/leaving j.

and we therefore omit it.

The following proposition states that for finite but large J, all robust equilibrium thresholds lie close to the informal solution: Let $x_1^{*J}, \ldots, x_J^{*J}$ denote the thresholds in a robust equilibrium of the mobile game with J sectors.

Proposition 3. *For any $\epsilon > 0$ there exist J' such that for all $J > J'$*

$$|x_j^{*J} - x^*| < \epsilon,$$

for all $j = 1, \ldots, J$, where x^* is the root of (7).

Each sector j faces an environment defined by the behavior in sectors $k \neq j$ sectors. If J is large, each sector faces approximately the same environment, as the influence of any particular sector is negligible. Hence the solution has to be approximately symmetric, which is the main idea of the proof (delegated to Appendix).

4 Comparative Statics

Proposition 3 implies that the informal solution is a good prediction for the equilibrium threshold under a setup with a large but finite number of sectors. With this interpretation in mind we return to examine the comparative statics of the solution of (7).

We start by examining the comparative statics with respect to changes in the distribution of fundamentals $\Phi(\cdot)$. The left hand side of (7) increases in $\Phi(x^*)$ and thus the implicit function theorem implies:

Finding 1. *Let x^* be the equilibrium threshold under the distribution of fundamentals $\Phi(\cdot)$, and x'^* under the distribution of fundamentals $\Phi'(\cdot)$. If $\Phi'(\cdot)$ stochastically dominates $\Phi(\cdot)$ then $x'^* > x^*$.*

Thus an improvement in the distribution of fundamentals increases the threshold. Intuitively, an improvement in $\Phi(\cdot)$ increases the expected payoff of emigration as the chance of being matched to an established sector increases, and decreases the expected threshold payoff for staying as the measure of immigrants decreases. The increase of the threshold x^* increases the probability $p = \Phi(x^*)$ of a sector's failure, which partially offsets the direct effect of the improvement in $\Phi(\cdot)$. However, the direct positive effect always prevails and the equilibrium probability of a sector's failure unambiguously decreases with an improvement in $\Phi(\cdot)$:

Finding 2. *Let x^* be the equilibrium threshold under the distribution of fundamentals $\Phi(\cdot)$, and x'^* under the distribution $\Phi'(\cdot)$. If $\Phi'(\cdot)$ stochastically dominates $\Phi(\cdot)$ then $\Phi'(x'^*) < \Phi(x^*)$.*

The proof of Finding 2 is the following: We substitute $x^* = \Phi^{-1}(p)$ into (7) and get

$$(1-c)(2p-1) + \begin{cases} \Phi^{-1}(p) & \text{if } \Phi^{-1}(p) \geq 0, \\ (1-c)\Phi^{-1}(p) & \text{if } \Phi^{-1}(p) < 0 \end{cases} = 0. \tag{10}$$

The left hand side of (10) increases in p and an improvement from $\Phi(\cdot)$ to $\Phi'(\cdot)$ causes an increase in the left hand side of (10) which, according to the implicit function theorem, implies Finding 2.

Finding 2 directly translates into comparative statics with respect to the population size. If the measure of players per sector is changed from 1 to $n > 0$ then the critical labor mass needed for the success of sector changes from $1-\theta$ to $\frac{1-\theta}{n}$. Thus the change of the population has the same effect as keeping the population size at 1 but shifting the fundamental in the following way:[7]

$$\theta \to \theta' = \vartheta_n(\theta) = \begin{cases} 1 - \frac{1}{n}(1-\theta) & \text{if } \theta < 1, \\ \theta & \text{if } \theta > 1. \end{cases}$$

The distribution of θ' is $\Phi'_n(\cdot) \equiv \Phi(\vartheta_n^{-1}(\cdot))$ and hence, as $\vartheta_n(\cdot)$ increases in n, $\Phi'_{n'}(\cdot)$ stochastically dominates $\Phi'_n(\cdot)$ for $n' > n$, which, together with Finding 2, implies:

Finding 3. *The equilibrium probability of a sector's failure decreases in the population size n per sector.*

Next we examine the comparative statics with respect to c. Equation (7) has different properties for $x^* < 0$ than for $x^* > 0$. In the case $x^* < 0$ it simplifies to

$$2\Phi(x^*) - 1 + x^* = 0,$$

and thus:

Finding 4. *If the equilibrium threshold x^* is negative, it is not sensitive to local changes in c.*

[7]For $\theta > 1$ the sector succeeds for any population size n and any l.

In the case $x^* > 0$, equation (7) simplifies to

$$(1-c)(2\Phi(x^*)-1) + x^* = 0. \tag{11}$$

The left hand side of (11) increases in c because $(2\Phi(x^*)-1) = \frac{-x^*}{1-c}$ is negative for $x^* > 0$. The implicit function theorem implies:

Finding 5. *If the equilibrium threshold x^* is positive then it increases in c.*

Intuitively, an increase in c by dc makes both staying in and leaving the sector less attractive. The expected payoff for leaving j is $(1-c)(1-\Phi(x^*))$ and it decreases by $dc\Phi(x^*)$. The expected payoff for staying in j under the Laplacian beliefs decreases by $dcn = dc\Phi(x^*)$.[8] For $\Phi(x^*) < \frac{1}{2}$, which is implied by $x^* > 0$, the adverse effect on the payoff for leaving prevails and hence the equilibrium threshold must decrease — this increases the payoff for leaving, decreases the Laplacian threshold payoff for staying, and thus restores the balance.

Further we analyze welfare consequences of changes in c where welfare is defined as expected payoff *before* the observation of signal:[9] (p denotes the equilibrium probability of a sector's failure.)

$$W = (1-p) + p(1-p)(1-c).$$

The total welfare effect $\frac{dW}{dc}$ consists of the direct effect $\frac{\partial W}{\partial c} = -p(1-p) \leq 0$ which is negative, and of the strategic effect $\frac{\partial W}{\partial p}\frac{dp}{dc}$. The strategic effect is positive, as $\frac{\partial W}{\partial p}$ is negative, and according to Finding 5 the derivative $\frac{dp}{dc} < 0$. The total effect is ambiguous, both the negative direct or the positive strategic effect can prevail, see Figure 1. Below we describe conditions under which the strategic effect dominates.

The equation (11) defines x^* (and hence p) only implicitly, and so it is generally possible to express $\frac{dW}{dc}$ only as a function of x^* (and p). However, it is possible to obtain an explicit expression for x^*, and hence for $\frac{dW}{dc}$ in the limit of a very narrow prior distribution. Formally, the prior distribution is set to be $\Phi(\frac{\theta-y}{\omega})$ and we examine the equilibrium in the limit $\omega \to 0$ in which the root of (7) can be expressed explicitly:

Finding 6. *The equilibrium threshold x^* and failure probability p as functions of y in the limit $\omega \to 0$ are*

[8] This is because according to (3) the Laplacian threshold payoff for staying is $cL(x^*) + (1-c)L(x^*+n) = x^* + (1-c)n$.

[9] We analyze only the case $x^* > 0$. The welfare analysis is straightforward for $x^* < 0$, as there is no indirect welfare effect.

	$y < -1$	$-1 \leq y < 0$	$0 \leq y < 1 - c$	$1 - c < y$
x^*	-1	y	y	$1 - c$
p	1	$\frac{1-y}{2}$	$\frac{1}{2} - \frac{y}{2(1-c)}$	*0*

.

Further we look for pairs of y and c for which the total welfare effect $\frac{dW}{dc}$ is positive (in the limit $\omega \to 0$). The first condition is $0 \leq y < 1 - c$ otherwise $\frac{dp}{dc} = 0$. If this condition is satisfied, then using the expression $p = \frac{1}{2} - \frac{y}{2(1-c)}$ from Finding 6, the total welfare effect $\frac{dW}{dc} = \frac{\partial W}{\partial c} + \frac{\partial W}{\partial p}\frac{dp}{dc}$ simplifies into

$$\frac{dW}{dc} = \frac{-(c-1)^2 - (y-1)^2 + 1}{4(1-c)^2},$$

which implies:

Finding 7. *The set of pairs (c, y) at which the total welfare effect is positive (in the limit $\omega \to 0$) is the one depicted in Figure 3.*

While a "partial" equilibrium analysis of a coordination problem would suggest that an improvement of the prior distribution by a slight increase in y would dramatically increase welfare because the probability of a sector's failure would decrease to 0, we find that this is not the case in a mobile world. The threshold increases with y, and the decrease in the probability of failure is only proportional to the increase in y, for $y \in (-1, 1)$.

We now return to the labor market interpretation introduced in Section 2.3. Welfare in the labor market is $\underline{w} + (\overline{w} - \underline{w})W_{1-\delta^T}$ where $W_{1-\delta^T}$ is the welfare in the abstract setup studied above if c is set to $c = 1 - \delta^T$. The function $1 - \delta^T$ increases in the length of the retraining period T and thus welfare at the labor market increases in T if and only if welfare in the abstract game increases at $c = 1 - \delta^T$. Note that the sign of the welfare effect is entirely independent of $\overline{w}$ and $\underline{w}$, and depends only on T.

The labor market interpretation of the abstract mobile game is not the only one possible. Alternatively we could stress the capital side of industrialization and interpret the players as investors. The penalty for the late investment or for the late success can originate in increased government regulations in post-industrialized society. Finding 7 suggests that the welfare effect of the regulations is ambiguous. While the regulations surely decrease the returns of late investors or of those whose sectors succeed late, the expectation of the regulations may increase welfare by enhancing efficient coordination at the beginning of the industrialization.

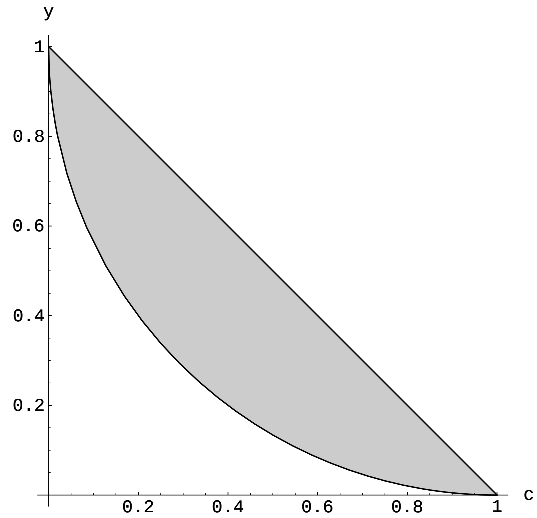

Figure 3: The total welfare effect is positive for pairs (c, y) in the shaded area.

In order to examine additional comparative statics, we consider the abstract game with general payoff parameters $U > U - \gamma > D$, instead of $1 > 1 - c > 0$ and vary U or D keeping other parameters constant.[10] The threshold x^* under payoffs U, $U - \gamma$ and D equals to the threshold solving (7) with $c = \frac{\gamma}{U-D}$, so the strategic effect of variation in U or D can be straightforwardly mapped to the strategic effects with respect to c in the abstract game. The direct effect of increasing U or D is positive; the indirect effect of increasing D is positive as well, because, according to Finding 5, an increase in D increases c which decreases equilibrium probability of a sector's failure. In contrast, c decreases in U, and thus an increase in U has a negative strategic effect which can override the positive direct effect. This is summarized by:

Finding 8. *The welfare effects of an increase in U or D have the following signs:*

Increase in:	U	D
Direct effect	+	+
Strategic effect	-	+
Total effect	±	+

.

Counterintuitively, more productive technologies do not necessarily facilitate industrialization. If the new inventions are highly productive, the temptation to leave the

[10]The payoffs U, $U - \gamma$ and D are not independent under the labor market interpretation, so such a comparative statics is useful only for alternative applications of the abstract mobile game.

native industry for a more promising one is high. The losses from the excessive mobility may override the direct advantage of the high productivity.

5 Concluding Remarks

While our main economic application is the labor aspect of industrialization, the model is abstract enough to accommodate other interpretations. We could reinterpret the industry sectors as different geographical locations and assume increasing returns to scale in local industries as discussed in Krugman [9]. The mobile game with its non-monotonic welfare effect of increased mobility would then be a simple model of globalization.

By focusing the discussion on the counterintuitive comparative statics we have exposed ourselves to the danger of overemphasizing the negative consequences of mobility. While we find that an increase in mobility *may* lead to a decrease in welfare, the finding is sensitive to the model's parameters. Also, though the presented model has a unique equilibrium in the limit of many sectors; a general payoff function may lead to equilibrium multiplicity, similar in the logic to the one we encountered in the case of a few sectors. See Steiner [18] who, in a related model, provides the examples of setups leading to such equilibrium multiplicity, the underlying intuition and sufficient conditions for equilibrium uniqueness in the limit of many sectors.

Thus, rather than in particular policy recommendations, the contribution of the model is in building intuition needed for judging the tension between mobility and ability of local coordination. More generally, the model transfers the understanding of the distinction between partial and general equilibrium analysis from market systems to coordination problems. The "partial equilibrium" approach to coordination problems, which has been prevalent in the existing research, is useful in focusing the model on a particular economic problem, but we should be aware of the "general equilibrium" effects it abstracts from.

In particular, the equilibrium threshold in a static global game is independent of the prior distribution $\Phi(\cdot)$ in the limit $\sigma \to 0$ and hence the global game theory can be used as an equilibrium selection in an isolated complete information coordination problem. In contrast, the prior has an influence on the equilibrium threshold in the mobile game even in the limit $\sigma \to 0$. Once the players have the means to choose the coordination problem they participate in, it may be misleading to select an equilibrium in a coordination problem in isolation from other problems because the whole set of coordination problems interacts as a result of the "general equilibrium" effects.

A Proofs

Proof. (Lemma 1) Each strategy profile s_{-j} induces some joint distribution $\Xi_{s_{-j}}(n_j, n_{-j}, \theta_{-j})$ over n_j, n_{-j} and θ_{-j}, where n_j denotes measure of $-j$-players who immigrate into j, and n_{-j} the measure of $-j$-players who stay in $-j$.[11] We define

$$\pi'(\theta_j, l_j; n_j, n_{-j}, \theta_{-j}) = c\pi(\theta_j, l_j) + (1-c)\pi(\theta_j, l_j + n_j) - (1-c)\pi(\theta_{-j}, 1 - l_j + n_{-j}),$$

which is the incentive to stay in j for a fixed triple n_j, n_{-j}, θ_{-j}.

The payoff difference between staying and leaving sector j is

$$\tilde{\pi}(\theta_j, l_j; s_{-j}) \equiv \int \pi'(\theta_j, l_j; n_j, n_{-j}, \theta_{-j}) d\Xi_{s_{-j}}(n_j, n_{-j}, \theta_{-j}).$$

We check that $\tilde{\pi}(\theta_j, l_j; s_{-j})$ satisfies all six assumptions of Proposition 2.2. in Morris and Shin [14]:

1. (Action Monotonicity) $\pi'(\theta_j, l_j; n_j, n_{-j}, \theta_{-j})$ is weakly increasing in l_j for any n_j, n_{-j}, θ_{-j} and thus $\tilde{\pi}(\theta_j, l_j; s_{-j})$ is weakly increasing in l_j after we integrate over n_j, n_{-j}, θ_{-j}.

2. (State Monotonicity) $\pi'(\theta_j, l_j; n_j, n_{-j}, \theta_{-j})$ is weakly increasing in θ_j for any n_j, n_{-j}, θ_{-j} and thus $\tilde{\pi}(\theta_j, l_j; s_{-j})$ is weakly increasing in θ_j after we integrate over n_j, n_{-j}, θ_{-j}.

3. (Unique Laplacian State) The Laplacian payoff $\int_0^1 \tilde{\pi}(\theta_j, l_j; s_{-j}) dl_j$ is *not strictly* monotone, as the payoff function $\tilde{\pi}(\cdot, \cdot; s_{-j})$ is only weakly monotone, however we prove single crossing — the Laplacian payoff is strictly increasing at the root of the Laplacian equation (with the slope bounded from zero uniformly over all s_{-j}):

   We call pair (θ, n) *unresolved* if $n < 1 - \theta < n + 1$. This terminology reflects that, for unresolved (θ, n), sector with fundamental θ and measure of immigrants n has a probability of success in the second round (under the uniform distribution of l) strictly between 0 and 1. Thus the derivative of the probability of the (late) success with respect to θ is 1, for unresolved (θ, n).[12]

   The probability of the late success of sector j (under the uniform distribution of l_j) is bounded between $\underline{p} \equiv \frac{\underline{V}}{1-c}$ and $\overline{p} \equiv \frac{\overline{V}}{1-c}$ at the root of the Laplacian equation $\int_0^1 \tilde{\pi}(\theta_j, l_j; s_{-j}) dl_j = 0$; the bounds $\underline{V}$ and $\overline{V}$ on the payoff for emigration were

[11] n_j and n_{-j} are trivially dependent as, in the two sectors setup, $n_j + n_{-j} = 1$. However, this notation facilitates the generalization of the proof to cases $J > 2$.

[12] This derivative is zero if (θ, n) is not unresolved, which is the reason why this property is useful.

established in Section 3.2. Otherwise the threshold player with Laplacian beliefs would strictly prefer leaving or staying in j and the indifference implied by the Laplacian equation would not hold.

Consider $\theta_j > 0$: $1 - \theta_j < n_j + 1$ for all $n_j \in [0,1]$ and hence the probability that the sector j succeeds (under uniform distribution of l_j) is strictly positive for any measure of immigrants n_j. Be θ_j the root of the Laplacian equation $\int_0^1 \tilde{\pi}(\theta_j, l_j; s_{-j}) dl_j = 0$. Then under the distribution of n_j induced by s_{-j}

$$Prob\left(n_j + 1 - (1 - \theta_j) > \frac{\overline{p}+1}{2}\right) \leq b,$$

where b solves $\frac{\overline{p}+1}{2} b + (1-b)0 = \overline{p}$. In words, if $n_j + 1 - (1-\theta_j) > \frac{\overline{p}+1}{2}$ then, under Laplacian beliefs, probability of success of j, which is $n_j + 1 - (1 - \theta_j)$, exceeds $\frac{\overline{p}+1}{2}$, and if the measure of immigrant n_j would be so high with a probability higher than b, the total probability of success, would exceed $\overline{p}$. Thus, if θ_j is the root of the Laplacian equation, then the probability that (θ_j, n_j) is unresolved is at least $1 - b > 0$. The derivative of the Laplacian payoff with respect to the threshold under a fixed n_j is one if (θ_j, n_j) is unresolved. Hence, after we integrate over n_j, $\frac{\partial}{\partial \theta_j} \int_0^1 \tilde{\pi}(\theta_j, l_j; s_{-j}) dl_j \geq (1-b)(1-c) > 0$.
The symmetric argument applies for $\theta_j < 0$. In the case of $\theta_j = 0$ we can apply the argument from the case $\theta_j > 0$ for the right hand side derivative and the argument from the case $\theta_j < 0$ for the left hand side derivative. Hence the derivative of the Laplacian expected payoff is positive at the root of the Laplacian equation, for any s_{-j}, which establishes the single-crossing.

4. (Limit Dominance) The payoff for leaving j is bounded by the bounds $0 < \underline{V} < \overline{V} < 1$. Measure n_j of immigrants to j is bounded by 0 and 1, and thus $\tilde{\pi}(\theta_j, l_j; s_{-j})$ is strictly negative for $\theta_j < -1$ and strictly positive for $\theta_j > 1$.

5. (Continuity)

$$\begin{aligned}
&\int_0^1 g(l_j)\pi'(\theta_j, l_j; n_j, n_{-j}, \theta_{-j}) dl_j = \\
&\quad Prob(l_j > 1 - \theta_j)c + Prob(l_j + n_j > 1 - \theta_j)(1-c) - Prob(1 - l_j + n_j > 1 - \theta_{-j}) = \\
&\quad (1 - G(1-\theta_j))c + (1 - G(1 - \theta_j - n_j))(1-c) - G(\theta_{-j} + n_{-j})(1-c),
\end{aligned}$$

where $G(\cdot)$ is the c.d.f. of $g(\cdot)$. Thus $\int g(l_j)\pi'(\theta_j, l_j; n_j, n_{-j}, \theta_{-j}) dl$ is continuous with respect to density $g(\cdot)$ and with respect to θ_j for any n_j, n_{-j}, θ_{-j}. This is preserved when we integrate over n_j, n_{-j}, θ_{-j}.

6. (Finite Expectation of Errors) Satisfied by the assumption on the error distribution.

□

Proof. (Lemma 2) We check that the proof in Morris and Shin [14] allows for the statement in Lemma 2 to be uniform over all fixed strategy profiles s_{-j} in sector $-j$:

Morris and Shin define $\tilde{\pi}_\sigma(x, k; s_{-j})$ to be the expected payoff for staying conditional on observing signal x when all other j-players have a threshold strategy with threshold k, and s_{-j} is fixed. The authors show in their Lemma 6.1 that, for a fixed s_{-j}, it is possible to find dominance regions for $\tilde{\pi}_\sigma(x, k; s_{-j})$ uniformly for σ below some $\overline{\sigma} > 0$. We add that the dominance regions can be specified uniformly across all s_{-j} because the expected payoff for leaving is bounded between $0 < \underline{V} < \overline{V} < 1$. Thus the left dominance region can be taken from the game which pays $\underline{V}$ for leaving and the right dominance region from the game which pays $\overline{V}$ for leaving.

In their Lemma 6.2 Morris and Shin prove that, fixing s_{-j}, $\tilde{\pi}_\sigma(x, x - \sigma\xi; s_{-j})$ converges to $\tilde{\pi}^*_\sigma(x, x - \sigma\xi; s_{-j})$ as $\sigma \to 0$, where $\tilde{\pi}^*_\sigma(x, k; s_{-j})$ is the variable corresponding to the uniform prior and private values, and the convergence is uniform for x in a compact interval.[13] We add, in the rest of the proof, that this convergence is uniform over all s_{-j}:

$$\tilde{\pi}_\sigma(x, k; s_{-j}) = \int \pi'_\sigma(x, k; n_j, n_{-j}, \theta_{-j}) d\Xi_{s_{-j}}(n_j, n_{-j}, \theta_{-j}),$$

where $\pi'_\sigma(x, k; n_j, n_{-j}, \theta_{-j})$ is the expected payoff conditional on signal x when all opponents use threshold k under the payoff function $\pi'(\theta_j, l_j; n_j, n_{-j}, \theta_{-j})$. Be $\pi'^*_\sigma(x, k; n_j, n_{-j}, \theta_{-j})$ the variable corresponding the uniform prior and private values under the same payoff function $\pi'(\theta_j, l_j; n_j, n_{-j}, \theta_{-j})$.

We have restricted ourselves to x from a compact interval, n_j and n_{-j} lie in $[0, 1]$, and we can restrict to θ_{-j} in $[-1, 1]$ because its further decrese/increase beyond $[-1, 1]$ does not influence the success of sector $-j$.

[13] We can constraint ourselves to a compact interval after eliminating the dominance regions.

Morris and Shin prove that

$$\pi'_\sigma(x,k;n_j,n_{-j},\theta_{-j}) = \int_{l=0}^{1} \pi'\left(k-\sigma F^{-1}(l), l; n_j, n_{-j}, \theta_{-j}\right)\psi_\sigma(l;x,k)dl,$$

where $\psi_\sigma(\cdot;x,k)$ is p.d.f. corresponding to c.d.f. $\Psi_\sigma(\cdot;x,k)$ and $\Psi_\sigma(l;x,x-\sigma\xi)$ uniformly converges to $\Psi^*_\sigma(l;x,x-\sigma\xi) \equiv 1-F\left(\xi+F^{-1}(1-l)\right)$ as $\sigma\to 0$. The c.d.f. $\Psi^*_\sigma(l;x,k)$ corresponds to a game with uniform prior distribution.

Hence $\pi'_\sigma(x,x-\sigma\xi;n_j,n_{-j},\theta_{-j})$ converges to $\pi'^*_\sigma(x,x-\sigma\xi;n_j,n_{-j},\theta_{-j})$ and the convergence is uniform over x from a compact interval and over ξ because variation in ξ generates a compact set of distributions over l. We only need to add, that because we consider a compact set of (n_j,n_{-j},θ_{-j}), the convergence is uniform also over (n_j,n_{-j},θ_{-j}). Hence after integrating over (n_j,n_{-j},θ_{-j}) with the distribution $\Xi_{s_{-j}}(n_j,n_{-j},\theta_{-j})$, we get uniform convergence over all s_{-j}.

The rest of the proof in Morris and Shin relies on the fact that $\tilde\pi_\sigma(x,x-\sigma\xi;s_{-j})$ converges uniformly to $\tilde\pi^*_\sigma(x,x-\sigma\xi;s_{-j})$ and needs not to be altered for our needs. □

Proof. (Proposition 2) Consider any equilibrium profile $s=(s_1,s_2)$ in the mobile game Γ^M_σ. The profile induces two thresholds $x^*_j(\sigma)$, $j=1,2$ that are solving the Laplacian equations in the global games $\Gamma_\sigma(s_j)$, $j=1,2$. Using Lemma 2, there exists a function $\delta(\sigma)>0$, $\lim_{\sigma\to 0}\delta(\sigma)=0$, such that the equilibrium strategies in Γ^M_σ satisfy $s^{(j,i)}(x)=1$ for $x>x^*_j(\sigma)+\delta(\sigma)$ and $s^{(j,i)}(x)=0$ for $x<x^*_j(\sigma)-\delta(\sigma)$; $j=1,2$.[14]

Lemma 3. *There exists a function* $\epsilon(\cdot)>0$ *such that* $\lim_{\sigma\to 0}\epsilon(\sigma)=0$, *and*

$$\left|g\left(x^*_j(\sigma), x^*_{-j}(\sigma)\right)\right| < \epsilon(\sigma)$$

for $j=1,2$.

Proof. $x^*_j(\sigma)$ satisfies

$$cL\left(x^*_j(\sigma)\right)+(1-c)\int_{-\infty}^{+\infty}\left[L\left(x^*_j(\sigma)+1-l_{-j,\sigma}(\theta_{-j})\right)-L\big(\theta_{-j}+l_{-j,\sigma}(\theta_{-j})\big)\right]d\Phi(\theta_{-j})=0, \tag{12}$$

[14]The equilibrium profile in Γ^M_σ, $\sigma>0$, need *not* consist of threshold strategies. Lemma 2 only guaranties that there exist thresholds $x^*_j(\sigma)$ such that the equilibrium strategies differ from the threshold strategy only on a neighborhood of thresholds thresholds $x^*_j(\sigma)$.

where $l_{-j,\sigma}(\theta_{-j})$ is the measure of $-j$-players who stay in $-j$ under the fundamental θ_{-j}, strategy profile $s_{-j}(\sigma)$, and error size σ. Be $l_{-j,0}(\theta_{-j})$ the function corresponding to the threshold strategy with threshold x^*_{-j} and $\sigma = 0$, that is $l_{-j,0}(\theta_{-j}) = 1$ for $\theta_{-j} > x^*_{-j}(\sigma)$ and 0 otherwise. By replacing $l_{-j,\sigma}$ with $l_{-j,0}$ in the left hand side of (12) we get $g\left(x^*_j(\sigma), x^*_{-j}(\sigma)\right)$. The difference caused by the replacement is smaller than

$$\epsilon(\sigma) \equiv 2\overline{\phi}(1-c)\left(\delta(\sigma)+\sqrt{\sigma}\right) + (1-c)\max\left(F\left(-\frac{1}{\sqrt{\sigma}}\right), 1-F\left(\frac{1}{\sqrt{\sigma}}\right)\right) \qquad (13)$$

The first term comes from integration over interval $\left[x^*_{-j}(\sigma)-\delta(\sigma)-\sqrt{\sigma}, x^*_{-j}(\sigma)+\delta(\sigma)+\sqrt{\sigma}\right]$. Outside of this interval, the difference between $l_{-j,\sigma}$ and $l_{-j,0}$ is bounded by

$$\max\left(F\left(-\frac{1}{\sqrt{\sigma}}\right), 1-F\left(\frac{1}{\sqrt{\sigma}}\right)\right)$$

which gives the second term in (13) $\square$

Function $g(\cdot,\cdot)$ is continuous, thresholds $x^*_j(\sigma)$ are from a closed interval and thus there exists a function $\epsilon'(\sigma) > 0$, $\lim_{\sigma\to 0}\epsilon'(\sigma) = 0$, such that $|x^*_j(\sigma) - x^*_j| < \epsilon'(\sigma)$, $j = 1, 2$ where (x^*_1, x^*_2) is a solution of (8).

Then, the j-players in Γ^M_σ stay in j for signals above $x^*_j + \delta'(\sigma)$ and leave below $x^*_j - \delta'(\sigma)$ where $\delta'(\sigma) = \epsilon'(\sigma) + \delta(\sigma)$. $\square$

Proof. (Proposition 3) Below, we omit asterisk from the notation of the threshold and instead of x^{*J}_j write simply x^J_j.

Let X^J denote $(x^J_1, \ldots, x^J_J)$. We introduce a system of *approximate* equations, $\tilde{g}^J(x^J_j, X^J) = 0$, where $\tilde{g}^J(x^J_j, X^J)$ equals

$$cL(x^J_j)+(1-c)E\left[L\left(x^J_j + \frac{\sum_{k=1}^J e_k}{J-1}\right) - \frac{1}{J-1}\sum_{k=1}^J \int_{l_j=0}^1 \pi\left(\theta_k, 1-e_k+\frac{\sum_{m\neq k} e_m + 1 - l_j}{J-1}\right) dl_j\right]. \qquad (14)$$

The difference between the correct equations (9) and the approximate equations (14) is that in (14) we allow index k and m to equal j.[15]

Approximate equations are a good approximation of the exact equations if J is large:

[15] In words in (14) we "add" one virtual sector with the threshold equal to x_j and with an independent realization of fundamental. This assures that in (14) all sectors $j = 1, \ldots, J$ face identical environment of other sectors.

Lemma 4. *There exists $\alpha > 0$ such that*

$$\left|\tilde{g}_j^J(x_j^J, X^J) - g_j^J(x_j^J, X_{-j}^J)\right| \leq \frac{\alpha}{J-1},$$

for all $j = 1, \ldots, J$ and all $X^J \in [-1, 1]^J$.

Proof. (Lemma 4) The first terms in (9) and (14) are identical. The difference between (9) and (14) in the second term is that the approximate equation overstates the measure of immigrants to j, but at most by $\frac{1}{J-1}$ and derivative of $L(\cdot)$ is bounded by 1 which leads to a difference at most $\frac{1-c}{J-1}$. There are two differences in the third term: the approximate equation has one additional summand in the sum over index k, with the value of the summand at most $\frac{1}{J-1}$ which leads to a difference at most $\frac{1-c}{J-1}$. The additional summand in the sum over the index m leads to a difference at most $(1-c)\frac{2}{J-1}\overline{\phi}$ because

$$\left|\int_{l_j=0}^{1} \pi\left(\theta_k, n_k + \frac{1-l_j}{J-1}\right) dl_j - \int_{l_j=0}^{1} \pi\left(\theta_k, n_k + \frac{1}{J-1} + \frac{1-l_j}{J-1}\right) dl_j\right|$$

is 0 if $1-\theta_k$ lies outside of $[n_k, n_k + \frac{2}{J-1}]$ and is at most 1 if $\theta_k \in [n_k, n_k + \frac{2}{J-1}]$. Thus, in expectation with respect to θ_k, the additional measure of immigrants in the approximate equation can cause a difference at most $(1-c)\frac{2}{J-1}\overline{\phi}$. The total difference between (9) and (14) is thus at most $\left[(1-c) + (1-c) + (1-c)2\overline{\phi}\right]\frac{1}{J-1}$. $\square$

Lemma 4 implies that any solution $x_1^{*J}, \ldots, x_J^{*J}$ of the correct system (9) approximately solves the approximate system (14):

$$\left|\tilde{g}_j^J(x_j^{*J}, X^{*J})\right| \leq \frac{\alpha}{J-1} \text{ for all } j = 1, \ldots, J. \tag{15}$$

The argument identical to the one in the proof of Lemma 1 point 3 (unique Laplacian state), establishes that the derivative $\frac{\partial}{\partial x_j^{*J}}\tilde{g}_j^J(x_j^{*J}, X^{*,J})$ is bounded from below at the root of the approximate equations with a positive bound denoted here as b which is uniform over all J. Hence

$$|x_j^{*J} - x_{j'}^{*J}| \leq \frac{2\alpha}{b(J-1)} \text{ for all } j, j' = 1, \ldots, J. \tag{16}$$

We introduce function $\gamma^J : \mathbf{R} \to \mathbf{R}$, where $\gamma^J(x) \equiv \tilde{g}_j^J(x, x, \ldots, x)$. Next, using the following Lemma, we establish that $\gamma^J(x_j^{*J})$, $j = 1, \ldots, J$, are close to zero:

Lemma 5. *There exists $\beta > 0$ such that*

$$\frac{\partial}{\partial x_{j'}^J} \left| \tilde{g}_j^J \left(x_j^J, x_1^J, x_2^J, \ldots, x_J^J \right) \right| < \frac{\beta}{J-1},\ j' \neq j. \tag{17}$$

Proof. (Lemma 5) The derivative of the second term in (14) with respect to $x_{j'}^J$ is, in absolute value, at most $(1-c)\frac{1}{J-1}\overline{\phi}$. The derivative of the third term is proportional to $\frac{1}{J-1}$ as shown in the rest of the proof. We will use the following:

$$\frac{\partial}{\partial n_{j'}} \int_{l_j=0}^{1} \pi \left(\theta_k, n_{j'} + \frac{1-l_j}{J-1} \right) dl_j$$

is $J-1$ if $n_{j'} \leq 1 - \theta_{j'} \leq n_{j'} + \frac{1}{J-1}$ and 0 otherwise. Thus

$$\frac{\partial}{\partial n_{j'}} E \left[\int_{l_j=0}^{1} \pi \left(\theta_k, n_{j'} + \frac{1-l_j}{J-1} \right) dl_j \right]$$

is at most $(J-1)\frac{1}{J-1}\overline{\phi} = \overline{\phi}$.

The derivative of the third term in (14) with respect to $x_{j'}^J$ consists of two parts: 1. The derivative of the summand when $k = j'$ is at most $(1-c)\frac{1}{J-1}\overline{\phi}$. 2. The inflow of immigrants to all sectors indexed by $k \neq j'$ changes, which leads to the derivative at most, in absolute value, $(1-c)\frac{1}{J-1}(J-1)\frac{\overline{\phi}}{J-1}$. ∎

Inequality (16) and Lemma 5 imply

$$|\tilde{g}_j^J(x_j^{*J}, x_j^{*J}, x_j^{*J}, \ldots, x_j^{*J}) - \tilde{g}_j^J(x_j^{*J}, x_1^{*J}, x_2^{*J}, \ldots, x_J^{*J})| \leq (J-1)\frac{\beta}{J-1}\frac{2\alpha}{b(J-1)}, \tag{18}$$

and hence, using (15)

$$|\gamma^J(x_j^{*J})| = \left| g^J \left(x_j^{*J}, x_j^{*J}, x_j^{*J}, \ldots, x_j^{*J} \right) \right| \leq \left(\frac{\beta 2\alpha}{b} + \alpha \right) \frac{1}{J-1},$$

for all $j = 1 \ldots J$.

$\gamma^J(x_j^{*J})$ differs from the left hand side of the informal equation (7) by the fact that the measures of immigrants to sector j and k are stochastic. The weak law of large numbers assures that $\gamma^J(x_j^{*J})$ converges to the left hand side of the informal equation (7), and the convergence is uniform, because the derivative of the second and third term in (14) with respect to the measure of immigrants are bounded.

We denote the left hand side of the informal equation by $\gamma(x^*)$. For any $\epsilon' > 0$

there exist J' such that $|\gamma(x_j^{*J})| < \epsilon'$ for all $J > J'$ and $j = 1, \dots J$. Function $\gamma(\cdot)$ is continuous and x_j^{*J} are from the closed interval $[-1, 1-c]$. Hence there must exist function $\epsilon(\epsilon')$ such that $|x_j^{*J} - x^*| < \epsilon(\epsilon')$, where x^* is the unique root of $\gamma(x^*) = 0$, and $\lim_{\epsilon' \to 0} \epsilon(\epsilon') = 0$. $\square$

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