Optimal Illusion of Control

and related perception biases

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predictable errors in perception of payoff-relevant parameters

- illusion of control
- overconfidence
- optimism

prevalence and persistency of these biases suggest that they have a function

this paper:
perception biases arise as optimal error management when errorless perception is infeasible

towards informed paternalism. . .

Outline

Decision Stage

$a \in \{1, \ldots, n\}$

$\theta \in \{l, h\}$

$u(a, \theta)$

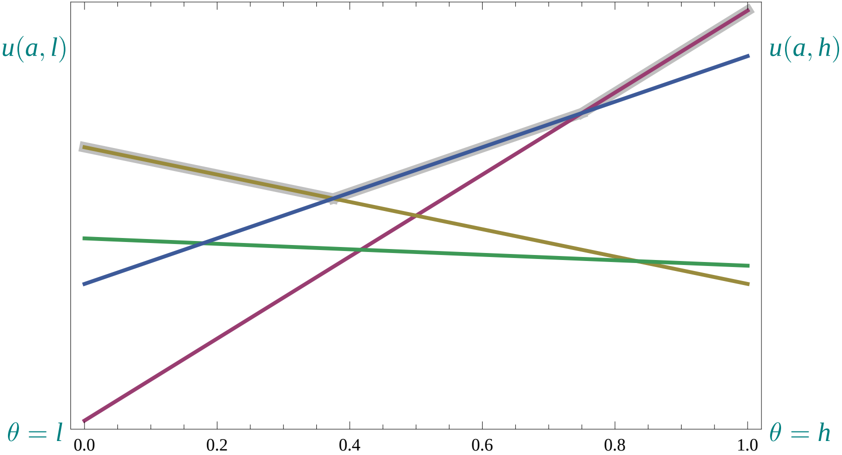
p objective probability of the state h

q subjective probability of the state h

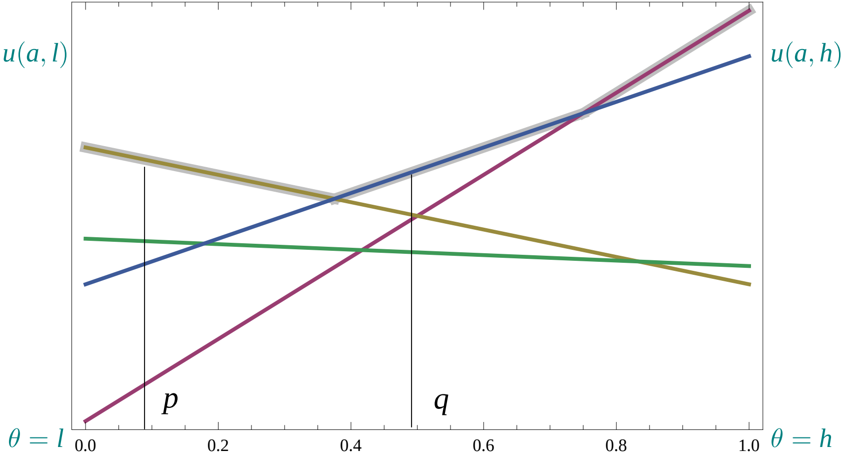
$u(a, p) = (1 - p)u(a, l) + pu(a, h)$

$a_q^* \in \arg\max_a u(a, q)$

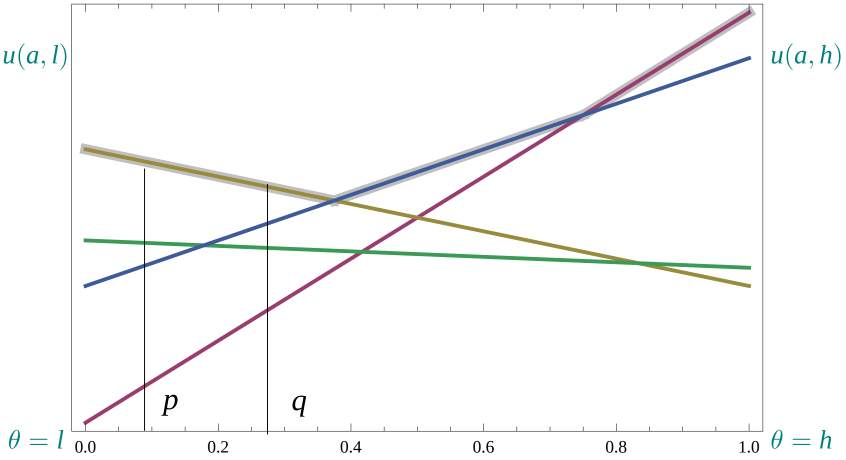
$\ell(p, q; u) = \max_a u(a, p) - u\left(a_q^*, p\right)$



$u(a, l)$
$u(a, h)$
$\theta = l$
$\theta = h$
0.0
0.2
0.4
0.6
0.8
1.0



loss



no loss

Ex Ante Stage

Perception is optimized across many decision problems

$u = (u(a, \theta))_{a \in A, \theta \in \Theta}$ is a random variable with support $\mathbf{R}^{2n}$

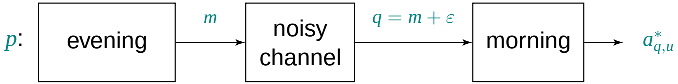
ex ante expected loss from perceiving true probability p as q:

$$L(p, q) = E_u[\ell(p, q; u)]$$

Perception

the first-best perception, $p = q$, is infeasible

perception technology:



unbiased perception is feasible: $m(p) = p$ leads to $E[q \mid p] = p$

Definition (Perception problem)

For each p,

$$m^*(p) \in \arg\min_{m \in [\sigma, 1-\sigma]} E_{q(m)} \left[L\big(p, q(m)\big) \right]$$

Outline

Overconfidence

talent

agent chooses one from n projects

each project a pays $t_\theta b_a - c_a$

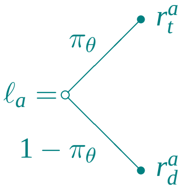
talent $t_h > t_l > 0$

$b_a, c_a \sim \mathcal{N}(0,1)$, observed at the choice stage

Overconfidence

knowledge

agent chooses one from n binary lotteries



expected value of each ℓ_a is $\pi_\theta r_t^a - (1 - \pi_\theta) r_d^a$

$\pi_h > \pi_l \geq 1/2$, agent is more informed at state h

r_t^a, r_d^a are i.i.d. $\sim \mathcal{N}(0, 1)$, observed at the choice stage

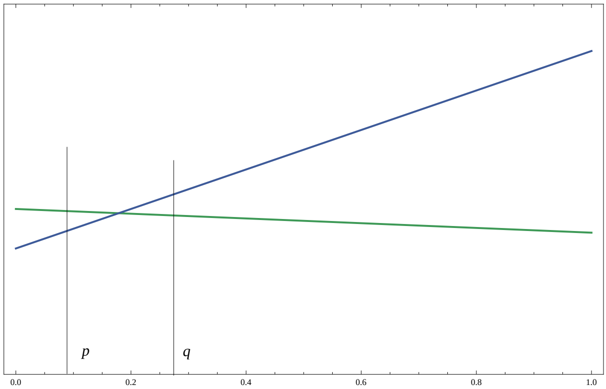
Optimal Perception

Proposition

In both examples, the second-best perception strategy is biased upwards; $m^*(p) > p$ *and* $E[q \mid p] > p$ *for all* p.

Preliminary Intuition

misperceiving p for q leads to a loss only if the optimal choice at p and q differs



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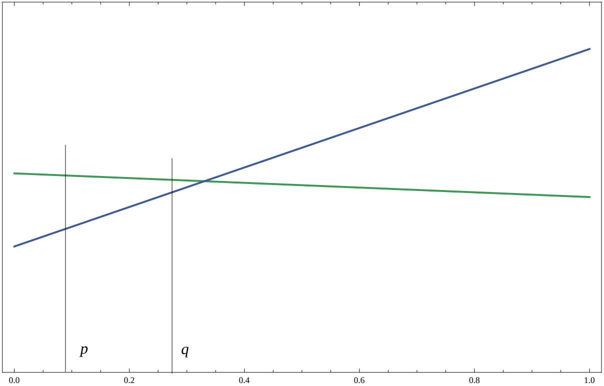


Where Do the Ties Arise?

loss arises only if a tie arises for some probability in between p and q

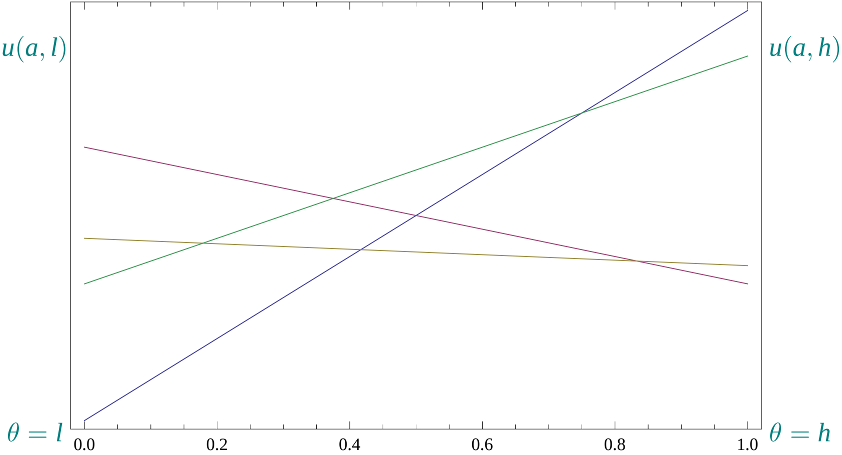
in all three examples, $Var[u(a,h)] > Var[u(a,l)]$

a likelihood that a tie arises at a probability s decreases in s

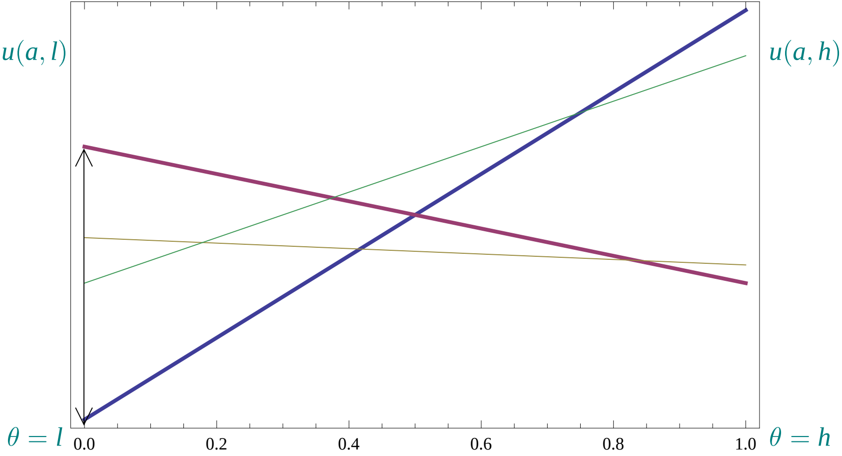
overvaluing the true probability is less consequential than undervaluing it

Outline

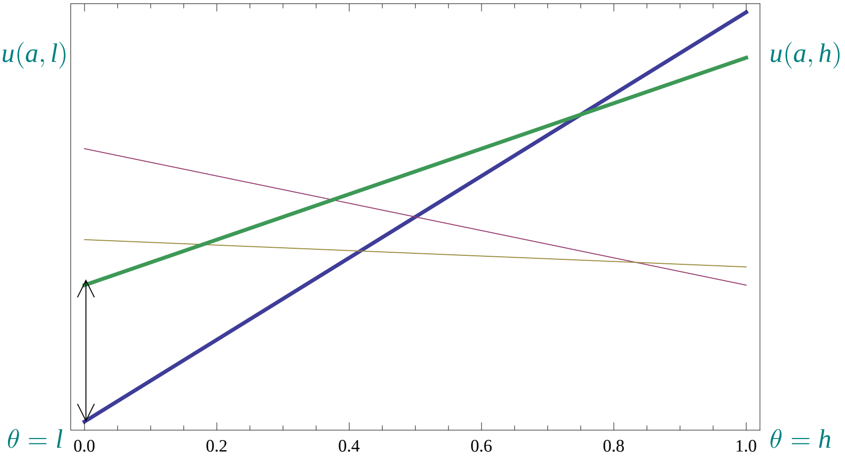
$\ell(l, h; u)$



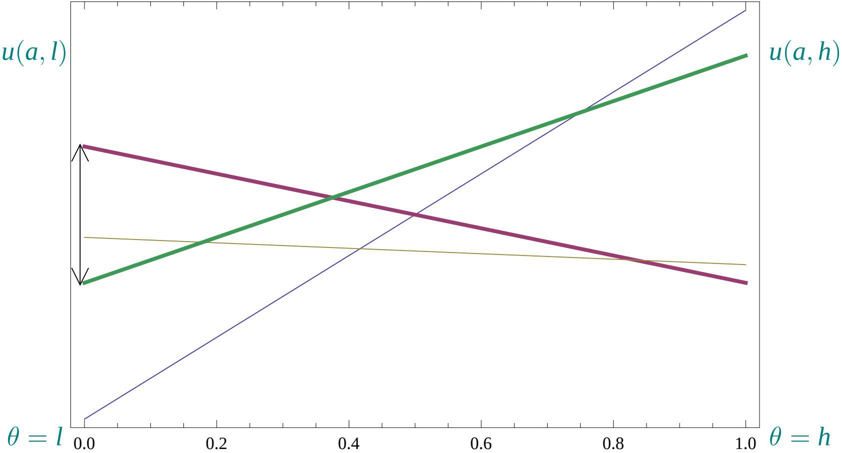
$\ell(l, h; u)$



$\ell(l, h; u)$



$\ell(l, h; u)$



Towards the Ex Ante Loss

we build a function $w(p)$ that captures

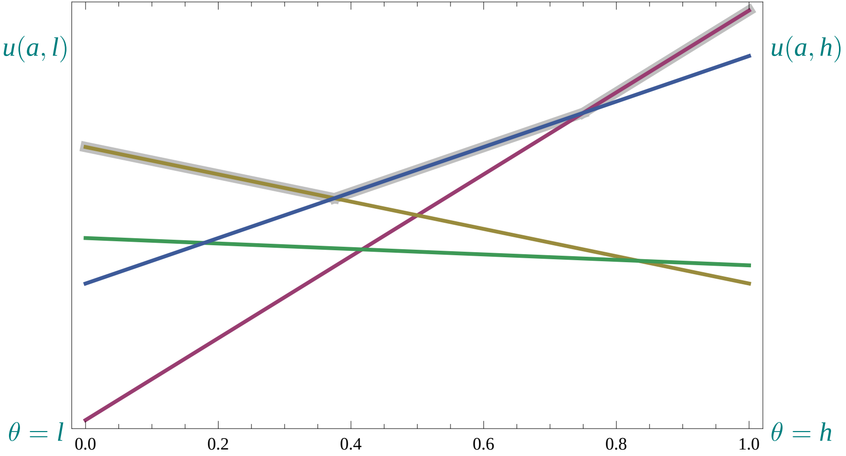
- likelihood of a tie between the best and second-best action
- angle of the intersection

then

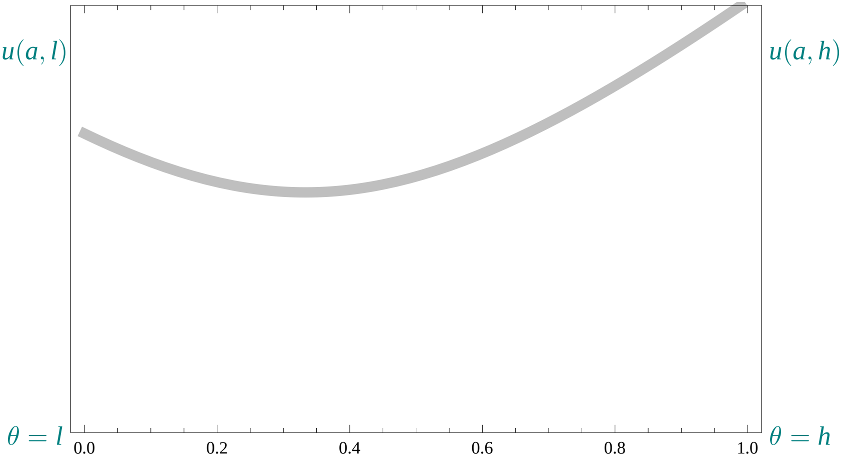
$$L(p,q) = \int_p^q (s-p)w(s)ds$$

tractable when payoffs are normally distributed

Value Function



Value Function



Value Function

$v(p) = E_u[\max_a u(a,p)]$

Lemma

$$L(p,q) = \int_p^q (s-p)v''(s)ds.$$

Outline

Specific Setting

fits all the three applications

$u(a, \theta) = u_a + \sigma_\theta \eta_a$

$u_a \sim N(0, 1)$, $\eta_a \sim N(0, 1)$, all independent

$u(a, p) \sim N(0, \omega^2(p))$

$v(p) \propto \omega(p)$

Lemma

$$v(p) = \textit{const.} \times \sqrt{1 + (\sigma_l(1 - p) + \sigma_h p)^2}.$$

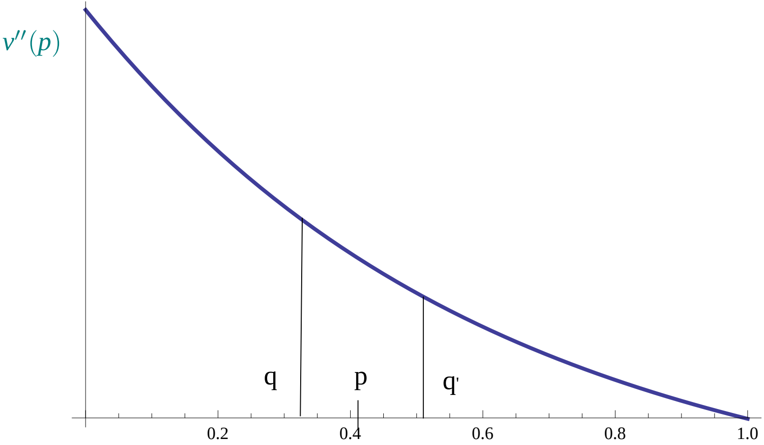
Illusion of Control

fits all the three applications

Lemma

If $\sigma_h > \sigma_l$ (so that choice matters more in the high state) then $v''(p)$ is decreasing.

The First-order Condition



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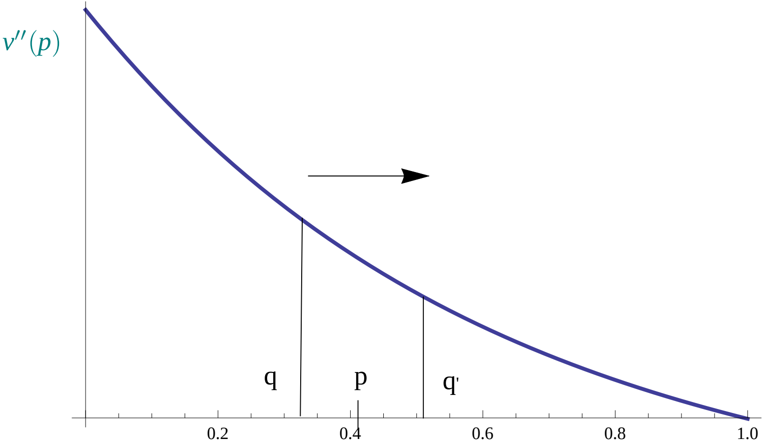

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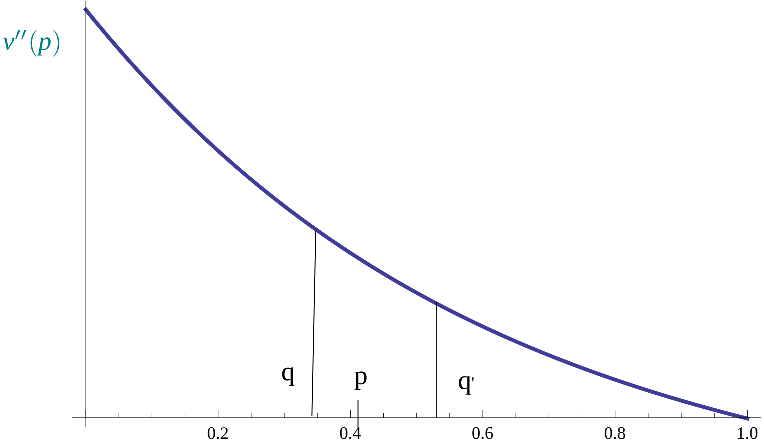

Optimal Illusion of Control

Proposition

When $\sigma_h > \sigma_l$ then the optimal perception strategy is biased towards the high state

$$m^*(p) > p \text{ and } E[q \mid p] > p.$$

Outline

Irrelevance of the Action-Independent Payoff Terms

consider two random payoffs $u(a,\theta)$ and $\tilde{u}(a,\theta)$ such that

$$\tilde{u}(a,\theta) = u(a,\theta) + \text{const}_\theta$$

Proposition

The optimal perception strategies under u and $\tilde{u}$ are identical.

argument:
optimal perception depends only on $v''(p)$
additive constants do not affect $v''(p)$

No hedonic beliefs

Some Optimism Nevertheless

illusion of control $\Rightarrow Var[u(a,h)] > Var[u(a,l)]$

if $E[u(a,l)] = E[u(a,h)]$ then $v(1) > v(0)$

$\Rightarrow$ perception is biased towards the better state

illusion of control generates optimism unless, incidentally, the state with high control is predictably bad

Outline

From Naivety to Sophistication

our naive agent

- hasn't used all the retained information at the choice stage
- has used a rigid decision rule

next, we examine sophisticated agent who

- forgets some information during the decision process
- forms bayesian posteriors using all the retained information
- can employ any decision rule

optimal bias in the same direction

caused by the same forces

Sophisticated Perception Technology

binary message space $M = \{0, 1\}$

perception function $m(p)$, $m : [0, 1] \longrightarrow M$

choice rule $c(u, m)$, $c : \mathbf{R}^{2n} \times M \longrightarrow A$

Definition (sophisticated perception problem)

$$\max_{m(\cdot), c(\cdot, \cdot)} E_{u,p} \left[u \left(c\big(u, m(p)\big), p \right) \right].$$

Optimal Bias

assumptions: density of p is

- symmetric around $1/2$
- independent from payoffs

Proposition

The agent partitions the probability interval into $\{[0, p^*), [p^*, 1]\}$*.*
If $v''(p)$ *is decreasing then* $p^* < 1/2$*.*

corollary: the high type is more prevalent than the low one

Outline

General Perception Technology

agent observes true p

chooses stochastic perception $\mathbf{q} \in Q$

each $\mathbf{q}$ is a random variable with support in $[0, 1]$.

optimal perception:

$$\mathbf{q}(p) \in \arg\min_{\mathbf{q} \in Q} E_{\mathbf{q}} \left[L(p, \mathbf{q}) \right]$$

General Bias

definition

an outsider observes joint density $\mu(p, q)$

$\Rightarrow$ the outsider knows $\mathbf{q}(p)$ for all p

the outsider compares the agent with a “statistician” who minimizes $E_{\mathbf{q}}\left[(\mathbf{q}-p)^2\right]$

the agent is revealed to be biased upwards if

$$\arg\min_{\hat{p}}\left[\left(\mathbf{q}\left(\hat{p}\right)-p\right)^2\right] < p$$

Proposition

If the set of the feasible perceptions is ordered with respect to the stochastic dominance order and $v''(p)$ *is decreasing then the agent is revealed to be biased upwards.*

Literature

hedonic beliefs:
Brunnermeier & Parker (2005), Caplin & Leahy (2001), Köszegi (2006)

evolution of preferences
Robson (2001), Samuelson & Swinkels (2006), Robson & Samuelson (2007), Rayo & Becker (2007), Netzer (2009)

rational inattention
Matějka & McKay (2015), Caplin and Dean (2013), Woodford (2012a,b)

second-best perception
Steiner & Stewart (2015)

Conclusion

are the perception biases mistakes?

the biases are optimal given the designer's information about the decision problem

what if the outside observer has finer information about the decision problem than the perception designer?