On the Cost of Misperception: General Results and Behavioral Applications

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Two Ideas Combined

Ecological Rationality

- Simon, Gigerenzer
- objective: performance across all problems in the DM’s environment
- optimization over rules

Error Management

- Haselton, Buss, Nettle
- objective: expected loss in a given decision problem
- optimization over distributions of errors

we:

- characterization: loss across the DM’s environment
- optimization: management of perception error
In This Paper

loss characterization:

distr. of decision problems $\times$ misperception $\rightarrow$ expected loss

second-best perception:

- errors are not completely avoidable
- biases help to avoid costly types of errors

applications:

overprecision, anchoring, illusion of control, prob. weighting
hedonic beliefs

evolution of preferences
Robson (2001), Netzer (2009)

rational inattention
Sims (1998, 2001), Woodford (2012a,b)

second-best perception
Steiner & Stewart (2016)
Outline

Loss from Misperception

Loss Characterization

Second-Best Perception

Applications

Bayesian Sophistication
A Decision Problem

\( a \in \{1, \ldots, n\} \)

\( u(a, \theta), \ \theta \in \{0, 1\} \)

\( p \ \text{objective probability of the state 1} \)

\( q \ \text{subjective probability of the state 1} \)

\( u(a, p) = (1 - p)u(a, 0) + pu(a, 1) \)

\( a_q^* \in \arg\max_a u(a, q) \)

\( \ell(p, q; u) = \max_a u(a, p) - u(a_q^*, p) \)
Example

the DM chooses one from \( n \) binary lotteries

e.g. financial bets on a binary event

\[
\ell_a = \begin{cases} 
  p & u(a, 1) \\
  1 - p & u(a, 0)
\end{cases}
\]

\[
\ell_{a'} = \begin{cases} 
  p & u(a', 1) \\
  1 - p & u(a', 0)
\end{cases}
\]
the DM chooses one from $n$ binary lotteries

e.g. financial bets on a binary event

\[ \ell_a = \begin{cases} q & u(a, 1) \\ 1 - q & u(a, 0) \end{cases} \]

\[ \ell_{a'} = \begin{cases} q & u(a', 1) \\ 1 - q & u(a', 0) \end{cases} \]
$u(a, 0)$

$u(a, 1)$

no loss
Environment

the DM encounters many decision problems in her environment

\[ u = (u(a, \theta))_{a \in A, \theta \in \Theta} \] is a random variable with support \( \mathbb{R}^{2n} \)

each draw of \( u \) is a decision problem in the DM’s environment

loss from misperception in the DM’s environment: ex ante expected loss from misperceiving true probability \( p \) as \( q \)

\[ L(p, q) = E_u[\ell(p, q; u)] \]
Example

the DM chooses one from $n$ binary lotteries

$$\ell_a = p u(a, 1) - (1 - p) u(a, 0)$$

$$\ell_{a'} = p u(a', 1) - (1 - p) u(a', 0)$$

$u(a, \theta)$ are iid $\sim \mathcal{N}(0, 1)$, observed at the choice stage

further away $q$ is from $1/2$, the more knowledgeable DM feels

loss from misperception of the precision?
Example

The DM chooses one from $n$ binary lotteries

\[
\ell_a = \frac{1}{2} + q(1-u(a,0))\quad \ell_{a'} = \frac{1}{2} + q(1-u(a',0))
\]

$u(a, \theta)$ are iid $\sim \mathcal{N}(0,1)$, observed at the choice stage.

Further away $q$ is from $1/2$, the more knowledgeable DM feels loss from misperception of the precision?
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Main Result

how much does the DM lose if she misperceives $p$ for $q$?

$$L(p, q) = \int_{p}^{q} (s - p)v''(s)ds$$

how well-off is the realistic DM who optimizes?

$$v(p) = E_u[\max_a u(a, p)]$$
$\ell(0, 1; u)$

\[ u(a, 0) \]

\[ u(a, 1) \]

\[ \theta = 0 \quad \text{to} \quad \theta = 1 \]

\[ 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 \]
\ell(0, 1; u)
\[ \ell(0, 1; u) \]
$\ell(0, 1; u)$

$u(a, 0)$

$u(a, 1)$

$\theta = 0$

$\theta = 1$
Towards the Ex Ante Loss

let us build a function $w(s)$ that captures:

- likelihood of a tie at $s$ between the best and the second-best action
- angle of the intersection

then:

$$L(p, q) = \int_p^q (s - p)w(s)ds$$
\[ u(a, 0) \]

\[ u(a, 1) \]

\[ v''(p) \] is the expected intensity of the kink at \( p \).
Value Function

\[ u(a, 0) \quad u(a, 1) \]

\[ \theta = 0 \quad \theta = 1 \]

\[ v''(p) \] is the expected intensity of the kink at \( p \)
$v(p)$ is the expectation of the maximum over $n$ draws of

$u(a, p) \text{ iid } \sim \mathcal{N}(0, p^2 + (1 - p)^2)$
Back to the Example

\[ \ell_a = p \cdot u(a, 1) - (1-p) \cdot u(a, 0) \]

\[ \ell_{a'} = p \cdot u(a', 1) - (1-p) \cdot u(a', 0) \]

\[ v(p) = \text{const}_n \times \sqrt{p^2 + (1-p)^2} \]

\[ u(a, p) \text{ iid } \sim \mathcal{N}(0, p^2 + (1-p)^2) \]
intuition:
- misperception of $p$ for $q$ is costly iff tie arises inbetween
- ties are concentrated near $1/2$
- under-precision is costlier than over-precision
Back to the Example

intuition:
- misperception of $p$ for $q$ is costly iff tie arises inbetween
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Perception Problem

the first-best perception, $p = q$, is infeasible
Perception Problem

the first-best perception, \( p = q \), is infeasible

\[ p: \text{memory input} \rightarrow m(p) \rightarrow \text{noisy channel} \rightarrow q = m(p) + \varepsilon \rightarrow \text{action choice} \rightarrow a_{q,u} \]

\( \varepsilon \) symmetrically distributed on \([−\sigma, \sigma]\)
Perception Problem

the first-best perception, $p = q$, is infeasible

$\varepsilon$ symmetrically distributed on $[-\sigma, \sigma]$

unbiased perception is feasible: $m(p) = p$ leads to $\mathbb{E}_\varepsilon[q \mid p] = p$
Perception Problem

the first-best perception, $p = q$, is infeasible

$\varepsilon$ symmetrically distributed on $[-\sigma, \sigma]$

unbiased perception is feasible: $m(p) = p$ leads to $E_{\varepsilon}[q | p] = p$

Definition (Naive Perception problem)

For each $p$,

$$\min_{m(p) \in [\sigma, 1-\sigma]} E_{\varepsilon} \left[ L(p, m(p) + \varepsilon) \right]$$
Proposition

Let \( q = m^*(p) + \varepsilon \) be the stochastic second-best perception when the true probability is \( p \). Then,

\[
E_q \left[ v''(q) (q - p) \right] = 0.
\]

(except of the corners)
Direction of the Bias

Proposition

Let \( p \in (\sigma, 1 - \sigma) \). 

1. If, for all \( \delta \in (0, \sigma) \),

\[
\nu''(p + \delta) < \nu''(p - \delta),
\]

then an **upward bias** occurs: \( m^*(p) > p \).

2. If, for all \( \delta \in (0, \sigma) \),

\[
\nu''(p + \delta) > \nu''(p - \delta),
\]

then a **downward bias** occurs: \( m^*(p) < p \).
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Overprecision

the DM chooses one from \( n \) binary lotteries

\[
l_a = \begin{cases} p & u(a, 1) \\ 1 - p & u(a, 0) \end{cases}
\]

\[
l_{a'} = \begin{cases} p & u(a', 1) \\ 1 - p & u(a', 0) \end{cases}
\]

\( u(a, \theta) \) are iid \( \sim \mathcal{N}(0, 1) \), observed at the choice stage
Overprecision

if $p < 1/2$, then $m^*(p) < p$
if $p > 1/2$, then $m^*(p) > p$
Anchoring

the DM chooses an object $a \in \{1, 2\}$

quality of each object $a$ is $x_1^a + x_2^a$

the DM observes:

- the first components of both objects in round 1
- the second components in round 2

$\Delta_t = x_2^t - x_1^t$

$\Delta_1$ is the first impression
Anchoring

the DM in round 1:

▶ observes the true $\Delta_1$
▶ memorizes a value $m(\Delta_1)$

the DM in round 2:

▶ recalls subjective $\hat{\Delta}_1 = m(\Delta_1) + \varepsilon$
▶ observes the true $\Delta_2$
▶ chooses object 2 iff $\hat{\Delta}_1 + \Delta_2 > 0$

expected loss from misperception:

$$L(\Delta_1, \hat{\Delta}_1) = E_{\Delta_2} \left[ (\Delta_1 + \Delta_2)1_{\Delta_1+\Delta_2>0} - (\Delta_1 + \Delta_2)1_{\hat{\Delta}_1+\Delta_2>0} \right]$$

what should the DM memorize to minimize the loss?
Anchoring

**assumption**: density of $\Delta_2$ is single-peaked, symmetric

**result**: optimal exaggeration of the first impression:

- if $\Delta_1 > 0$, then $m^*(\Delta_1) > \Delta_1$
- if $\Delta_1 < 0$, then $m^*(\Delta_1) < \Delta_1$

**intuition**:
- misperception of $\Delta_1$ for $\hat{\Delta}_1$ causes loss iff it reverts choice
- reversal arises iff tie happens at $\tilde{\Delta}_1$ between $\Delta_1$ and $\hat{\Delta}_1$
- ties are more likely when the first impression is near 0
- $\Rightarrow$ asymmetry in the cost of the errors
Anchoring

\( v(\Delta_1) \) — value of the realistic DM across all draws of \( \Delta_2 \)

loss from misperception:

\[
L(\Delta_1, \hat{\Delta}_1) = \int_{\Delta_1}^{\hat{\Delta}_1} v''(s)(s - \Delta_1) d\Delta_1
\]

\( v'' \) is single-peaked, symmetric around 0:

\[
v''(\Delta_1) = \psi(-\Delta_1) = \psi(\Delta_1)
\]
Illusion of Control

the DM is of either:

- **low** type—choice has a low impact, or
- **high** type—choice has a high impact

on her own well-being in most (but not all) decision problems

the high type may stand for:

- high talent
- more informed type
- less randomness in the impact of choice

formalization:

\[ u(a, \theta) = u_a + \tau_\theta \eta_a \]

\[ u_a, \eta_a \text{ are normal r.v.s, iid across actions, } \tau_1 > \tau_0 > 0 \]
Illusion of Control

$v''(q)$

$q$
Illusion of Control

\[ v''(q) \]

The diagram illustrates the function \( v''(q) \) with values at points \( q, p, q' \) on the horizontal axis from 0.2 to 1.0.
Illusion of Control

$v''(q)$

$q$

$p$

$q'$

$q$

0.2 0.4 0.6 0.8 1.0
Illusion of Control

intuition:

- variance of $u(a, p)$ increases in $p$

- $\Rightarrow$ likelihood of tie decreases in $p$

- $\Rightarrow$ upward error is more likely consequential than the downward one of a same size
Probability Weighting
Steiner and Stewart (2016)

\[ b \]

\[ p \]

\[ 1 - p \]

\[ b \]

\[ u(1, 1) \]

\[ vs \]

\[ s \]

\[ u(1, 0) \]

\[ u(1, \theta) \text{ are iid } \sim \mathcal{N}(0, 1) \]

if \( s > 3^{1/4} \), then \( v''(p) \) is U-shaped
Probability Weighting

Steiner and Stewart (2016)
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Bayesian Sophistication
From Naivety to Sophistication

our **naive** DM

- hasn’t used all the retained information at the choice stage
- has used a rigid decision rule

we now examine a **sophisticated** DM who

- forgets some information during the decision process
- forms bayesian posteriors using all the retained information
- can employ any decision rule

we’ll find

- optimal bias in the same direction
- caused by the same forces
Sophisticated Perception Technology

\( \theta \in \{0, 1\} \), uniform prior

the DM:

- observes signal about \( \theta \) and updates her belief to \( p \)
- chooses to memorize whether \( p \) is “low” or “high”
- forgets \( p \)
- observes \( u(., .) \)
- chooses \( a \) to maximize \( \mathbb{E}[u(a, \theta) | \text{her information set}] \)
Sophisticated Perception Technology

binary message space $M = \{0, 1\}$

perception function $m(p), m : [0, 1] \rightarrow M$

choice rule $c(u, m), c : \mathbb{R}^{2n} \times M \rightarrow A$

**Definition (sophisticated perception problem)**

$$\max_{m(\cdot), c(\cdot, \cdot)} E_{u,p} \left[ u \left( c(u, m(p)), p \right) \right].$$
Optimal Partition

assumptions: density of $p$ is
  ▶ symmetric around $1/2$
  ▶ independent from payoffs

Proposition

The DM partitions the probability interval into $\{[0, p^*), [p^*, 1]\}$.
If $v''(p)$ is decreasing, then $p^* < 1/2$.

⇒ the high type is more prevalent than the low one
Intuition

$v''(p)$ measures the local value of information:

- consider a DM whose current belief is $p$
- her value in absence of further information is $v(p)$
- how helpful is a weakly informative signal?

$$E_{\text{posterior}}[v(\text{posterior})] - v(p)$$

if $v''$ decreases, then

- distinguishing among high probabilities is less important than among the low ones
- the optimal partition reflects that
Intuition

$v''(p)$ measures the local value of information:

- consider a DM whose current belief is $p$
- her value in absence of further information is $v(p)$
- how helpful is a weakly informative signal?

$$v''(p) \text{Var( posterior)}/2$$

if $v''$ decreases, then

- distinguishing among high probabilities is less important than among the low ones
- the optimal partition reflects that
Discussion

are the perception biases mistakes?

the biases are optimal given the designer’s information about the decision problem

what if the outside observer has finer information about the decision problem than the perception designer?