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Note

The effects of risk preferences in mixed-strategy equilibria of 2 × 2 games

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Abstract

We consider the effects of risk preferences in mixed-strategy equilibria of 2 × 2 games, provided such equilibria exist. We identify sufficient conditions under which the expected payoff in the mixed equilibrium increases or decreases with the degree of risk aversion. We find that (at least moderate degrees of) risk aversion will frequently be beneficial in mixed equilibria.
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1. Introduction

General economic intuition is that risk aversion as well as risk loving hampers expected payoffs, yet both risk averse and risk loving preferences are observed in different contexts. Robson (1992, 1996a, 1996b) and To (1999) provide evolutionary explanations for the prevalence of specific risk preferences. Dekel and Scotchmer (1999) study conditions for the selection of risk loving in winner-take-all games. Strobel (2001) shows that for chicken games a payoff monotone dynamic would lead to ever increasing risk loving.

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Table 1
Monetary payoffs

	L	R
T	a, e	b, f
B	c, g	d, h

Huck et al. (1999) have simulated an evolutionary process selecting between agents with different risk preferences based on the equilibrium payoffs in randomly generated 2×2 games. These simulations indicated an advantage for risk averse players. The strength of the effect depended on the equilibrium selection criterion applied to games with multiple equilibria. However, the effect of higher long term propagation of risk averse players was particularly pronounced if attention was restricted to the class of 2×2 games with no pure equilibrium. These results yield the intuition that risk averse players receive higher equilibrium payoffs in this class, which has been confirmed for the special case of uniformly distributed payoffs in Engelmann (2003).

In the study at hand we investigate conditions under which a player's expected payoff in a mixed-strategy equilibrium of a 2×2 game increases or decreases with his degree of risk aversion. Compared to Engelmann (2003), we consider also 2×2 games with two pure equilibria and one mixed but we always restrict players to play the mixed equilibrium.[1] We make substantially less restrictive assumptions about the distribution of payoffs than in Engelmann (2003).

Consider a game G with payoffs as in Table 1 and player 1 choosing rows and player 2 choosing columns. Note that the payoffs are understood as "material" payoffs, i.e. they are not utility levels and hence if we introduce risk aversion or risk loving, this will not change this payoff matrix. The players maximize expected utility, not the material payoffs. We are looking for conditions under which the equilibrium material payoff increases or decreases with a unilateral change of risk preferences.[2]

The model follows the logic of the indirect evolutionary approach (Güth and Yaari, 1992), which considers the evolution of preferences and assumes that behavior is driven by utility maximization, while evolution is driven by underlying material payoffs. The main argument of the indirect evolutionary approach—that players may benefit from divergence between the actual material payoffs and preferences—has appeared already in Schelling (1960). Koçkesen et al. (2000a, 2000b) generalized the idea of indirect evolution to a larger class of games. Dekel et al. (in press) examine the sensitivity of the theory to the (in)observability of preferences. Heifetz et al. (in press) examine indirect evolution under very broad classes of distortions. See Samuelson (2001) for a critical introduction to the indirect evolutionary approach.

Without loss of generality we make the following assumptions on the payoff parameters: $|a-b| > |c-d|$ and $a-b > 0$. We assume that G has a mixed-strategy equilibrium, which holds if and only if $\operatorname{sign}(a-c) = \operatorname{sign}(d-b)$ and $\operatorname{sign}(e-f) = \operatorname{sign}(h-g)$.[3] Let q and p denote the equilibrium probabilities of player 1 to choose T and of player 2 to choose L, respectively.

[1] The effects of risk preferences on the payoffs via the effect of equilibrium selection if pure strategy equilibria are played is the case analyzed by Strobel (2001).

[2] We only study the effect of risk preferences on the expected material payoff, we cannot say anything about the utility. Henceforth, with expected payoff we will always refer to the expected material payoff.

[3] More precisely, G has only a mixed-strategy equilibrium if and only if $\operatorname{sign}(a-c) = \operatorname{sign}(d-b) = -\operatorname{sign}(e-f) = -\operatorname{sign}(h-g) = 1$ and G has two pure-strategy equilibria (T, L) and (B, R) as well as a mixed-strategy equilibrium if and only if $\operatorname{sign}(a-c) = \operatorname{sign}(d-b) = \operatorname{sign}(e-f) = \operatorname{sign}(h-g) = 1$. We ignore any cases where one of the relevant differences equals zero, as these would happen only with measure zero under general assumptions on the distribution of payoffs and would lead to degenerate mixed-strategy equilibria.

In the analysis presented in the next section, we will proceed in three steps. First, we identify a class of games where risk aversion unambiguously increases player 1's expected payoff (and indeed the more risk aversion the better) for any distribution of payoffs of player 2. Second, we state a general condition on both players' payoffs that determines whether risk aversion or risk loving increases player 1's expected payoff. We will see in a straightforward way that the first class is a special case where the condition for risk aversion to be beneficial is satisfied for any combination of payoffs of player 2. Third, we will consider the special case that the distribution of the payoffs of player 2 conditional on the payoffs of player 1 is symmetric across rows. We show that under this assumption player 1's ex-ante (i.e. before payoffs are drawn) expected payoff increases with risk aversion.

The basic intuition why a change in risk preferences of player 1 changes his expected payoff is that player 2 changes his strategy in order to keep player 1 indifferent. For example, the attractiveness of T for player 1 can decrease relative to the attractiveness of B. In such a case, player 2 has to increase p, by which he increases the attractiveness of T, and thus reestablishes player 1's indifference. The increase of the probability of player 2 choosing L in turn increases or decreases the expected payoff of player 1, according to the conditions described below.

The result that risk aversion can be beneficial is in contrast to the effects of risk aversion in bargaining games. As outlined e.g. by Binmore et al. (1986), a concave transformation of a player's utility function, i.e. if he becomes more risk averse, changes the Nash bargaining solution in favor of the other player. Increasing a player's risk aversion weakens his bargaining position because the risk of not reaching an agreement becomes more threatening to him.

2. Analysis

In the set of games we analyze, i.e. $|a-b| > |c-d|$, $a-b>0$, $\mathrm{sign}(a-c) = \mathrm{sign}(d-b)$ and $\mathrm{sign}(e-f) = \mathrm{sign}(h-g)$, we can distinguish three classes that, as we shall see, differ with respect to the effects of risk preferences:

(1) $a > c > d > b$,
(2) $a > d > c > b$,
(3) $a > d > b > c$ or $d > a > c > b$.

The first crucial result is

Lemma 1. (i) *In classes* 1 *and* 2*, the equilibrium probability p that player* 2 *chooses L increases in the degree of risk aversion of player* 1*.*

(ii) *In class* 3*, suppose that player* 1 *has a constant absolute risk aversion utility function $u(x) = -\frac{1}{r}e^{-rx}$. The equilibrium probability p that player* 2 *chooses L increases in r, in a neighborhood of* 0*.*

Note that the statements in Lemma 1 depend only on the payoffs of player 1, but not on those of player 2.

Proof. (i) Consider the mixed equilibrium for a risk-neutral player 1. Lottery L_T associated with strategy T is a mean-preserving spread of L_B because L_T has to have the same expected payoff as L_B to keep player 1 indifferent and L_T is a spread of L_B. Hence, if player 1 were risk averse, his risk premium for L_T would be larger than the risk premium for L_B. This requires that in equilibrium p is larger for a risk-averse player 1 than for a risk-neutral one. To see this, note that if $a-b>c-d>0$, then an increase of p has a larger impact on the expected payoff for T than on that for B, while if $a-b>0>c-d$, an increase of p will increase the expected payoff for T and decrease that for B.

The above argument generalizes to the consideration of increases in risk aversion, i.e. considering utility functions $u(\cdot)$ and $v(\cdot)$, where the latter is more risk averse than the former, so $v(\cdot)=T(u(\cdot))$ with $T(\cdot)$ concave. This is equivalent to comparing a risk-neutral player with a risk-averse player with utility function $T(\cdot)$ who both face a material payoff matrix with $u(a), u(b), u(c), u(d)$. The same argument as above yields that p is higher for $v(\cdot)$ than for $u(\cdot)$. Note that the classification into the three classes is preserved under the transformation $u(\cdot)$ because $u(\cdot)$ is increasing. The latter argument obviously applies equally for risk loving preferences, completing the proof.

(ii) The proof consists of an explicit computation of the sign of $\lim_{r\to 0}\frac{\mathrm{d}p}{\mathrm{d}r}(r)$. Player 1 is indifferent between the two strategies in a mixed equilibrium which implies $p(r)=\frac{u(d)-u(b)}{u(a)-u(b)+u(d)-u(c)}$. The sign of the derivative $\frac{\mathrm{d}p}{\mathrm{d}r}$ is therefore the same as the sign of

$$\begin{aligned}&\big(u_r(d)-u_r(b)\big)\big(u(a)-u(b)+u(d)-u(c)\big)\\&\quad-\big(u(d)-u(b)\big)\big(u_r(a)-u_r(b)+u_r(d)-u_r(c)\big).\end{aligned}\tag{1}$$

The derivative of the utility function is $u_r(x)=\frac{1}{r^2}\mathrm{e}^{-rx}+\frac{x}{r}\mathrm{e}^{-rx}=-\frac{1}{r}u(x)-xu(x)$. By substituting the derivative $u_r(x)$ into (1) we get that the sign of $\frac{\mathrm{d}p}{\mathrm{d}r}(r)$ is equal to the sign of

$$\begin{aligned}&-\big[du(d)-bu(b)\big]\big[u(a)-u(b)+u(d)-u(c)\big]\\&\quad+\big[u(d)-u(b)\big]\big[au(a)-bu(b)+du(d)-cu(c)\big].\end{aligned}\tag{2}$$

We find the sign of (2) for small r with the help of the Taylor expansion of e^{-rx} with respect to r around $r=0$:

$$u(x;r)=-\frac{1}{r}\mathrm{e}^{-rx}=-\frac{1}{r}\left(1-rx+\frac{1}{2}r^2x^2+o\big(r^3\big)\right).$$

Using this expansion we can rewrite (2) as

$$=0\frac{1}{r^2}+0\frac{1}{r}+(a-c)(d-b)\big(a-b-(d-c)\big)+o(r).\tag{3}$$

The terms $o(\frac{1}{r^2})$ and $o(\frac{1}{r})$ have canceled out and the constant term is positive by the assumption that $\operatorname{sign}(a-c)=\operatorname{sign}(d-b)$ and $(a-b)>|d-c|$. Hence (3) is positive for sufficiently small r, and therefore the sign of the derivative $\frac{\mathrm{d}p}{\mathrm{d}r}(r)$ is positive as well in a neighborhood of 0. □

The reason for the weaker result in class 3 compared to classes 1 and 2 is that L_T is not a spread of L_B in class 3. In contrast to classes 1 and 2, the interval $[c,d]$ is not contained in $[a,b]$, so the impact of a change in risk preferences may be larger for the lottery L_B associated with the smaller interval. Yet, as $r\to 0$, this problem disappears.

Now for $a>c>d>b$ it is obvious that independent of his own equilibrium strategy and hence independent of player 2's payoffs, player 1 always prefers that player 2 chooses L and hence his expected payoff increases in p. We conclude

Proposition 1. *In any mixed-strategy equilibrium of a* 2×2 *game, if* $a>c>d>b$ (*class* 1) *then the expected payoff of player* 1 *increases in his degree of risk aversion.*

Note that Proposition 1 is independent of the payoffs of player 2 as long as they satisfy the necessary condition for the existence of a mixed-strategy equilibrium. This result has an interesting implication. Whenever player 1 has a strict preference about the choice of player 2, i.e. independent of his own choice, he prefers player 2 to choose the same strategy, he will always benefit from being risk averse. The logic is that in order to keep player 1 indifferent, player 2 has to "sweeten" the action of player 1 with higher risk by increasing the probability for the strategy that player 1 wants him to choose.

We now turn to the case when player 1 does not have a strict preference over player 2's choice, i.e. $a-b>0>c-d$. According to Lemma 1, the probability of player 2 to choose L increases in the degree of risk aversion of player 1, at least in a neighborhood of risk neutrality, which increases player 1's expected payoff if he chooses T, but lowers it if he chooses B. Hence the overall effect depends on his equilibrium (mixed) strategy which in turn depends on the payoffs of player 2. We find

Lemma 2. *The expected payoff of player* 1 *in the mixed equilibrium increases in the probability* p *that player* 2 *chooses* L *if and only if*[4]

$$a-b>\frac{e-f}{h-g}(d-c). \tag{4}$$

Proof. The proof is straightforward. Let q be the equilibrium probability of player 1 to choose T. Player 2 is indifferent between L and R in the mixed equilibrium, hence $qe+(1-q)g=qf+(1-q)h$ or equivalently $\frac{1-q}{q}=\frac{e-f}{h-g}$. The expected payoff of player 1 in equilibrium is

$$\begin{aligned} &q\big(pa+(1-p)b\big)+(1-q)\big(pc+(1-p)d\big)\\ &\quad=pq(a-b)+p(1-q)(c-d)+qb+(1-q)d. \end{aligned}$$

This is increasing in p if and only if $q(a-b)+(1-q)(c-d)>0$ or equivalently $(a-b)>\frac{1-q}{q}(d-c)=\frac{e-f}{h-g}(d-c)$. $\square$

Combining Lemmas 1 and 2 immediately yields

Proposition 2. (a) (i) *In class* 2, *the expected payoff of player* 1 *in the mixed equilibrium increases in his degree of risk aversion if and only if* (4) *holds.*

(ii) *In particular, in class* 2, *the expected payoff of player* 1 *increases if he becomes more risk loving if and only if the inverse of* (4) *holds.*

(b) (i) *In class* 3, *for constant absolute risk aversion, the expected payoff of player* 1 *in the mixed equilibrium increases in his degree of risk aversion* r *in a neighborhood of* 0, *if and only if* (4) *holds.*

(ii) *In particular, in class* 3, *for constant absolute risk aversion, player* 1 *benefits from at least small degrees of risk loving if and only if the inverse of* (4) *holds.*

Obviously, Proposition 1 is a corollary of Proposition 2 since under the assumptions of Proposition 1 the left-hand side of (4) is positive and the right-hand side is negative. Thus (4) is always

[4] We assume here that player 2 is risk neutral. In case player 2 is risk averse or risk loving, we have to replace his material payoffs e, f, g, h with his utility payoffs $u_2(e), u_2(f), u_2(g), u_2(h)$ in condition (4).

satisfied in class 1. For classes 2 and 3, $d - c > 0$, so we can rewrite (4) as $\frac{a-b}{d-c} > \frac{e-f}{h-g}$. Considering $a - b$ and $d - c$ the risk of decisions T and B for player 1, we can call $\frac{a-b}{d-c}$ the relative risk of player 1's strategies. Similarly, $e - f$ and $h - g$ are the incentives for player 2 to choose L instead of R, conditional on player 1 choosing T and B, respectively. Hence we can call $\frac{e-f}{h-g}$ the relative incentive for player 2. Thus in addition to class 1 where player 1 has a strict preference for player 2's choice, he benefits from risk aversion if his relative risk is larger than the relative incentive of player 2. Put differently, if for choice T relative to B the decision of player 2 matters more to player 1 than to player 2 himself, then the expected payoff of player 1 is increasing in his degree of risk aversion, at least in a neighborhood of 0.

Condition (4) holds for quite a large range of games. For example, if we assume that the payoffs $\{a, b, c, d, e, f, g, h\}$ are ex ante all independently drawn from the same distribution (and then restrict attention to the games that satisfy our assumptions), then for some of the games $d - c < 0$, so (4) holds trivially. Of the other games, for half of them $\frac{e-f}{h-g} < 1$ and hence (4) holds by assumption on the payoffs of player 1.[5] Finally, among the remaining games, (4) would still hold for half of the games.[6] Hence in the case of payoffs independently drawn from the same distribution, in more than 3 out of 4 games, the expected payoff of player 1 increases in his degree of risk aversion (in some of these only in a neighborhood of 0, but in many globally), whereas only in the remaining games it decreases. As an illustrative example, we formulate a simple condition on the payoff distribution that assures that an increase in the degree of risk aversion increases the expected payoff over the whole class of games:

Result 1. *Suppose that the distribution of player* 2*'s payoff conditional on player* 1*'s payoff is symmetric across rows, i.e. games* $G = \begin{array}{c|c|c|} & L & R \\ T & a, e & b, f \\ B & c, g & d, h \end{array}$ *and* $G' = \begin{array}{c|c|c|} & L & R \\ T & a, g & b, h \\ B & c, e & d, f \end{array}$ *have the same density. Then across the class of all games with a mixed-strategy equilibrium* (*or across any subclass of games consisting of such pairs of games*)*, the ex ante* (*i.e. before the game is drawn*) *expected payoff of player* 1 *increases in his degree of risk aversion in a neighborhood of* 0.[7,8]

Proof. Consider again that in the mixed equilibrium of G, player 1 will choose T with probability q such that $qe + (1 - q)g = qf + (1 - q)h$. Since G' is derived from G by switching player 2's payoffs across rows, in the equilibrium of G' player 1 will choose T with probability $1 - q$. Games G and G' have the same density and the probability p of player 2 choosing L is the same in both games as it depends on the payoffs of player 1. Hence the expected payoff for player 1 across both games is

$$q\big(pa + (1 - p)b\big) + (1 - q)\big(pc + (1 - p)d\big) + (1 - q)\big(pa + (1 - p)b\big) \\ + q\big(pc + (1 - p)d\big)$$

[5] Recall that $e - f$ and $h - g$ have the same sign by the assumption of the existence of the mixed equilibrium. The probability that the numerator is greater than the denominator is $\frac{1}{2}$ because they are i.i.d.

[6] $\frac{a-b}{d-c}$ and $\frac{e-f}{h-g}$ now have the same condition imposed, they are both > 1. Thus their conditional distributions are the same, hence the probability that one fraction is greater than the other is $\frac{1}{2}$.

[7] In classes 1 and 2 the payoff will again increase in the degree of risk aversion throughout.

[8] Note that if G only has a mixed-strategy equilibrium, then G' has two pure-strategy equilibria as well as a mixed-strategy equilibrium and vice versa. Remember that we assume that the mixed-strategy equilibrium will be played whenever it exists.

$$= pa + (1-p)b + pc + (1-p)d$$
$$= b + d + p\big[(a-b) - (d-c)\big].$$

This increases in p because by assumption $a - b > |d - c|$. By Lemma 1, hence player 1's overall expected payoff for G and G' together increases in his degree of risk aversion (in class 3 at least in a neighborhood of 0). Since the class of all games that have a mixed-strategy equilibrium falls into such pairs of games, the same holds for the ex ante expected payoff across this whole class (or any subclass consisting of such pairs of games) if the assumption on the distribution of payoffs holds. □

3. Concluding remarks

Consider player 1 facing a constant environment that is captured by a game of class 1 or a game of class 2 where (4) holds. Then the population of players 1 would evolve towards ever increasing risk aversion, while if the interaction is described by a game of class 2 where the inverse of (4) holds, the population would evolve towards ever increasing risk loving. In interactions captured by a game of class 3, evolution would drive the result at least towards some degree of risk aversion or risk loving (depending on whether (4) or its inverse holds).

In our analysis, a change in risk preferences can be beneficial because it triggers a beneficial change in the behavior of the opponent. A similar logic is underlying the indirect evolutionary analysis of other types of preferences. For example, Fershtman and Weiss (1998a) consider a case where it is beneficial to have an intrinsic motivation to increase an action when there are strategic complementarities and when the opponent's high action is beneficial. Because of the strategic complementarities, the opponent increases his action in response to the player's anticipated increase of his action. The analogical logic holds if there are strategic substitutes and a player benefits from a low action of the opponent. The analysis (see Fershtman and Weiss, 1998b, for a more general treatment) can be divided into two steps. First, an increase in the intrinsic motivation may lead to an increase or decrease of his opponent's action (depending on whether the game exhibits strategic complements or strategic substitutes). Second, an increase of his opponent's action can be beneficial or detrimental to a player's payoff. The overall effect of an increase of the intrinsic motivation results from the combination of both steps.

Our analysis has the same structure. For comparison, we adopt (somewhat arbitrarily) a terminology that T and L are high and B and R low actions. A shift of probability from the low to the high action is denoted as an *increase* of a player's action. Lemma 1 then specifies the conditions under which player 2 increases his action, and Lemma 2 specifies whether this increase is beneficial. The second step, examining whether a player profits from a high action of his opponent, is entirely identical in the Fershtman and Weiss (1998b) analysis and ours. Thus our Lemma 2 is identical to their condition (and the one identified by Heifetz et al., in press). Our results differ from theirs in the first step. The notable difference stems from the fact that we deal with mixed equilibria. Under the conditions of Lemma 1, an increase of his risk aversion would induce player 1 to decrease his action if player 2 left his strategy unchanged. In Fershtman and Weiss player 2 changes his action because he anticipates this *change* in the behavior of player 1. By contrast, in our model player 2 increases his action in order to keep player 1's behavior *unchanged*, which is necessary in a mixed equilibrium. Consequently, his own payoffs are irrelevant in determining player 2's reaction in our model, while they matter in Fershtman and Weiss (1998a, 1998b).

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