Asset Prices and Business Cycles with Liquidity Shocks

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Abstract

We develop a production based asset pricing model with financially constrained firms to explain the observed high equity premium and low risk-free rate volatility. Investment opportunities are scarce and firms face productivity and liquidity shocks. A negative liquidity shock forces firms to liquidate a fraction of their assets. We calibrate the model to U.S. data and find that it generates an equity premium and a level and volatility of risk-free rate comparable to those observed in the data. The model also fits key aspects of the behavior of aggregate quantities, in particular, the volatility of aggregate consumption and investment.

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1 Introduction

The equity premium puzzle and the risk-free rate puzzle (Mehra and Prescott 1985 and Weil 1989) are two fundamental challenges to theoretical models that have been developed in the finance and macroeconomics literature. Building a production economy model that would satisfactorily account for both the dynamics of asset prices and business cycle fluctuations has proven to be rather difficult.

In this paper, we build a production economy model and show that in the presence of scarce real investment opportunities, liquidity constraints and liquidity shocks play an important role in explaining not only business cycle fluctuations, but also the high equity premium. Our model, calibrated to U.S. data, generates an equity premium, an average risk-free rate, and a risk-free rate volatility comparable to those observed in the data. The model also matches the time-series properties of aggregate macroeconomic quantities, in terms of matching the volatility of aggregate investment and consumption growth relative to output growth. In contrast to existing literature (see Shi 2015 and Bigio and Schneider 2017, among others), negative liquidity shocks in our model are associated with drops in investment and equity prices, implying a positive correlation between asset prices and investment, as observed in the data. In addition, the model generates a time varying equity premium that is countercyclical and a risk-free rate that is procyclical.

Our model is a dynamic stochastic general equilibrium model with hand-to-mouth workers, heterogeneous entrepreneurs, financial frictions, and liquidity shocks. A unit measure of ex ante identical entrepreneurs with Epstein-Zin preferences produce, consume, and trade financial assets. In every period, entrepreneurs face a common productivity shock. In addition, in every period, only a fraction of entrepreneurs find new investment projects. These entrepreneurs face a financial friction. They can pledge only a fraction of the returns to their newly produced capital, i.e., sell only a fraction of a new project as equity. Entrepreneurs who cannot find a new investment project may buy claims—we call these claims *equity*—to returns on other entrepreneurs' projects to replace their depreciated capital. Markets are incomplete, and equity is the only financial asset that is explicitly traded in the economy. Finally, there is a liquidity shock, which is common to all noninvesting entrepreneurs. Due to the liquidity shock, the noninvesting entrepreneurs may not be able to buy equity to replace their depreciated capital, and, they may even be forced to liquidate some of their equities.

Our model allows for aggregation because entrepreneurs have a homothetic utility function and their budget constraints are linear. We show that the dynamics of aggregate quantities and prices in the model can be described by a system of first order difference equations. We also derive asset pricing formulas for the equity return and the (implicit) risk-free return. The risk-free return is calculated using the stochastic discount factor of the noninvesting entrepreneurs.

To assess the quantitative significance of liquidity and productivity shocks, we solve the model numerically. We calibrate the model to match several moments of the U.S. macroeconomic variables and asset returns. The model matches the relative volatility of aggregate investment growth, overcoming a shortcoming of previous papers with convex adjustment costs or factor immobility (see, e.g., Jermann 1998; Boldrin et al. 2001; Guvenen 2009; and Campanale et al. 2010). Moreover, because of the random arrival of investment opportunities, the model produces investment spikes at the firm level observed in the data (see, e.g., Favilukis and Lin 2013 and Khan and Thomas 2013 for an alternative mechanism that generates investment spikes at the firm level). The model also matches the volatility of aggregate consumption growth. This is important, since for a production based model to offer a plausible mechanism to explain asset prices, it should be consistent with the consumption volatility observed in the data.[1]

The calibrated model generates a quarterly equity premium of 1.46%, an average quar-

[1]In our model, entrepreneurs bear all the asset market risk, as workers do not hold equities. Hence, it is critical that the model does not overstate the magnitude of consumption risk that entrepreneurs face. The literature finds that the volatility of consumption growth of stock market participants is 1.5 to 5 times larger than that of non-participants (see Mankiw and Zeldes 1991, Attanasio et al. 2002, and Ait-Sahalia et al. 2004). In our model, this ratio is about 1.5, which is consistent with the data.

terly risk-free of 0.25% and a quarterly risk-free rate volatility of 0.72%. The empirical counterparts are 1.52%., 0.25% and 0.72%, respectively. Two forces contribute to the high equity premium in our model. First, holding a risk-free asset would allow entrepreneurs to smooth consumption over time, which would increase the demand for the risk-free asset and decrease the risk-free return. Second, holding equity exposes the entrepreneur to a liquidity shock in the next period. A negative liquidity shock next period forces the entrepreneur to liquidate assets, which leads to a sub-optimal consumption-saving choice. This decreases the demand for equity, increasing its equilibrium return. We further show that, in our model, the equity premium is strongly countercyclical, and the risk-free rate is procyclical, consistent with the data.

In our model, liquidity shocks generate strongly procyclical asset prices. The reason is as follows: A negative liquidity shock reduces the demand for equity by the noninvesting entrepreneurs, decreasing its price. Since a negative liquidity shock dries up the funds available to investing entrepreneurs, it also reduces aggregate investment, creating to a positive correlation between asset prices and investment. This is an important step forward relative to the literature based on the seminal contributions of Kiyotaki and Moore (2005) and Kiyotaki and Moore (2019).

In that literature, financial/liquidity shocks affect asset supply. In particular, financial/liquidity shocks determine how easily an entrepreneur can obtain funds for investment. However, these models have often been criticized, since they generate counterfactual asset-price dynamics (see Shi 2015 and Bigio and Schneider 2017). In particular, an adverse financial/liquidity shock in these models implies a stock market boom. The logic is clear: a negative financial/liquidity shock reduces investment, reducing the supply of new capital. This makes capital more valuable, which increases its price.[2] This prediction is clearly not in line with data, where the correlation between investment and asset prices is positive. As Shi (2015) shows, this feature is robust across a variety of model specifications. Our paper

[2]The logic is similar in models with investment-specific technology (IST) shocks. See, for example, Christiano and Fisher (2003) and Papanikolaou (2011).

proposes a rethink of liquidity shocks as shocks that affect asset demand rather than asset supply, resulting in a co-movement between the stock market and aggregate investment consistent with the data.

The calibrated model generates predictable excess equity returns. Empirical equity return predictability by financial variables and macroeconomic variables is well documented in the literature (see, e.g., Campbell and Shiller 1988; Fama and French 1988; Lettau and Ludvigson 2001; Cooper and Priestley 2009, among many others). However, standard business cycle models do not produce economically significant predictability in excess returns (see Kaltenbrunner and Lochstoer 2010). Our model produces predictable excess returns because the expected excess equity returns vary systematically over the business cycle. This is due to a systematic variation in both expected consumption growth and the volatility of consumption growth of investing and noninvesting entrepreneurs.

Various approaches other than introducing financial frictions and liquidity shocks have been taken to explain the observed asset price dynamics using production economy models (see, e.g., Jermann 1998; Tallarini 2000; Boldrin et al. 2001; Kuehn 2007; Guvenen 2009; Kaltenbrunner and Lochstoer 2010; Campanale et al. 2010; Papanikolaou 2011; Gourio 2012; Croce 2014; Jaccard 2014; Chen 2017; Bai and Zhang 2021). Our contribution to this literature is twofold. First, we employ an alternative notion of liquidity shocks, where liquidity shocks affect asset demand rather than asset supply. We show that this alternative notion of liquidity shocks leads to more realistic dynamics in the model and addresses a major shortcoming of the extensive literature that builds on Kiyotaki and Moore (2019). Second, using a calibrated model, we show that liquidity constraints and liquidity shocks can lead to an equity premium comparable to the U.S. data. At the same time, the model matches the level and volatility of the risk-free rate and key aspects of the behavior of macroeconomic quantities. In addition, we show the roles that liquidity constraints and liquidity shocks play in generating predictability in asset returns.

2 The Model

Time is discrete and infinite. There are two types of agents: a unit measure of ex ante identical entrepreneurs who consume, produce, and hold financial assets, but do not work, and a unit measure of identical hand-to-mouth workers who work and consume, but do not hold assets. There are two types of goods and two production technologies: a consumption good and a capital good, and a technology to produce the consumption good and a technology to produce the capital good. One type of financial asset traded: claims to returns on capital. Each period is divided into two subperiods. In the first subperiod, the consumption good is produced. In the second subperiod, the capital good is produced, and consumption and asset trading take place.

Next, we describe the details of the two production technologies, the asset trading structure, and the financial frictions that entrepreneurs face. We then present the optimization problems of workers and entrepreneurs, and define competitive equilibrium.

2.1 Production Technologies

In the first subperiod of each time period t, the production of the consumption good takes place. All entrepreneurs have access to the consumption good production technology (workers do not have this access). Entrepreneurs face a productivity shock, which is common to all of them. The productivity shock consists of two components. The first component, X_t, is deterministic and grows at rate g, i.e., $X_t = (1+g)X_{t-1}$. We normalize X_0 to 1 and hence $X_t = (1+g)^t$. The second component, A_t, is stochastic, and follows an AR(1) process in logs:

$$\log A_{t+1} = \rho_A \log A_t + \epsilon_{A,t}.$$

Entrepreneur j enters period t with capital k_t^j, hires labor l_t^j, and produces the consumption good y_t^j using the following technology (α is the capital share parameter):

$$y_t^j = A_t \left(k_t^j\right)^{\alpha} \left(X_t l_t^j\right)^{1-\alpha}.$$

Capital depreciates at rate δ during the production of the consumption good, i.e., entrepreneur j enters the second subperiod with capital holdings $(1-\delta)k_t^j$.

In the second subperiod, only a fraction π of entrepreneurs have the opportunity to start new projects. This "investment opportunity" is modeled as the entrepreneurs' ability to access the capital good production technology. This technology enables the entrepreneurs to produce new capital one-to-one from the consumption good, which is standard in the real business cycle literature.[3] In practice, apart from investment in the depreciated capital, firms adjust their capital stock by starting new projects. However, new projects are not always available. This technological constraint implies that an individual entrepreneur's investment also responds to the entrepreneurs' specific real opportunities rather than only to the aggregate productivity shocks. This assumption is further motivated by the empirical observation that only a small fraction of firms invest a lot in a given year.

The arrival of the opportunity to access the capital good production technology is assumed to be i.i.d. over time and over entrepreneurs. The i.i.d. assumption is made for simplicity and is common in the literature. Entrepreneurs with access to the capital good production technology are called *investing entrepreneurs* and entrepreneurs without this access are called *noninvesting entrepreneurs*.

[3] Although the arrival of the opportunity to access the capital good production technology could be thought of as a version of an investment-specific technology (IST) shock, we should emphasize that this shock is quite different from the IST shock present in, for example, Papanikolaou (2011), and in New Keynesian models, see, for example, Smets and Wouters (2007) and Christiano et al. (2014). In these models, the IST shock is aggregate and affects the economy's production possibility frontier, whereas in our model, the investment shock, i.e., the arrival of an investment opportunity, is idiosyncratic and does not affect the economy's production possibility frontier.

2.2 Asset Market Structure

In the second subperiod, consumption, capital good production, and asset trading take place. One type of financial asset is traded: claims to capital returns (referred to simply as assets or equities).

Before we discuss the asset trading structure, it should be emphasized that the return per unit of capital is equal across entrepreneurs, independent of their capital holdings and independent of their opportunity to access the capital good production technology. Therefore, entrepreneurs are indifferent as to whose equity they hold. To understand this claim, consider entrepreneur j with capital k_t^j. In the first subperiod, he hires labor on a competitive labor market at wage w_t to maximize his profit, which can be written as

$$\text{profit}(k_t^j; A_t, w_t) := A_t \left(k_t^j\right)^\alpha \left(X_t l_t^j\right)^{1-\alpha} - w_t l_t^j.$$

The optimal behavior of entrepreneur j implies that he hires labor $l_t^j = \left(\frac{(1-\alpha)A_t X_t^{1-\alpha}}{w_t}\right)^{\frac{1}{\alpha}} k_t^j$. This amount of labor equalizes the wage rate with the marginal product of labor (MPL):

$$w_t = MPL_t = (1-\alpha)A_t X_t^{1-\alpha} \left(k_t^j\right)^\alpha (l_t^j)^{-\alpha}.$$

Therefore, $\text{profit}(k_t^j; A_t, w_t) = \alpha A_t X_t^{1-\alpha} \left((1-\alpha)A_t X_t^{1-\alpha}/w_t\right)^{\frac{1-\alpha}{\alpha}} \cdot k_t^j = r_t k_t^j$, where $r_t = \alpha A_t X_t^{1-\alpha} \left(\frac{(1-\alpha)A_t X_t^{1-\alpha}}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$ denotes the return per unit of capital, i.e., the marginal return on capital. Since all entrepreneurs face the same productivity shock, and hire labor at the same wage, which is determined by aggregate labor market clearing, the return on capital, r_t, is the same for all entrepreneurs.

To explain the trading structure in the economy, we first describe the capital and asset holdings of the entrepreneurs. Entrepreneurs can hold physical capital and equity to other entrepreneurs' capital returns. We define the individual state of entrepreneur j by $\{k_t^j, e_t^j, s_t^j\}$, where k_t^j is the physical capital held by the entrepreneur, e_t^j is the equity to other entrepreneurs' capital, and s_t^j is equity to entrepreneur j's capital sold to other entrepreneurs.

Physical capital k_t^j is used by entrepreneur j in the production of the consumption good and depreciates at rate δ. Physical capital is not traded in the economy. Equity e_t^j entitles entrepreneur j to the stream of returns of e_t^j units of other entrepreneurs' capital. Since the underlying capital depreciates at rate δ, one can think of e_t^j as depreciating at rate δ. Finally, s_t^j denotes claims to capital returns sold by entrepreneur j, and one can think of these claims as depreciating at rate δ as well. Therefore, an entrepreneur with an individual state $\{k_t^j, e_t^j, s_t^j\}$ is entitled to returns from $k_t^j - s_t^j + e_t^j$ units of capital. As a result, the budget constraint of entrepreneur j can be written as:

$$c_t^j + i_t^j + q_t^j(k_{t+1}^j - s_{t+1}^j + e_{t+1}^j) \ \leq \ (k_t^j - s_t^j + e_t^j)(r_t + (1-\delta)q_t) + q_t i_t^j,$$

where c_t^j is consumption, q_t is the equilibrium price of equity and i_t^j is investment, which is 0 for noninvesting entrepreneurs.

Each investing entrepreneur faces a financial constraint as in Kiyotaki and Moore (2005). Specifically, an investing entrepreneur who produces i_t^j units of new capital can sell at most a fraction ϕ of returns from i_t^j. This means that an entrepreneur is able to finance only a fraction of his investment externally. This assumption is motivated by the empirical observation that firms do not fully finance their investments externally. The assumption could be microfounded by a moral hazard problem in which an investing entrepreneur has the ability to walk away with fraction $(1-\phi)$ of the capital he promised to deliver to a buyer.

These assumptions imply that the total amount of equity sold up until period t by entrepreneur j (denoted by s_{t+1}^j) can be at most the sum of a fraction ϕ of period t investment i_t^j and the depreciated period t capital holdings $(1-\delta)k_t^j$, i.e.,

$$s_{t+1}^j \leq \phi i_t^j + (1-\delta)k_t^j. \tag{1}$$

To understand this constraint, define $k^j_{t+1} = (1-\delta)k^j_t + i^j_t$ and rewrite inequality (1) as

$$k^j_{t+1} - s^j_{t+1} \geq (1-\phi)i^j_t. \tag{2}$$

The left-hand side of inequality (2) captures the net amount of claims to entrepreneur j's own capital returns that he must carry into period $t+1$. Since he can sell at most ϕi^j_t of his "new" equity, he must keep at least $(1-\phi_t)i^j_t$ of the newly produced capital unsold, which is captured in the right-hand side of inequality (2).

Each noninvesting entrepreneur faces a liquidity shock, denoted by τ_t, which follows an AR(1) process in logs:

$$\log \tau_{t+1} = \rho_\tau \log \tau_t + \epsilon_{\tau,t}.$$

The liquidity shock is common to all noninvesting entrepreneurs and forces noninvesting entrepreneur j to liquidate a fraction $(1-\tau_t)$ of his period t networth, which can be written as $(k^j_t - s^j_t + e^j_t)(r_t + (1-\delta)q_t)$. As a result, entrepreneur j faces the following constraint, which we refer to as the liquidity constraint:

$$c^j_t \geq (1-\tau_t)(k^j_t - s^j_t + e^j_t)(r_t + (1-\delta)q_t).$$

One can rewrite this constraint by using the noninvesting entrepreneur's budget constraint as:

$$q_t(k^j_{t+1} - s^j_{t+1} + e^j_{t+1}) \leq \tau_t(k^j_t - s^j_t + e^j_t)(r_t + (1-\delta)q_t). \tag{3}$$

Equation (3) provides an alternative interpretation of the liquidity constraint by making it clear that the liquidity shock τ_t limits the amount of an entrepreneur's asset holdings. Since investing entrepreneurs can invest in their own firms by producing new capital, we assume that they are not subject to the liquidity shock.

The liquidity shock in our model causes a distortion in the entrepreneurs' consumption/investment decisions. In particular, a negative liquidity shock forces an entrepreneur to liquidate a larger fraction of his equity holdings than optimal. Although a theory that endogenizes the time variation in τ_t would be of interest, this paper does not attempt to incorporate such a theory in the model. While research on endogenous time-variation in illiquidity is sparse, Eisfeldt (2004) presents a model in which real-sector liquidity fluctuates with productivity, Brunnermeier and Pedersen (2004b) show how predatory trading can lead to illiquidity when liquidity is most needed, and Brunnermeier and Pedersen (2004a) show how market liquidity varies with dealers' "funding liquidity."

Several observations are important. First, liquidity shocks imply time variation in how constrained entrepreneurs' consumption is. As a result, the effects of liquidity shocks will be similar to effects of shocks to preferences. However, if a negative liquidity shock hits, the demand for equity decreases, while the demand for alternative assets (the risk-free asset) increases. A negative preference shock decreases the demand for both the risk-free asset and equity.

Second, τ_t cannot be interpreted as a transaction cost (see Ajello 2016 for a related model with transaction costs). This is because the resources are not lost or transferred to other market participants. Third, we interpret τ_t as a liquidity shock. However, this liquidity shock differs from the one that is typically used in the literature; see, for instance Kiyotaki and Moore (2019) and Del Negro et al. (2017). In these papers, a liquidity shock changes the amount of equity that the investing entrepreneurs can sell in a given period, leading to a counterfactual negative correlation between equity prices and investment. In our model, in contrast, the liquidity shock changes the amount of equity that noninvesting can entrepreneurs buy in a given period.

2.3 Agents' Optimization Problems

There is no heterogeneity among the workers. The preferences of the representative worker are of the Greenwood–Hercowitz–Huffman form:

$$\sum_{t=0}^{\infty} \beta^t U\left(c'_t - \frac{\omega}{1+\eta} X_t (l'_t)^{1+\eta}\right),$$

where c'_t is the consumption of the representative worker in period t, l'_t is the labor provided by the representative worker in period t, the function $U(.)$ is increasing and strictly concave, $\beta > 0, \omega > 0$ and $\eta > 0$. X_t is the labor-augmenting technological progress, defined above. Its inclusion in the utility function guarantees the existence of a balanced growth path.

For simplicity, workers are assumed not to participate in asset trading. Their maximization problem is thus static and can be written as

$$\max_{c'_t, l'_t} \quad U\left(c'_t - \frac{\omega}{1+\eta} X_t (l'_t)^{1+\eta}\right) \text{ s.t. } c'_t \leq w_t l'_t.$$

The preferences of the entrepreneurs are of the recursive Epstein-Zin form (dropping the j superscript for simplicity):

$$v_t = \left[(1-\beta) c_t^{\frac{1-\gamma}{\theta}} + \beta \left(\mathbb{E}_t \left[v_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}},$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ and ψ is the coefficient of the (constant) elasticity of intertemporal substitution and γ is the coefficient of the (constant) relative risk aversion.

Ex post, entrepreneurs will differ in their capital and asset holdings. The budget constraint of an entrepreneur with capital and asset holdings (k_t, e_t, s_t) can be written as

$$c_t + i_t + q_t(k_{t+1} - s_{t+1} + e_{t+1}) \leq r_t(k_t - s_t + e_t) + (1-\delta) q_t (k_t - s_t + e_t) + q_t i_t,$$

where r_t is the return on capital. The first term on the right-hand side is the return to which the entrepreneur is entitled. The second term is the market value of his depreciated unsold capital and asset holdings. The third term is the market value of equity to his newly installed capital at market price q_t. The left-hand side sums up his expenditures. He can consume $c_t \geq 0$, invest i_t with investment being generated one-to-one from the consumption good, and carry unsold equity to his own capital $k_{t+1} - s_{t+1}$ and outside equity e_{t+1} into period $t+1$. These are traded at market price q_t.

IO_t is a random variable in period t with $\mathrm{IO}_t = 0$ if the entrepreneur does not have an investment opportunity in period t and $\mathrm{IO}_t = 1$ if he does have the opportunity. The maximization problem of an entrepreneur can then be written as:

$$\max_{\{(c_t, i_t, k_{t+1}, s_{t+1}, e_{t+1}) \geq 0\}_{t=0}^{\infty}} \quad \left[(1-\beta) c_t^{\frac{1-\gamma}{\theta}} + \beta \left(\mathbb{E}_t \left[v_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \qquad \text{s.t.}$$

$$\begin{aligned}
&\text{(IC)} && i_t = 0 \quad \text{if} \quad \mathrm{IO}_t = 0,\\
&\text{(BC)} && c_t + i_t + q_t[k_{t+1} - s_{t+1} + e_{t+1}] \leq [k_t - s_t + e_t][r_t + (1-\delta)q_t] + q_t i_t,\\
&\text{(FC1)} && k_{t+1} - s_{t+1} \geq (1-\phi) i_t,\\
&\text{(FC2)} && e_{t+1} \geq 0,\\
&\text{(LC)} && c_t \geq (1 - \mathrm{IO}_t) \cdot (1-\tau_t)[k_t - s_t + e_t][r_t + (1-\delta)q_t].
\end{aligned}$$

In this problem, expectations are taken over the stochastic processes for τ_t and A_t, equilibrium processes for prices (taken as given and correctly forecasted by the entrepreneur), and the arrival of the investment opportunity IO_t.

The liquidity constraint (LC) is not binding for the investing entrepreneurs. In addition, the return on the unsold capital $k_{t+1} - s_{t+1}$ and the return from claims to other entrepreneurs' capital e_{t+1} are the same, given the state of the economy. Moreover, trades in these assets in period $t+1$ are not subject to any restrictions that differ by asset type. Therefore, inside equity $k_{t+1} - s_{t+1}$ and outside equity e_{t+1} are perfect substitutes, and (FC1) binding is equivalent to the no-short-sales (FC2) binding, and they can be summed up without loss of generality. As a result, the maximization problem can be simplified by defining net asset holdings $n_t \equiv k_t - s_t + e_t$, the gross return on equity $R_t \equiv r_t + (1-\delta)q_t$ and writing the entrepreneur's problem as

$$\max_{\{(c_t, i_t, n_{t+1}) \geq 0\}_{t=0}^{\infty}} \quad \left[(1-\beta)c_t^{\frac{1-\gamma}{\theta}} + \beta \left(\mathbb{E}_t\left[v_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \qquad \text{s.t.}$$

$$\text{(IC)} \qquad i_t = 0 \quad \text{if} \quad \mathrm{IO}_t = 0,$$

$$\text{(BC)} \qquad c_t + i_t + q_t n_{t+1} \leq n_t R_t + q_t i_t,$$

$$\text{(FC)} \qquad n_{t+1} \geq (1-\phi) i_t,$$

$$\text{(LC)} \qquad c_t \geq (1 - \mathrm{IO}_t) \cdot (1-\tau_t) n_t R_t.$$

2.4 Equilibrium

A competitive equilibrium is quantities for entrepreneurs $[\{c_t^j, i_t^j, n_{t+1}^j\}_{t=0}^{\infty}]_{j\in[0,1]}$, quantities for the representative worker $\{c_t', l_t'\}_{t=0}^{\infty}$, and prices $\{q_t, r_t, w_t\}_{t=0}^{\infty}$, such that quantities solve workers' and entrepreneurs' problems given prices, input prices w_t, r_t are determined competitively, and markets clear.

3 Solution of the Model

This section first characterizes the solution to the entrepreneurs' optimization problem and then studies the equilibrium aggregate dynamics. Finally, it discusses the model's asset pricing implications. All proofs are provided in Appendix A.

3.1 Solution to the Entrepreneurs' Problem

In our model, entrepreneurs are heterogeneous because their wealth depends on their individual sequences of the idiosyncratic investment opportunity shocks. However, one can solve for aggregate dynamics without having to keep track of the whole wealth distribution. This is because the homotheticity of the Epstein-Zin utility function and the linearity of the budget constraint imply linear decision rules. We omit the proof of this well-known result summarized in Lemma 1.[4]

In this section, we use superscript i to denote entrepreneurs with an investment opportunity in period t and superscript s to denote entrepreneurs with no investment opportunity in period t; rather than denoting an individual entrepreneur.

Lemma 1. *Suppose that an entrepreneur has asset holdings n_t at the beginning of period t. The policy functions describing the entrepreneur's optimal decisions conditional on whether he is an investing or noninvesting entrepreneur are as follows.*

$$c_t^i = \zeta_t^i n_t R_t, \tag{4}$$

$$q_t^R n_{t+1}^i = (1 - \zeta_t^i) n_t R_t, \tag{5}$$

$$c_t^s = \zeta_t^s n_t R_t, \tag{6}$$

$$q_t n_{t+1}^s = (1 - \zeta_t^s) n_t R_t, \tag{7}$$

where ζ_t^i and ζ_t^s are the period t consumption-to-wealth ratios of the investing and noninvesting entrepreneurs, respectively, and $q_t^R \equiv (1 - \phi q_t)/(1 - \phi)$.

Equation (5) follows from the fact that an investing entrepreneurs' budget constraint can be expressed as $c_t^i + q_t^R n_{t+1}^i \leq n_t[r_t + (1 - \delta) q_t]$, where q_t^R is the replacement cost of capital which satisfies $q_t^R \leq 1 \leq q_t$. The difference between q_t^R and q_t means that investing and noninvesting entrepreneurs face different effective prices of the next period's assets n_{t+1}.

Lemma 1 implies that all entrepreneurs of the same type $j \in \{i, s\}$ have the same consumption-to-wealth ratio $\zeta_t^j = c_t^j/(n_t R_t)$ independent of individual wealth, which is defined as $n_t R_t$. This property implies that the model allows for exact aggregation. For noninvesting entrepreneurs, $\zeta_t^s \leq (1 - \tau_t)$, with equality if the liquidity constraint (LC) binds. The lemma also implies that the associated value functions V_t^i and V_t^s are linear in wealth and that ξ_t^i and ξ_t^s exist such that $V_t^i = \xi_t^i \cdot n_t^i R_t$ and $V_t^s = \xi_t^s \cdot n_t^s R_t$. The next proposition

[4]This result was first derived by Samuelson (1969). For an extension to an environment with entrepreneurial investment risk and Epstein-Zin utility similar to ours, see Angeletos (2007).

derives the equations that determine the dynamic behavior of consumption-to-wealth ratios ζ_t^i and ζ_t^s.

Proposition 1. *The entrepreneurs' consumption-to-wealth ratios ζ_t^i and ζ_t^s are described recursively by the following system of equations:*

$$(1-\beta)\left(\frac{\zeta_t^i}{1-\zeta_t^i}\right)^{\frac{1-\gamma-\theta}{\theta}}(q_t^R)^{\frac{1-\gamma}{\theta}} = \beta J_t^{\frac{1}{\theta}}, \tag{8}$$

$$(1-\beta)\left(\frac{\zeta_t^s}{1-\zeta_t^s}\right)^{\frac{1-\gamma}{\theta}}\frac{M_t-\zeta_t^s}{\zeta_t^s}(q_t)^{\frac{1-\gamma}{\theta}} = \beta J_t^{\frac{1}{\theta}}, \tag{9}$$

where we define $M_t \equiv 1+\frac{\mu_t^s\tau_t}{1-\beta}\left[\frac{\zeta_t^s}{\xi_t^s}\right]^{1-\frac{1-\gamma}{\theta}}$ with μ_t^s being the Lagrange multiplier on the liquidity constraint (LC) *in period t and*

$$J_t \equiv \pi\mathbb{E}_t\left[\left((1-\beta)^{\frac{\theta}{1-\gamma}}(\zeta_{t+1}^i)^{\frac{1-\gamma-\theta}{1-\gamma}}R_{t+1}\right)^{1-\gamma}\right]+(1-\pi)\mathbb{E}\left[\left((1-\beta)^{\frac{\theta}{1-\gamma}}(\zeta_{t+1}^s)^{\frac{1-\gamma-\theta}{1-\gamma}}(M_{t+1})^{\frac{\theta}{1-\gamma}}R_{t+1}\right)^{1-\gamma}\right].$$

Equations (8) and (9) describe the dynamic behavior of the optimal policies ζ_t^s and ζ_t^i as a system of first order difference equations.

3.2 Aggregation and Equilibrium Asset Price Dynamics

Given that workers do not own capital or hold assets, their utility maximization problems are static, and for space considerations, we next provide only the aggregate solution to workers' problems.

In what follows, for a given variable D_t, the variable $\tilde{D}_t$ denotes its detrended counterpart defined as $\frac{D_t}{X_t}$.

Lemma 2. *The equilibrium aggregate labor, denoted by L_t, the equilibrium wage rate, denoted by w_t, and the equilibrium aggregate consumption by workers, denoted by C_t', are*

$$L_t = \left(\frac{(1-\alpha)A_t}{\omega}\right)^{\frac{1}{\alpha+\eta}}\tilde{N}_t^{\frac{\alpha}{\alpha+\eta}}, w_t = \omega^{\frac{\alpha}{\eta+\alpha}}((1-\alpha)A_t)^{\frac{\eta}{\eta+\alpha}}X_t\tilde{N}_t^{\frac{\eta\alpha}{\eta+\alpha}}, C_t' = (1-\alpha)Y_t,$$

where N_t denotes aggregate capital stock/equity holdings and Y_t denotes aggregate output.

This lemma shows that the aggregate labor is a function of workers' utility parameters, production function parameters, $\tilde{N}_t$ and A_t. Specifically, aggregate labor in period t does not depend on the liquidity shock τ_t in period t. Therefore, in period t, aggregate output $Y_t = A_t N_t^{\alpha}(X_t L_t)^{1-\alpha}$ is not a function of τ_t, and detrended output $\tilde{Y}_t$ is a function of detrended aggregate capital $\tilde{N}_t$ and the TFP shock A_t: $\tilde{Y}_t = Y(\tilde{N}_t, A_t)$. Similar reasoning holds for the return to capital $r_t = MPK_t = \frac{\partial Y_t}{\partial N_t}$, i.e., $r_t = r(\tilde{N}_t, A_t)$. Specifically:

$$\tilde{Y}_t = A_t \tilde{N}_t^{\alpha} \left(\left(\frac{A_t(1-\alpha)}{\omega} \right)^{\frac{1}{\alpha+\eta}} \tilde{N}_t^{\frac{\alpha}{\alpha+\eta}} \right)^{1-\alpha} \tag{10}$$

$$r_t = A_t \alpha \tilde{N}_t^{\alpha-1} \left(\left(\frac{A_t(1-\alpha)}{\omega} \right)^{\frac{1}{\alpha+\eta}} \tilde{N}_t^{\frac{\alpha}{\alpha+\eta}} \right)^{1-\alpha}. \tag{11}$$

Corollary 1 in the Appendix describes the joint dynamics of detrended output and labor, and output and workers' consumption.

Characterizing Aggregate Dynamics. The previous subsection derives optimal entrepreneur policies. To solve for equilibrium, they need to be aggregated over all entrepreneurs and combined with market clearing conditions. Given the fact that the arrival of the investment opportunity is i.i.d., entrepreneurs with an investment opportunity hold a fraction π of aggregate assets in the economy at the beginning of period t. Investors without an investment opportunity hold a fraction $1-\pi$ of aggregate assets at the beginning of period t. The evolution of aggregate asset holdings and aggregate entrepreneur consumption is characterized by the following lemma.

Lemma 3. *The dynamics of detrended aggregate asset holdings and aggregate entrepreneur consumption C_t are characterized by:*

$$(1+g)\tilde{N}_{t+1} = (1-\delta)\tilde{N}_t + \alpha \tilde{Y}_t - [\zeta_t^i \pi + \zeta_t^s (1-\pi)] R_t \tilde{N}_t, \tag{12}$$

$$C_t = [\zeta_t^i \pi + \zeta_t^s (1-\pi)] R_t N_t.$$

Equation (12) is a rewrite of the goods market clearing condition, and hence guarantees that the goods market clears. Equation (10) defines $\tilde{Y}_t$ as a function of $(\tilde{N}_t, A_t)$. Equation

(12) then implies that the $t+1$ period's detrended aggregate assets $\tilde{N}_{t+1}$ are a time-invariant function of the period t aggregate states $(\tilde{N}_t, A_t, \tau_t)$.

Equilibrium Price of Equity. The equilibrium price of equity is determined by a market clearing condition, which equates the demand for equity by noninvesting entrepreneurs with the supply of equity by investing entrepreneurs. The properties of the equilibrium price of equity q_t are summarized by the following proposition.

Proposition 2. *The equilibrium price of equity is*

$$q_t = \max(1, q_t^*), \tag{13}$$

where q_t^* *is the solution to a quadratic equation:* $a_2 q_t^2 + a_1 q_t + a_0 = 0$, *where* $a_0 = -(1-\zeta_t^s)(1-\pi)r_t$, $a_1 = (1-\delta)\left[1-(1-\zeta_t^s)(1-\pi)\right] + \phi r_t \left[(1-\zeta_t^i)\pi + (1-\zeta_t^s)(1-\pi)\right]$, *and* $a_2 = (1-\delta)\phi\left[(1-\zeta_t^i)\pi + (1-\zeta_t^s)(1-\pi) - 1\right]$. *The relevant root is* $q_t^* = \frac{-a_1+\sqrt{a_1^2-4a_0a_2}}{2a_2}$.

The proposition implies that q_t is a time invariant function of the aggregate states, i.e., $q_t = q(\tilde{N}_t, A_t, \tau_t)$. $q_t = 1$ iff the financial constraint (FC) is slack. Otherwise $q_t > 1$.

Return on Equity. To compare the model with the data in the next section, the return on equity in the model is defined as

$$r_t^e = [r_t + (1-\delta)q_t]/q_{t-1} - 1. \tag{14}$$

Implicit Risk-Free Rate. We wrap up this section by discussing how we compute the risk-free rate. The risk-free asset is not traded in our model. We use the shadow risk-free rate of noninvesting entrepreneurs as a measure of the risk-free rate. This approach is similar to that taken in Gomes et al. (2003). One could rationalize this choice by assuming that (investing) entrepreneurs are not allowed to issue the risk-free asset. In equilibrium, the risk-free asset would not be traded and the risk-free rate would be determined by the shadow risk-free rate of the noninvesting entrepreneurs. This is because investing entrepreneurs find investing and selling equity more profitable than buying the risk-free asset. The (shadow) risk-free return is defined as: $r_t^f = 1/\mathbb{E}_t\left[\frac{v'(c_{t+1}^s)}{v'(c_t^s)}\right]$. The following Proposition characterizes the implicit risk free rate.

Proposition 3. *Suppose that the risk-free asset cannot be used for equity purchases by noninvesting entrepreneurs. Then the implicit risk free rate is*

$$r_t^f = \frac{1}{\beta}\left[\frac{\zeta_t^s q_t}{(1-\zeta_t^s)}\right]^{\frac{1-\gamma}{\theta}-1} \cdot \frac{\left(\mathbb{E}_t[(\xi_{t+1}R_{t+1})^{1-\gamma}]\right)^{-\frac{1-\theta}{\theta}}}{\mathbb{E}_t\left[(R_{t+1}\xi_{t+1})^{-\gamma}\left(\frac{\xi_{t+1}}{\zeta_{t+1}}\right)^{1-\frac{1-\gamma}{\theta}}\right]}.$$

4 Numerical Model Solution and Model Calibration

This section discusses the calibration procedure and the numerical approach that we use to solve the model.

4.1 Model Calibration

One period in the model corresponds to one quarter. In the benchmark calibration, the share of capital in output production is $\alpha = 0.4$, in line with the decline in labor share which occurred over the last several decades. The quarterly depreciation rate is $\delta = 2.1\%$ implying that the annual investment to capital ratio is 10% on the balanced growth path of the model.[5] This is approximately the value in the data for the period from 1964 to 2013. For the workers' utility function parameters, the inverse labor supply elasticity parameter is $\eta = 2$. The scaling parameter of the workers' utility function is $\omega = 23.6$, so that the labor supply on the balanced growth path is $l_s = 1/3$ (ω is only a scaling parameter, and none of the statistics reported are affected by its value). For the entrepreneurs' utility function parameters, the quarterly discount factor is $\beta = 0.995$.[6] The intertemporal elasticity

[5] On the balanced growth path of our model, the quarterly investment-to-capital ratio is $I^{SS}/N^{SS} = \delta + g$, where g is the growth rate of TFP, which is set to 0.4% as discussed below. To see this argument, note that at any point in time, the law of motion for capital is $N_{t+1} = (1-\delta)N_t + I_t$. The detrended aggregate capital stock thus satisfies $\tilde{N}_{t+1}(1+g) = (1-\delta)\tilde{N}_t + \tilde{I}_t$ and at the balanced growth path $\tilde{N}^{SS}(\delta+g) = \tilde{I}^{SS}$.

[6] With this parameterization, the balanced-growth-path quarterly capital-to-output ratio equals approximately 10, which is close to the data.

of substitution parameter is $\psi = 0.4$, close to the empirical estimate for stockholders in Vissing-Jorgensen (2002), and the risk aversion parameter is $\gamma = 2$. This implies that, in the benchmark parameterization, entrepreneurs have a utility close to a time separable CRRA utility function.

The literature has documented several aspects of "infrequent" and "large" capital adjustment (see, e.g., Doms and Dunne 1998). Although this type of capital adjustment has typically been taken as evidence of the existence of fixed costs of investment, it can also be thought of as evidence of infrequent arrival of investment opportunities. Therefore, π is calibrated by matching it to the percentage of firms with an investment spike in the data. We made this choice because, in our model, firms with an investment opportunity generally invest a lot relative to their size. Given that the definition of an investment spike in the literature is not unique, similar to Gourio and Kashyap (2007), we use two definitions: investment exceeding 20% and investment exceeding 35% of capital at the beginning of the period. The time series for investment is constructed as an increase in "Net property, plant and equipment," i.e., variable *ppent* in the COMPUSTAT database, $investment_t = ppent_t - ppent_{t-1}$. We then determine the percentage of firms whose investment at time t exceeds a given fraction of $ppent_{t-1}$. As in Gourio and Kashyap (2007), firms are weighted by the amount of capital available at the beginning of the period $ppent_{t-1}$. We find that in 1965-2013, on average, 4.3% (10.6%) percent of firms' investment exceeds 35% (20%) of their initial capital. We set the annual π to an intermediate level of 6% in the benchmark (i.e., quarterly $\pi = 1.5\%$).

We parameterize the variable ϕ using Flow-of-Funds data as follows. In our model, the variable ϕ represents the fraction of investment in period t that is financed externally. In the model, equity financing is the only external financing option that firms have, but in reality firms use equity, debt and loans to raise capital. Therefore, we bring our model to the data based on the total amount of outside financing. Specifically, we construct the time series of ϕ for the nonfinancial corporate sector using Flow of Funds data (for precise definitions of these variables see Appendix B). We define

$$\phi_t = \frac{(\text{Debt securities} + \text{Loans} + \text{Corporate equities})_t}{(\text{Fixed investment})_t}.$$

We calculate that the average ϕ over the period from 1964 to 2013 is 26.6%.

The Solow residual (total factor productivity or TFP) in our model consists of two components, a deterministic growth component X_t and a stochastic component A_t. Jointly, the total factor productivity equals $A_t(X_t)^{1-\alpha}$ since output is defined as $Y_t = A_t\,(K_t)^{\alpha}\,(X_t L_t)^{1-\alpha}$. The corresponding time series for productivity is constructed using the time series of output, capital, and labor with the assumption of the Cobb-Douglas production function above with the capital output share of $\alpha = 0.4$. We find that the average growth rate of TFP is 0.4% at a quarterly frequency and thus set $g = 0.004$. To calibrate the stochastic component, A_t, and the liquidity shock, τ_t, we assume that they follow the following processes:

$$\log A_{t+1} = \rho_A \log A_t + \varepsilon_{A,t}, \tag{15}$$

$$\log \tau_{t+1} = \log \mu_\tau + \rho_\tau (\log \tau_t - \log \mu_\tau) + \varepsilon_{\tau,t}, \tag{16}$$

where $\varepsilon_{A,t}$ and $\varepsilon_{\tau,t}$ are normally distributed random variables with standard deviations σ_{ε_A} and $\sigma_{\varepsilon_\tau}$, respectively, which are i.i.d. over time. The correlation coefficient between $\varepsilon_{A,t}$ and $\varepsilon_{\tau,t}$ is denoted by $\rho_{A,\tau}$. Mean A_t is normalized to 1 without loss of generality and ρ_A and $\varepsilon_{A,t}$ are estimated using equation (15) outside of the model. Similar to previous studies, we find that ρ_A is close to 0.95. Our estimate of σ_{ε_A} is approximately 0.006, implying that the productivity shock A_t varies approximately by 1.5% on a quarterly basis.

The remaining parameters $\rho_\tau, \mu_\tau, \sigma_{\varepsilon_\tau}$ and $\rho_{A,\tau}$ are then calibrated so that the simulated model generates moments consistent with the data.[7] In general, all endogeneous variables

[7] To compare model generated statistics with the data, we simulate the model starting from the balanced growth path for 100 years (400 periods) and discard the first 50 years (200 periods), so as to eliminate the effect of initial conditions. This way, the model-generated data has the same length as the true data. We then repeat this procedure 10,000 times and report the means and standard deviations over the 10,000 repetitions.

depend on all parameters, but we find that $\sigma_{\varepsilon_\tau}$ is well identified by the volatility of aggregate investment growth (relative to output growth), $\rho_{A,\tau}$ by the volatility of aggregate consumption growth (relative to output growth), ρ_τ by risk-free-rate volatility, and μ_τ by the average (implicit) risk-free rate. Therefore, we use these four endogeneous variables as targets in our calibration procedure. With these targets, we ensure that the volatilities of the main macroeconomic variables are in line with the data. In addition, the basic time series properties of the risk-free rate are also consistent with the data. This is an important starting point for a sensible discussion regarding the model's performance with respect to other macroeconomic and financial statistics. Table 1 reports the benchmark parameters and Table 2 reports the calibration moments.

4.2 Numerical Model Solution

Propositions 1 and 2 and Lemma 3 imply that the aggregate dynamics of the economy are fully characterized by equations (8), (9), (12) and (13) (along with the definitions of $\tilde{Y}$ and r which are functions of the aggregate states $(\tilde{N}_t, A_t)$; see equations (10) and (11)). Equations (8) and (9) guarantee that the entrepreneurs' utility maximization problem is solved, equations (12) and (13) guarantee that goods and equity markets clear, and equations (10) and (11) imply that the workers' utility maximization problem is solved and that the labor and capital markets clear. Solving the system of equations (8), (9), (12) and (13) yields the equilibrium policies $\zeta^i(\tilde{N}_t, A_t, \tau_t)$ and $\zeta^s(\tilde{N}_t, A_t, \tau_t)$, the equilibrium equity price $q(\tilde{N}_t, A_t, \tau_t)$, and the law of motion for the evolution of the detrended aggregate capital stock $\tilde{N}_{t+1}(\tilde{N}_t, A_t, \tau_t)$. Given an initial state $(\tilde{N}_0, A_0, \tau_0)$, these are sufficient to determine the aggregate equilibrium dynamics. The dynamics of the remaining variables can easily be recovered. We next briefly discuss how we solve the system of equations (8), (9), (12) and (13).

1. We make an initial guess of the pattern of the occasionally binding liquidity constraint (LC) on the discretized state space, i.e., we make an initial guess about the time

invariant functions $\mu^s(\tilde{N}, A, \tau)$ and $M(\tilde{N}, A, \tau)$. One possible initial guess is that the liquidity constraint never binds: $\mu^s = 0$, $M = 1$ over the state space.

2. We use this guess as M_{t+1} in the expression denoted by J_t and solve the system of equations (8), (9). We solve this system by plugging in for $\tilde{N}$ and q from (12) and (13) into (8) and (9). When expressing $\tilde{N}$ and q, we implement the restriction that $\zeta^s \leq (1-\tau)$. The substitution yields a two-dimensional system of first-order difference equations in ζ^s, ζ^i and M, which can be solved iteratively.

3. We iterate on the functions $\zeta^s(\tilde{N}, A, \tau), \zeta^i(\tilde{N}, A, \tau)$ and $M(\tilde{N}, A, \tau)$ as follows. We start with an initial guess on $\zeta^s_{t+1}, \zeta^i_{t+1}$ and M_{t+1} assuming that $M_t = 0$ for all states. Given this assumption, we use (8) and (9) to solve for ζ^s_t and ζ^i_t.

   Next, we check whether the (LC) constraint $\zeta^s_t(\tilde{N}, A, \tau) \leq (1-\tau)$ holds. If it does, then $M_t = 0$. If not, then the (LC) binds, and we set $\zeta^s_t(\tilde{N}, A, \tau) = (1-\tau)$, and we use equation (9) to compute M_t.

   Once this has been done for all states $(\tilde{N}, A, \tau)$, we have $\zeta^s_t(\tilde{N}, A, \tau), \zeta^i_t(\tilde{N}, A, \tau)$ and $M_t(\tilde{N}, A, \tau)$ which we use as our new $t+1$ guesses. We iterate until ζ^s_{t+1} and ζ^i_{t+1} and ζ^s_t and ζ^i_t are close enough. Once they are, M_{t+1} and M_t are also close enough.

4. Next, using the calculated (time-invariant) functions $\zeta^s(\tilde{N}, A, \tau)$ and $\zeta^i(\tilde{N}, A, \tau)$, we compute $\xi^i(\tilde{N}, A, \tau), \xi^s(\tilde{N}, A, \tau)$ using the formulas (18) and (19) in Appendix A. These are used to simplify our computation of the implicit risk-free rate.

5 Main Quantitative Results

This section studies the quantitative implications of the model. It shows that the model matches both macroeconomic quantities and asset prices well.

5.1 Macroeconomic Quantities

We start by studying the implications of the model for standard business cycle statistics. These results are reported in Table 3. The data column reports the U.S. statistics for 1964-2013. Details of the construction of the time series can be found in Appendix B.

Column (1) reports the statistics for the benchmark model. To explain the roles of the financial constraint, the liquidity constraint, and liquidity shocks in the model, Table 3 reports the results from four additional versions of the model. Column (2) reports the statistics for a version of the model with the financial constraint, a constant liquidity constraint, and productivity shocks. In this model, there are no liquidity shocks, i.e., τ_t is constant at its mean level of 0.984. Column (3) reports the statistics for a version of the model with liquidity and productivity shocks. In this model, there is no financial constraint. Column (4) reports the statistics for a version of the model with the financial constraint and productivity shocks. In this model, there is no liquidity constraint and no liquidity shocks. This model is very similar to the one presented in Kiyotaki and Moore (2005) in which a productivity shock, amplified by a financial constraint, is the only source of business cycle fluctuations. Column (5) reports the statistics for a version of the model without the financial and liquidity constraint (since there is no liquidity constraint, liquidity shocks are irrelevant). This model is closely related to the standard stochastic one sector growth model. Each of these alternatives highlights the role of a specific constraint or shock in generating our findings.

Consumption Volatility. Table 3 shows that the benchmark model matches the relative volatility of aggregate consumption growth. This is not surprising, given that this is a targeted moment. In the model, workers do not participate in asset markets and hence entrepreneurs bear all the asset market risk. Hence, for the model to be quantitatively plausible, it is critical that it also matches the magnitude of consumption risk that entrepreneurs face. In the benchmark model, the standard deviation of entrepreneurs' consumption growth is 1.23%, while the standard deviation of workers' consumption growth is 0.80%, implying that the ratio $\sigma_{\Delta C^e}/\sigma_{\Delta C^w}$ is about 1.5. In the data, consumption growth of stock market participants is significantly more volatile than that of non-participants (see Mankiw and Zeldes 1991, Attanasio et al. 2002, and Ait-Sahalia et al. 2004). This literature finds that, depending on the particular definitions of participation and consumption, the volatility of the consumption growth of stock market participants is 1.5 to 5 times larger than that of non-participants. Our benchmark model is thus in line with this empirical evidence.

A comparison of column (1) with columns (2) through (5) suggests that adding a liquidity shock to the model (compare columns (1) and (2)) dampens the volatility of aggregate consumption growth. The reason is that 'good' liquidity shocks, which typically accompany 'good' TFP shocks (recall that the two shocks are positively correlated) imply that noninvesting entrepreneurs are less constrained in their investment behavior. Therefore, investment increases more while entrepreneurs' consumption increases less or decreases in good times. As a result, aggregate consumption volatility is lower with liquidity shocks. On the other hand, adding the liquidity constraint (compare column (2) to column (4)) and adding the financial constraint (compare column (1) to column (3)) increases consumption volatility. This confirms that, in our model, financial and liquidity constraints function as amplifiers of productivity shocks, a common finding in the literature.

Investment Volatility. In column (1) of Table 3, one can see that the benchmark model matches the relative volatility of aggregate investment (which was a targeted moment). This is an improvement relative to earlier models with convex-adjustment costs (see, e.g., Jermann 1998; Guvenen 2009; and Campanale et al. 2010). To create a sizable equity premium, the adjustment costs in these models need to be quite severe. This implies a counterfactually low aggregate investment volatility, which has been one of the main criticisms of these models. Our model improves upon the adjustment cost models by matching the relative volatility of aggregate investment. Because of the random arrival of investment opportunities, our model also produces investment spikes at the firm level, which are present in the data.

In the absence of liquidity shocks, column (2), the aggregate investment growth volatility is only 0.42 times its value in the benchmark model. This finding suggests that adding a liquidity shock to the model increases the volatility of aggregate investment growth. The reason is that with 'good' ('bad') liquidity shocks, investment demand can increase (decrease) substantially, which increases the volatility of aggregate investment. In the absence of the financial constraint, column (3), aggregate investment growth is 1.36 times more volatile than in the benchmark model. In the absence of a liquidity constraint and liquidity shocks, column (4), aggregate investment growth is approximately 0.47 times as volatile as in the benchmark model. Comparing column (2) to column (4), however, suggests that adding the liquidity constraint dampens aggregate investment volatility, similar to the results of adding the financial constraint. These results are intuitive, as adding these constraints restricts either investment supply (the financial constraint) or investment demand (the liquidity constraint). In the absence of financial and liquidity constraints, column (5), aggregate investment growth is about 0.52 as volatile as in the benchmark model.

Output and Labor Volatility. Column (1) in Table 3 shows that the benchmark model explains 96% of the volatility of output growth. All alternative versions of the model yield a similar result, which suggests that productivity shocks are the main determinant of the volatility of output. In addition, the financial constraint, the liquidity constraint, and liquidity shocks do not play a significant role in output volatility. All versions of the model generate a lower volatility of aggregate labor supply growth relative to the data.

Persistence and Cyclicality. The benchmark model generates high persistence in major macroeconomic variables (these results are not reported in Table 3). However, it falls short of fully accounting for the persistence of growth rates observed in the data. Table 3 shows that all versions of the model generate high (unconditional) correlations between growth rates of output and growth rates of investment, consumption, and labor, consistent with the correlations observed in the data.

5.2 Asset Prices

We next study the implications of the model for asset prices. Table 4 reports asset price statistics for the benchmark model and the four versions of the model discussed in the previous subsection. The data column reports U.S. statistics for the period of 1964-2013. Details of the construction of the time series can be found in Appendix B.

Expected Asset Returns. Column (1) of Table 4 shows that the benchmark model matches the mean risk-free rate, which was a target in the calibration procedure. The model generates a sizable equity premium: a quarterly equity premium of 1.46%, compared to 1.52% in the data. This is an important result: The benchmark model can explain the equity premium puzzle while matching the (relative) volatility of aggregate consumption and investment.

The mechanism that generates a high equity premium in our model consists of two forces. First, holding a risk-free asset would allow entrepreneurs to smooth consumption over time, which would increase the demand for the risk-free asset and decrease the risk-free return. Second, holding equity exposes an entrepreneur to a liquidity shock in next period. A negative liquidity shock next period forces the entrepreneur to liquidate some of his equity, which leads to a sub-optimal consumption-saving choice. This decreases the demand for equity, increasing its equilibrium return.

Table 4 also shows that, in the absence of liquidity shocks, column (2), the equity premium is 1.38%, in the absence of the financial constraint, column (3), the equity premium is 1.45%, in the absence of the liquidity constraint, column (4), the equity premium is 0.01%, and in the absence of both financial and liquidity constraints, column (5), the equity premium 0.00%.

We next discuss the roles of the constraints and shocks on the equity return and the risk-free rate to better understand why the model generates a large equity premium. Comparing columns (2) and (4) suggests that the existence of liquidity constraints increases the equity return and decreases the risk-free rate. This happens because the liquidity constraint increases the supply of equity in the asset market, as the constraint forces noninvesting entrepreneurs to sell some of their equity. In addition, the equilibrium amount of capital declines which increases the marginal product of capital, which also increases the equity return (see equations (11) and (14)). The liquidity constraint does not impose any constraint on risk-free asset holdings. In fact, the liquidity constraint increases the demand for the risk-free asset and thus reduces its return, as holding the risk-free asset would allow the entrepreneurs to smooth their consumption.

Comparing columns (1) and (2) suggests that adding a liquidity shock to the model increases the equity premium and, as Table 4 shows, the increase arises because of the reduction in the risk-free rate. The risk-free asset would be more valuable with liquidity shocks, decreasing its return for the following reason. An entrepreneur hit by a negative liquidity shock is forced to liquidate some of his equity holdings and to consume more than would otherwise be optimal. He would prefer to partially mitigate the effect of the liquidity shock by purchasing more risk-free assets (rather than consuming), which would increase the price of the risk-free asset and thus decrease its return.

Comparing columns (4) and (5) suggest that the financial constraint on its own does not generate a substantial equity premium. This implies that financial frictions on their own do not strongly propagate productivity shocks (see also Gomes et al. 2003; and Cordoba and Ripoll 2004 for a similar finding in environments based on Carlstrom and Fuerst 1997; and Kiyotaki and Moore 1997).

Comparing columns (1) and (2), it is tempting to conclude that the liquidity shocks do not matter for the equity premium. However, we caution against this interpretation of this finding. The high equity premium obtained in this specification of the model is associated with a volatility of consumption growth that is too high. Moreover, as we will discuss later, the volatility of the risk-free rate and the cyclical properties of the risk premium in this specification of the model are not in line with the data.

Volatility of Asset Returns. The benchmark model of column (1) falls short of fully explaining the volatility in equity returns, generating about one fifth of the observed volatility in asset returns.[8] All the other specifications of our model generate substantially less volatility of equity returns.

Comparing column (4) to column (5) highlights that the financial constraint plays an important role in generating the volatility in asset returns. This is because, without the financial constraint, the price of equity is constant at $q_t = 1$, there are no capital gains, and no volatility in capital gains. Volatility in equity returns can only come from time variation in the marginal product of capital, which is limited, as capital is a slowly moving (state) variable and labor also does not vary much. Comparing column (3) to column (5) suggests that the liquidity shocks (and the liquidity constraint) on their own do not contribute significantly to asset price volatility. The reason is the same as before, without the financial constraint, $q_t = 1$ at all times whether or not liquidity shocks (and the liquidity constraint) are present. However, in the presence of the financial constraint, liquidity shocks do increase asset price volatility significantly; compare columns (1) and (2).

We next discuss the volatility of the risk-free rate. Table 4 shows that the benchmark model matches the volatility of the risk-free rate in the data.[9] This is not surprising, given that the volatility of the risk-free rate was one of our calibration targets.

The volatility of the risk-free asset is closely related to the volatility of the entrepreneurs' consumption. Therefore, the comparison of the volatility of the risk-free rate across the various specifications of the model (see Table 4) is similar to the comparison of the volatility of the entrepreneurs' consumption (see Table 3). Comparing column (4) to column (5) in Table 4 highlights that, in contrast to the equity return, the financial constraint does not play an

[8] As discussed in an earlier version of this paper, one could generate substantial asset return volatility by introducing time-varying financial shocks, i.e., one would assume that ϕ in the financial constraint (FC) is a stochastic process common to all entrepreneurs. This modification, however, would lead to a negative correlation between asset prices and investment, contrary to the data, as we discusss in detail below.

[9] In an endowment economy, Bansal and Yaron (2004) argue that it is necessary to choose an EIS>1 to explain the behavior of asset prices and, in particular, the low risk-free rate volatility. However, the empirical evidence suggests that the EIS is well below one (as in our benchmark calibration); see, for example, Vissing-Jorgensen (2002).

important role in generating risk-free rate volatility. However, the liquidity shocks generate significant volatility in the risk-free rate (compare columns (1) and (2)). This is intuitive, since adding liquidity shocks increases the volatility of entrepreneurs' consumption. Moreover, the financial constraint tends to reduce the effect of liquidity shocks on the volatility of the risk-free rate, as one can see by comparing columns (1) and (3). As noted earlier, adding the financial constraint decreases the volatility of investment and decreases the volatility of entrepreneurs' consumption. This, in turn, results in a decline in the volatility of the risk-free rate.

Asset Price Cyclicality. An extensive literature has studied the role of financial shocks. In many of these models, changes in the tightness of the financial constraint, which could be modeled as time variation in ϕ in our model, directly affect the amounts of investment, but do not affect the productivity of the existing capital. Tighter constraints imply less investment and less new capital, which makes old capital (and new capital) more valuable to agents in the economy. Therefore, tighter constraints imply less investment and higher asset prices.[10] However, the correlation between asset prices and investment is positive in the data; see Table 4.

As Table 4 shows, our model generates a positive correlation between asset prices and investment and asset prices and output. This is an important improvement relative to the existing literature. As we show via impulse responses in Section 6.1, both productivity and liquidity shocks imply procyclical asset prices and investment. The logic is standard for the productivity shock: A negative productivity shock decreases the productivity of capital, leading to a drop in its price, in investment and in output. A negative liquidity shock reduces

[10]We also find this to be true in the original Kiyotaki and Moore (2019) model, in which the friction takes the form of limited resaleability. In this model, an entrepreneur can sell only a fraction of his assets at a point in time to finance new investments. Tightening this constraint implies a decrease in investment and an increase in the asset price by the same logic. Shi (2015) and Bigio and Schneider (2017) discuss this result in detail in several versions of Kiyotaki and Moore (2019). A similar result appears in models with investment-specific technological shocks: a positive investment-specific technological shock implies higher investment and lower price of capital/equity. However, in a recent paper, Guerron-Quintana and Jinnai (2019) show that negative liquidity shocks are associated with equity price drops in an environment with endogeneous growth.

the demand for equity by the noninvesting entrepreneurs, decreasing its price. The reduction in capital demand decreases aggregate investment, leading to a positive correlation between asset prices and investment. As investment decreases, so does output, implying that both investment and asset prices are procyclical.

Asset Return Predictability. Although controversial, equity return predictability by financial variables and macroeconomic variables is documented in the literature (see, e.g., Campbell and Shiller 1988; Fama and French 1988; Lettau and Ludvigson 2001; Cooper and Priestley 2009, among many others). Next, we show that the equity premium varies systematically over the business cycle and is, therefore, predictable in our production economy. The ability to generate predictable excess returns is important because standard business cycle models do not generate economically significant predictability in excess returns (see Kaltenbrunner and Lochstoer 2010).

We investigate whether the price-to-capital return ratio predicts the (cumulative) excess equity returns one year, two years, and five years ahead. Frequently, researchers ask whether future equity returns are predictable by the log of the current price-to-dividend ratio. However, our model does not explicitly include dividends. Therefore, we use the marginal product of capital, i.e., the return on capital r_t, instead of dividends to investigate the predictability of equity returns. One can also think of the price-to-return on capital ratio as a measure of the value-to-earnings ratio (this ratio is also used in the predictability literature). This is because, in our model, the firm value is $q_t k_t$ and earnings is $r_t k_t$. The value-to-earnings ratio is therefore equal to the price-to-return on capital ratio.

Table 5 reports the results of the predictability regressions and the cyclical properties of asset returns and consumption growth of investing and noninvesting entrepreneurs. Each correlation coefficient reports the correlation between logged output (net off the balanced growth path) in period t and a value of another variable between period t and $t+1$. As an example, the correlation between output and $\mathbb{E}[r^e]$ is the correlation between output in period t and the expected equity return between period t and $t+1$. The consumption growth statistics take into account that a period t noninvesting entrepreneur can become an investing entrepreneur in period $t+1$ and vice versa.

Columns (1), (2) and (3) in Table 5 suggest that these models generate a substantial equity premium, which is predictable and varies systematically over the business cycle. As we explain next, the time variation in equity premium arises from systematic changes in expected consumption growth and the volatility of consumption growth of investing and noninvesting entrepreneurs.

In the benchmark model, presented in column (1), the expected equity return is countercyclical, the risk-free rate is procyclical, and the risk premium is countercyclical. The expected consumption growth of noninvesting entrepreneurs is procyclical and the standard deviation of their consumption growth is weakly countercyclical. A noninvesting entrepreneur can remain noninvesting or become an investing entrepreneur in the next period. Given that the liquidity shock and productivity shock are highly correlated, the liquidity constraint is loose in good times and the next period consumption levels in the two idiosyncratic states (investing vs. noninvesting) differ less than in bad times when the liquidity constraint is tight. This makes consumption growth risk countercyclical.

The expected and the standard deviation of consumption growth of investing entrepreneurs is countercyclical and weakly countercyclical, respectively. In our calibrated model, in the next period, an investing entrepreneur will become a noninvesting entrepreneur with high probability. During bad times, an investing entrepreneur who becomes a noninvesting entrepreneur will face a tighter liquidity constraint than in good times. Since investing entrepreneurs are not facing a liquidity constraint in the current period, a tight liquidity constraint in the next period would imply higher consumption growth in bad times than in good times. This explains why consumption growth of investing entrepreneurs is countercyclical. A similar argument as the one for the noninvesting entrepreneurs explains the countercyclicality of the consumption risk of investing entrepreneurs.

Recall that we determine the (implicit) risk-free rate using the stochastic discount factor of noninvesting entrepreneurs. The procyclicality of expected consumption growth of noninvesting entrepreneurs and the countercyclicality of the volatility of their consumption make the risk-free rate procyclical. This is because during good times, noninvesting entrepreneurs do not want to invest in the risk-free asset, given that they expect a high consumption growth and a low consumption risk. The reduction in demand for the risk-free asset increases its return during good times. The opposite holds during bad times.

Investing entrepreneurs' consumption dynamics mainly determines equity returns, because noninvesting entrepreneurs' equity holdings are constrained by the liquidity constraint. The countercyclicality of the expected consumption growth of investing entrepreneurs and the countercyclicality of the volatility of their consumption growth make the expected equity return countercyclical. This is because, during bad times, they reduce their equity holdings because they expect high consumption growth and high consumption risk. The reduction in equity holdings reduces the equity price and thus increases its expected return in bad times. The opposite holds during good times. The behavior of the risk-free rate and the equity return combined imply that the excess return is time-varying and countercyclical, as in the data. This systematic time variation over the business cycle, in turn, implies that excess returns are predictable; see the second panel of Table 5.

A comparison of columns (1), (2), and (3) provides several interesting findings regarding the time variation in the equity premium over the business cycle. Both the liquidity constraint and liquidity shocks play important roles in generating a countercyclical risk premium in the presence of the financial constraint. In the absence of liquidity shocks, column (2), the risk premium is procyclical. Without the financial constraint, column (3), the time variation in the equity returns is negligible. These properties are not in line with the data. In addition, column (4) of Table 5 highlights that the financial constraint on its own does not lead to excess return predictability. Consistent with the literature, column (5) shows that the standard real business cycle model does not generate any excess return predictability. This is because, as Table 5 shows, the risk premium in these two models is negligible, the time variation in the equity premium is minuscule and the variation is not systematic over the business cycle.

6 Analyzing the Mechanisms

This section analyzes the mechanisms through which the productivity and liquidity shocks operate. It also analyzes the roles of various assumptions in generating high equity premium and low risk-free rate volatility.

6.1 Impulse Responses

To shed light on the mechanisms through which the two aggregate shocks operate, Figures 1 and 2 plot the impulse responses of major macroeconomic variables, asset prices, and Lagrange multipliers on the financial and liquidity constraints to the two shocks in the model. Figure 1 shows that a productivity shock generates co-movement between major macroeconomic variables. Figure 2 shows that a liquidity shock also generates co-movement between major macroeconomic variables. The figures also show that, in response to a negative productivity shock or a negative liquidity shock, the price of equity, Tobin's Q (defined as q_t/q_t^R) and the risk-free rate drop.

Although both investing and noninvesting entrepreneurs' consumption drops in response to a drop in productivity, Figure 2 shows that investing entrepreneurs' consumption drops, whereas noninvesting entrepreneurs' consumption increases in response to a negative liquidity shock. This is because, after a negative liquidity shock, the price of equity drops, reducing the wealth of investing entrepreneurs (having an investment opportunity becomes less valuable) and their consumption, whereas noninvesting entrepreneurs are forced to liquidate some of their equity holdings and consume.

The Lagrange multiplier on the financial constraint measures the marginal value to an investing entrepreneur of being able to sell an additional unit of equity. As shown in Figures 1 and 2, the value of the Lagrange multiplier decreases with both negative productivity and negative liquidity shock. This is because the price of equity drops after both of these shocks, and so it is less beneficial for the investing entrepreneur to sell an additional unit of equity.

The Lagrange multiplier on the liquidity constraint measures the marginal value of the noninvesting entrepreneurs being able to buy an additional unit of equity, because the constraint restricts the entrepreneur's consumption-saving choice. The value of being able to buy an additional unit of equity is high when productivity is high, and low when productivity is low. This is why, in Figure 1, the multiplier drops after a negative productivity shock. As shown in Figure 2, the Lagrange multiplier on the liquidity constraint increases when a negative liquidity shock hits. A negative liquidity shock restricts the noninvesting entrepreneurs' equity purchases. An additional unit of investment (resulting from a relaxation of the constraint) is more valuable when investment is low, implying that the multiplier increases after a negative liquidity shock.

6.2 Comparative Statics

This section analyzes the extent to which the high equity premium and low risk-free rate and its volatility in the model depend on certain assumptions about preferences, shock processes, etc. To analyze the role of a particular parameter, we perform a comparative statics exercise with respect to the parameter. All comparative statistics results are based on equilibrium quantities and prices. This means that the results can be interpreted as comparisons between economies that differ in one parameter of interest.[11]

Tightness of the Liquidity Constraint. As discussed in the previous section, eliminating the liquidity constraint completely implies that the model generates lower investment volatility, higher aggregate consumption volatility, lower equity and higher risk-free return, negligible equity premium, and lower equity and risk-free rate volatility. Therefore, changing the tightness of the liquidity constraint affects the volatility of macroeconomic quantities, asset prices, and asset returns. Table 6 reports the results for the benchmark parametrization with $\mu_\tau = 0.984$ and two cases of low $\mu_\tau = 0.980$, and high $\mu_\tau = 0.988$. Higher μ_τ

[11]The only inconsequential exception is the labor disutility parameter ω which is recalibrated so that labor supply on the balanced growth path remains 1/3.

corresponds to a looser liquidity constraint, as $1-\tau$ measures the percentage of his wealth that an entrepreneur is forced to consume because of a liquidity shock. This table shows that relaxing the liquidity constraint (increasing μ_τ) around the benchmark affects the volatility of most macroeconomic variables in a non-monotonic way. Specifically, increasing μ_τ to 0.988 increases the volatility of entrepreneurs' consumption substantially (while increasing μ_τ further would lead to a full elimination of the LC and lower volatility of entrepreneurs' consumption). This results in an increase in the volatility of the risk-free rate. As μ_τ increases, the equity return decreases and the risk-free rate increases, decreasing the equity premium, which is consistent with full elimination of the liquidity constraint (compare column (4) to columns (1) and (2) in Table 4).

Size of the Liquidity Shocks. We already know that shutting down liquidity shocks completely implies that the model generates low investment volatility, high consumption volatility, low equity return, and low risk-free rate volatility. Therefore, it is true that, in general, changing the volatility of liquidity shocks affects the properties of variables of interest. Table 6 reports the results for the benchmark parametrization with $\sigma_\tau = 4.6 \cdot 10^{-4}$ and two cases of low $\sigma_\tau = \frac{1}{2} \cdot 4.6 \cdot 10^{-4}$, half of the benchmark, and high $\sigma_\tau = 2 \cdot 4.6 \cdot 10^{-4}$, double the benchmark. This table shows that changing the volatility of the liquidity shocks around the benchmark value affects the equity premium mainly through affecting the level of the risk-free rate. As expected, an increase in the volatility of liquidity shocks is associated with an increase in the volatility of investment, entrepreneurs' consumption, equity returns, and the risk-free rate and with a reduction in the volatility of aggregate consumption. It is worth noting, that the model generates high equity premia for a range of liquidity-shock volatilities.

Scarcity of Investment Opportunity. Table 6 reports the results for the benchmark parameterizations with quarterly $\pi = 1.5\%$ and two cases of low $\pi = 1\%$ and high $\pi = 2\%$. This table shows that an increase in π is associated an increase in the risk-free rate, and an increase in the equity return. With higher π, there are fewer noninvesting entrepreneurs, hence, the demand for both equity and the risk-free asset decreases, decreasing their price and increasing their returns. As the increase in the equity return is smaller, higher π also implies a lower equity premium. Since there are now more investing entrepreneurs, the equity price also becomes less price-sensitive, implying a reduction in the volatility of the equity returny. As for the risk-free rate, its volatility increases, because the volatility of entrepreneurs' consumption increases. Finally, there is a non-monotonic relationship between the scarcity of investment opportunities and the volatility of aggregate consumption and investment. Importantly, we see that our main result - a high equity premium - survives for a range of values of π.

Tightness of the Financial Constraint. We next analyze how the tightness of the financial constraint, measured by the level of ϕ, affects the macroeconomic quantities and asset prices. Table 6 reports the results for the benchmark parameterizations with $\phi = 0.266$ and two cases of low $\phi = 0.20$ and high $\phi = .33$. A lower ϕ means a tighter financial constraint. The table shows that increasing ϕ leaves the volatility of most macroeconomic variables by and large unaffected. The only exception is the volatility of entrepreneurs' consumption, which increases with ϕ, which is related to the increase in risk-free rate volatility. Similarly to a larger π, a larger ϕ implies higher equity and risk-free return and a higher equity premium. The volatility of the equity return is lower with a higher ϕ as well, as shocks propagate less into equity with a looser financial constraint.

Elasticity of Intertemporal Substitution. Table 6 reports the results for the benchmark parameterizations with $\psi = 0.4$ and two cases of low $\psi = 0.3$ and high $\psi = 0.5$. Table 6 shows that changing the EIS does not have a significant impact on output, but it does affect consumption and investment. As expected, the EIS particularly affects the volatility of entrepreneurs' consumption, resulting in changes in the risk-free rate and its volatility. The risk-free rate increases with a higher EIS, since entrepreneurs are more willing to accept fluctuating consumption profiles. This decreases the demand for the risk-free asset, increasing its return.

Risk Aversion. In our model, entrepreneurs have access to a single asset (equity) and thus do not solve a portfolio allocation problem. As a result, changing the degree of risk aversion does not affect the properties of macroeconomic quantities and asset prices, as can be seen in Table 6.

Labor Supply Elasticity. Changing the level of workers' elasticity of labor supply does not result in substantial changes in financial variables. It does affect macroeconomic variables, however. Higher labor supply elasticity implies higher output volatility (and higher investment volatility), as shown in the last two columns of Table 6.

7 Conclusion

This paper studies the role of liquidity shocks in business cycle fluctuations and asset prices. Specifically, the paper develops a production economy model with heterogeneous entrepreneurs, financial frictions, and liquidity and productivity shocks. To assess the quantitative importance of liquidity and productivity shocks, the model is calibrated to U.S. data. The calibrated model matches the equity premium and both the level and volatility of the risk-free rate. The model also fits key aspects of the behavior of aggregate quantities. In particular, the model matches the volatility of aggregate investment and consumption growth relative to the volatility of output growth.

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A Appendix A: Proofs

Proof of Lemma 2: A worker solves (we drop the index j for simplicity)

$$\max_{c'_t, l'_t} U\left(c'_t - \frac{\omega}{1+\eta} X_t (l'_t)^{1+\eta}\right) \text{ s.t. } c'_t \leq w_t l'_t,$$

where c'_t denotes the consumption of the worker, l'_t denotes the labor provided by the worker, and w_t denotes the wage rate. Therefore $l'_t = \left(\frac{w_t}{\omega X_t}\right)^{1/\eta}$, which holds for each worker.

Therefore, the aggregate labor supply, denoted by L'_t, can be written as

$$L'_t = \left(\frac{w_t}{\omega X_t}\right)^{1/\eta}. \tag{17}$$

The aggregate labor demand of the entrepreneurs, denoted by L_t, is determined by $w_t = (1-\alpha) A_t X_t^{1-\alpha} N_t^{\alpha} L_t^{-\alpha}$, where we use the fact that aggregate capital equals aggregate equity holdings N_t. In equilibrium, the supply of labor by the workers is equal to the demand for labor by the entrepreneurs, i.e., $L'_t = L_t$. Therefore, $w_t = \omega^{\frac{\alpha}{\eta+\alpha}} [(1-\alpha)A_t]^{\frac{\eta}{\eta+\alpha}} X_t^{\frac{\eta+\alpha-\eta\alpha}{\eta+\alpha}} N_t^{\frac{\eta\alpha}{\eta+\alpha}}$ and $L_t = \left(\frac{(1-\alpha)A_t}{\omega}\right)^{\frac{1}{\alpha+\eta}} \left(\tilde{N}_t\right)^{\frac{\alpha}{\alpha+\eta}}$.

Given that workers cannot save, their aggregate consumption, denoted by C'_t, equals the labor share in output, i.e., $C'_t = (1-\alpha) Y_t$. □

Corollary 1. *The joint dynamics of detrended output and labor and output and workers' consumptions satisfy*

$$\rho(\log L_t, \log \tilde{Y}_t) = 1, \qquad var(\log L_t) = \frac{1}{(1+\eta)^2} var(\log \tilde{Y}_t),$$
$$\rho(\log C'_t, \log Y_t) = 1, \qquad var(\log C'_t) = var(\log Y_t),$$

where $var(x)$ denotes the variance of variable x and $\rho(x,y)$ denotes the correlation between variables x and y.

Proof: The joint dynamics of output and labor are determined by equation (17). Using equilibrium conditions, this equation can be rewritten as

$$L_t = \left(\frac{w_t}{\omega X_t}\right)^{1/\eta} = \left(\frac{MPL_t}{\omega X_t}\right)^{1/\eta} = \left(\frac{(1-\alpha)Y_t}{\omega X_t L_t}\right)^{1/\eta}.$$

This implies that $(1+\eta)\log L_t = \log \tilde{Y}_t + \log\left(\frac{1-\alpha}{\omega}\right)$, which implies $\rho(\log L_t, \log \tilde{Y}_t) = 1$ and $(1+\eta)^2 var(\log L_t) = var(\log \tilde{Y}_t)$, which establishes the first two claims made in Corollary 1. In Lemma 2, we established that $C'_t = (1-\alpha)Y_t$. Therefore, $\rho(\log C'_t, \log Y_t) = 1$ and $var(\log C'_t) = var(\log Y_t)$. □

This corollary shows that the relative variance of labor and detrended output is determined by the labor supply elasticity parameter η, whereas the variance of workers' consumption is the same as the variance of output. Given that workers account for a large fraction of aggregate consumption in the economy (the combined workers' and entrepreneurs' consumption), the dynamics of workers' consumption will significantly affect the dynamics of aggregate consumption relative to output.

Proof of Proposition 1: Let V_t^i denote the value function of an investing entrepreneur and V_t^s denote the value function of a noninvesting entrepreneur. These value functions are functions of the aggregate state $(A_t, \tilde{N}_t, \tau_t)$ and the individual state n_t. They satisfy

$$\begin{aligned} V_t^i &= \left((1-\beta)(c_t^i)^{\frac{1-\gamma}{\theta}} + \beta\left(\pi\mathbb{E}[(V_{t+1}^i)^{1-\gamma}] + (1-\pi)\mathbb{E}[(V_{t+1}^s)^{1-\gamma}]\right)^{\frac{1}{\theta}}\right)^{\frac{\theta}{1-\gamma}}, \\ V_t^s &= \left((1-\beta)(c_t^s)^{\frac{1-\gamma}{\theta}} + \beta\left(\pi\mathbb{E}[(V_{t+1}^i)^{1-\gamma}] + (1-\pi)\mathbb{E}[(V_{t+1}^s)^{1-\gamma}]\right)^{\frac{1}{\theta}}\right)^{\frac{\theta}{1-\gamma}}. \end{aligned}$$

Lemma 1 implies that V_t^i and V_t^s are linear in wealth and, in particular, $V_t^i = \xi_t^i \cdot n_t^i R_t$ and $V_t^s = \xi_t^s \cdot n_t^s R_t$, where ξ_t^i and ξ_t^s are time-invariant functions of the aggregate state $(A_t, \tilde{N}_t, \tau_t)$. Therefore,

$$\begin{aligned} (\xi_t^i)^{\frac{1-\gamma}{\theta}} &= (1-\beta)(\zeta_t^i)^{\frac{1-\gamma}{\theta}} + \beta\left(\pi\mathbb{E}\left[\left(\xi_{t+1}^i \frac{n_{t+1}^i R_{t+1}}{n_t^i R_t}\right)^{1-\gamma}\right] + (1-\pi)\mathbb{E}\left[\left(\xi_{t+1}^s \frac{n_{t+1}^s R_{t+1}}{n_t^i R_t}\right)^{1-\gamma}\right]\right)^{\frac{1}{\theta}}, \\ (\xi_t^s)^{\frac{1-\gamma}{\theta}} &= (1-\beta)(\zeta_t^s)^{\frac{1-\gamma}{\theta}} + \beta\left(\pi\mathbb{E}\left[\left(\xi_{t+1}^i \frac{n_{t+1}^i R_{t+1}}{n_t^s R_t}\right)^{1-\gamma}\right] + (1-\pi)\mathbb{E}\left[\left(\xi_{t+1}^s \frac{n_{t+1}^s R_{t+1}}{n_t^s R_t}\right)^{1-\gamma}\right]\right)^{\frac{1}{\theta}}, \end{aligned}$$

which can be simplified to

$$(\xi_t^i)^{\frac{1-\gamma}{\theta}} = (1-\beta)(\zeta_t^i)^{\frac{1-\gamma}{\theta}} + \beta\left(\frac{1-\zeta_t^i}{q_t^R}\right)^{\frac{1-\gamma}{\theta}} \left(\pi\mathbb{E}\left[\left(\xi_{t+1}^i R_{t+1}\right)^{1-\gamma}\right] + (1-\pi)\mathbb{E}\left[\left(\xi_{t+1}^s R_{t+1}\right)^{1-\gamma}\right]\right)^{\frac{1}{\theta}}, \quad (18)$$

$$(\xi_t^s)^{\frac{1-\gamma}{\theta}} = (1-\beta)(\zeta_t^i)^{\frac{1-\gamma}{\theta}} + \beta\left(\frac{1-\zeta_t^s}{q_t}\right)^{\frac{1-\gamma}{\theta}} \left(\pi\mathbb{E}\left[\left(\xi_{t+1}^i R_{t+1}\right)^{1-\gamma}\right] + (1-\pi)\mathbb{E}\left[\left(\xi_{t+1}^s R_{t+1}\right)^{1-\gamma}\right]\right)^{\frac{1}{\theta}}. \quad (19)$$

Next, we show that a simple relationship holds between ξ_t's and ζ_t's. We write the problem of the noninvesting entrepreneur recursively (note that the value function is in fact time invariant, but depends on the aggregate states):

$$V_t^s(n_t) = \max_{(c_t, n_{t+1}) \geq 0} \left[(1-\beta)c_t^{\frac{1-\gamma}{\theta}} + \beta\left(\mathbb{E}_t\left[V_{t+1}(n_{t+1})^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \qquad \text{s.t.}$$

$$\begin{aligned} &\text{(BC)} \qquad c_t + q_t n_{t+1} \leq n_t[r_t + (1-\delta)q_t], \\ &\text{(LC)} \qquad q_t n_{t+1} \leq \tau_t[r_t + (1-\delta)q_t]n_t. \end{aligned}$$

The first order condition with respect to c_t and the envelope theorem are:

$$\begin{aligned}\frac{\theta}{1-\gamma}V_t^s(n_t)^{1-\frac{1-\gamma}{\theta}} \cdot (1-\beta)\frac{1-\gamma}{\theta}c_t^{s\,\frac{1-\gamma}{\theta}-1} &= \lambda_t^s, \\ \frac{\partial V_t^s(n_t)}{\partial n_t} &= \lambda_t^s R_t + \mu_t^s \tau_t R_t,\end{aligned}$$

where λ_t^s and μ_t^s denote the Lagrange multipliers on (BC) and (LC), respectively.

Rewriting, and using the fact that $V_t^s = \xi_t^s R_t n_t$ we obtain:

$$\begin{aligned}V_t^{s\,1-\frac{1-\gamma}{\theta}} \cdot (1-\beta)c_t^{s\,\frac{1-\gamma}{\theta}-1} &= \lambda_t^s \\ \xi_t^s R_t &= \lambda_t^s R_t + \mu_t^s \tau_t R_t\end{aligned}$$

Combining:

$$\xi_t^s = \left(\frac{V_t^s}{c_s^t}\right)^{1-\frac{1-\gamma}{\theta}} \cdot (1-\beta) + \mu_t^s \tau_t$$

Rewriting, and using the fact that $V_t^s = \xi_t^s R_t n_t, c_t^s = \zeta_t^s R_t n_t$ we obtain:

$$\xi_t^s = \left(\frac{\xi_t^s}{\zeta_t^s}\right)^{1-\frac{1-\gamma}{\theta}} \cdot (1-\beta) + \mu_t^s \tau_t, \tag{20}$$

or alternatively

$$\xi_t^s = (1-\beta)^{\frac{\theta}{1-\gamma}} (\zeta_t^s)^{\frac{1-\gamma-\theta}{1-\gamma}} \cdot M_t^{\frac{\theta}{1-\gamma}}, \tag{21}$$

where we define $M_t = 1 + \frac{\mu_t^s \tau_t}{1-\beta}\left[\frac{\zeta_t^s}{\xi_t^s}\right]^{1-\frac{1-\gamma}{\theta}}$.

Equation (21) can be written for period $t+1$ as:

$$\xi_{t+1}^s = (1-\beta)^{\frac{\theta}{1-\gamma}} (\zeta_{t+1}^s)^{\frac{1-\gamma-\theta}{1-\gamma}} \cdot M_{t+1}^{\frac{\theta}{1-\gamma}}, \tag{22}$$

The investing entrepreneur does not face a liquidity constraint and, hence, for him/her we have for any t:

$$\xi_t^i = (1-\beta)^{\frac{\theta}{1-\gamma}} (\zeta_t^i)^{\frac{1-\gamma-\theta}{1-\gamma}}, \tag{23}$$

Now, we plug in from equations (21) and (22) into equation (18) and from equation (23) into equation (19)

$$
\begin{aligned}
(\xi_t^i)^{\frac{1-\gamma}{\theta}} &= (1-\beta)(\zeta_t^i)^{\frac{1-\gamma}{\theta}} + \beta J^{\frac{1}{\theta}} \frac{(1-\zeta_t^i)^{\frac{1-\gamma}{\theta}}}{q_t^{R\,\frac{1-\gamma}{\theta}}}, \\
(\xi_t^s)^{\frac{1-\gamma}{\theta}} &= (1-\beta)(\zeta_t^i)^{\frac{1-\gamma}{\theta}} + \beta J^{\frac{1}{\theta}} \frac{(1-\zeta_t^s)^{\frac{1-\gamma}{\theta}}}{q_t^{\frac{1-\gamma}{\theta}}}.
\end{aligned}
$$

These can be written as:

$$
\begin{aligned}
(1-\beta)\left(\frac{\zeta_t^i}{1-\zeta_t^i}\right)^{\frac{1-\gamma-\theta}{\theta}} (q_t^R)^{\frac{1-\gamma}{\theta}} &= \beta J^{\frac{1}{\theta}}, \\
(1-\beta)\left(\frac{\zeta_t^s}{1-\zeta_t^s}\right)^{\frac{1-\gamma}{\theta}} \frac{M_t-\zeta_t^s}{\zeta_t^s}(q_t)^{\frac{1-\gamma}{\theta}} &= \beta J^{\frac{1}{\theta}}.
\end{aligned}
$$

□

Proof of Lemma 3: Equation (12) is a rewrite of the goods market clearing condition $C_t + C_t' + I_t = Y_t$. Using the fact that workers' aggregate consumption C_t' is a fraction $(1-\alpha)$ of output Y_t, and the fact that $N_{t+1} = (1-\delta)N_t + I_t$, one can write

$$
N_{t+1} = (1-\delta)N_t + \alpha Y_t - C_t,
$$

or alternatively

$$
(1+g)\tilde{N}_{t+1} = (1-\delta)\tilde{N}_t + \alpha \tilde{Y}_t - \tilde{C}_t. \tag{24}
$$

Aggregate entrepreneurs' consumption C_t can be computed by aggregating the consumption of investing entrepreneurs in equation (4) and the consumption of noninvesting entrepreneurs in equation (6). The fact that the initial asset holdings of investing entrepreneurs are πN_t and of the noninvesting entrepreneurs $(1-\pi)N_t$ (this follows from the fact that investment opportunity arrival is i.i.d.) implies

$$
C_t = [\zeta_t^i \pi + \zeta_t^s (1-\pi)] R_t N_t,
$$

and by plugging into equation (24)

$$
(1+g)\tilde{N}_{t+1} = (1-\delta)\tilde{N}_t + \alpha \tilde{Y}_t - [\zeta_t^i \pi + \zeta_t^s (1-\pi)] R_t \tilde{N}_t.
$$

□

Proof of Proposition 2: The equilibrium price of equity is defined as one which clears the equity market, i.e., implies that demand for and supply of equity are equal. Aggregating over all noninvesting entrepreneurs using equation (7) their net demand for (new) equity can be written as

$$
D_t^e := N_{t+1}^s - (1-\delta)(1-\pi)N_t = (1-\zeta_t^s)(1-\pi)N_t[\frac{r_t}{q_t} + 1 - \delta] - (1-\delta)(1-\pi)N_t.
$$

Similarly, aggregating over all investing entrepreneurs using equation (5), the net supply of equity of the investing entrepreneurs is given by

$$\begin{aligned} S_t^e : &= \pi(1-\delta)N_t + I_t - N_{t+1}^i = \pi(1-\delta)N_t + \frac{\phi}{(1-\phi)} N_{t+1}^i \\ &= \pi(1-\delta)N_t + \frac{\phi}{(1-\phi q_t)}(1-\zeta_t^i)\pi N_t[r_t + (1-\delta)q_t], \end{aligned}$$

where I_t denotes aggregate investment, and we use the fact that the (FC) implies that $I_t = \frac{N_{t+1}^i}{1-\phi}$. Combining the two equations, we obtain a quadratic equation in q_t : $a_2 q_t^2 + a_1 q_t + a_0 = 0$, with the following definitions: $a_2 = (1-\delta)\phi\left[(1-\zeta_t^i)\pi + (1-\zeta_t^s)(1-\pi) - 1\right]$, $a_1 = (1-\delta)\left[1-(1-\zeta_t^s)(1-\pi)\right] + \phi r_t\left[(1-\zeta_t^i)\pi + (1-\zeta_t^s)(1-\pi)\right]$, $a_0 = -(1-\zeta_t^s)(1-\pi) r_t$. To select the correct root of the quadratic equation, note that $a_2 < 0, a_1 > 0, a_0 < 0$. In addition, there is exactly one solution to the equation $a_2 q_t^2 + a_1 q_t + a_0 = 0$ in the interval $(0, \frac{1}{\phi})$.[12] To see this point, define $f(q_t) = a_2 q_t^2 + a_1 q_t + a_0$ and notice that it has the following properties. $f(0) < 0, f(\frac{1}{\phi}) > 0$. These two facts along with $a_2 < 0$ imply that one root (the smaller one) of the quadratic equation lies in $(0, \frac{1}{\phi})$ and the other root is larger than $\frac{1}{\phi}$. Finally, if $q_t^* < 1$, then the equilibrium price of equity $q_t = 1$.[13]

□

Proof of Proposition 3: We assume that if the risk-free asset were traded in the model, it could not be used for equity purchases. In this case, the liquidity constraint and the budget constraint would need to be adjusted and the recursive problem of the noninvesting entrepreneur would be (expressing the value function as a function of the risk-free asset holdings b_t andn_t and suppressing its dependence on aggregate states):

$$V_t^s(n_t, b_t) = \max_{(c_t, n_{t+1}, b_{t+1}) \geq 0} \left[(1-\beta)c_t^{\frac{1-\gamma}{\theta}} + \beta\left(\mathbb{E}_t\left[V_{t+1}(n_{t+1}, b_{t+1})^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \quad \text{s.t.}$$

$$\begin{aligned} &\text{(BC)} \qquad c_t + q_t n_{t+1} + b_{t+1} \leq n_t[r_t + (1-\delta)q_t] + r_{t-1}^f b_t, \\ &\text{(LC)} \qquad q_t n_{t+1} \leq \tau_t[r_t + (1-\delta)q_t] n_t. \end{aligned}$$

The first order conditions with respect to c_t and b_{t+1} and the envelope theorem applied

[12] q_t cannot be larger than $\frac{1}{\phi}$. If it were, then investing entrepreneurs could make their consumption arbitrarily large. Mathematically, the argument comes from the fact that q_t^R would be negative.

[13] If capital can be converted back to consumption $q_t < 1$ is not an equilibrium outcome, since the value of one unit of capital is bounded below by the value of one unit of consumption, which is normalized to one. With capital irreversibility, i.e., an $I_t \geq 0$ constraint, q_t would be smaller than one if and only if this constraint binds. This could happen only if capital levels were very high, which does not happen in our quantitative analysis. We therefore ignore the possibility that $q_t < 1$ and do not take a stand on capital reversibility.

to b_t and moved forward by one period are:

$$\begin{aligned}
\frac{\theta}{1-\gamma}[V_t^s(n_t,b_t)]^{1-\frac{1-\gamma}{\theta}}\cdot(1-\beta)\frac{1-\gamma}{\theta}{c_t^s}^{\frac{1-\gamma}{\theta}-1} &= \lambda_t^s, \\
[V_t^s(n_t,b_t)]^{1-\frac{1-\gamma}{\theta}}\cdot\beta(\mathbb{E}_t\left[V_{t+1}(n_{t+1},b_{t+1})^{1-\gamma}\right])^{\frac{1}{\theta}-1}\mathbb{E}_t[V_{t+1}(n_{t+1},b_{t+1})^{-\gamma}\cdot\frac{\partial V_{t+1}(n_{t+1},b_{t+1})}{\partial b_{t+1}}] &= \lambda_t^s, \\
\frac{\partial V_{t+1}(n_{t+1},b_{t+1})}{\partial b_{t+1}} &= r_t^f(\lambda_{t+1}+\mu_{t+1}),
\end{aligned}$$

where λ_t^s is the Lagrange multiplier on (BC) of the noninvesting entrepreneurs in period t. λ_{t+1} is the multiplier on period $t+11$ (BC) for a period t noninvesting entrepreneur, who can become either an investing or noninvesting entrepreneur in period $t+1$. Substituting in for the multiplier λ_t^s from the first equation and for $\frac{\partial V_{t+1}(n_{t+1},b_{t+1})}{\partial b_{t+1}}]$ from the third into the second implies:

$$r_t^f = \frac{(1-\beta){c_t^s}^{\frac{1-\gamma}{\theta}-1}}{\beta(\mathbb{E}_t\left[{V_{t+1}}^{1-\gamma}\right])^{\frac{1}{\theta}-1}\mathbb{E}_t[{V_{t+1}}^{(1-\gamma)(1-\frac{1}{\theta})}\cdot(1-\beta){c_{t+1}}^{\frac{1-\gamma}{\theta}-1}]}.$$

Using the fact that the risk-free asset is not traded (in equilibrium) and the linearity of the value and policy functions: $V_t=\xi_t\cdot n_tR_t, c_t=\zeta_t\cdot n_tR_t$ implies:

$$r_t^f = \frac{1}{\beta}\left[\frac{\zeta_t^s q_t}{(1-\zeta_t^s)}\right]^{\frac{1-\gamma}{\theta}-1}\cdot\frac{(\mathbb{E}_t[(\xi_{t+1}R_{t+1})^{1-\gamma}])^{-\frac{1-\theta}{\theta}}}{\mathbb{E}_t\left[(R_{t+1}\xi_{t+1})^{-\gamma}\left(\frac{\xi_{t+1}}{\zeta_{t+1}}\right)^{1-\frac{1-\gamma}{\theta}}\right]}.$$

□

B Appendix B: Data Construction

We restrict our attention to the time period 1964Q1 - 2013Q4. We use the following databases to construct the macroeconomic variables.

1. CES-BLS: Current Employment Statistics survey by the Bureau of Labor Statistics.
2. FAT-BEA: Fixed Asset Tables published by the Bureau of Economic Analysis.
3. NIPA-BEA: National Income and Product Accounts published by the Bureau of Economic Analysis.

Hours, denoted by L, is constructed from CES-BLS as L = Average weekly hours × Average number of workers. Real capital, denoted by K, is constructed by generating quarterly data by interpolating the yearly "Fixed assets and consumer durable goods," in Table 1.2 in FAT-BEA. Output, denoted by Y, is the real GDP, in Table 1.1.6 in NIPA-BEA. Productivity series A_t is computed from the series of output, capital and hours series as $A_t=Y_t/(L_t^{0.64}\times K_t^{0.36})$.

To construct a measure for real investment, denoted by I, we first compute nominal investment as NI = Nominal private fixed investment + Nominal durable consumption good expenditure, where nominal private fixed investment is from Table 1.1.5 in NIPA-BEA and nominal durable consumption good expenditure is from Table 1.1.5 in NIPA-BEA.

Real investment is then constructed by deflating NI with the deflator for gross private domestic investment, constructed using Tables 1.1.5 and 1.1.6 in NIPA-BEA. The time-series properties of alternative real investment measures are very similar to those we consider, with a correlation around 0.95. Including government investment or inventories, or excluding durable consumption makes the series slightly more volatile.

To construct a measure for real consumption, denoted by C, we first compute nominal consumption as $NC =$ Nondurable goods + Services, where nondurable goods is from Table 1.1.5 in NIPA-BEA and services is from Table 1.1.5 in NIPA-BEA. The real counterparts of these nominal series are only reported starting in 1995. Therefore, to generate the real series, these nominal series are deflated using a personal consumption expenditure deflator constructed using Tables 1.1.5 and 1.1.6 in NIPA-BEA.

All nominal prices and returns are deflated using the CPI series from the BLS database. Asset price, denoted by q, is the S&P500 Composite Price Index from the CRSP database (Center for Research in Security Prices), where quarterly data is generated as the mean of monthly observations. We have computed the relevant statistics for the Wilshire 5000 Total Market Index from the St. Louis FED database. The time-series properties of the HP-filtered logged real versions of these indexes are very similar (the correlation is 0.99). The Wilshire 5000 is slightly more volatile than the S&P500 (11.0% vs. 10.6%). Asset return, denoted by r^e, is constructed using the series *vwretd* from the CRSP database, value-weighted returns including distributions from NYSE, AMEX, and NASDAQ. Quarterly data is constructed as the geometric mean of monthly observations. Total market value, denoted by val, is constructed using the series *totval* from the CRSP database. Quarterly data is constructed as average over monthly observations. The real risk-free rate, denoted by r^f, is the three-month return on a three month T-bill (Fama risk-free in the CRSP database). Quarterly data is constructed as the geometric mean of monthly observations.

Table 1: Benchmark Parameters

This table shows the parameters used in the benchmark quantitative exercise.

	Symbol	Value
Parameters set outside the model		
Capital share in output	α	0.4
Capital depreciation rate	δ	0.021
Growth rate of TFP	g	0.004
Quarterly discount factor	β	0.995
Entrepreneurs' risk aversion parameter	γ	2
Entrepreneurs' intertemporal elasticity of substitution	ψ	0.4
Workers' inverse Frisch elasticity	η	2
Disutility of labor	ω	23.6
Fraction of firms with investment opportunity	π	0.015
Borrowing constraint parameter	θ	0.266
Standard deviation of log productivity shock	σ_{ε_A}	0.006
Persistence of log productivity shock	ρ_z	0.95
Calibrated parameters		
Mean of liquidity shock	μ_τ	0.984
Persistence of log liquidity shock	ρ_τ	0.9795
Standard deviation of log liquidity shock	$\sigma_{\varepsilon_\tau}$	$4.6 \cdot 10^{-4}$
Correlation of innovations of log τ_t and log A_t	$\rho_{A,\tau}$	0.79

Table 2: Calibration Moments

This table reports the calibration moments.

Target	Symbol	Data	Model Mean	Model STD
Volatility of relative investment growth	$\frac{\sigma_{\Delta I}}{\sigma_{\Delta Y}}$	2.71	2.71	0.07
Volatility of relative consumption growth	$\frac{\sigma_{\Delta C}}{\sigma_{\Delta Y}}$	0.56	0.56	0.03
Volatility of risk-free rate	σ_{r^f}	0.73	0.72	0.15
Mean quarterly risk-free rate	r^f	0.25	0.25	0.35

Table 3: Standard Business Cycle Statistics

This table reports the business cycle statistics for three versions of our model. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded. The symbol $\sigma_{\Delta x}$ represents the standard deviation of the growth rate of variable x, $\rho_{\Delta x}$ represents the autocorrelation of the growth rateg of x, and $\rho(\Delta x, \Delta y)$ represents the correlation between growth rates of x and y. The 'Data' column reports statistics for quarterly U.S. data for the period 1964:1-2013:4. Column (1) reports statistics for the calibrated benchmark model with the financial constraint, liquidity constraint, and both liquidity and productivity shocks. Column (2) reports the statistics for a version of the model with the financial constraint, liquidity constraint, and no liquidity shocks. Column (3) reports the statistics for a version of the model with liquidity constraint but no financial constraint. Column (4) reports the statistics for a version of the model with financial constraint but no liquidity constraint. Finally, column (5) reports the statistics for a version of the model with neither financial nor liquidity constraints.

Statistic	Data	(1) With FC and LC τ stochastic A stochastic	(2) With FC and LC τ constant A stochastic	(3) LC only τ stochastic A stochastic	(4) FC only τ irrelevant A stochastic	(5) No LC, no FC τ irrelevant A stochastic
$\sigma_{\Delta Y}$	0.83	0.80	0.80	0.80	0.80	0.80
$\sigma_{\Delta I}/\sigma_{\Delta Y}$	2.71	2.71	1.15	3.69	1.28	1.42
$\sigma_{\Delta C}/\sigma_{\Delta Y}$	0.56	0.56	0.96	0.37	0.89	0.84
$\sigma_{\Delta L}/\sigma_{\Delta Y}$	0.98	0.33	0.33	0.33	0.33	0.33
$\sigma_{\Delta C^e}/\sigma_{\Delta C^w}$		1.54	0.80	3.28	0.40	0.07
$\rho_{\Delta Y}$	0.34	-0.04	-0.04	-0.03	-0.04	-0.04
$\rho_{\Delta I}$	0.39	-0.04	-0.04	-0.04	-0.04	-0.04
$\rho_{\Delta C}$	0.54	-0.02	-0.04	0.08	-0.04	-0.04
$\rho_{\Delta L}$	0.68	-0.04	-0.04	-0.03	-0.04	-0.04
$\rho(\Delta Y, \Delta I)$	0.77	0.96	1.00	0.96	1.00	1.00
$\rho(\Delta Y, \Delta C)$	0.57	0.92	1.00	0.54	1.00	1.00
$\rho(\Delta Y, \Delta L)$	0.70	1.00	1.00	1.00	1.00	1.00

Table 4: Asset Price Statistics

This table reports the business cycle statistics for three versions of our model. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded. The first panel reports the means of the statistics. The symbol σ_x represents the standard deviation of variable x and $\rho(\Delta x, \Delta y)$ represents the correlation between growth rates of x and y. The returns r^e and r^f are averages over the simulated paths. The 'Data' column reports statistics for quarterly U.S. data for the period 1964:1-2013:4. Column (1) reports statistics for the calibrated benchmark model with the financial constraint, liquidity constraint, and both liquidity and productivity shocks. Column (2) reports the statistics for a version of the model with the financial constraint, liquidity constraint, and no liquidity shocks. Column (3) reports the statistics for a version of the model with liquidity constraint but no financial constraint. Column (4) reports the statistics for a version of the model with financial constraint but no liquidity constraint. Finally, column (5) reports the statistics for a version of the model with neither financial nor liquidity constraints.

Statistic	Data	(1) With FC and LC τ stochastic A stochastic	(2) With FC and LC τ constant A stochastic	(3) LC only τ stochastic A stochastic	(4) FC only τ irrelevant A stochastic	(5) No LC, no FC τ irrelevant A stochastic
r^f	0.25	0.25	0.32	0.58	1.08	1.51
r^e	1.76	1.70	1.70	2.03	1.08	1.51
$r^e - r^f$	1.52	1.46	1.38	1.45	0.01	0.00
σ_r^f	0.73	0.72	0.34	1.80	0.16	0.06
σ_{r^e}	8.68	1.56	0.66	0.09	0.73	0.06
$\rho(\Delta q, \Delta Y)$	0.25	0.96	1.00	0.00	1.00	0.00
$\rho(\Delta q, \Delta I)$	0.28	1.00	1.00	0.00	1.00	0.00

Table 5: Asset Return Cyclicalities and Asset Return Predictability

The first panel of this table reports the correlations between logged output (net off the balanced growth path) at time t and a variable of interest computed between t and $t+1$ for various versions of our model. $\mathbb{E}$ denotes the expectations and σ the standard deviation as of time t. Standard deviations of the statistics over the 10,000 simulations are reported in parantheses. The second panel of this table reports the asset return predictability for the U.S. data and various versions of our model. The data column is taken from Guvenen (2009) and contains the predictability of the log of the cumulative excess stock return by the log of the price-to-dividend ratio. Columns (1) - (5) report the predictability of the log of the cumulative excess equity return by the log of the price-to-capital return ratio. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded.

Statistic	Data	(1) With FC and LC τ stochastic A stochastic	(2) With FC and LC τ constant A stochastic	(3) LC only τ stochastic A stochastic	(4) FC only τ irrelevant A stochastic	(5) No LC, no FC τ irrelevant A stochastic
Correlation with output						
$\mathbb{E}[r^e]$		-0.83	-0.95	0.02	-0.95	0.80
		(0.09)	(0.02)	(0.37)	(0.02)	(0.12)
$\mathbb{E}[r^e] - r^f$		-0.81	0.97	-0.77	-0.04	-0.01
		(0.09)	(0.01)	(0.09)	(0.32)	(0.19)
r^f		0.79	-0.96	0.75	-0.95	0.80
		(0.10)	(0.01)	(0.10)	(0.02)	(0.12)
$\sigma[\mathbb{E}[r^e] - r^f]$		-0.05	-0.04	-0.04	-0.06	0.05
		(0.28)	(0.32)	(0.27)	(0.31)	(0.31)
$\mathbb{E}[\Delta c^s]$		0.79	-0.96	0.76	-0.95	0.80
		(0.10)	(0.01)	(0.10)	(0.02)	(0.12)
$\sigma_{\Delta c^s}$		-0.39	-0.32	-0.44	0.87	0.04
		(0.37)	(0.27)	(0.19)	(0.08)	(0.31)
$\mathbb{E}[\Delta c^i]$		-0.66	0.96	-0.68	0.87	0.80
		(0.22)	(0.02)	(0.15)	(0.08)	(0.12)
$\sigma_{\Delta c^i}$		-0.48	-0.13	-0.58	0.87	0.04
		(0.31)	(0.30)	(0.17)	(0.08)	(0.31)
Excess return predictability						
1-year β	-0.22	-1.51	-1.65	1.67	-0.19	0.00
R^2	0.09	0.64	0.16	0.36	0.02	0.02
2-year β	-0.39	-2.55	-2.72	2.84	-0.38	0.00
R^2	0.14	0.71	0.16	0.31	0.03	0.03
5-year β	-0.77	-4.01	-4.14	4.83	-0.92	0.00
R^2	0.26	0.52	0.12	0.24	0.07	0.06

Table 6: Comparative Statics

This table reports the business cycle statistics for several versions of our model. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded. The symbol σ_x ($\sigma_{\Delta x}$) represents the standard deviation of variable x (standard deviation of growth rate of x), ρ_x represents the autocorrelation of x, and $\rho(x,y)$ represents the correlation between x and y. The returns r^e and r^f are averages over the simulated paths. The 'Data' column reports statistics for quarterly U.S. data for the period 1964:1-2013:4. Column 'Bench' reports statistics for the benchmark version of the model with the financial constraint and both liquidity and productivity shocks. The remaning columns report the results for various comparative statics exercises, in which we change a parameter of interest.

Statistic	Data	Bench	Liquidity Constraint		Liquidity Shock		Inv. Opportunity		Financial Friction		EIS		Risk Aversion		Frisch Elasticity	
			Low τ $= 0.98$	High τ $= 0.988$	Low $\sigma_{\varepsilon_\tau}$ 0.00023	High $\sigma_{\varepsilon_\tau}$ 0.00092	Low π $= 1\%$	High π $= 2\%$	Low ϕ $= 0.20$	High ϕ $= 0.33$	Low ψ $= 0.3$	High ψ $= 0.5$	Low γ $= 1.5$	High γ $= 2.5$	Low η $= 1$	High η $= 3$
Macro variables																
$\sigma_{\Delta Y}$	0.83	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.91	0.75
$\sigma_{\Delta I}/\sigma_{\Delta Y}$	2.71	2.71	2.60	2.52	1.91	4.14	3.21	3.64	2.72	2.71	2.59	2.71	2.71	2.71	2.51	2.81
$\sigma_{\Delta C}/\sigma_{\Delta Y}$	0.56	0.56	0.63	0.60	0.75	0.49	0.53	0.37	0.58	0.55	0.60	0.56	0.56	0.56	0.61	0.55
$\sigma_{\Delta C^e}/\sigma_{\Delta C^w}$		1.54	1.52	3.33	0.50	3.55	1.51	3.19	1.38	1.70	1.36	1.55	1.54	1.54	1.27	1.67
$\rho(\Delta Y, \Delta I)$	0.77	0.96	0.97	0.94	0.98	0.93	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.97	0.96
$\rho(\Delta Y, \Delta C)$	0.57	0.92	0.96	0.89	0.99	0.34	0.89	0.57	0.93	0.91	0.94	0.92	0.92	0.92	0.95	0.91
Financial variables																
r^f	0.25	0.25	-0.22	0.71	0.31	-0.03	-1.12	0.34	-0.14	0.53	0.95	-0.30	0.25	0.26	0.25	0.25
r^e	1.76	1.70	2.12	1.31	1.70	1.73	1.19	2.03	1.54	1.87	1.71	1.70	1.70	1.70	1.71	1.70
$r^e - r^f$	1.52	1.46	2.34	0.60	1.38	1.76	2.31	1.69	1.68	1.34	0.76	2.00	1.46	1.45	1.45	1.46
σ_r^f	0.73	0.72	0.51	0.99	0.21	1.76	0.66	1.86	0.63	0.83	0.83	0.64	0.73	0.71	0.70	0.73
σ_{r^e}	8.68	1.56	1.50	1.44	1.10	2.36	1.58	0.25	1.69	1.42	1.49	1.56	1.56	1.56	1.65	1.52
$\rho(\Delta q, \Delta Y)$	0.25	0.96	0.96	0.94	0.98	0.92	0.95	0.29	0.96	0.96	0.95	0.96	0.96	0.96	0.96	0.96
$\rho(\Delta q, \Delta I)$	0.28	1.00	0.97	0.94	1.00	0.99	1.00	0.25	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00

Figure 1: Impulse Responses to a Productivity Shock

This figure shows the impulse responses to a negative productivity shock.

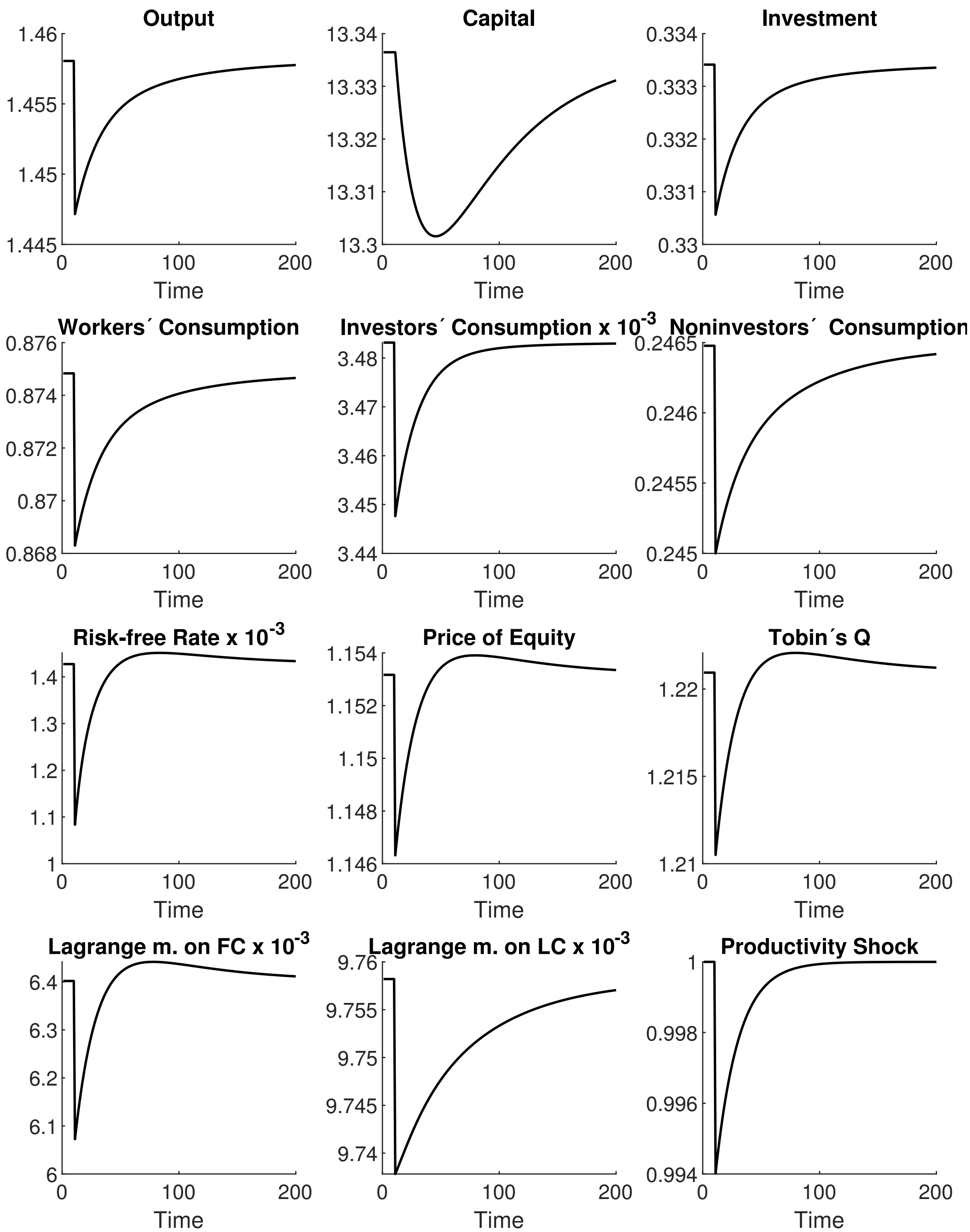

Figure 2: Impulse Responses to a Liquidity Shock

This figure shows the impulse responses to a negative liquidity shock.