The Implications of Financial Frictions and Imperfect Knowledge in the Estimated Model of the US Economy*

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Abstract

In this paper I study how alternative assumptions about the expectation formation can modify the implications of financial frictions for the real economy. I incorporate financial accelerator mechanism into a version of Smets and Wouters (2007) DSGE model and perform a set of estimation and simulation exercises assuming, on the one hand, complete rationality of expectations and, alternatively, several learning algorithms that differ in terms of the information set used by agents to produce the forecasts. I show that implications of financial accelerator for the busyness cycle may vary depending on the approach to modeling the expectations. The results suggest that the learning scheme based on small forecasting functions is able to amplify the effects of financial frictions relative to the model with Rational Expectations. Specifically, I show that the dynamics of real variables under learning is driven to a significant extent by the time variation of agents' beliefs about financial sector variables. During periods when agents perceive asset prices as being relatively more persistent, financial shocks lead to more pronounced macroeconomic outcomes. The amplification effect raises as financial frictions become more severe. At the same time, learning specification in which agents use more information to generate predictions (close to MSV learning) produces very different asset price and investment dynamics. In such a framework, learning cannot significantly alter the real effects of financial frictions implied by the Rational Expectations model.

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1 Introduction

Economists generally admit that public's expectations greatly influence actual macroeconomic outcomes and thus may impact the ability of the central bank to maintain stable monetary and financial environment as well as high employment. Therefore, when forming macroeconomic projections and policy reactions, policymakers constantly monitor and analyze the dynamics of expectations formed by different market participants – households, professional economists, credit institutions, and businesses, who produce their own perceptions of future market developments. Effective expectation channel, which implies the ability of the bank to anchor the public's expectations, may decrease the costs of implementing the policy in the presence of the trade-offs between several competing objectives and prevent the economy from following a self-fulfilling path leading to periods of high inflation or prolonged recessions.

Despite the significant implications for actual macroeconomic dynamics, the process of modeling the expectation formation mechanism has not received a sufficient attention in the literature. The current generation of macroeconomic models is based on the strongest form of rationality, which implies that agents possess complete knowledge about the economy (the model and its parameters) and therefore rely on "true" forecasts in their decision making process. At the same time, modern economies face various uncertainties and feature unstable and constantly evolving structure. In particular, dynamic growth of financial markets and implementation of more sophisticated financial instruments, which requires instant analysis and adjustment to new information, have complicated the task of efficient and up to date pricing and credit decisions. In other words, in reality, agents possess only limited information about the economy and have to rest their choices on the basis of forecasts produced in environment with incomplete information. Therefore, allowing the public to learn the underlying economic structure is more realistic and enables generating more reasonable conclusions about the factors affecting the evolution of public's predictions, the way the expectations may affect actual economic activity and how they, in turn, are influenced by policy actions and communications. Models with more realistic forms of rationality could add to better understanding the economic linkages and risks originating from uncertain environment with imperfectly efficient financial markets.

In my research, I would like to contribute to studying the macro-financial linkages by focusing on adaptively formed expectations as a mechanism that can potentially amplify and propagate shocks to the real economy and introduce additional challenges to the policy conduct. The rationale for combining the "expectation" and "macro-financial" factors is twofold. First of all, the latest financial turmoil has demonstrated that the impact of imbalances in financial sector on the real economy and wealth can be far more influential than many economists have anticipated. Even the most recent studies that analyze DSGE

models with financial frictions have documented the problems in replicating the observed boom-bust cycle, explaining the "surprising" origin of the crisis, and its propagation channels. Therefore, it might be useful to consider other features of the transmission mechanism that might interact with/affect the dynamics of financial and real sectors. Secondly, expectations play an important role in driving asset prices, risk premium, and investments – the key financial markets variables. Thus, two frameworks can be naturally combined.

More specifically, I add to the existing literature in two aspects. Firstly, I incorporate adaptively formed expectations (Evans and Honkapohja, 2001) into a version of the Smets and Wouters (2007) DSGE model with financial accelerator. I estimate the model using Bayesian methods and assess the joint role of financial frictions and the departure from the complete rationality assumption for the US business cycle. I evaluate and compare the model fit, estimated parameters, and the transmission mechanism in models with Rational Expectations (RE) and adaptive learning (AL). Several AL schemes that differ in terms of the information set used by agents to form their expectations are considered. Discussing the estimation results, I evaluate the role of alternative sources of inertia - structural rigidities (such as habit formation, Calvo pricing, indexation,etc.) and learning in propagating financial and non-financial shocks. Secondly, on the basis of the estimated model as well as simulation exercises, I assess the ability of alternative learning algorithms to modify the transmission mechanism relative the RE model with financial frictions and generate additional macroeconomic fluctuations in line with real data. To my best knowledge, this paper is the first one which evaluates the effects of financial frictions under adaptive learning within the *estimated* model. The recent paper by Soto et al. (2010) studies how financial accelerator mechanism combined with adaptive learning influences the business cycle fluctuations in a *calibrated* model. Another important difference of their paper from my work is the information set that learning agents are assumed to use in order to form their predictions. The results of Soto et al. are derived for so-called "MSV" learning. This means that agents use the full set of endogenous and exogenous variables in their forecasting functions. The same set of variables is used to form forecasts under RE. In this paper I assume that agents may use very limited information set. In fact, I compare the results for alternative information sets and demonstrate that the learning scheme is an important determinant of the effects of the financial accelerator for the real economy.

1.1 Related literature

In the recent literature, the "financial accelerator" represents the most common approach to incorporate financial frictions into DSGE models. This framework implies that endogenous developments in the credit markets work to amplify and propagate shocks to the real economy. Depending on the origin/type of such an acceleration mechanism, two main strands in the literature can be distinguished. The first one implies capturing the firms' balance sheet effects on investment by relying on one-period stochastic optimal debt contract with costly state verification (Bernanke and Gertler, 1989; Carlstrom and Fuerst, 1997; and Cespedes et. al, 2004). The key aspect is that such a framework allows modeling of endogenous, positive interest rate spread. The second approach emphasizes another aspect of many possible frictions - the role of endogenous collateral constraint that links the credit capacity of borrowers to the value of their asset holdings (Kiyotaki and Moore, 1997; Iacoviello, 2005; and Iacoviello and Neri, 2008).

In this paper, I follow the first approach and incorporate financial frictions in the form of financial accelerator of Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999). They introduce the agency problem with asymmetric information in order to model the positive interest rate spread, i.e. "external finance premium" defined as the difference between the cost of external sources of funding and the opportunity cost of funds internal to the firm. Due to the agency problem in lending, the external finance premium depends inversely on the borrowers' net wealth, and thus will be countercyclical, enhancing the swings in real variables and the effects of monetary and financial shocks. Bernanke, Gertler and Gilchrist (1999) incorporate an "external finance premium" into a dynamic New Keynesian model with nominal rigidities to study how the credit market frictions may influence the transmission of monetary policy. They show that under reasonable parameterization of the model, the financial accelerator significantly amplifies the effects of shocks to the economy. In terms of its empirical relevance, recent research has found that for the Euro Area and for the US the financial accelerator plays a relevant role in amplifying shocks that move prices and output in the same direction (e.g. monetary policy shocks) as well as in explaining the business cycle (Christiano et al, 2007). De Graeve (2008) estimates external finance premium for the US economy incorporating financial accelerator into the Smets and Wouters (2003) model. He finds that model-consistent estimate of this unobservable financial variable has substantial realistic content (the estimate strongly commoves with the proxies for the premium). Another important result of his study is that incorporating financial frictions improves the empirical performance of an otherwise standard DSGE model.

In modeling the departures from the complete rationality assumptions, I follow the most influential contributions in the adaptive learning literature such as Evans and Honkapo-

hja (2001), Milani (2007), Orphanides and (2007). In particular, I assume that agents know the structure but they are uncertain about the parameters of the model. To learn the parameters, they formulate models based on their economic perceptions and re-estimate these models as soon as new information arrives. A number of studies have demonstrated that adaptive learning can improve the fit of macroeconomic models. In particular, Milani (2007, 2008), Sargent, Williams, and Zha (2005) have shown that introducing adaptive learning can generate levels of persistence observed in the US data. Slobodyan and Wouters (2008, 2010) incorporate less-than-rational beliefs into Smets and Wouters (2007) model and find that impact of adaptive learning on the macro dynamics is more pronounced when agents' information set is more restrictive than the one implied by rational expectations. In small forecasting models learning can explain episodes of inflation dynamics in the US and lowers persistence of some of the exogenous shocks. Rychalovska and Slobodyan (2010) estimate a set of DSGE models of various complexity for the Euro Area. They also find that assuming adaptive expectations results in better model fit than if RE is used, especially when the agents use very little information to form their beliefs. Therefore, the conclusion that adaptive learning based on small forecasting models outperforms MSV and RE models seems to be a robust one, at least for the US and European data. In this paper, I follow Slobodyan and Wouters (2010) and assume that agents' forecasts can be based on very small forecasting models, in particular on a model where expected value of a forward-looking variable depends on a constant and two lags of this variable. Agents estimate and update simple forecasting models using the Kalman filter algorithm. Thus, the learning represents an alternative source of endogenous inertia and influences the degree of economic persistence through the time variation in agents' beliefs.

The rest of the paper is organized as follows: in Section 2 I present the model; Section 3 contains the estimation methodology and results; Section 4 describes the effects of financial frictions on the transmission mechanism in the model with adaptive learning, and Section 5 concludes.

2 The model

In this paper, I consider a medium-scale DSGE model based on Smets and Wouters (2007). The model contains a number of nominal and real rigidities widely used in order to match the observed persistence of main macroeconomic series (Smets and Wouters, 2003, 2007; Christiano et al., 2005). In this section I outline the main features and present a log-linearized version of the model (for more detailed description of micro-foundations see the original papers). The economy consists of households, final and intermediate goods pro-

ducers, and a monetary authority. Households choose consumption, hours worked, bonds, capital stock, investment, and capital utilization so as to maximize a utility function, non-separable in two arguments - consumption goods and labour effort. Consumption term in the utility function incorporates an external habit variable, which is proportional to the lagged aggregate consumption. Households rent capital services to firms and decide how much capital to accumulate given the capital adjustment costs they face. As the rental price of capital changes, the utilization of the capital stock can be adjusted at increasing cost. Intermediate sector firms are monopolistically competitive. They produce differentiated goods, decide on labour and capital inputs, and set prices according to the Calvo model (1983). Households supply the homogenous labour to an intermediate labour union, which differentiates the labour services. Thus there is some monopoly power over wages and à la Calvo sticky nominal wages are introduced. In addition, nominal rigidities in wage and price setting are augmented by the assumption that prices that are not re-optimised are partially indexed to past inflation rates.

In a specification with financial frictions, I follow Bernanke et al. (1999) and De Graeve (2008) and introduce capital producers, financial intermediary, and entrepreneurs.

2.1 The linearized model.

The dynamics of consumption follows from the consumption Euler equation and is given by:

$$\begin{aligned}\widehat{c}_t = \ & \frac{1}{(1+(\eta/\gamma))} E_t\left[\widehat{c}_{t+1}\right] \ + \frac{(\eta/\gamma)}{(1+(\eta/\gamma))}\widehat{c}_{t-1} \\ & -\frac{(1-\eta/\gamma)}{\sigma_c(1+(\eta/\gamma))}(\widehat{b}_t + \widehat{R}^n_t - E_t[\widehat{\pi}_{t+1}]) \ - \frac{(\sigma_c-1)(w^h_* L/c_*)}{\sigma_c(1+(\eta/\gamma))}(E_t\left[\widehat{L}_{t+1}\right] - \widehat{L}_t).\end{aligned} \tag{1}$$

The backward looking term arises in the consumption equation due to the assumptions of external habit formation captured by the parameter η.Therefore, current consumption $(\widehat{c}_t)$ depends on a weighted average of past and expected future consumption. Consumption process is also affected by the expected growth in hours worked $(E_t\left[\widehat{L}_{t+1}\right] - \widehat{L}_t)$ (due to non-separable in consumption and labour form of utility function), the ex–ante real interest rate $(\widehat{R}^n_t - E_t[\widehat{\pi}_{t+1}])$, and a disturbance term $\widehat{b}_t$. γ is the deterministic trend. $\widehat{b}_t$ is equity premium shock, which is assumed to follow a first–order autoregressive process with an iid–Normal error term: $\widehat{b}_t \ = \rho_b \widehat{b}_{t-1} + \epsilon^b_t$. Variables with stars denote the steady state values.

Solution to the profit maximization problem of intermediate labour unions and labour

packers results in the following wage equation:

$$\widehat{w}_t = \frac{1}{(1+\overline{\beta}\gamma)}(\widehat{w}_{t-1} + \overline{\beta}\gamma E_t[\widehat{w}_{t+1}] - (1+\overline{\beta}\gamma\iota_w)\widehat{\pi}_t + \iota_w\widehat{\pi}_{t-1} + \overline{\beta}\gamma E_t[\widehat{\pi}_{t+1}] \quad (2)$$
$$+\frac{(1-\xi_w\overline{\beta}\gamma)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}[\frac{1}{1-\eta/\gamma}\widehat{c}_t - \frac{\eta/\gamma}{1-\eta/\gamma}\widehat{c}_{t-1} + \sigma_l\widehat{L}_t - \widehat{w}_t] + \widehat{\lambda_{w,t}}$$

Due to nominal wage stickiness and partial indexation of wages to inflation, real wages adjust only gradually to the desired wage mark–up. ξ_w is a wage stickiness parameter, thus every period only $(1-\xi_w)$ fraction of intermediate labour unions can set wages optimally. Parameter ι_w measures the degree of indexation. If wages are perfectly flexible ($\xi_w = 0$), the real wage is a constant mark–up over the marginal rate of substitution between consumption and leisure. When wage indexation is zero (ι_w), real wages do not depend on lagged inflation. The wage–mark up disturbance ($\widehat{\lambda_{w,t}}$) is assumed to follow an ARMA (1,1) process with an iid–Normal error term: $\widehat{\lambda_{w,t}} = \rho_w\widehat{\lambda_{w,t-1}} - \mu_w\epsilon_{w,t-1} + \epsilon_t^w$.

Profit maximization by price–setting firms gives rise to the following New–Keynesian Phillips curve:

$$\widehat{\pi}_t = \frac{1}{(1+\overline{\beta}\gamma\iota_p)}(\iota_p\widehat{\pi}_{t-1} + \overline{\beta}\gamma E_t[\widehat{\pi}_{t+1}] + \frac{1}{((\phi_p-1)\varepsilon_p+1)}\frac{(1-\xi_p\overline{\beta}\gamma)(1-\xi_p)}{\xi_p}(\widehat{mc}_t)) + \widehat{\lambda_{p,t}} \quad (3)$$

Similar to wages, each period only a fraction of firms $(1-\xi_p)$ can re-optimize prices. Non-reoptimized prices are partially indexed to past inflation thus determining the presence of the backward-looking term in the inflation equation; ι_p denotes the indexation coefficient.

The price mark–up disturbance ($\widehat{\lambda_{p,t}}$) is assumed to follow an ARMA(1,1) process: $\widehat{\lambda_{p,t}} = \rho_p\widehat{\lambda_{p,t-1}} - \mu_p\epsilon_{p,t-1} + \epsilon_t^p$, where ϵ_t^p is an iid–Normal price mark–up shock.

The marginal cost is given by:

$$\widehat{mc}_t = (1-\alpha)\,\widehat{w}_t + \alpha\,\widehat{r}_t^k - \widehat{A}_t \quad (4)$$

Cost minimization by firms will also imply that the rental rate of capital is negatively related to the capital–labour ratio and positively to the real wage (both with unitary elasticity):

$$\widehat{k}_t = \widehat{w}_t - \widehat{r}_t^k + \widehat{L}_t. \quad (5)$$

The production sector consists of a continuum of monopolistically competitive firms producing intermediate goods. Their output is combined to produce final goods, which are sold on a perfectly competitive market. The aggregate supply is represented by a

typical Cobb-Douglas production function augmented with fixed costs and variable capital utilization:

$$\widehat{y}_t \ = \Phi(\ \alpha(\frac{1-\psi}{\psi}\widehat{r}_t^k + \ \widehat{\overline{k}}_{t-1}) \ + \ (1-\alpha)\widehat{L}_t + \widehat{A}_t) \tag{6}$$

where α is the share of capital in production, parameter Φ is one plus the share of fixed costs in production, $\widehat{r}_t^k$ is the rental rate of capital, ψ is a positive function of the elasticity of the capital utilization adjustment cost function and normalized to be between zero and one. $\widehat{\overline{k}}_t$ denotes the installed capital. As newly installed capital becomes only effective with a one–quarter lag, current capital services used in production are a function of capital installed in the previous period ($\widehat{\overline{k}}_{t-1}$) and the degree of capital utilization ($\widehat{u}_t$):

$$\widehat{k}_t \ = \ \widehat{u}_t \ + \ \widehat{\overline{k}}_{t-1}. \tag{7}$$

Total factor productivity ($\widehat{A}_t$) is assumed to follow a first-order autoregressive process: $\widehat{A}_t \ = \rho_a \widehat{A}_{t-1} + \epsilon_t^a$.

The evolution of installed capital is represented by the following expression:

$$\widehat{\overline{k}}_t = \ (1 - \frac{i_*}{\overline{k}_*})\ \widehat{\overline{k}}_{t-1} \ + \ \frac{i_*}{\overline{k}_*}\widehat{i}_t + \frac{i_*}{\overline{k}_*}(1+\overline{\beta}\gamma)\gamma^2 S''\widehat{q}_t. \tag{8}$$

The dynamics of investment is given by:

$$\widehat{i}_t = \frac{1}{(1+\overline{\beta}\gamma)}(\widehat{i}_{t-1} + (\overline{\beta}\gamma)\widehat{i}_{t+1} + \frac{1}{\gamma^2 S''}\widehat{Q}_t^k\ ) + \widehat{q}_t, \tag{9}$$

where S'' is the steady–state elasticity of the capital adjustment cost function and $\overline{\beta} = (\beta/\gamma^{\sigma_c})$ where β is the discount factor applied by households. As in CEE (2005), a higher elasticity of the cost of adjusting capital reduces the sensitivity of investment ($\widehat{i}_t$) to the real value of the existing capital stock ($\widehat{Q}_t^k$). Finally, $\widehat{q}_t$ represents a disturbance to the investment–specific technology process and is assumed to follow a first–order autoregressive process with an iid–Normal error term: $\widehat{q}_t \ = \rho_q \widehat{q}_{t-1} + \epsilon_t^{\mu}$.

In the original Smets and Wouters (2007) model financial markets do not incorporate endogenous forms of inefficiency. In particular, households can deposit funds and take loans in any quantity at the rate that might exceed the risk free rate R_t set by the central bank due to the exogenous premium. Modeling endogeous credit imperfections requires distinguishing between borrowers and lenders, and the existence of a conflict between the two parts. Therefore, a new type of agents - entrepreneurs, who take loans from the banking sector, has to be introduced. Banks finance the loans by accepting deposits from the households at the risk free rate. The financial intermediation between households and entrepreneurs is subject to frictions, which result in interest rate spread. In this paper I

follow works by Bernanke et al. (1999) and De Graeve (2008) in modeling the financial frictions.

At the end of period t, entrepreneurs purchase capital k_{t+1} from capital goods producers at price Q_t. They choose the degree of capital utilization and rent the capital services to intermediate goods firms at a rate $\widehat{r}_t^k$. The aggregate expected real return to capital is given by:

$$E_t\widehat{R}_{t+1}^K = \frac{1-\tau}{\overline{R}^K}E_t\widehat{Q}_{t+1}^k + \frac{\overline{r}^k}{\overline{R}^K}E_t\widehat{r}_{t+1}^k - \widehat{Q}_t^k \tag{10}$$

where τ is the depreciation rate, $\overline{R}^K$ denotes the steady state return to capital and $\overline{r}^k$ is the steady state rental rate.

In order to finance the capital stock, entrepreneurs can use both internal funds (net worth N_{t+1}) and loans from the bank ($B_{t+1} = Q_t k_{t+1} - N_{t+1}$). After the purchase of the capital stock, each entrepreneur experiences an idiosyncratic shock that affect their capital holdings $R_{t+1}^K Q_t k_{t+1}$. Following the financial accelerator framework of Bernanke et al. (1999), entrepreneurs have to pay a premium over the riskless rate in order to borrow funds. The difference between the cost of external finance and the risk-free rate arises because financial intermediary cannot observe the entrepreneurs' realized return and has to pay a "state verification" (monitoring) cost. The equilibrium condition is derived from the optimal debt contract problem, which maximizes welfare of the entrepreneur, combined with the zero profit condition of the bank. The log-linearized version is represented by the following equation, which relates the interest rate spread to the capital expenditures and the entrepreneurial financial position:

$$E_t\widehat{R}_{t+1}^K = -el\left\{E_t\left[\widehat{N}_{t+1} - \widehat{Q}_t^k - \widehat{k}_{t+1}\right]\right\} + \widehat{R}_t + \widehat{b}_t \tag{11}$$

where $\widehat{R}_t = (\widehat{R}_t^n - E_t[\widehat{\pi}_{t+1}])$, is the risk-free real interest rate; el represents the elasticity of the external finance premium to the change in the financial conditions. The equation above indicates that in equilibrium an entrepreneur purchases capital up to the point where the expected real return to capital is equated to the marginal cost of external finance. The higher fraction of the project value is financed by the entrepreneur's internal funds (the higher is the net worth N relative to the gross value of capital $Q^k k$), the lower the capital market friction, and the lower is the corresponding risk premium. Absence of financial frictions implies the case when entrepreneurs have sufficient net worth to finance the demand for capital stock. In such a situation, the risk of default associated with borrowing the external funds vanishes, risk free rate and the real return to capital coincide, and the model reduces to the model of Smets and Wouters (2007). $\widehat{b}_t$ describes exogenous fluctuations in the risk premium, not captured by financial frictions of Bernanke.

Therefore, in this model, financial accelerator mechanism consists of both endogenous and exogenous components.

Financial wealth of entrepreneurs is given by the difference between the revenue from capital holding in time t and the cost of borrowing (rate of return paid by entrepreneurs on time $t-1$ loans):

$$\widehat{N}_{t+1} = \varkappa \overline{R}^K \left[\frac{\overline{k}}{\overline{N}} \widehat{R}_t^K - \left(\frac{\overline{k}}{\overline{N}} - 1 \right) \widehat{R}_t - el \left(\frac{\overline{k}}{\overline{N}} - 1 \right) \left(\widehat{k}_t + \widehat{Q}_{t-1}^k - \widehat{N}_t \right) + \widehat{N}_t \right] \tag{12}$$

Combining equations (11) and (12), entrepreneurial net worth can be also expressed in the form of the following accumulation equation:

$$\widehat{N}_{t+1} = \varkappa \overline{R}^K \left[\frac{\overline{k}}{\overline{N}} \left(\widehat{R}_t^K - E_{t-1} \widehat{R}_t^K \right) + E_{t-1} \widehat{R}_t^K + \widehat{N}_t \right] \tag{12a}$$

where $\varkappa$ is the entrepreneurial survival rate and $\overline{k}/\overline{N}$ is the steady state ratio of capital to net worth, i.e. the inverse of the leverage ratio. The equation (12a) demonstrates that, in general terms, the endogenous variations in the next period net worth come from the unexpected changes in the real return to capital. In this model, variability of asset prices is one of the main sources of such a volatility. The values of the parameters $\varkappa$, $\overline{k}/\overline{N}$, and el determine the impact of financial frictions on the real economy. The higher the entrepreneurial survival rate and the capital to net worth steady state ratio, the more persistent the evolution of the net worth will be. Combined with higher elasticity of the external finance premium, this would imply stronger adjustment of the wedge between the expected return to capital and the risk-free rate. Therefore, the shocks affecting the entrepreneurial net worth would have greater real effects.

The aggregate resource constraint is given by:

$$\widehat{y}_t \;=\; \widehat{g}_t \;+\; \frac{c_*}{y_*}\widehat{c}_t \;+\; \frac{i_*}{y_*}\widehat{i}_t \;+\; \frac{r_*^k k_*}{y_*}\widehat{u}_t + u_{bankr,t}. \tag{13}$$

where $u_{bankr,t}$ measures the bankruptcy cost (small under reasonable parametrization, and therefore typically neglected). $\widehat{g}_t$ is exogenous spending, which is assumed to follow a first–order autoregressive process with an iid–Normal error term and is also affected by the productivity shock as follows: $\widehat{g}_t \;= \rho_g \widehat{g}_{t-1} \;+ \rho_{ga}\epsilon_t^a + \epsilon_t^g$. The latter is empirically motivated by the fact that in estimation exogenous spending also includes net exports, which may be affected by domestic productivity developments.

Finally, the model is closed by adding the following empirical monetary policy reaction

function:

$$\begin{aligned}\widehat{R}_t^n &= \rho_R \widehat{R}_{t-1}^n + (1-\rho_R)(r_\pi \widehat{\pi}_t + r_y \widehat{ygap}_t) \\ &\quad + r_{\Delta y}(\widehat{ygap}_t - \widehat{ygap}_{t-1}) + r_t\end{aligned} \tag{14}$$

The monetary authority follows a generalized Taylor rule responding to inflation and the output gap terms (current and lagged). The latter one is defined as the difference between actual and potential output. The output gap is given by $\widehat{ygap}_t = \widehat{y}_t - \widehat{A}_t$. The parameter ρ_R captures the degree of interest rate smoothing. I assume that the monetary policy shock (r_t) follows a first–order autoregressive process with an iid–Normal error term: $\widehat{r}_t = \rho_r \widehat{r}_{t-1} + \epsilon_t^r$.

3 Estimation strategy and results

I estimate several model specifications. In particular, I estimate versions with and without financial frictions in order to assess the empirical validity of the financial accelerator mechanism. In addition, I estimate each model under the assumption of RE and with learning. Thus I have two dimensions of comparison – the effect of financial frictions and the impact of expectations. Moreover, when assessing the effects of financial accelerator under learning I experimented with alternative adaptive learning schemes, which differ in terms of variables used by agents to form the forecasts. The log-linearized versions of the models are estimated using Bayesian methods. These methods combine likelihood function of the data with a prior density to derive the posterior distribution of the structural parameters. A prior density contains information about the model parameters from other sources (microeconometric and calibration evidence). The estimation procedure included: at first, the estimation of the mode of the posterior distribution by maximizing the log posterior function; secondly, the Metropolis–Hastings algorithm was used to compute the posterior distribution and to evaluate the marginal likelihood of the model. Typically, 300 000 to 500 000 MCMC draws were performed, using three chains. For more details on Bayesian estimation of DGSE models, see An and Schorfheide (2007). In order to speed up the convergence, I employed so-called Adaptive Metropolis–Hastings algorithm for the estimation of the most complicated version of the model, i.e. adaptive learning with financial frictions. This method was proposed by H.Haario, E.Saksman, and J.Tamminen (2001). They show that in some cases, the performance of the Adaptive MH is significantly better than the standard random-walk Metropolis Hastings when dealing with DSGE models, in the sense that it explores the posterior distribution more efficiently and accurately.

I choose priors following Smets and Wouters (2003 and 2007). These papers present a careful description of the estimation methodology as well as the justification for the choice of priors. The priors for additional parameters related to the financial frictions are based on calibration exercises and previous literature (Bernanke et al., 1999; De Graeve, 2008). In particular, $\overline{R}^K \sim Normal(1.0149, 0.002)$. $\varkappa$, $\overline{k}/\overline{N}$, and el are assumed to have Uniform priors with sufficient standard deviations. The choice of the flat and disperse priors enables checking whether macroeconomic time series used in the estimation contain the information about financial frictions parameters.

3.1 Data and measurement equations

The model is estimated using seven key macro–economic quarterly US time series as observable variables: real GDP, real consumption, real investment, real wage, hours worked, GDP deflator and the federal funds rate. Nominal variables are deflated by GDP-deflator. Aggregate variables are expressed in per capita terms. All variables except hours, inflation, and interest rate - are taken in first differences. Thus the data set is the same as in Smets and Wouters (2003, 2007). Financial variables are not included in the set of observables in order to facilitate the comparison with the results from the previous studies. Moreover, De Graeve (2008) points out that it is rather problematic to find the proxy for the net worth or external finance premium that would be consistent with the model dynamics. The sample period is from 1954:1 till 2008:3. Therefore, the estimated model is augmented with a set of the following measurement equations:

$$\begin{bmatrix} dlGdp_t \\ dlCons_t \\ dlInv_t \\ dlWage_t \\ lHours_t \\ dlP_t \\ FedFundsR_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma_y} \\ \overline{\gamma_c} \\ \overline{\gamma_i} \\ \overline{\gamma_w} \\ \bar{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} \widehat{y}_t - \widehat{y}_{t-1} \\ \widehat{c}_t - \widehat{c}_{t-1} \\ \widehat{i}_t - \widehat{i}_{t-1} \\ \widehat{w}_t - \widehat{w}_{t-1} \\ \widehat{l}_t \\ \widehat{\pi}_t \\ \widehat{R}^n_t \end{bmatrix}, \tag{15}$$

where l and dl stand for log and log difference respectively. Unlike Smets and Wouters (2007), I estimate separately the trends for output, consumption, investment , and wages growth rates, instead of imposing a common trend on these variables. $\overline{\pi} = 100(\Pi_* - 1)$ is the quarterly steady–state inflation rate and is $\overline{r} = 100(\overline{\gamma}^{\sigma_c}\Pi_*/\beta - 1)$ the steady–state nominal interest rate. Given the estimates of the average trend growth rate and the steady–state inflation rate, the latter will be determined by the estimated discount rate. Finally, $\bar{l}$ is steady–state hours–worked.

3.2 Estimation under adaptive learning

I implement the adaptive learning within the Dynare 3.064 Matlab toolbox which is used to estimate and simulate DSGE models. I use the toolbox developed by Slobodyan and Wouters (2009, 2010). Agents learn the model parameters using the Kalman filter algorithm. The alternative, widely used learning method is the constant gain Recursive Least Squares (RLS). Sargent and Williams (2005) demonstrated that both learning methods mentioned above are asymptotically equivalent on average. However, their transitory behavior may differ significantly. In particular, Kalman filter tends to result in much faster adjustment of agents' beliefs. Therefore, I opt for Kalman filter and estimate several adaptive learning specifications, which differ in terms of the information sets used by agents to form their beliefs about the forward looking variables:

- "AR(2)+constant": forecasting equation for every forward-looking variable includes two own lags and a constant. Thus, agents form and update their beliefs about the persistence and expected mean of endogenous variables.
- "AR(2)" : forecasting equation for every forward-looking variable includes only two own lags (without a constant).
- "All states": forecasting equation for every forward-looking variable includes all the state variables. Therefore, functional form of the relationship between forward and state variables is very similar to MSV Rational Expectation Equilibrium (REE) reduced form.

3.2.1 Learning Setup

In this section I present a general description of Kalman filter learning setup. For more details see Slobodyan and Wouters (2010).

It is assumed that the model is driven by the exogenous process w_t, which follows an AR(1) process:

$$w_t = \Gamma w_{t-1} + \Pi \epsilon_t. \tag{16}$$

Dynare represents the model in the following way:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+1} + B_0 \epsilon_t = 0, \tag{17}$$

where the vector y_t includes endogenous variables of the model. The solution of the model is provided by Dynare as

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu + T \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R\epsilon_t. \tag{18}$$

The vector y contains state variables y^s (those appearing with a lag), forward variables y^f that appear with a lead in the model, and the so–called static variables.[1] Deviation from the rational equilibrium assumption implies that agents form the predictions of the lead variables using a linear function of the endogenous model variables (Marcet and Sargent (1989); Evans and Honkapohja (2001)).

$$y_j^f = X_j \beta_j + u_j \tag{19}$$

In the adaptive learning literature, this equation is called the Perceived Law of Motion (PLM). The agents then use the linear model (19) for forecasting, with forecasts given as

$$y_{j,t}^f = X_{j,t-1} \beta_{j,t-1} + u_{j,t}.$$

Data matrix $X_{j,t-1}$ includes a set of variables which are used to form predictions about forward-looking variable j. In particular, $X_{j,t-1}$ may consist of all the state variables of the model. In my estimations, such a specification would correspond to "All states" learning model. Simpler forms of forecasting equations may imply the presence of the subset of endogenous variables, for example only one or two lags of the corresponding forward-looking variable on the RHS (as in the "AR(2)" model). In addition, $X_{j,t-1}$ may also incorporate a constant. The errors $u_{j,t}$ are different linear combinations of true model errors with variance-covariance matrix $\Sigma = E\left[u_t u_t^T\right]$. In all learning specifications considered in this paper, I assume that agents do not access values of exogenous process parameters when forming the predictions.

The agents believe that the coefficients β follow a vector autoregressive process:

$$vec\left(\beta_t\right) = F \cdot vec\left(\beta_{t-1}\right) + v_t, \tag{20}$$

where F is a diagonal matrix with $\rho \leq 1$ on the main diagonal, and use Kalman filter to update their beliefs about β. Updating of the beliefs at any t depends on the data (best estimates of the state, the lead and the exogenous variables at time $t-1$) and on the initial beliefs. I assume that initial beliefs are consistent with the REE. Thus initial values for β $(\beta_{1|0})$, variance-covariance matrix as well as starting values for the Kalman filter parameters are derived based on the correlations between the model variables, implied by the rational expectations equilibrium for the currently evaluated parameter vector. Specifically, $\beta_{1|0}$ is given by the projection of X on y :

$$\beta_{1|0} = E\left[X^T X\right]^{-1} \cdot E\left[X^T y\right].$$

[1] y^f and y^s could intersect.

Given $\beta_{1|0}$, variance-covariance matrix is calculated as:

$$\Sigma = E\left[\left(y_t^f - X_{t-1}\beta_{1|0}\right)\left(y_t^f - X_{t-1}\beta_{1|0}\right)^T\right],$$

More detailed description of the procedure can be found in Slobodyn and Wouters (2010).

The beliefs generated in the Kalman filter step are then used to generate expectations of forward–looking variables according to forecasting equations. Plugging these expectations into (17), we can solve the purely backward–looking equations to obtain a representation

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t\epsilon_t. \tag{21}$$

This equation represents the Actual Law of Motion (ALM). Thus, the estimation of a DSGE model under adaptive learning reduces to calculating a time–varying law of motion. The values of μ_t, T_t, and R_t are then used to form expectations of the next period model variables in the main Kalman filter step, used to calculate the model likelihood. The rest of the standard Dynare toolbox stays unchanged. The time-varying procedure makes T_t a complicated function of the data, current parameters, and beliefs that could easily become unstable for one or several periods. Such discontinuities in the evolution of beliefs lead to numerical problems during estimation and deterioration of the estimation results. In particular, allowing T_t to be explosive for some periods leads to the increase of forecasting errors and thus to much worse likelihood. In this paper, I have to deal with explosive dynamics of T_t for some time periods when estimating the DSGE model under non-MSV learning (when agents use limited information set to form predictions). This problem seems to be more important for estimations of a model with financial accelerator. In particular, financial frictions introduce additional volatility, which in turn may lead to more frequent and sizable adjustments of beliefs. Thus the probability of the eigenvalues of T_t to jump outside of the unit circle also increases. However, in all the estimations I performed, the number of periods with unstable eigenvalues does not exceed 5. As is common in the learning literature, I use a projection facility that skips an updating in such cases.

3.3 Estimation results

3.3.1 Model fit

The fit of a model estimated using Bayesian methods can be ascertained using marginal data density, defined as

$$p\left(Y|\mathcal{M}\right) = \int \mathcal{L}\left(\theta|Y\right)p(\theta)d\theta,$$

where $\mathcal{L}(\theta|Y)$ is the likelihood function of the data Y given parameters of the model θ, and $p(\theta)$ is the prior density. This measure allows a straightforward comparison of several models estimated on the same data with respect to a reference model. Posterior odds ratio, a measure of how much more likely a model $\mathcal{M}_1$ is when compared to the model $\mathcal{M}_2$, is given by

$$\frac{\pi(\mathcal{M}_1)}{\pi(\mathcal{M}_2)} \cdot \frac{p(Y|\mathcal{M}_1)}{p(Y|\mathcal{M}_2)},$$

where $\pi(\mathcal{M}_i)$ represents prior probability of a model $\mathcal{M}_i$. The first term in the above expression is known as prior odds, and the second as Bayes factor. Usually, the prior probabilities are taken to be equal, and thus a posterior odds ratio equals the corresponding Bayes factor. For more details on model comparison, consult An and Schorfheide (2007).

Table 1 reports logarithms of marginal data densities for the various specifications I have estimated. I compare the results for models with RE and AL. In addition, each version was estimated with financial accelerator mechanism (FA) and without ($noFA$).

Table 1: Model Comparison in terms of Marginal Likelihood

Model specification	*FA*	*noFA*
REE	-1207.8	-1231.52
Kalman Filter AL :		
AR(2)+constant	-1198.39	-1209.8
AR(2)	-1201.44	-1212.46
All states (near MSV)	-1206.11	-1234.55

Log marginal data densities for the models with and without financial accelerator. Learning specifications vary depending on the information set used to form predictions; initial beliefs are REE-consistent. Bayes factor — a relative probability of one model over another, equals exp of the difference between the corresponding log densities.

The estimation results suggest that the REE model with financial accelerator fits data much better compared to a version that does not incorporate financial frictions (similar to findings of De Graeve, 2008 and Christensen and Dib, 2008). Ability of AL to improve the data fit can be clearly observed for "$noFA$" specifications. In particular, Table 1 indicates that RE hypothesis in this case is definitely restrictive. Relaxing the rationality assumption through introduction of Kalman filter AR(2) learning significantly improves the fit of the model. Such a result can imply that additional volatility and time variation introduced by adaptive learning can correct for some of the model misspecification. Performance of adaptive learning model based on more complicated forecasting functions (i.e. "All states") is essentially the same as the RE model. Table 1 also demonstrates that the improvement in the data fit under learning is lower once financial frictions are introduced. In order to shed more light on this result, I analyze the relative likelihood (evaluated at the posterior mode) of alternative model specifications as a function of time.

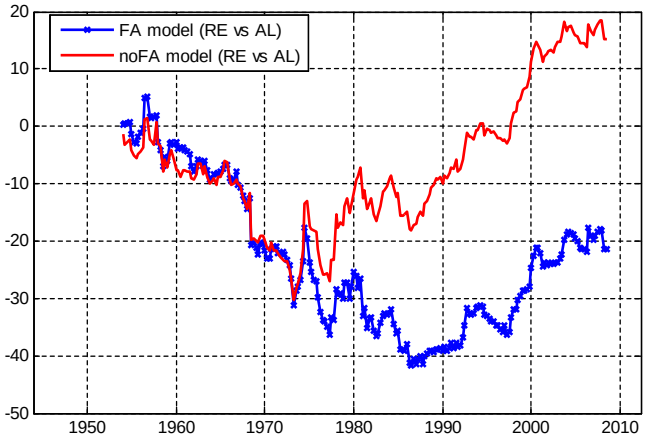

Figure 1: Cumulative likelihood for "AR(2)+const." learning specification with and without financial accelerator relative to corresponding RE model.

I would like to find out how introducing financial accelerator mechanism changes the relative performance of RE and AL models over time, i.e. for which time periods the "best" learning model outperforms the RE version with financial accelerator and why. Firstly, I will illustrate graphically how departure from the RE hypothesis affects the data fit in models with and without financial accelerator. Figure 1 shows the cumulative likelihood for "AR(2)+constant" learning models with and without financial accelerator *relative* to the corresponding RE models.[2] The upward trend of the cumulative difference line indicates that on average the likelihood of the learning model on this time interval is better relative to the RE one.

Figure 1 indicates that the RE and learning models estimated with or without financial accelerator delivered very similar relative data fit before the beginning of 70-s. In particular, on average the RE model fitted data better than the learning model. At the same time, the model with adaptive expectations and frictionless financial markets significantly outperformed the corresponding RE model in 1973-1974, the late 70-s, and the beginning of 80-s. The aptitude of the learning model to describe the data generating process has been improving gradually but surely since late 80-s. Graphs presented in the Figure 1 also show that the relative performance of the RE and learning models in "FA" and "noFA" specifications differ the most during middle 70-s and 80-s. This period was characterized by two severe recessions accompanied by increased volatility of inflation, consumption, investment growth as well as labor hours. In a version without financial accelerator, time variation introduced by AL enables capturing varying economic processes

[2] I compute the difference in likelihood for AL and RE models, and plot the cumulative sum of this difference.

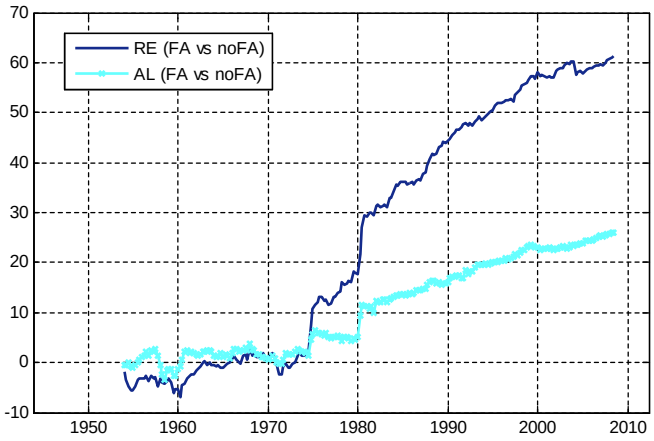

Figure 2: Cumulative likelihood for RE and AL models with "FA" relative to corresponding "noFA" models.

and thus contributes to explaining the increased instability. In other words, learning can partially substitute for the absence of financial market inefficiencies. The inclusion of financial frictions considerably improves the performance of the RE model especially during middle 70-s and the beginning of 80-s. Thus, the relative gain from modeling the time varying transmission is somewhat reduced. Figure 1 also demonstrates that learning introduced in a model with financial accelerator steadily adds to the improved data fit after late 1980-s. The overall gain is, however, more modest than for the "noFA" model.[3]

In order to illustrate the contribution of the financial accelerator mechanism to the data fit under alternative expectation assumptions, I calculate the cumulative likelihood for the RE model with financial accelerator relative to the RE version which does not incorporate financial frictions. The same cumulative difference in likelihood is computed for the model with "AR(2)+const." adaptive learning scheme. Figure 2 compares the results. Upward trend of the likelihood differences indicates that "FA" specification (with RE or AL) fits the data better than "noFA" model. Figure 2 indicates that integrating financial frictions into the DSGE model with either rational or adaptive expectations improves the data fit. The highest gain in likelihood is observed in middle 70-s and beginning of 80-s and then gradually increases. The overall gain is much greater for the model with rational expectations, whose performance appears to be more sensitive with respect to inclusion of financial frictions.

Forecasting performance is an important criterion in model selection. In fact, likelihood function can be interpreted as a measure of one period ahead prediction error. Figure

[3]In the next subsections we will compare the implied persistence of the main macroeconomic variables under assumptions of RE and AL, and will add to the explaining the results presented above.

2a compares one step ahead forecasting performance of the RE and the best performing AL model. In particular, I contrast the forecasts and the actual dynamics of inflation, output growth, and investment growth. Moreover, the graph contains the results up to the latest period, i.e. 2011q3. The analysis of the model performance on the data interval covering the recent financial crisis is of particular interest. However, this period is not included in the baseline estimation sample due to the fact that "crisis" observations produce too adverse model dynamics and significantly distort the estimation results. Therefore, in order to check the forecasting performance on the period starting from 2008 q3, I fix the parameters at the corresponding posterior mode values and run the forecast forward without the systematic update. Figure 2a illustrates that AL model was more successful in predicting inflation in the middle of 70-s and the beginning of 80-s. Output and investment growth was better predicted by AL model during the recessions in the beginning of 80-s and 90-s. In addition, learning model is particularly successful in capturing the most recent economic downturn, characterized by a significant and persistent drop in the real sector.

Therefore, the main results of this section can be summarized as follows. Introducing both adaptive learning and financial accelerator mechanism into the otherwise standard DSGE model can contribute to capturing the properties of real data on certain time intervals, especially on the second half of the data sample.

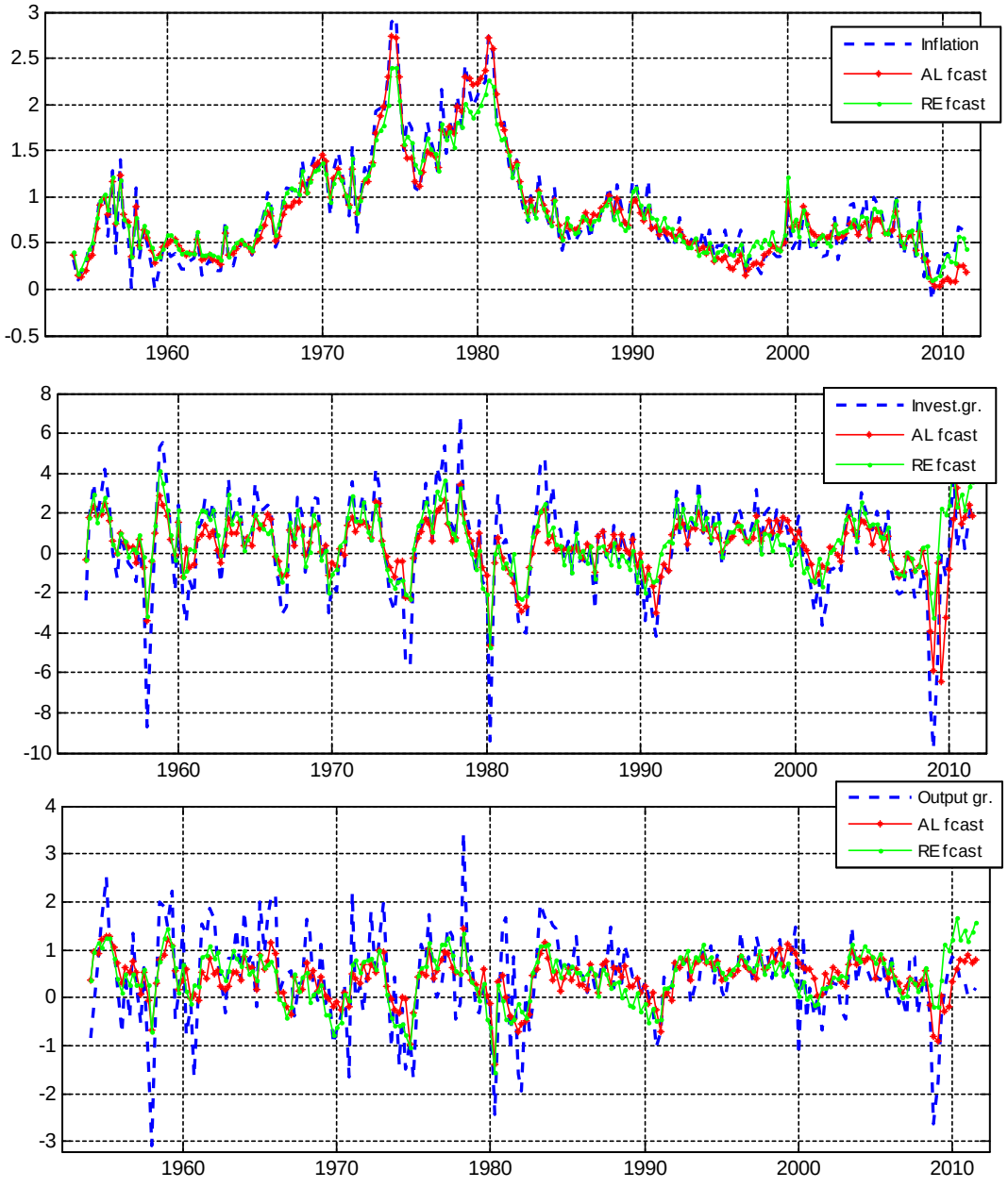

Figure 2a: One step ahead forecasting performance under RE and AL.

3.3.2 Parameter estimates

In this section I will present the MCMC results for the RE and "AR(2)+constant" learning models estimated with and without financial frictions. I will also report the results from the posterior maximization of all adaptive learning models and contrast them with RE. The results of the model comparison in terms of the estimated parameters are presented in Tables 2 and 3. Apart from the financial sector, the model is based on Smets and Wouters (2007) and Slobodyan and Wouters (2010). Therefore, in a version without financial accelerator, I can observe very similar pattern in deviations of the parameters estimated under "AR(2)+const" learning from the parameters obtained under RE. In particular, I find that some of the estimated structural rigidities and shock persistence decrease. More specifically, autoregressive components of exogenous processes for price and wage mark up shocks fall significantly. The decline in the persistence of the shock to investment technology is not so dramatic, but still notable. The confidence bounds for this parameter clearly shift left and do not overlap with the range of possible values implied by the posterior distribution of the RE model. In addition, the variance of the investment shock declines under learning. These results imply that learning is helpful in explaining inflation, wage, and investment dynamics. Modeling adaptive expectations of these variables introduces "endogenous" persistence, which has empirically appealing economic interpretation. At the same time, the autoregressive coefficient of the exogenous equity premium shock, which can be viewed as a proxy for financial markets inefficiencies, tends to increase under learning. This may imply that learning cannot substitute for financial frictions. The parameters of structural rigidities do not show a consistent change. The degree of price rigidities, wage indexation, and to some extent wage stickiness decline. For AL model without financial accelerator, a significant decline in the investment adjustment cost parameter is observed. The degree of habit formation is also estimated at somewhat lower level. At the same time, price indexation tends to increase under learning. We may conclude that learning is an important source of endogenous inertia, but it can only partially substitute for "mechanical" source of rigidities and persistence of some of the disturbances.

Analyzing the estimated parameters of the model that incorporates financial frictions, I would like to highlight several interesting details. Three parameters - capital to net worth ratio, entrepreneurial survival rate, and the elasticity of the external finance premium, are jointly responsible for financial accelerator effects in the model. Higher value of these parameters strengthens the impact of financial frictions on the real economy. Comparing the results for RE and AL models presented in the Table 2, I can see that estimated capital to net worth ratio tends to increase under learning from 2.89 to 3.02 at the posterior mean (the confidence bounds, however, very much overlap). The confidence

bounds of the elasticity of the risk premium also rise slightly under learning. Thus, it appears that there might be the tendency for these financial parameters to trend up under learning. Entrepreneurial survival rate stays essentially the same. The posterior distributions for the financial parameters have nice shapes (uni-modal). This suggests that data is quite informative about the degree of financial markets frictions. Our estimates of the elasticity (0.0186 in the posterior mean for the RE model , and 0.0196 for AL) are somewhat lower comparing to the regression and calibration results from the previous literature.[4] In particular, Bernanke et al. (1999) calibrates $el = 0.05$ based on realistic value of monitoring costs and bankruptcy rates. Christensen and Dib (2008) estimate this parameter at the level 0.042. However, they calibrate the remaining financial parameters at the lower level. De Graeve (2008) reports the value of the elasticity 0.1047. At the same time, his estimated k/N ratio is twice lower comparing to my results. Therefore, the estimated overall impact of financial frictions has comparable magnitude across different studies.

Results presented in Tables 2 and 3 demonstrate that introducing financial accelerator influences some of the structural parameters. Comparing the regression results for RE models I notice that investment adjustment costs increase slightly in the specification with financial frictions; the autoregressive parameter of the exogenous equity premium shock declines. The latter fact may suggest that introducing financial accelerator captures some of the persistence in fluctuations of the external risk premium. However, the variability of the exogenous premium shock is not falling, which implies that the type of financial friction considered in this paper cannot fully explain the equity premium dynamics. In AL specifications, introducing financial accelerator leads to even more pronounced increase in the investment adjustment cost parameter (φ). The autoregressive component of the exogenous risk premium shock shows some decline comparing to its value estimated in models without FA. I can explain the rise in k/N and φ as follows. Equation (11) demonstrates that external finance premium can be decomposed into endogenous and exogenous components: $(E_t\widehat{R}^K_{t+1} - \widehat{R}_t) = -el\left\{E_t\left[\widehat{N}_{t+1} - \widehat{Q}^k_t - \widehat{k}_{t+1}\right]\right\} + \widehat{b}_t$. Modeling adaptive expectations of asset prices, rental rate,and inflation (all are forward-looking variables) lead to more persistent evolution of $\widehat{R}^K$ (see equation (10)) and $\widehat{R}$, and thus of the risk premium. At the same time, in this very simple form of financial frictions, endogenous component of the risk premium does not incorporate sufficient persistence mechanism related either to expectations formation or other financial markets features. Agents do not form predictions about the net worth or capital (both are state variables). Stronger (on impact) and more persistent response of these variables can be achieved only via increase of "mechanical" factors - adjustment costs (for capital) and k/N (for

[4] In fact, we can compare only with estimation results for models with rational expectations.

net worth). It would be interesting to check how adaptive learning affects the parameters in the model with more elaborated types of financial frictions.

Table 2: Comparison of RE and AR(2) learning models in terms of the estimated parameters

Parameters		Prior distribution			Posterior, RE model			Posterior, AL model		
		Type	Mean	St.err	5%	Mean	95%	5%	Mean	95%
Shocks										
exo.risk prem.	σ_b	I.Gam	0.1	2	2.3479	2.9965	3.5874	1.4758	2.0498	2.7821
investment	σ_q	I.Gam	0.1	2	0.4536	0.5396	0.6221	0.3752	0.4309	0.4972
price markup	σ_p	I.Gam	0.1	2	0.1344	0.1598	0.1844	0.1654	0.1849	0.2054
wage markup	σ_w	I.Gam	0.1	2	0.1962	0.2245	0.2547	0.2042	0.2269	0.2476
AR.coeff-s										
exo.risk prem.	ρ_b	Beta	0.5	0.2	0.0288	0.1253	0.2174	0.2175	0.3243	0.4441
investment	ρ_q	Beta	0.5	0.2	0.5812	0.6709	0.7615	0.3003	0.4030	0.5272
price markup	ρ_p	Beta	0.5	0.2	0.8372	0.9002	0.9634	0.0357	0.2018	0.3522
wage markup	ρ_w	Beta	0.5	0.2	0.8985	0.9315	0.9699	0.2964	0.5279	0.7721
pr.markup,ma	μ_p	Beta	0.5	0.2	0.6458	0.7752	0.9020	0.2763	0.4260	0.5637
w.markup,ma	μ_w	Beta	0.5	0.2	0.7944	0.8566	0.9303	0.1551	0.4146	0.6833
Str.params										
adjust. cost	φ	Norm	4	1.5	4.7261	6.3475	7.9322	4.3979	5.9552	7.4496
habit	η	Beta	0.7	0.1	0.6258	0.6873	0.7510	0.6850	0.7335	0.7967
Calvo wages	ξ_w	Beta	0.5	0.1	0.6737	0.7633	0.8500	0.6974	0.7500	0.8043
Calvo prices	ξ_p	Beta	0.5	0.1	0.6255	0.6914	0.7579	0.5259	0.5904	0.6455
index. wages	ι_w	Beta	0.5	0.15	0.2534	0.4582	0.6378	0.0921	0.1914	0.2912
index. prices	ι_p	Beta	0.5	0.15	0.0772	0.1933	0.3052	0.2382	0.4139	0.5849
int.rate smooth	ρ_r	Beta	0.75	0.1	0.8352	0.8603	0.8859	0.8982	0.9193	0.9392
pol rule, inflat.	r_π	Norm	1.5	0.25	1.6977	1.9354	2.1733	1.4630	1.7521	2.0222
cap./net worth	k/N	U	1	3.5	2.3413	2.8907	3.5000	2.4914	3.0231	3.5000
survival rate	$\varkappa$	Norm	0.9	1	0.9142	0.9451	0.9787	0.9039	0.9466	0.9672
elast.risk prem	el	Norm	0	0.5	0.0031	0.0186	0.0318	0.0039	0.0196	0.0351
gain (AL)	g	Beta	0.5	0.289	-	-	-	0.9751	0.9842	0.9931

4 Financial frictions under learning. Time variation and transmission mechanism

4.1 Evolution of agents' beliefs and implied persistence

Adaptive learning can affect model fit in several ways. First, time variation of beliefs allows the model to become time varying (21). This could improve the model fit if the process that generates time series of observed variables is itself time-varying. On the other hand, if the beliefs updating process is too volatile, parameter uncertainty could

Table 3: Comparison of RE and alternative learning models in terms of the estimated parameters (Posterior Mode)

Parameters / Poster.Mode		RE		AR(2)+const		AR(2)		All states	
		FA	noFA	FA	noFA	FA	noFA	FA	noFA
St.er shocks									
exo.risk prem.	σ_b	2.9133	2.6325	1.7918	1.3622	1.8087	1.6461	3.7498	2.4496
investment	σ_q	0.5283	0.4761	0.3959	0.4452	0.4041	0.4525	0.9005	0.911
price markup	σ_p	0.1624	0.1642	0.1852	0.1926	0.188	0.1895	0.1347	0.1198
wage markup	σ_w	0.2255	0.2279	0.2277	0.2238	0.2279	0.2233	0.2564	0.2631
AR. coeff-s									
exo.risk prem.	ρ_b	0.1091	0.2197	0.3881	0.4515	0.3897	0.4293	0.2318	0.368
investment	ρ_q	0.6753	0.6887	0.4265	0.4303	0.4192	0.3924	0.4233	0.4633
price markup	ρ_p	0.9006	0.8783	0.3206	0.1698	0.1628	0.1637	0.1518	0.1514
wage markup	ρ_w	0.944	0.9681	0.5702	0.5357	0.553	0.7273	0.8625	0.7815
pr.markup,ma	μ_p	0.7881	0.757	0.5020	0.3886	0.4123	0.3919	0.4447	0.4937
w.markup,ma	μ_w	0.8896	0.9234	0.4520	0.4265	0.4334	0.594	0.7772	0.6359
Str. params									
adjust. cost	φ	6.0523	5.2727	5.9774	3.299	5.7383	3.8392	5.6242	4.4685
habit	η	0.6765	0.7477	0.7194	0.6631	0.7201	0.6895	0.8347	0.7855
Calvo wages	ξ_w	0.7807	0.7825	0.7329	0.7498	0.7375	0.7373	0.8262	0.8452
Calvo prices	ξ_p	0.6947	0.6822	0.5756	0.5603	0.575	0.5546	0.7295	0.7041
index. wages	ι_w	0.4472	0.424	0.2020	0.2015	0.1933	0.2014	0.3695	0.4453
index. prices	ι_p	0.1822	0.1727	0.4184	0.4316	0.4501	0.4529	0.5504	0.5057
int.rate smooth	ρ_r	0.8580	0.8603	0.9209	0.9216	0.9198	0.9241	0.8814	0.8814
pol rule, inflat.	r_π	1.9251	1.8771	1.6692	1.7382	1.6609	1.6409	1.6396	1.6545
cap./net worth	k/N	3.4950	-	3.4525	-	3.499	-	3.50	-
survival rate	$\varkappa$	0.937	-	0.9447	-	0.9431	-	0.90	-
elast.risk prem	el	0.013	-	0.0197	-	0.0187	-	0.0047	-
gain (AL)	g	-	-	0.9881	0.9868	0.9872	0.9917	0.8458	0.8896

lead to deterioration of the fit. Another channel through which adaptive learning operates is through change in the transmission mechanism. Even when beliefs are consistent with a REE and are not time varying, the changes in the information set used by the agents to form the expectations (which will introduce differences from the MSV set) will lead to the divergence of the transmission mechanism from that under the RE. From the estimation results I can see that both factors mentioned above matter. Some of the parameters in adaptive learning models differ from those obtained under RE. Moreover, there is significant divergence in parameters estimated under alternative learning schemes (both versions of AR(2) and "All states"). In particular, the elasticity of the external finance premium parameter in "All states" model is several times lower than in "AR(2)+const" model. The discrepancy is also observed across the structural parameters. As a result, financial markets frictions may have very different macroeconomic implications depending on the assumptions about the rationality of expectations and the information set.

Figure 3 plots time variation of the agents' beliefs given by the coefficients of forecasting functions for "AR(2)+constant" adaptive learning model. I present the evolution of the autoregressive component, given as a sum of AR(1) and AR(2) coefficients, and a constant. In other words I plot agents' Perceived Law of Motion (PLM) given by (19).

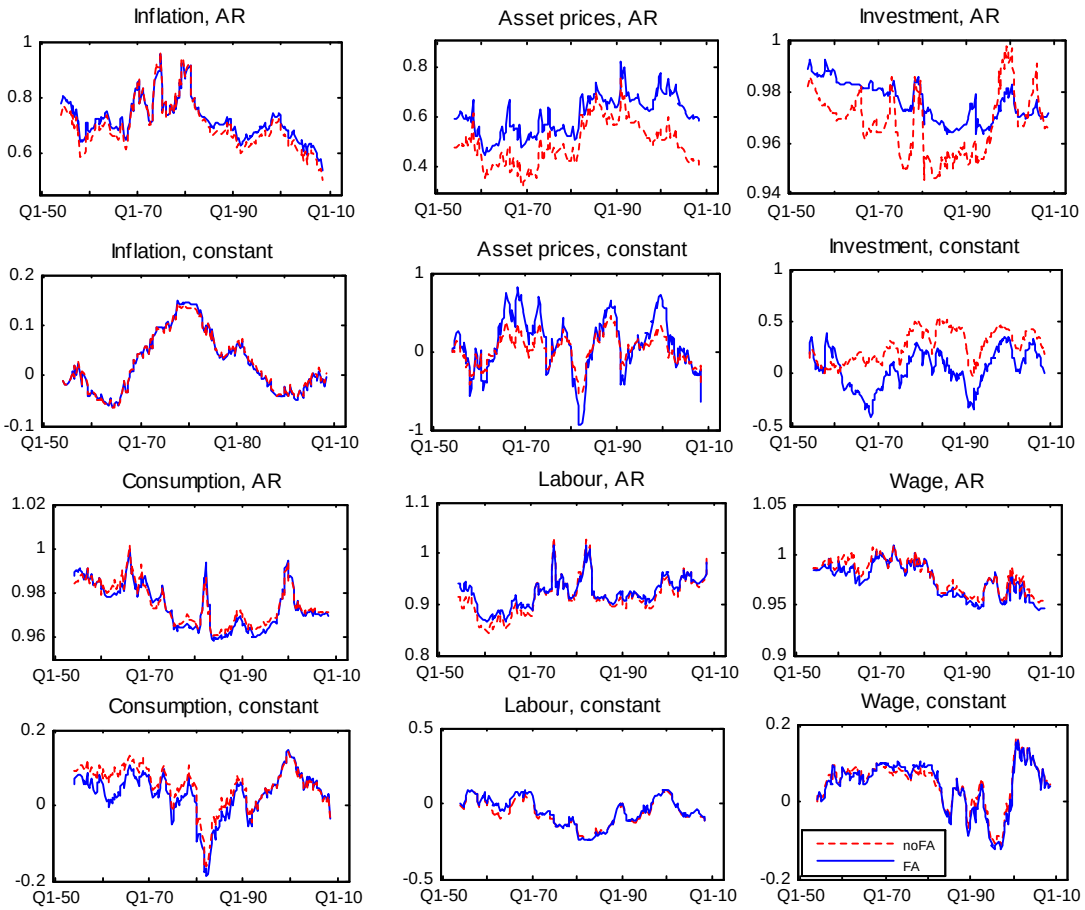

Figure 3: Beliefs about key macroeconomic variables: persistence and constant

Figure 3 illustrates that agents perceive real consumption, labour, wages, and investment as highly persistent processes with relatively stable autoregressive parameters. At the same time, Kalman filter learning introduces significant time variation in agents beliefs about inflation and asset prices. The perceived inflation persistence displayed peaks around mid 1970-s and again around 1980, then gradually declined to the level around 0.6 since middle of 1980-s. The perceived asset price persistence evolved in the range 0.4-0.8, with noticeable spikes in 1980-s, 90-s and 2000-s. In the model with financial accelerator, the perceived asset price persistence is higher compared to the persistence estimated in a version without financial frictions. Perceived inflation persistence is not significantly affected by the presence of financial frictions (it is just slightly higher after 80-s). Agents also believe that imperfect financial markets make investment process more persistent in

50-s, 70-s and again in 80s-90s compared to a frictionless economy. In addition to the autoregressive components, agents also revise their beliefs about the expected means of forward-looking variables (given by a constant in the forecasting equations). The presence of a constant brings additional changes in the transmission mechanism compared to the one implied by RE model, where all the real variables are assumed to have the common trend growth rate and inflation is centered around the fixed inflation objective. Slobodyan and Wouters (2010) interpret the variations of the constant as deviations of agents' expectations from these steady state values. Figure 3 illustrates that constants vary the most for asset price and investment, and reflect rather cyclical pattern of a change in these variables. The fluctuations of the constant for expected asset price and investment rate are more pronounced in a model with financial accelerator. Significant shifts in the expected means can add to the macroeconomic volatility and contribute to over-optimistic or pessimistic developments in agents' expectations. Financial frictions do not have important implications for the perceived persistence of other variables - real consumption, wage, and labour, making our results in this respect very similar to Slobodyan and Wouters (2010).

In order to demonstrate more explicitly the joint impact of the financial frictions and adaptive expectations on the transmission mechanism, I compute the persistence implied by the Actual Law of Motion, given by (21). The results are presented in Figure 4. The horizontal solid line shows the persistence implied by the RE model with financial accelerator, whereas the horizontal dotted line depicts the corresponding value for the model without financial frictions. Thus I compare the implied persistence for RE and "AR(2)+constant" learning models with and without financial accelerator. Figure 4 demonstrates that implied inflation persistence in a model with learning and financial accelerator was higher in 1950-s-60-s, and after 1980-s compared to analogous model without financial frictions. Introducing financial accelerator resulted in higher implied asset price persistence during all the time span. Implied investment growth persistence was also generally higher in a model with financial frictions and differentiated the most from the "noFA" level in 1970-s and after 1990-s. The similar pattern is observed for implied output growth persistence. The dynamics presented in Figure 4 can add to explaining the results of the Table 1 and Figures 1 and 2. Specifically, I notice that, on average, the difference in persistence between the RE and the learning models with financial accelerator is lower than the corresponding difference which arises when financial frictions are shut off. In other words, it appears that RE model with financial frictions does better in capturing the "true" data generating process and delivers the level of persistence which is closer to the average agents' perceptions about the economy. Indeed, the implied inflation persistence under RE was very close to (time-varying) implied persistence under learning from the middle of 1980-s to beginning of 1990-s and 2000. The same is true for invest-

ment and output. Thus, the gain from modeling the time-varying transmission mechanism declines on some time intervals of the second half of the sample. This explains why the improvement in the data fit under learning relative to RE model declined compared to "noFA" specification.Finally, Figure 5 compares the implied persistence for alternative

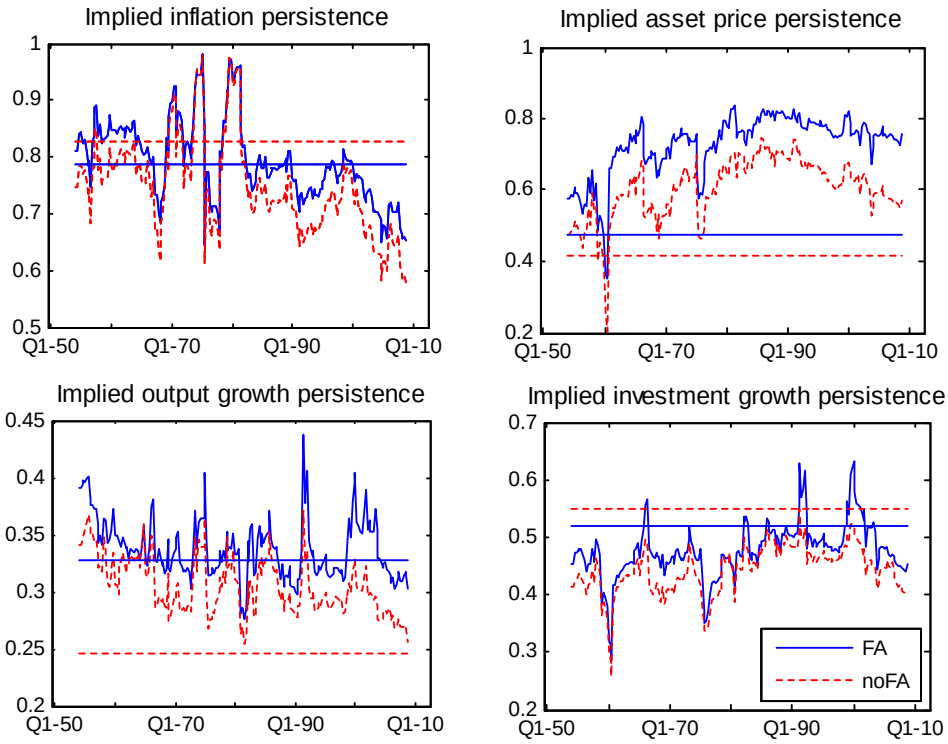

Figure 4: Implied persistence under RE and learning

adaptive learning schemes, which differ in terms of the variables on the RHS of the forecasting equations: "AR(2)+constant", "AR(2)", and "All states" (near MSV). Figure 5 demonstrates that the information set used by agents to form the forecasts has important implications for the implied persistence and thus for the transmission mechanism and the business cycle. In particular, forecasts based on small learning models lead to higher implied persistence. The major difference is observed for persistence of asset price and investment, the variables which play a crucial role in generating financial accelerator effects. In particular, forecasting model which incorporates all state variables on the RHS implies very low persistence of asset prices and thus would fail to generate significant real effects following financial shocks. The results presented in the Figure 5 also indicate that introducing a constant into the forecasting functions leads to a smoother transition of agents' beliefs and implied persistence. In a learning model without a constant, agents will associate any developments in observable variables with a change in their persistence,

whereas in the "AR(2)+constant" model, some of the volatility may be attributed to variation of the expected mean. As a result, the "AR(2)" model will generate greater swings in implied persistence and thus more volatile model dynamics. In a specification with financial accelerator such an extra volatility can make the problem with projection facilities (mentioned in the previous sections) more severe. Therefore, the learning model that incorporates a constant is also preferable from the computational point of view.

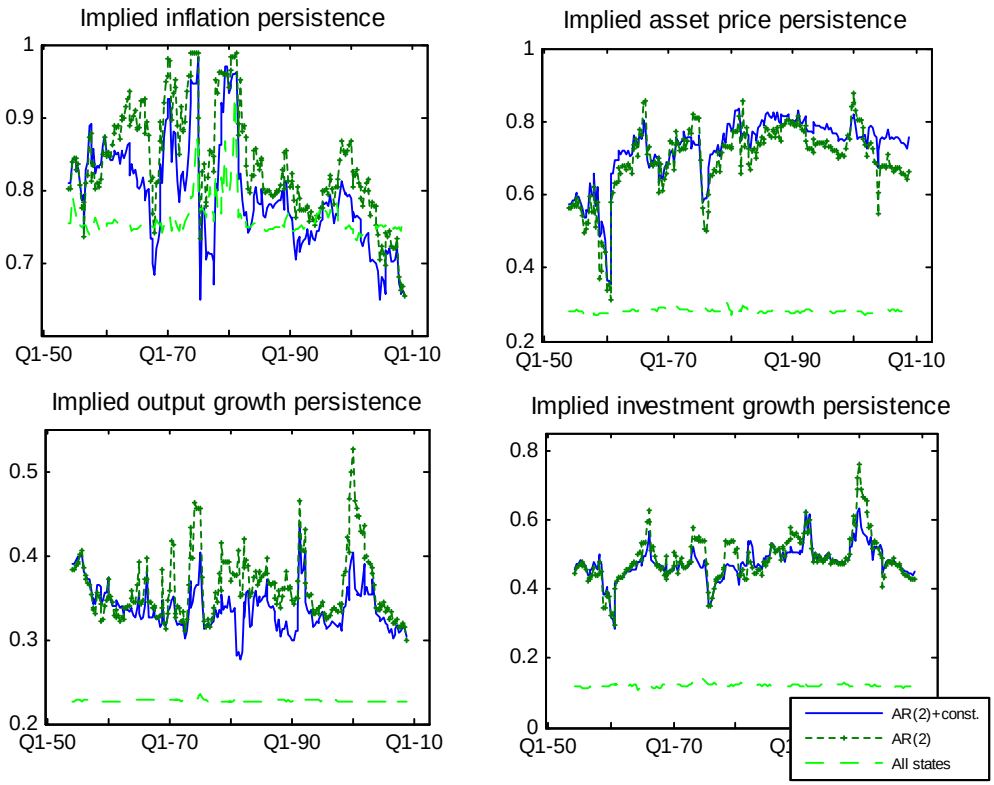

Figure 5: Implied persistence for alternative learning models with financial accelerator

4.2 Financial accelerator under learning and the transmission mechanism

Implied persistence is an important determinant of real effects of shocks hitting the economy. In particular, shocks to inflation rate, which is perceived as a highly persistent process, will lead to stronger and long-lasting responses of inflation. For inflation targeting central bank, such a dynamics would imply more aggressive monetary policy reaction which would affect real output to a greater extent. In the financial accelerator framework, agents' perceptions about financial variables such as asset prices may have additional macroeconomic implications. If agents perceive asset prices to be more persistent,

financial shocks will result in stronger and more gradual responses of this variable and, hence, greater impact on the households' financial position (net worth) and the external finance premium. Therefore, financial disturbances will entail higher cumulative effects on investment and output. The results presented in the previous subsection indicate that learning model with "AR(2)+constant" beliefs may have significant implications for the shock transmission due to the higher implied persistence of asset prices, inflation (after 80-s) as well as real variables relative to the model with RE and also compared to the version without financial frictions.

Previous literature have already provided some insights about the transmission mechanism in models with financial frictions. Christensen and Dib (2008) study the transmission of shocks in the estimated model with RE and financial accelerator. Unlike Bernanke et al. (1999) their model incorporates the nominal debt contract, allowing for debt deflation effects. Christensen and Dib find that financial accelerator mechanism considerably amplifies and propagates the impact of demand-side shocks - monetary policy, money demand and preference shock - on investment and the price of capital. The implications of financial frictions for inflation and output are found to be relatively minor. De Graeve (2008) reports similar effects of the financial accelerator. In particular, investment response to a preference and monetary policy shocks is stronger relative to the model without financial frictions. In both studies, financial accelerator mechanism dampens the rise of investment following positive technology and investment supply shocks. This contrasts sharply with results in Bernanke et al. (1999) and Walentin (2005), in which favorable productivity shocks reduce the premium and therefore boost investment relative to a model without financial frictions. In addition, in De Graeve model, dynamics of investment following investment supply shocks somewhat differs from results documented in Bernanke et al. (1999) and other existing studies (Walentin, 2005; Christensen and Dib). He explains the difference in responses by the form of adjustment costs.[5]

In this paper, I compare the implications of financial frictions for the transmission mechanism in the RE and adaptive learning model based on "AR(2)+constant" forecasting functions. Figures 6, 7 , and 8 show the impulse responses under the productivity, risk premium, and monetary policy shocks respectively. In fact, figures present the time variation of impulse responses and thus reflect the time varying transmission mechanism under learning. In particular, inflation responded much stronger to shocks around 70-s, when perceived inflation was very persistent. The dynamics of inflation is similar to the one documented in Slobodyan and Wouters (2010) because financial accelerator did not significantly affect inflation persistence in seventies. The peaks of the responses of asset

[5]Bernanke et al. works with capital adjustment costs, whereas F.De Graeve assumes investment adjustment costs. This implies more gradual and hump-shaped response of investment.

prices happen around 80-s, 90-s and 2000. This corresponds to the dates when agents revised the perceived asset price persistence upwards (see Figure 3 which shows the evolution of beliefs). The very first impulse response (denoted by the thick line) corresponds to the reaction under RE. Figure 6 shows the response under 1% positive technology shock. The immediate response of output, asset price, and investment is lower relative to the model with RE, but becomes more persistent and sizable afterwards. The dynamics of financial variables, i.e. asset prices, net worth, and risk premium, differ sharply from the responses under RE. The reduction of the external risk premium is very persistent and therefore explains more gradual increase in investment and output. The responses of inflation, asset prices, and investment exhibit most volatile dynamics. The reaction of the external finance premium would display more of time variation if the estimated elasticity of the risk premium was higher.Figure 7 shows the responses to a risk premium

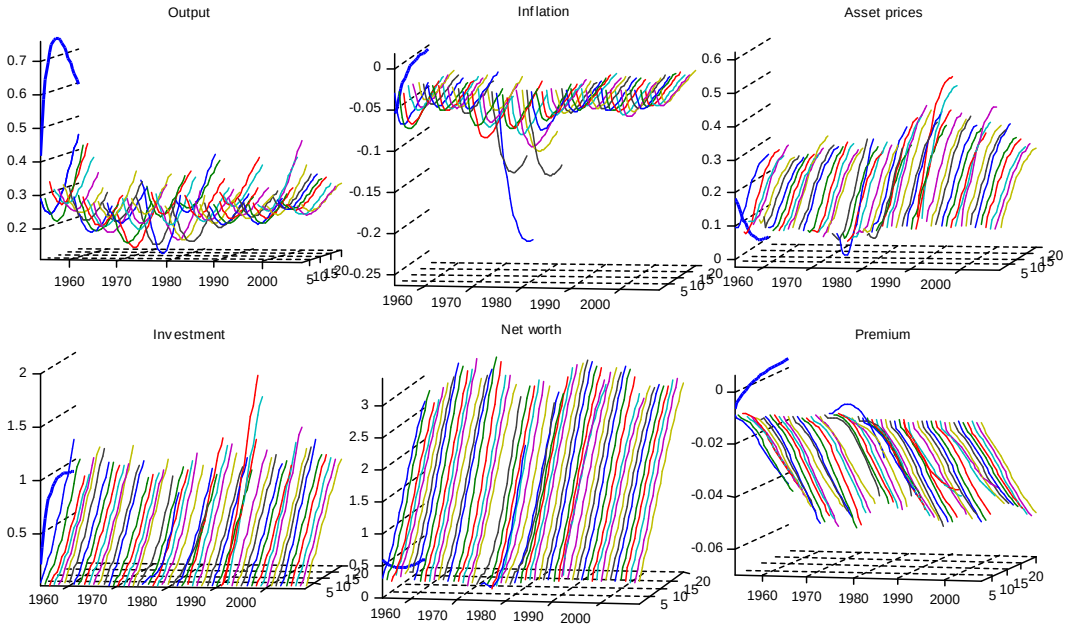

Figure 6: Impulse responses to a productivity shock

shock. Reaction of output, investment as well as financial variables in the adaptive learning model is stronger relative to the model with rational expectations. Specifically, sharp fall in asset prices reduces the net worth and thus raises the external finance premium, whose immediate reaction is stronger compared to the model with RE. Therefore, responses of investment and output are also amplified. Figure 7 displays significant time variation in responses to a shock of both financial and real variables, thus demonstrating the implications of the departure from the complete rationality assumption. The peaks in the perceived asset price persistence observed in 1974, 80-s, and 90-s leads to a dramatic fall in asset prices, which sharply reduces the net worth. As a result, even under rela-

tively low estimated sensitivity of the premium to changes in the entrepreneurial financial health, the gap between the cost of external financing and the risk-free rate shows a significant increase and therefore leads to stronger impact of financial accelerator on the real economy. This example clearly illustrates the mutually reinforcing interaction between financial accelerator and adaptive learning.Finally, I investigate effects of the monetary

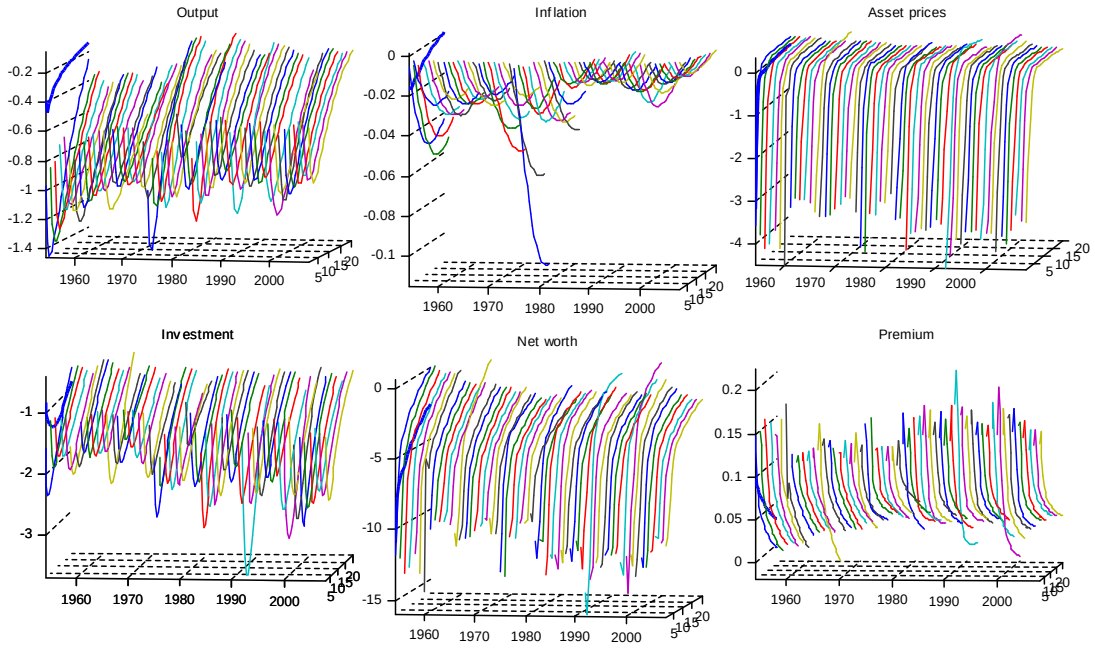

Figure 7: Impulse responses to a risk premium shock

policy shock, presented in Figure 8. Following the monetary tightening, inflation, asset prices, investment and output decline. The immediate reaction of variables under learning is generally lower but much more persistent. At the peak, the responses are considerably amplified relative to the model where financial frictions interact with rational expectations. Again, responses show significant time variation reflecting the evolution of agents beliefs about the macroeconomy.

4.3 Simulation exercises and sensitivity analysis

In the previous subsection I analyzed the implications of introducing financial frictions into the model with adaptive learning on the basis of the estimated values of the parameters. At the same time, the reported empirical values of the level of financial frictions vary across different studies and may depend on the estimation sample and modeling assumptions. Moreover, the inclusion of observations that cover the most recent financial distress followed by adverse macroeconomic consequences would definitely imply higher estimated values of financial parameters (the observations of the last several years are not

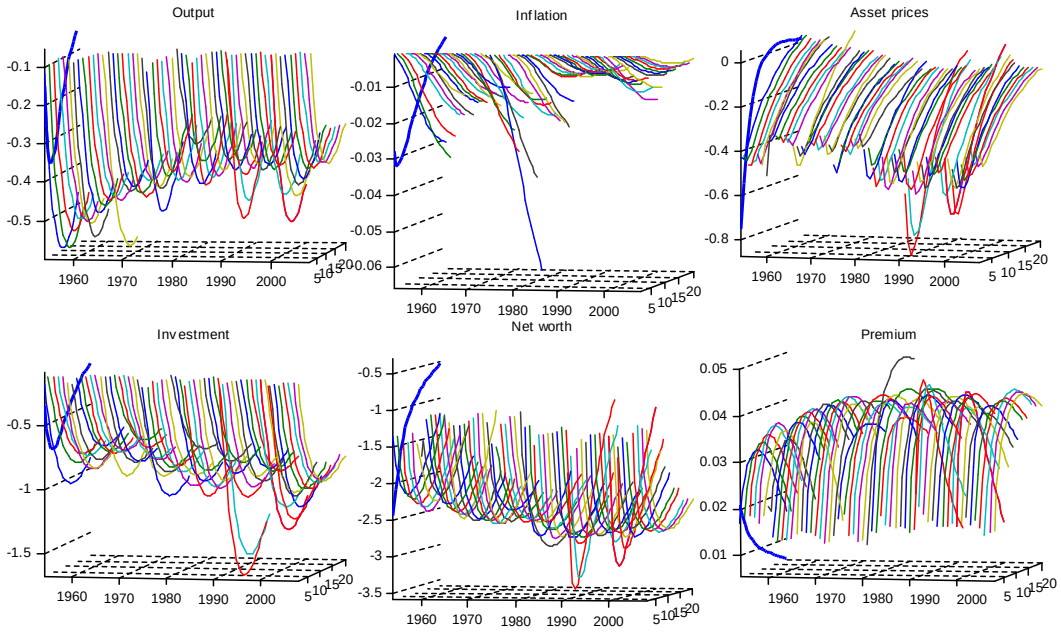

Figure 8: Impulse responses to a monetary policy shock

included in the sample). Therefore, in this part of the paper I conduct the sensitivity analysis and investigate the impact of the gradual increase of financial frictions on the macroeconomy. In order to perform this task, I fix the parameters for each model at the corresponding posterior mode value and simulate the models for 219 periods, which coincides with the duration of the estimation sample. Additionally, I assume $k/N = 2.5$ (somewhat lower than the estimated value) in order to avoid the problem with projection facilities, which may arise due to excessive volatility of beliefs under high values of financial frictions parameters. I vary the elasticity of the risk premium from 0.02 (relatively low) to 0.04 (average) and 0.06 (relatively high). In each case, I simulate agents' beliefs and calculate implied (simulated) persistence of inflation, asset prices, growth of output and investment. In order to assess the potential ability of learning to amplify the business cycle fluctuations, I compute the difference in the impulse responses to a risk premium shock between the rational expectations and adaptive learning models. I contrast the results for two alternative learning models which differ in terms of the information sets used by agents to forecast forward-looking variables.

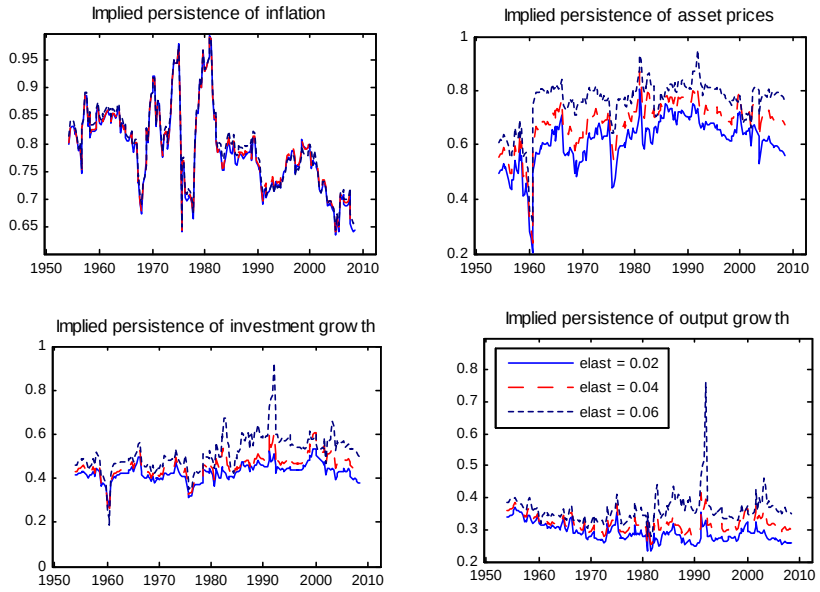

Figure 9: Implied persistence for alternative degrees of financial frictions in "AR(2)+const." learning model

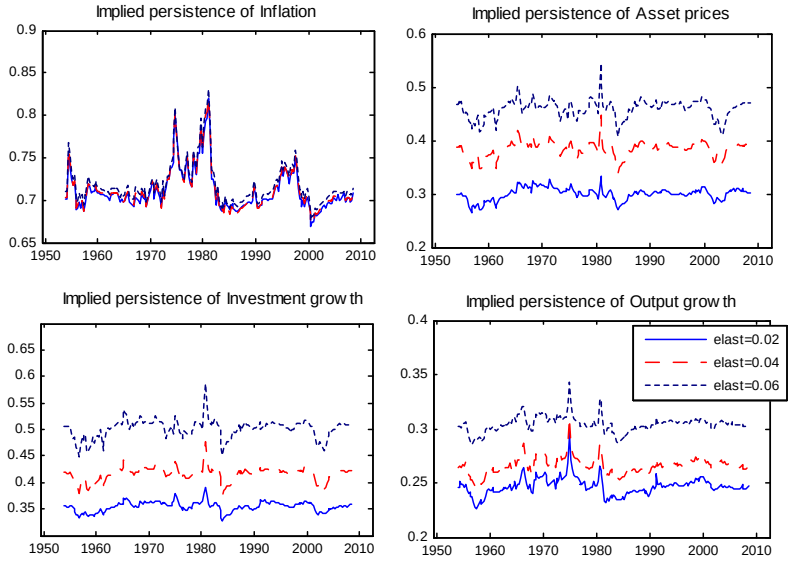

Figure 10: Implied persistence for alternative degrees of financial frictions in "All states" learning model

Figures 9 and 10 show the implied persistence for "AR(2)+const." and "All states" learning models for different degrees of financial frictions. The graphs illustrate that implied inflation persistence is almost independent on the elasticity of the risk premium parameter. At the same time, for AR(2)+const. learning model, higher financial frictions

significantly affect the persistence of asset prices, investment,and output growth. In particular, greater elasticity of the risk premium increases the level and /or time variation of the implied persistence of these variables, especially on the second half of the sample. Implied persistence simulated for the learning model in which agents employ all the state variables in forecasting functions is generally lower and more stable over time relative to the learning scheme based on small forecasting functions. Specifically, Figure 10 demonstrates that increase of financial frictions does lead to somewhat higher level but hardly affects the time variation of the implied persistence in "All states" model. For impulse responses, such a result would imply their lower variability and less pronounced business cycle fluctuations.

Figures 11 and 12 present the difference in the impulse responses to a risk premium shock between the rational expectations and two alternative learning models. The greater is the deviation of the line from the zero level, the stronger is the reaction of the economy under learning relative to the model with RE. For example, negative values of the differences in the responses of investment (Fig.11) mean that the fall of investment was more pronounced under learning. Figure 11 illustrates that the peak response of all the variables under learning was stronger relative to the responses under RE. The difference in the responses increases as financial frictions become stronger. For the highest value of the parameter elasticity (0.06), the increase of the external finance premium and the corresponding decline of investment is much more sizable under learning compared to the model with RE. Figure 12 demonstrates that "All states" learning algorithm is rather unsuccessful in amplifying the business cycle fluctuations even for the highest degree of financial frictions. In particular, under this learning scheme, the risk premium shock leads to the fall of investment that is lower (in absolute value) relative to the negative investment response under RE. Even assuming the highest value of the elasticity parameter does not allow reversing of the results. Therefore, it appears that the type of the learning model, and in particular the information set used by learning agents in forecasting, is more fundamental factor in generating additional macroeconomic volatility than the degree of financial frictions as such.

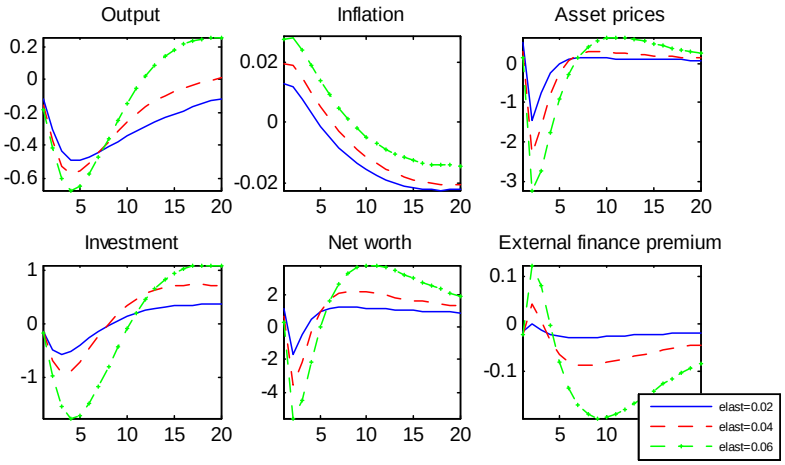

Figure 11: *Difference* in responses to a risk premium shock between "AR(2)+const." learning and RE model for alternative degrees of financial frictions

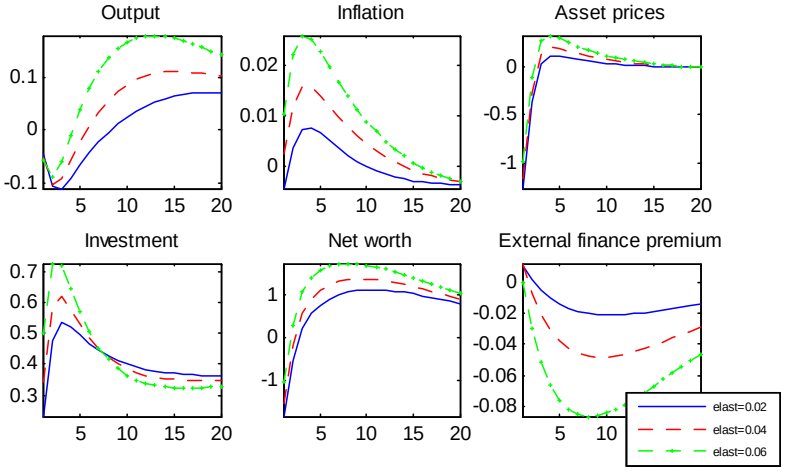

Figure 12: *Difference* in responses to a risk premium shock between "All states" learning and RE model for alternative degrees of financial frictions

5 Conclusions and future research

In this paper, I compare the implications of financial accelerator mechanism for the real economy in models with alternative assumptions about the expectation formation. I perform Bayesian estimation of a medium-scale DSGE model with financial frictions assuming, on the one hand, complete rationality of expectations and, alternatively, several forms of AL that differ in terms of the information set used by agents to form their predictions. I evaluate and compare the model fit, estimated parameters, and the transmission mechanism. The estimation results suggest that both financial frictions and adaptively formed expectations based on very simple forecasting functions add to the improved model fit, at least on certain time intervals.

I show that implications of financial accelerator for the busyness cycle may vary depending on the expectation assumptions (RE or forms of learning). The results suggest that the learning scheme based on small forecasting functions is able to amplify the effects of financial frictions relative to the model with RE. I show that the model dynamics under learning is driven to a significant extent by the time variation of agents' beliefs about the evolution of financial variables. Specifically, I demonstrate that perceived asset price persistence in a learning model with simple forecasting equations varies through the cycle and thus differs significantly from the levels implied by the RE and alternative learning schemes. During periods when agents perceive asset prices as being relatively more persistent, shocks that affect this variable lead to more pronounced macroeconomic outcomes. The asset price persistence appears to be particularly important for explaining the investment dynamics. This effect is clearly observed in impulse responses. In particular, increased asset price persistence implies more pronounced (and persistent) response of investment under the risk premium or monetary policy shocks. Therefore, I argue that certain forms of AL may play a significant role in driving and amplifying the macroeconomic fluctuations; it introduces important time variation and strengthens the real effects of the financial accelerator compared to the assumption of RE. Simulation exercises illustrate, that the amplification effect raises more than proportionally as financial frictions become more severe. At the same time, learning specification in which agents use more information to generate predictions (close to MSV learning) produces very different asset price and investment dynamics. In such a framework, learning cannot significantly alter the real effects of financial frictions implied by the RE model.

The results of the paper allow drawing several conclusions relevant for DSGE modeling and policy analysis. In particular, due to the ability to amplify the macroeconomic fluctuations learning can be a suitable framework to simulate financial crisis scenarios and various policy reactions. Comparison of the data fit for alternative models suggests that AL with financial accelerator represents the best specification to describe the data generating process and analyze the shock transmission in the second half of the sample. In addition, the results imply that the link between the asset prices and the real economy has become more important and the sensitivity of the economy to financial shocks increased after middle of 80-s. Such an empirical conclusion is supported by impulse responses of real variables that show higher time variation in 90-s and 2000-s following monetary and financial shocks. Which economic processes or policy reactions could contribute to the increased propagation of financial shocks is an important question for further research. There exists an opinion that stable economic environment with low interest rates and inflation could be partly responsible for the adverse dynamics of asset prices and the development of a bubble leading to the crisis. In the near future, I would like to comple-

ment this paper with the analysis of the monetary policy in the economy with adaptive learning and financial frictions. I plan to study how strong anti-inflationary stance or too expansionary monetary policy can impact the implied persistence of asset prices as well as variation of real variables and inflation. In addition, I would like to experiment with the policy rules which incorporate the response to asset prices and examine whether such rules could deliver better macroeconomic outcomes in terms of monetary and financial stability.

References:

An, S., and F. Schorfheide (2007): "Bayesian Analysis of DSGE Models," Econometrics Reviews, Taylor and Francis Journals, vol. 26(2-4), pp. 113-172.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations", American Economic Review, Vol. 79, No. 1, (Mar., 1989), pp. 14-31

Bernanke, B., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework" in J Taylor and M Woodford (eds) Handbook of Macroeconomics.Volume 1C, North Holland.

Carlstrom, C., and T. Fuerst (1997): "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis", American Economic Review, vol. 87(5), pages 893-910, December.

Cespedes, L., R. Chang and A. Velasco (2004): "Balance sheets and exchange rate policy", The American Economic Review, Vol. 94, No. 4, September.

Christiano, L.J., M. Eichenbaum and C.L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, Vol. 113(1), p. 1–45.

Christensen, I., and A. Dib (2008): "The financial accelerator in an estimated New Keynesian model," Review of Economic Dynamics 11, p.155-178.

De Graeve, F., (2008): "The external finance premium and the macroeconomy: US post-WWII evidence," Journal of Economic Dynamics and Control 32.

Evans, G., and S. Honkapohja (2001): "Learning and Expectations in Macroeconomics," Princeton University Press, Princeton, NJ.

Haario, H., E.Saksman, and J.Tamminen (2001): "An Adaptive Metropolis Algorithm," Bernoulli, Vol.7, No.2, p.223-242.

Kiyotaki, N., and J.Moore (1997): "Credit cycles", Journal of Political Economy, 105:211-248.

Milani, F., (2006): "A Bayesian DSGE Model with Infinite-Horizon Learning: Do 'Mechanical' Sources of Persistence Become Superfluous?" *International Journal of Central*

Banking, Iss. 6.

Milani, F., (2007): "Expectations, Learning and Macroeconomic Persistence", *Journal of Monetary Economics*, 54, 2065–2082.

Milani, F., (2008): "Learning, Monetary Policy rules, and Macroeconomic Stability," *Journal of Economic Dynamics and Control*, 32(10), 3148-3165.

Iacoviello, M., (2005): "House prices, borrowing constraints and monetary policy in the business cycle", American Economic Review, 95(3):739-764.

Iacoviello, M., and S.Neri (2008): "Housing market spillovers: evidence from the estimated DSGE model", Bank of Italy, WP 659

Orphanides, A., and J. Williams (2007): "Inflation targeting under imperfect knowledge," *Economic Review*, Federal Reserve Bank of San Francisco, pages 1-23.

Rychalovska, Y., and S.Slobodyan (2010): "Bayesian estimation of DSGE models for the Euro Area: robustness issues", mimeo.

Sargent, T.J., N. Williams, and T. Zha (2005): "Shock and Government Beliefs: The Rise and Fall of American Inflation," *American Economic Review*, forthcoming.

Smets, F., and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," Journal of the European Economic Association, 1(5), 1123-75.

Smets, F., and R. Wouters (2004): "Forecasting with a Bayesian DSGE Model: An Application to the Euro Area," Journal of Common Market Studies, 42(4), 841-67, 2004.

Smets, F., and R. Wouters (2007): "Shocks and frictions in US business cycle: A Bayesian DSGE approach", American Economic Review, 97(3).

Slobodyan,S., and R. Wouters (2009): "Learning in an Estimated Medium-Scale DSGE Model," CERGE-EI WP 396.

Slobodyan,S., and R. Wouters (2010): "Estimating a Medium-Scale DSGE Model with Expectations Based on Small Forecasting Models," mimeo.

Soto, C., J.P. Medina, and R. Caputo (2010): "The financial accelerator under learning and the role of monetary policy," Central Bank of Chile, Working Paper 590.