Capital Accumulation in Private Information Economies

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Abstract

This paper provides a general methodology for introducing capital accumulation into economies with private information and heterogeneous agents. The agents operate a stochastic neoclassical production technology with capital and labor input. I study a moral hazard economy with unobservable input (hidden action). I characterize the efficient allocation of capital, labor, and consumption in a stationary recursive competitive equilibrium. The economy is decentralized by the component planner approach developed by Atkeson and Lucas (1995). Accumulation of capital is facilitated by a "capital planner" who serves as a financial intermediary for the component planners. In the unique, feasible and non-degenerate stationary equilibrium, private information lowers the market-clearing interest rate below agents' discount rate.

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1 Introduction

This paper develops a general methodology for incorporating capital accumulation into economies with private information and heterogeneous agents. The agents operate neoclassical production technologies that are subject to idiosyncratic output shocks. Since the agents are risk averse, they would like to smooth their consumption by entering into insurance contracts. Suppose, however, that these insurance contracts are subject to one of the basic moral hazard problems: the principal can control agents' output and consumption but cannot observe the input in the production technology.

I study optimal allocation and distribution of resources in a stationary recursive competitive equilibrium for this type of closed economy with feasible, constant aggregate level of capital stock, labor, output, and consumption, implied by a time invariant distribution of agents' utility entitlements. I characterize incentive compatible allocations that ensure the agents obey the labor effort recommended by the principal.

The contribution of the paper is in the incorporation of the neoclassical production technology and capital accumulation into the general equilibrium, dynamic private information economies with heterogeneous agents. I decentralize the economy using the concept of component planners developed by Atkeson and Lucas (1995). These component planners minimize the cost of resources needed to provide incentive-compatible insurance contracts to agents with a given level of utility entitlements. In order to provide an efficient amount of observable capital input in agents' production technologies, the component planners accumulate capital and trade one-period capital loans at a market-clearing interest rate. The market for these one period loans can be imagined as organized by a "capital planner" who provides financial intermediation to the component planners. The component planners lend and borrow capital at an interest rate at which the market for capital clears and the overall surplus of all component planners is zero.

This decentralization does not impose any special restrictions on preferences, production technology, information structure, or contracts. Capital stock is an inherent part of the insurance contracts as a market clearing device in the decentralized economy. Previous models with private information and capital accumulation have imposed restrictions on preferences and technology (as in Khan and Ravikumar (1997a) or Khan and Ravikumar (1997b) with privately owned capital). A more related paper of Aiyagari and Williamson (1999) studies a private information endowment economy where the capital is not used in production of the agents and is not part of the principal-agent contract. Similarly, Golosov, Kocherlakota, and Tsyvinski (2003) study an economy with capital only in the aggregate production technology.

In this paper the principal-agent problem is modelled in a standard way used in the private information and contract theory literature. The principal (social planner, government) minimizes the cost of providing insurance subject to the promise keeping and incentive compatibility constraints. Because of moral hazard, the planner cannot provide full insurance but must condition all allocations on the history of output realizations. The planner rewards agents with a high realized output with higher consumption and increased utility entitlement from the next period on (and vice versa for agents with a low output). In the limit, the sequence of such allocations results in a stationary distribution of utility entitlements for a given information constraint faced by the planner. The standard result in the private information literature is that without an imposed lower bound, the limiting distribution of agents' wealth and consumption implied by the efficient allocation spreads forever over time (see Thomas and Worrall (1990)). Aiyagari and Alvarez (1995) show that distributions with mobility exist only if agents cannot be driven to the lower bound of their consumption sets in an incentive compatible way. I follow their approach assuming a utility function in which agents cannot be driven to "misery".

I characterize the optimal allocations in the stationary recursive competitive equilibrium and show conditions under which there exists a non-degenerate stationary distribution of agents in the steady state. This paper provides a simple prototype model suited to the study of the tradeoff between equality and efficiency in economies with private information and heterogeneous agents. Numerical simulations can determine the allocation and distribution of resources observable in real world economies. One can study the distribution and aggregate levels of labor hours, capital, income from labor and capital, output, consumption, and investment under different contractual and insurance frameworks as well as information structure of the economy. In general equilibrium, the possibility of accumulating capital stock plays a significant role for individual and aggregate allocations by endogenously determining the equilibrium prices. For examples where the equilibrium interest rate plays a crucial role in economies with heterogeneous agents, see a social security reform in Conesa and Krueger (1999) or a model of optimal social insurance with moral hazard in Bohacek (2003).

The paper is organized as follows. In the next Section, I describe the hidden action economy following the social planner model of Phelan and Townsend (1991). In Section 3, I decentralize the economy and introduce the concept of capital planner and define a stationary recursive equilibrium. In Section 4, I characterize optimal allocations and the properties of the invariant distribution. Section 5 concludes. Proofs are in the Appendix.

2 The Economy

In this Section I describe an economy with moral hazard related to unobserved labor supply. I consider an economy populated by a continuum of agents. Each agent operates a stochastic neoclassical technology with capital and labor inputs. In order to insure against bad realizations of output, the risk-averse agents enter into insurance contracts with a zero profit insurance agency, modelled as a social planner.

Since labor supply is private information of each agent, the social planner must overcome the moral hazard problem by conditioning each agent's insurance transfer on the entire history of his output realizations, rewarding high realizations of output with high current, as well as future, consumption levels. There is no private storage technology and the social planner prevents each agent from trading with other agents.

In each period each agent is endowed with one unit of time and derives a utility from consumption, c_t, and leisure, $1 - l_t$, $U(c_t, 1 - l_t) = u(c_t) + v(1 - l_t)$, discounted over time at $\beta \in (0, 1)$. The utility function is bounded, separable in consumption and leisure, u is strictly increasing and strictly concave in c, and v is strictly decreasing in labor supply l.

Each agent operates a constant returns to scale, Cobb-Douglas production technology $f(k_t, l_t) = k_t^{\alpha} l_t^{1-\alpha}$, with $\alpha \in (0, 1)$. For each possible choice of incentive compatible $l \in L = [0, 1]$, the capital input k is assigned according to the efficient capital-labor ratio, x, derived from the technology.[1] Choosing a labor effort l determines a probability distribution over output realizations, $y \in Y$, where Y is a finite set of possible outputs determined by the production technology. Let $P(y|k, l)$ denote the probability that the realization of output is y, given that the capital and labor inputs are (k, l). I assume that $P(y|k, l) > 0$ for all $y \in Y$ and all positive pairs (k, l), and that more effort implies greater expected output so that the planner is not able to exactly derive the agent's effort from the observed output realization.[2] The output realizations drawn from this stochastic technology are independent across agents and time periods. Finally, the capital stock depreciates each period at a rate $\delta \in (0, 1)$.

The risk-neutral, cost-minimizing social planner and risk-averse agents write dynamic, incentive compatible contracts in which the planner effectively controls all allocations of each agent. In the aggregate, the social planner accumulates the capital stock over time from the surpluses obtained from individual agents, i.e., from all the goods produced but not consumed by the agents.

The insurance contract is an allocation sequence specifying each agent's inputs and consumption for each period $t \geq 0$. The sequence must satisfy the promise keeping and incentive compatibility constraints. As it is usual in the private-information literature, the social planner identifies each agent by an initial entitlement to expected, discounted utility $w_0 \in W = [\underline{w}, \overline{w}] \subset \mathbb{R}$ and by a history of output realizations, $y^t = \{y_0, y_1, \ldots, y_t\} \in Y^{t+1}$. All agents identified with the same w_0 and the same history receive the same treatment.

The contract for an agent $w_0 \in W$ is an allocation sequence $\sigma = \{k_t(w_0, y^{t-1}), l_t(w_0, y^{t-1}), c_t(w_0, y^t)\}_{t=0}^{\infty}$, weighted by the conditional probability

[1]As inputs are chosen before the output uncertainty is realized, and the capital input is not informative for the moral hazard problem, the social planner uses the same capital-labor ratio for all agents. This will be shown to be true for the decentralized economy at a constant, market clearing interest rate r.

[2]Naturally, zero effort leads to zero output with probability one. In Section 4, I will assume that the probability distribution satisfies the conditions for the first-order approach.

P^{t+1} that a history $y^t \in Y^{t+1}$, conditional on inputs $(k_t(w_0, y^{t-1}), l_t(w_0, y^{t-1}))$ employed in the production technology, occurs. An agent w_0's initial expected discounted utility can be written as

$$U(w_0, \sigma) = \sum_{t=0}^{\infty} \beta^t \int_{Y^{t+1}} \left\{ u\left(c_t(w_0, y^t)\right) + v\left(1 - l_t(w_0, y^{t-1}\right)\right\} \cdot \\ \cdot P^{t+1}\left(dy^t | k_t(w_0, y^{t-1}), l_t(w_0, y^{t-1})\right).$$

The participation constraint requires the planner to deliver the expected discounted utility w_0 to the agent entitled to w_0,

$$U(w_0, \sigma) = w_0 \quad \text{for all } w_0 \in W. \tag{1}$$

Second, the incentive compatibility constraint ensures that each agent prefers the recommended labor supply to any labor supply deviation, for all $w_0 \in W$,

$$U(w_0, \sigma) \geq U(w_0, \hat{\sigma}), \tag{2}$$

where $\hat{\sigma}$ contains any labor supply deviation $\hat{l} \in L$ from the recommended $l \in L$ in any period of time.

From the economy-wide point of view, an efficient social planner minimizes the cost of the insurance scheme subject to the aggregate feasibility conditions for a closed economy. These conditions are obtained by aggregating the individual allocations according to the distribution of agents. Let λ_0 denote an arbitrary initial distribution of expected utility entitlements on $(W, \mathcal{B}(W))$, with Borel sets $\mathcal{B}(W)$. I will interpret $\lambda_0(A)$ as a fraction of the population entitled to expected discounted utility in $A \in \mathcal{B}(W)$.

The aggregate feasibility conditions require that in each period the social planner divides the accumulated aggregate capital stock, $\bar{K}_t$, into capital input assignments for all agents $w_0 \in W$,

$$\bar{K}_t \geq \int_{W \times Y^t} k_t(w_0, y^{t-1})\, P^t\left(dy^{t-1} | k_{t-1}(w_0, y^{t-2}), l_{t-1}(w_0, y^{t-2})\right)\, \lambda_0(dw_0), \tag{3}$$

and that the allocations to all agents are feasible,

$$\int_{W \times Y^{t+1}} \left\{ y_t - c_t(w_0, y^t) \right\}\, P^{t+1}\left(dy^t | k_t(w_0, y^{t-1}), l_t(w_0, y^{t-1})\right)\, \lambda_0(dw_0) \\ \geq \bar{K}_{t+1} - (1 - \delta)\bar{K}_t. \tag{4}$$

Note that the last equation serves as the law of motion for the capital stock: all goods produced but not consumed by the agents are added by the social planner to the depreciated current capital stock.

The goal of this paper is to study this economy in a *stationary recursive equilibrium.* For this purpose in the next Section I formulate the principal-agent problem recursively and study a *steady state* of a *closed* economy in which the distribution of utility entitlements is time invariant, the levels of aggregate variables are constant and the allocations are feasible.

3 A Decentralized Economy in Stationary Recursive Equilibrium

It is standard in the literature (see Atkeson and Lucas (1995) or Phelan and Townsend (1991)) to show that the sequential planning problem is equivalent to a recursive formulation with a utility entitlement $w \in W$ as a state variable for each agent. The utility entitlement w summarizes the history of output realizations of each agent at the beginning of each period.

Following Atkeson and Lucas (1995) or Albanesi and Sleet (2003), the problem of finding efficient allocations in an economy with heterogeneous agents can be solved by a partial decentralization with prices and "component planners" each responsible for allocating resources only to agents entitled to a particular utility entitlement w. Each w-component planner chooses an allocation that minimizes the cost of attaining w evaluated at equilibrium prices of goods traded among the component planners.

The component planners trade goods in the following way. They borrow capital according to the willingness of their agents to supply labor effort. Agents who supply high labor effort are assigned more capital input in their production function according to the optimal capital-labor ratio, and vice versa for the less working agents. One can imagine the capital trading intermediated by a zero-profit financial intermediary called "capital planner". The capital planner manages the accumulated aggregate stock of capital which he lends to the component planners in each period at the interest rate r. Given the interest rate r, a w-component planner assigns an agent w inputs $k(w)$ and $l(w)$ as functions of the current utility entitlement before the output uncertainty is realized.[3] The incentive compatibility requires the consumption $c(w,y)$ and the continuation utility entitlement $w'(w,y)$, the next-period state variable, to be also functions of the current output realization. At the end of each period, each component planner repays the capital loan $(r+\delta)k(w)$ and deposits all remaining surplus $y-c(w,y)$ with the capital planner.

For a constant interest rate r, define an *allocation policy* of a component planner associated with subpopulation $w \in W$ as $\sigma_r \equiv \{k(w), l(w), c(w,y), w'(w,y)\}$. The objective of each component planner is to minimize the present value of resources evaluated at the intertemporal price of resources $1/(1+r)$ subject to the promise keeping, incentive compatibility, and the minimal guaranteed consumption constraints. For all $w \in W$ define a value function $V_r : W \to \mathbb{R}$ for the component planning problem and an operator T_r on the space of bounded, continuous functions $C(W)$ as

$$(T_r V_r)(w) = \min_{\sigma_r} \sum_{Y} \left\{ c(w,y) + (r+\delta)k(w) - y + \frac{1}{1+r} V_r(w'(w,y)) \right\} P(y|k(w), l(w)), \quad (5)$$

subject to the promise keeping constraint,

$$w = \sum_{Y} \{u(c(w,y)) + v(1-l(w)) + \beta w'(w,y)\} P(y|k(w), l(w)), \quad (6)$$

and the incentive constraint,

$$l(w) \in \arg\max_{\hat{l} \in L} \sum_{Y} \left\{ u(c(w,y)) + v(1-\hat{l}) + \beta w'(w,y) \right\} P(y|k(w), \hat{l}). \quad (7)$$

The steady state for a closed economy will exhibit a constant market clearing interest rate, r, time invariant decision rules for all agents, σ_r, and a time invariant distribution of utility entitlements, λ_r. The aggregated allocations of all component planners must satisfy the market clearing conditions for a closed economy, namely

[3] From profit maximization of the Cobb-Douglas production function at a constant interest rate r, all component planners use the same optimal capital-labor ratio $x = (\alpha/(r+\delta))^{1/(1-\alpha)}$. Then for each level of labor, $y = l(w)x^{\alpha}$ so that for $L = [0,1]$, $Y = \{0, \ldots, 1 \cdot x^{\alpha}\}$. In each case, capital input is assigned proportionately to the labor input.

that all the capital stock is lent out and the aggregate surplus exactly equals the depreciated capital stock.

Definition 1 *A stationary recursive equilibrium for a decentralized economy is a constant interest rate r, a value function V_r, an allocation policy σ_r, a probability measure λ_r, and a law of motion for aggregate capital stock $\bar{K}_r$, such that*

1. *at interest rate r, for all $w \in W$, the allocation policy σ_r minimizes the objective function of each component planner (5) subject to the promise keeping constraint (6), and the incentive constraint (7);*

2. *the probability measure $\lambda_r \in \Lambda(W, \mathcal{B}(W))$ is time invariant,*

$$\lambda_r(A) = \sum_Y \int_{\{w:\, w'(w,y) \in A\}} P(y|k(w), l(w))\, \lambda_r(dw) \quad \text{for all } A \in \mathcal{B}(W);$$

3. *the aggregate capital stock is constant and finite,*

$$\bar{K}_r \equiv \int_W k(w)\, \lambda_r(dw) < \infty;$$

4. *and the aggregate feasibility condition holds,*

$$\sum_Y \int_W \{y - c(w,y)\}\ P(y|k(w), l(w))\, \lambda_r(dw) = \delta \bar{K}_r.$$

It follows from Theorems 9.2 in Stokey, Lucas, and Prescott (1989) that at a constant interest rate, the optimal allocations of the recursive and sequential formulations are equivalent. It is straightforward to apply the First Welfare Theorem as in in order to establish the efficiency outcome of the component planning problem.

4 Characterization of Optimal Allocations

In this section I study the existence and properties of the optimal allocations and the existence of a nondegenerate stationary distribution of utility entitlements in the steady state of the decentralized economy. All proofs are in the Appendix.

In order to characterize the optimal allocations analytically by the first-order conditions with respect to the effort choice, I follow Rogerson (1985) in imposing the following assumptions on the probability distribution. Let $G(y_j|k,l)$ to denote the corresponding distribution function $G(y_j|k,l) = \sum_{i=1}^{j} P(y_i|k,l)$.

Assumption 1 *For all $y_i \in Y$,*

1. *$P(y_i|k,l)$ is twice continuously differentiable in l;*
2. *$G_l(y_i|k,l) \leq 0$ for all $l \in L$;*
3. *$G_{ll}(y_i|k,l) \geq 0$ for all $l \in L$.*

The second assumption represents the stochastic dominance condition so that higher effort implies higher expected output. The third assumption guarantees the convexity of the distribution function in the form of diminishing returns to labor effort.[4]

Lemma 1

1. *The operator T_r has a unique fixed point, V_r in $C(W)$, and for all $V \in C(W)$, $\lim_{n\to\infty} T_r^n V = V_r$. The function V_r is convex and continuously differentiable;*
2. *$w'(w,y)$ and $c(w,y)$ are nondecreasing functions of w for all $y \in Y$;*
3. *$w'(w,y)$ and $c(w,y)$ are nondecreasing functions of y for all $w \in W$.*

The minimum of the component planner problem is attained by unique policy functions $(k(w), l(w), c(w,y), w'(w,y))$ for each agent $w \in W$ and all possible realizations of output, $y \in Y$. That an agent's consumption and the continuation utility entitlement are nondecreasing functions of the current utility entitlement and that agents are rewarded for a high output realization are standard results of the private information literature (see Rogerson (1985) or Mirrlees (1976)).

The existence of a stationary equilibrium defined in the previous Section depends on the existence of a nondegenerate stationary distribution. Recall that for

[4]The last condition might be too restrictive. In that case, linearization of the component planner's problem by lotteries is a well-known approach demonstrated in Phelan and Townsend (1991), for example.

private information economies Green (1987), Thomas and Worrall (1990) or Atkeson and Lucas (1995) showed that without an exogenously imposed lower bound the utility entitlements would spread out forever over time and no long run stationary distribution exists. Aiyagari and Alvarez (1995) study private information economies in which "misery" is or is not attainable.[5] They find that if misery is not attainable it is possible to have stationary distributions with mobility. I follow their strategy and assume a bounded, separable utility function for which misery is not attainable as the agents can always enjoy their leisure which the planner cannot control.

Thus, suppose that the consumption set $C = [0, \overline{c}]$ so that $W = [\underline{w}, \overline{w}]$, where the greatest utility entitlement is defined as $\overline{w} = (1-\beta)^{-1}\{u(\overline{c}) + v(1)\}$. The associated cost to the $\overline{w}$-component planner $V(\overline{w}) = (1-\beta)^{-1}\overline{c} > 0$. The exogenous lower bound must equal $\underline{w} = (1-\beta)^{-1}\{u(0) + v(1)\}$, i.e. a repeated allocation of zero consumption and full leisure. It is easy to verify that any lower utility entitlement would not be incentive compatible (it is not possible to motivate a positive labor effort without rewarding the agent by positive consumption). Since the agent neither consumes nor works, the value of such an allocation for the $\underline{w}$-component planner is $V(\underline{w}) = 0$.

The allocations and feasibility of the stationary equilibrium for a closed economy depends on the interest rate at which the component planners borrow and lend capital. First, it is useful to identify the values of the interest rate at which there is no feasible stationary equilibrium. Without loss of generality I consider only high and low realizations of output, $\overline{y} = x^{\alpha}$ and $\underline{y} = 0$, respectively. Finally, define the time preference rate as $\rho = (1-\beta)/\beta$.

Lemma 2 *Stationary equilibria with $r \geq \rho$ are not feasible in a closed economy.*

The analysis of the existence of a nondegenerate stationary distribution is based on component planner's intertemporal first order condition,

$$V_r'(w'(w,y)) = (1+r)\beta \left[V_r'(w) + \xi(w) \left(1 - \frac{P(y|k(w), \hat{l})}{P(y|k(w), l(w))} \right) \right],$$

[5] Misery is attainable if agents can be driven to the lower bound of their consumption sets in an incentive compatible way.

where the value function is convex, the Lagrange multiplier $\xi(w)$ on the incentive constraint is positive for a positive labor effort, and the likelihood ratio $P(y|k(w),\hat{l})/P(y|k(w),l(w))$ is a decreasing function of output for any downward deviation on labor effort $\hat{l} \in L$.

If the planners were less patient than the agents, i.e. if $r > \rho$, the planners would promise too much of future resources for current incentives. In the limit, all agents would amass at the exogenous upper bound. If $r = \rho$, all agents that do not work will keep the same utility entitlement forever. There is a positive probability of agents to arrive in finite time at a utility entitlement associated with zero labor supply, namely at least at the exogenous bounds $\{\underline{w}, \overline{w}\}$. Since in such equilibria no agent works but a positive fraction of them consumes, steady states with $r \geq \rho$ are not feasible in a closed economy. This Lemma confirms the results of Kehoe and Levine (1993), Huggett (1997) and Atkeson and Lucas (1995) that in a private information economy the market clearing interest rate is less than the agents' time preference parameter.

Therefore, I will focus on $r \in (0, \rho)$ and show that for such an interest rate the upper bound on the utility entitlements is always endogenous at $w^* \in (\underline{w}, \overline{w})$. The following Lemmas study the optimal continuation utility policy functions for the low and the high realization of output.

Lemma 3 *For interest rate $r \in (0, \rho)$, all inputs $(k, l) \in K \times L$ and a low output $\underline{y} \in Y$, there exist some $\gamma > 0$ such that*

1. $w'(w, \underline{y}) = \underline{w}$ *for* $w \in [\underline{w}, \underline{w} + \gamma]$; *and*

2. $w'(w, \underline{y}) < w$ *for* $w \in (\underline{w} + \gamma, \overline{w}]$.

Lemma 4 *For interest rate $r \in (0, \rho)$, all inputs $(k, l) \in K \times L$ and a high output $\overline{y} \in Y$, there exists utility entitlement $w^* \in (\underline{w}, \overline{w})$ where $l(w^*) > 0$, such that*

1. $w'(w, \overline{y}) > w$ *for* $w < w^*$;

2. $w'(w, \overline{y}) = w$ *for* $w = w^*$; *and*

3. $w'(w, \overline{y}) < w$ *for* $w > w^*$.

For $r \in (0, \rho)$ and all $w \in (\underline{w} + \gamma, w^*)$, the planners punish a low output realization by a lower continuation utility and reward a high output realization by increasing the utility entitlement from tomorrow on. The planners cannot punish the agents below $\underline{w}$ and it is not cost efficient to reward them beyond the endogenous w^*.

The remaining question is whether the exogenous lower bound is an absorbing point of the stationary distribution. The lower bound will not be the absorbing point if the technology is sufficiently productive so that planners associated with agents entitled to low utility agents will ask their agents to work and make positive profits. In other words, the nondegenerate equilibrium exists if the principal-agent relationship delivers a positive surplus at least at some levels of utility entitlements, i.e. for some $w \in W$ it must be the case that at the optimal policies

$$\sum_Y \{c(w, y) + (r + \delta)k(w) - y\} P(y|k(w), l(w)) < 0.$$

Because $V(\underline{w}) = 0$, $V(\overline{w}) > 0$, the convex value function attains its minimum at some w_* where $V(w_*) < 0$, and it is decreasing at low utility entitlements in $w \in [\underline{w}, w_*)$. An example of such a value function is shown in Figure 1.

INSERT FIGURE 1

Provided that the moral hazard is not too severe and the production function is sufficiently productive, an optimal credit system lends the poor agents a positive amount of capital to work with. From the economy-wide point of view, planners associated with relatively poor agents have a surplus while those associated with wealthy agents have a deficit. In equilibrium, positive and negative profits of component planners must be equal for the aggregate feasibility constraint to hold. Because no component planner would assign a continuation utility higher than w^*, there exists a unique, nondegenerate stationary equilibrium with a unique ergodic set $E = [\underline{w}, w^*]$ determined by policy functions shown in Figure 2.

INSERT FIGURE 2

The distribution is degenerate if the technology function is not productive and/or informative enough to overcome the moral hazard problem. In this case, the cost function of the component planners is nondecreasing for all $w \in [\underline{w}, \overline{w}]$. The $\underline{w}$-planner has a zero cost by allocating $l(\underline{w}) = 0$ and $c(\underline{w}, 0) = \underline{c}$, while all the other planners have positive cost $V(w) > 0$ even if their agents were working. Such a stationary equilibrium is feasible as the whole insurance problem collapses to the lower bound utility entitlement at which no agent works or consumes.

The following Proposition summarizes the two cases.

Proposition 1 *Suppose that for some $w \in W$ the optimal allocations $(k(w), l(w), c(w, y))$ imply $\sum_Y \{c(w, y) + (r + \delta)k(w) - y\} P(y|k(w), l(w)) < 0$. Then there exists a stationary recursive competitive equilibrium with a market clearing interest rate r and a unique nondegenerate, stationary distribution λ_r.*

These results are illustrated by numerical simulations in Phelan and Townsend (1991) or Bohacek (2003). The algorithm for finding the equilibrium interest rate is relatively simple. After fixing the parameters of the model, the algorithm iterates on the interest rate: 1. Guess an initial interest rate; 2. Calculate the optimal policies for all component planners and the invariant distribution of utility entitlements; 3. If the feasibility condition does not hold, repeat the previous step with an updated interest rate (lower the interest rate for a deficit).

5 Conclusions

This paper develops a general methodology how to incorporate capital accumulation and allocation into moral hazard economies with heterogeneous agents. Using the decentralization of the economy developed by Atkeson and Lucas (1995) and the concept of "capital planner", it studied one of the basic models of insurance with moral hazard. In the dynamic, general equilibrium model, capital accumulation serves as a tool of finding the stationary equilibrium in a closed economy. I confirm the previous results in the literature that the interest rate must be lower than the discount rate of the agents for the existence of a non-degenerate distribution of resources in the steady state.

This framework allows studying a wide variety of general equilibrium economies with private information without restrictive assumptions on functional forms of production technology or preferences. Incorporating capital to the private information environment is important for policy issues and aggregate results. The allocation of capital among the agents can be interpreted as an optimal credit system with a financial intermediary that allocates capital loans to productive agents. One can work backwards from the system of component planners to find the ownership of assets in the economy. In particular, the difference between an agent's consumption and output can be interpreted as income from assets owned by that agent. Agents who hold a large stock of capital lend the capital to agents who lack resources of their own but are willing to supply high labor effort.

The endogenous accumulation of capital is important for analyzing the general equilibrium effects in private information economies. In Bohacek (2003) I show how important is the accumulation of capital for the equilibrium prices and aggregate levels in economies with moral hazard and optimal social insurance policies. Similar numerical simulations can determine and compare efficiency and equality in economies with different contractual or informational assumptions.

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Appendix

Proof of Lemma 1

Under Assumption 1, the component planner's problem is a minimization of a continuous function V over a compact, nonempty set by applying the operator T_r. With $r > 0$, the operator T_r satisfies the hypotheses of Blackwell's theorem and is a contraction mapping of modulus $1/(1+r)$. $T_r V$ is continuous by the Theorem of the Maximum. The value function is strictly convex as each risk-neutral planner minimizes a cost function by delivering the promised utility entitlements to agents with strictly concave utility function. The differentiability follows from the assumptions on the preferences and the stochastic technology.

The intertemporal first order condition with respect to $w'(w,y)$ is

$$V_r'(w'(w,y)) = (1+r)\beta \left[V_r'(w) + \xi(w) \left(1 - \frac{P(y|k(w),\hat{l})}{P(y|k(w),l(w))} \right) \right], \tag{8}$$

where $\xi(w) \geq 0$ is the Lagrange multiplier on the incentive constraint (the envelope condition has been substituted for the Lagrange multiplier on the promise-keeping constraint). For $l(w) > 0$ the Lagrange multiplier on incentive constraint is positive (see Rogerson (1985)). The first order condition with respect to consumption $c(w,y)$ is

$$\frac{1}{u'(c(w,y))} = V_r'(w) + \xi(w) \left(1 - \frac{P(y|k(w),\hat{l})}{P(y|k(w),l(w))} \right). \tag{9}$$

The combination of the intertemporal and consumption first order conditions for both economies gives

$$u'(c(w,y))V_r'(w'(w,y)) = (1+r)\beta.$$

Under the assumptions on the probability distribution, with concave utility function and convex cost function, policies $c(w,y)$ and $w'(w,y)$ move in the same direction as w changes. Suppose, as a contradiction, that $c(w,y)$ and $w'(w,y)$ are decreasing functions of w, so that for $\underline{w} \leq \hat{w} < w \leq \overline{w}$, $c(\hat{w},y) > c(w,y)$ and $w'(\hat{w},y) > w'(w,y)$ for all $y \in Y$. Then from the promise-keeping constraint

$$w = u(c(w,y)) + v(1-l) + \beta w'(w,y) < u(c(\hat{w},y)) + v(1-l) + \beta w'(\hat{w},y) = \hat{w}$$

which contradicts the initial assumption that $\hat{w} < w$. Therefore, the policies $c(w,y)$ and $w'(w,y)$ are both nondecreasing functions of w for all $y \in Y$.

To prove the last claim of the Lemma, note first that for $l = 0$, there is no incentive problem and the first order conditions become $u'(c(w,y))V_r'(w) = 1$ and $V_r'(w'(w,y)) = (1+r)\beta V_r'(w)$, respectively. Thus for zero labor effort $c(w,y)$ and $w'(w,y)$ are independent of output realization. When $l(w) > 0$, $\xi(w)$ is positive. Recall that $P(y|k,\hat{l})/P(y|k,l)$ ratio is a decreasing function of y for any deviating strategy $\hat{l} < l$. Then for all $w \in W$, $w'(w,\overline{y}) > w'(w,\underline{y})$ and $c(w,\overline{y}) > c(w,\underline{y})$ for all $w \in W$ and $\underline{y}, \overline{y} \in Y$, such that $\overline{y} > \underline{y}$. □

Proof of Lemma 2

First consider the case when $r = \rho$. As $\beta = (1+\rho)^{-1}$, then $(1+r)\beta = 1$ and the intertemporal first order condition simplifies to

$$V_r'(w'(w,y)) = V_r'(w) + \xi(w)\left(1 - \frac{P(y|k(w),\hat{l})}{P(y|k(w),l(w))}\right), \tag{10}$$

At zero labor effort the intertemporal first order condition reduces to $V_r'(w'(w,y)) = V_r'(w)$. Therefore, if $l(w) = 0$ and $r = \rho$, $w'(w,y) = w$ for all $w \in W$ and all $y \in Y$. There exist as many ergodic sets as there are $w \in W$ for which $l(w) = 0$. There are at least two such utility entitlements, namely, the exogenous bounds $\{\underline{w}, \overline{w}\}$.

At utility entitlements where planners assign positive labor effort $l(w) > 0$ the Lagrange multiplier is positive. From the assumptions on the probability distribution, the likelihood ratio $P(y|k(w),\hat{l})/P(y|k(w),l(w))$ is less than one if a high output $\overline{y}$ is realized and greater then one if a low output $\underline{y}$ is realized. Then the continuation utility entitlement policies follow $w'(w,\overline{y}) > w$ for all $w < \overline{w}$ and $w'(w,\underline{y}) < w$ for all $w > \underline{w}$.

Agents assigned nonzero labor effort face positive probability of realizing either output. Then for all agents there exists a positive probability of reaching in finite time a utility entitlement where they do not work and where $w'(w,y) = w$. There are at least two such utility entitlements, namely $\{\underline{w}, \overline{w}\}$. Eventually, all agents amass at such utility entitlements and the distribution becomes degenerate without mobility.

When $r > \rho$, $(1+r)\beta > 1$, and it follows that even for zero labor effort the value function increases $V_r'(w'(w,y)) > (1+r)\beta V_r'(w)$. Then the continuation utility entitlement must increase when a high output is reported or when zero labor effort is assigned. Therefore, all agents must reach the exogenous upper bound $\overline{w}$ in finite time. At $\overline{w}$, $w'(\overline{w},y) = \overline{w}$ and the distribution is degenerate at the exogenous upper bound. Since in such equilibria no agent works but a positive fraction of them consumes, steady states with $r \geq \rho$ are not feasible in closed economies. □

Proof of Lemma 3

At the exogenous lower bound the result is trivial since $\underline{w}$ can be attained only by repeated allocations of the lowest utility entitlement $\underline{w}$. For $w > \underline{w}$, consider first the case when $l(w) = 0$. There is no incentive problem and the first order intertemporal condition equals $V_r'(w'(w,\underline{y})) = (1+r)\beta V_r'(w)$ and $w'(w,\underline{y}) < w$ if $r \in (0,\rho)$.

In case that $l(w) > 0$, the intertemporal first order condition is

$$V_r'(w'(w,\underline{y})) = (1+r)\beta\left[V_r'(w) + \xi(w)\left(1 - \frac{P(\underline{y}|k(w),\hat{l})}{P(\underline{y}|k(w),l(w))}\right)\right]. \tag{11}$$

For $y = \underline{y}$, the ratio $P(\underline{y}|k(w),\hat{l})/P(\underline{y}|k(w),l(w)) > 1$ for any downward deviation $\hat{l} \in L$. Hence the term multiplying the positive Lagrange multiplier is negative and for $r \in (0,\rho)$ the continuation utility entitlement decreases, $w'(w,\underline{y}) < w$ for all $w > \underline{w}$.

By assumption a continuation utility below the lower bound $\underline{w}$ cannot be assigned. By the continuity of $V'(\cdot)$, there is some $\gamma > 0$, such that $w'(w, \underline{y}) = w_*$ for $w \in [\underline{w}, \underline{w}+\gamma]$. For $w \in (\underline{w} + \gamma, \overline{w}]$ the continuation utility policy $w'(w, \underline{y}) < w$. □

Proof of Lemma 4

At the highest utility entitlement $\overline{w}$ labor effort is zero and $w'(\overline{w}, \underline{y}) < \overline{w}$ for $r \in (0, \rho)$. Thus the highest possible utility entitlement is not sustainable if $l(\overline{w}) = 0$ and is not attainable if $l(\overline{w}) > 0$.

For $w \in [\underline{w}, \overline{w})$ and $l(w) > 0$ the incentive constraint binds and $\xi(w) > 0$. For the high output level the ratio $P(\overline{y}|k(w), \hat{l})/P(\overline{y}|k(w), l(w)) < 1$ for any downward deviation $\hat{l} \in L$. Suppose that an agent begins with some arbitrary initial entitlement $w < \overline{w}$, corresponding to some $c \leq \overline{c}$ and $l > 0$. Since the highest continuation utility $\overline{w}$ is not attainable, he can be promised $w' = \overline{w} - \epsilon$ for some $\epsilon > 0$, i.e., he begins with a promised utility $w = u(c) + v(1-l) + \beta(\overline{w} - \epsilon)$. The component planner's first order intertemporal condition (8) becomes

$$V_r'(\overline{w} - \epsilon, \overline{y}) = (1 + r)\beta \left[V_r'(w) + \xi(w) \left(1 - \frac{P(\overline{y}|k(w), \hat{l})}{P(\overline{y}|k(w), l(w))} \right) \right].$$

For establishing the endogenous upper bound on utility entitlement, suppose that the agent's utility entitlement increased to $w'(w, \overline{y}) = \overline{w} - \epsilon > w$. By the continuity of the cost function, the argument can be repeated for a sequence of periods until an entitlement $w = \overline{w} - \epsilon^*$, with $\epsilon^* > 0$, is reached at which $w' = \overline{w} - \epsilon^* = w$, i.e., $w = \overline{w} - \epsilon^* = u(c) + v(1 - l) + \beta(\overline{w} - \epsilon^*)$. Since $c \in C$ and $l \in (0, 1)$ were arbitrary optimal allocations, for $r \in (0, \rho)$ there exists a utility entitlement $w^* \in (w_*, \overline{w})$ for which $w'(w^*, \overline{y}) = w^*$. For utility entitlements $w < w^*$ the continuation utility entitlement increases to $w'(w^*, \overline{y}) > w$, and for $w > w^*$ decreases to $w'(w, \overline{y}) < w$.

Note that at the endogenous bound agents must supply positive labor effort since at w^* the continuation utility equals $w'(w^*, \overline{y}) = w^*$ and $\overline{y}$ can only be realized from positive labor supply. Zero labor effort is not possible at w^* because for $r \in (0, \rho)$ continuations utility $w'(w^*, 0) < w^*$. □

Proof of Proposition 1

Denote the utility entitlement where the cost function has its minimum as w_*. Recall that at the exogenous upper bound $\overline{w} > w_*$ with $V(\overline{w}) = (1 - \beta)^{-1}\overline{c} > 0$.

First, if for all $w \in W$ $\sum_Y \{c(w, y) + (r + \delta)k(w) - y\} P(y|k(w), l(w)) \geq 0$. Then the utility entitlement with the lowest cost must be the exogenous lower bound, $w_* = \underline{w}$, with associated zero labor effort, zero consumption, and value $V(\underline{w}) = 0$. For $r \in (0, \rho)$, Lemma 3 implies that $w'(\underline{w}, 0) = \underline{w}$. Then, $\underline{w}$ is the absorbing point since the probability of reaching $\underline{w}$ in finite time is strictly positive. Such an equilibrium is feasible only if no agent works, no agent consumes and the distribution is degenerate with $\lambda_r(\underline{w}) = 1$.

Second, consider the case in which it is efficient to recommend a positive labor supply at least at some $w \in W$ where for a positive $l(w) > 0$, $\sum_Y \{c(w,y) + (r+\delta)k(w) - y\} P(y|k(w), l(w)) < 0$. From the exogenous $V(\underline{w}) = 0$, the value function reaches its minimum at some $w_* > \underline{w}$ where $l(w_*) > 0$, $V'(w_*) = 0$, and $V(w_*) < 0$, then it increases towards the maximum at $V(\overline{w}) > 0$. It follows from Lemma 4 that for a positive labor supply $w'(w,\overline{y}) > w \geq w'(w,\underline{y})$ for $w \in [\underline{w}, w^*)$.

That the time-invariant distribution is unique follows from the conditions of Theorem 11.12 in Stokey, Lucas, and Prescott (1989). Define the adjoint T^* operator for a probability measure λ_r on $(W, \mathcal{B}(W))$ as

$$(T^* \lambda_r)(A) = \sum_Y \int_{\{w:\, w'(w,y) \in A\}} P(y|k(w), l(w))\, \lambda_r(dw) \quad \text{for all } A \in \mathcal{B}(W).$$

I will show that there exists $\epsilon > 0$ and $N \geq 1$ such that $(T^{*N}\lambda_r)(\underline{w}) \geq \epsilon$ for all $w \in W$. For interest rate $r \in (0, \rho)$, Lemma 3 shows that there exists $\gamma > 0$ such that $w'(w,\underline{y}) = \underline{w}$ for $w \in [\underline{w}, \underline{w} + \gamma]$ or $w'(w,\underline{y}) < w$ for $w \in (\underline{w} + \gamma, \overline{w}]$. Assume that the probability of realizing low output is at least $\underline{p} > 0$ (it is one when no effort is assigned) and choose a large number N so that $w^* - N\underline{p} \leq \underline{w}$. As $w'(w,\underline{y})$ is nondecreasing in w, the transition to $\underline{w}$ is at least as probable as from any other $w \in W$ and $\epsilon = \underline{p}^N$. Similarly, assume that the probability of realizing the high output for $l(w) > 0$ is at least $\overline{p} > 0$. Then Lemma 4 establishes $w'(w,\overline{y}) > w$ for $w < w^*$ and $\epsilon = \overline{p}^N$.

As the Condition M holds for $N \geq 1$ and $\epsilon > 0$, then by Lemma 11.11 in Stokey, Lucas, and Prescott (1989) the operator $T^{*N} : \Lambda(W, \mathcal{B}(W)) \to \Lambda(W, \mathcal{B}(W))$ is a contraction mapping on the probability space $(W, \mathcal{B}(W))$ with the total variation norm. There exists a unique probability measure $\lambda \in \Lambda(W, \mathcal{B}(W))$ such that

$$\|\hat{T}_r^{Nk}\lambda - \lambda_r\| \leq (1-\epsilon)^k \|\lambda - \lambda_r\|$$

for all $\lambda_r \in \Lambda(W, \mathcal{B}(W))$ and $k = 1, 2, \ldots$. □

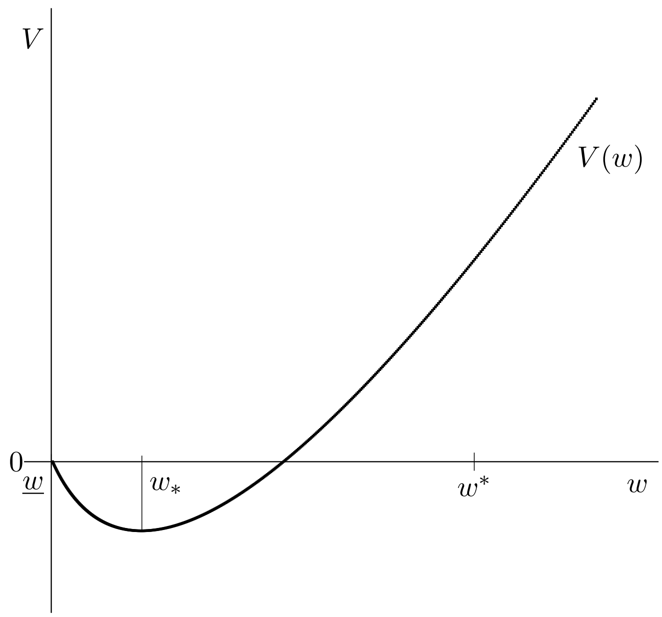

Figure 1: Value function of component planners for an equilibrium interest rate $r \in (0, \rho)$, corresponding to a stationary distribution of utility entitlements with a unique ergodic set $E = [\underline{w}, w^*]$.

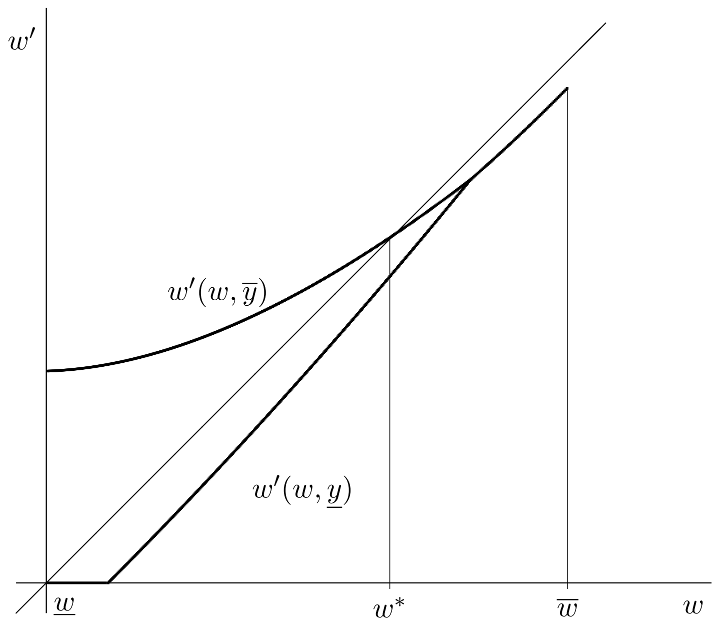

Figure 2: Continuation utility policy functions for high and low output realizations, for an equilibrium interest rate $r \in (0, \rho)$. Endogenous ergodic set $E = [\underline{w}, w^*]$.