













# SELF-SELECTION model (Borjas, 1987)

• *Set-up of the model:*

Two country model of migration:

0 – country of origin

1 – destination country

The wage of individual *i* in country *0* is:

$$
\ln(w_{i0}) = \mu_0 + \varepsilon_{i0} \tag{1}
$$

If everyone from country 0 were to migrate to the host country, their earnings would be:

$$
\ln(w_{i1}) = \mu_1 + \varepsilon_{i1} \tag{2}
$$

Where  $\varepsilon_{i0}$  and  $\varepsilon_{i1}$  are jointly normally distributed, and  $\sigma_{0,1}$  denotes cov  $(\varepsilon_{i0},\varepsilon_{i1})$ :

$$
\begin{pmatrix} \varepsilon_{i0} \\ \varepsilon_{i1} \end{pmatrix} \sim N \begin{bmatrix} 0 & \sigma_0^2 & \sigma_{1,0} \\ 0 & \sigma_{0,1} & \sigma_1^2 \end{bmatrix}
$$

Borjas assumes that  $\mu_1$  also gives the earnings of the average native worker in the US.

We can think of these expressions as decomposing earnings into the part explained by observable characteristics such as age and completed education ( $\mu_0$  and  $\mu_1$ ) and a part due to **unobserved characteristics** ( $\varepsilon_{i0}$  and  $\varepsilon_{i1}$ ).

# SELF-SELECTION model (Borjas, 1987)

• Let  $\rho_{0,1}$  denote the correlation coefficient of  $\varepsilon_0$  and  $\varepsilon_1$ , which represents the correlation of productive ability in the home country with productive ability in the US (skill transferability):

• 
$$
\rho_{0,1} = \frac{\text{cov}(\varepsilon_{i0}, \varepsilon_{i1})}{\sigma_0 \sigma_1} = \frac{\sigma_{0,1}}{\sigma_0 \sigma_1}
$$
 (3)

This correlation coefficient can range from 1 to -1. Bigger numbers in absolute value mean a stronger relationship between earnings in the origin and the destination.

If  $\rho$  is positive, then people who have higher-than-average earnings in the origin also have higher-than-average earnings in destination; and people with lowerthan-average earnings in the origin have lower-than-average earnings in the destination -> in case of skills that are valuable in origin being valuable in destination too (Indian IT expert in India and the US; …). The more similar the countries are, the higher  $\rho$  is likely to be.

If  $\rho$  is negative, then people who have higher-than-average earnings in the origin have lower-than-average earnings in the destination.  $\rho$  can be negative for immigrants moving from some developing countries to industrialized countries.

#### **Migration Decision**

Let C denote the cost of moving. Borjas defines  $C = \pi w_0$  so that moving costs are expressed relative to the home country wage;  $\pi = C/w_0$ . Borjas calls  $\pi$  a "time-equivalent" measure of migration costs.

People choose to migrate if their earnings will be higher in the destination than in the origin, net migration costs, (or an individual  $i$  will choose to migrate if  $I > 0$ ):

$$
I = \ln\left(\frac{w_1}{w_0 + C}\right) = \ln(w_1) - \ln(w_0 + C) =
$$

$$
\ln(w_1) - \ln(w_0 + \pi w_0) = \ln(w_1) - \ln(w_0(1 + \pi)) =
$$

$$
\mu_1 + \varepsilon_1 - \mu_0 - \varepsilon_0 - \ln(1 + \pi) \approx (\mu_1 - \mu_0 - \pi) + (\varepsilon_1 - \varepsilon_0)
$$

Where we use the first order Taylor approximation to approximate  $\ln(1 + \pi) \approx \pi$ .

Define 
$$
v = \varepsilon_1 - \varepsilon_0
$$
. Since migration occurs if  $I > 0$ , we can write the migration rate P as:

$$
P = Pr[I > 0] = Pr[(\mu_1 - \mu_0 - \pi) + (\varepsilon_1 - \varepsilon_0) > 0] =
$$

$$
Pr[(\varepsilon_1 - \varepsilon_0) > -(\mu_1 - \mu_0 - \pi)] =
$$

$$
Pr[v > -(\mu_1 - \mu_0 - \pi)] =
$$

$$
Pr[v > (\mu_0 - \mu_1 + \pi)] \qquad (4)
$$

#### **Migration Decision**

Define  $z = \frac{\mu_0 - \mu_1 + \pi}{\sigma_v}$ , and let  $\Phi$  denote the CDF of the standard normal distribution. Note that *v* has a normal distribution, as the sum of two normal distributions is also normal and  $\frac{v}{\sigma_v}$  has a standard normal distribution (joint normality). Therefore we can rewrite equation 4 as:

$$
P = Pr\left[\frac{v}{\sigma_v} > \frac{\mu_0 - \mu_1 + \pi}{\sigma_v}\right] =
$$

$$
1 - Pr\left[\frac{v}{\sigma_v} \le \frac{\mu_0 - \mu_1 + \pi}{\sigma_v}\right] =
$$

$$
1 - \Phi\left[\frac{\mu_0 - \mu_1 + \pi}{\sigma_v}\right] = 1 - \Phi[z] \qquad (5)
$$

For higher values of z, P is lower - implying migration is less likely. The migration rate P is:

- increasing in mean US wages  $\left(\frac{\partial P}{\partial \mu_1} > 0\right)$ ,
- decreasing in mean home country wages  $\left(\frac{\partial P}{\partial \mu_0} < 0\right)$ , and
- decreasing in moving costs  $\left(\frac{\partial P}{\partial \pi} < 0\right)$ .

Borjas assumes that  $P < 1$ , so that at least part of the population in the country of origin is better off not migrating. He also assumes  $\mu_1 \approx \mu_0$ .

#### **Some Properties**

Deriving Borjas's expressions for self-selection requires applying some properties of the normal distribution and a version of the law of iterated expectations:

*Property1: If a vector of random variables*  $X \sim N(\mu, \Sigma)$ , then  $AX + b \sim N(A\mu + b, A\Sigma A')$ 

**Property2:** If 
$$
\begin{pmatrix} X \\ Y \end{pmatrix} \sim N \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}
$$
,  $\begin{pmatrix} \sigma_x^2 & \sigma_{X,Y} \\ \sigma_{X,Y} & \sigma_y^2 \end{pmatrix}$ , then  $(Y | X = x) \sim N \begin{pmatrix} \mu_y + \rho_{X,Y} \begin{pmatrix} \sigma_Y \\ \sigma_X \end{pmatrix} (X - \mu_x)$ ,  $\sigma_y^2 (1 - \rho_{X,Y}^2)$ 

*Property3: Law of iterated expectations: For any non-stochastic function*  $f(.)$  *and* $X =$  $f(W)$ ,  $E(Y|X) = E(E(Y|W)|X)$ 

*Property4: Inverse Mills Ratio: Let*  $\varphi(z)$  *and*  $\varphi(z)$  *denote the PDF and CDF of the standard normal distribution respectively. If*  $\frac{v}{\sigma_v} \sim N(0,1)$ *, then*  $E\left[\frac{v}{\sigma_v} | \frac{v}{\sigma_v} > z\right] = \frac{\varphi(z)}{1-\varphi(z)}$ . *We refer to this expression as the Inverse Mills Ratio. Because*  $\varphi(z) = \varphi(-z)$  and  $1 - \varphi(z)$  $\Phi(z) = \Phi(-z)$ , we can also write Inverse Mills Ratio as  $\lambda(z) = \frac{\varphi(-z)}{\Phi(-z)}$ 

### **Selection**

Given propositions 1-4 we can now analyze self-selection. We want to derive expressions that let us compare  $E(\ln(w_0)|I > 0)$  and  $E(\ln(w_1)|I > 0)$ ; i.e. for individuals who immigrate we'd like to compare average log earnings in country 0 and average log earnings in country1. Let's start with home country:

$$
E(\ln(w_0)|I>0) = E\left[\mu_0 + \varepsilon_0 | \frac{v}{\sigma_v} > z\right] =
$$

$$
\mu_0 + E\left[\varepsilon_0 | \frac{v}{\sigma_v} > z\right] = \mu_0 + \sigma_0 E\left[\frac{\varepsilon_0}{\sigma_0} | \frac{v}{\sigma_v} > z\right] \quad (6)
$$

First, note that  $\varepsilon_0$  and  $v$  are jointly normally distributed and using property l

$$
\binom{\varepsilon_0}{v} \sim N \left( \binom{0}{0}, \binom{\sigma_0^2}{\sigma_{0,1} - \sigma_0^2} \quad \begin{array}{c} \sigma_{0,1} - \sigma_0^2 \\ \sigma_0^2 + \sigma_1^2 - 2\sigma_{0,1} \end{array} \right) \tag{7}
$$

**Second,** given that  $\varepsilon_0$  and  $v$  are jointly normally distributed, applying property2 we can show that:

$$
E(\varepsilon_0|v) = \rho_{0,v} \left(\frac{\sigma_0}{\sigma_v}\right) v \qquad (8)
$$

Further using the fact that  $\rho_{0,v} = \frac{\sigma_{0,v}}{\sigma_0 \sigma_v}$ , we can simplify equation 8 further:

$$
E(\varepsilon_0|v) = \frac{\sigma_{0,v}}{\sigma_0 \sigma_v} \frac{\sigma_0}{\sigma_v} v = \frac{\sigma_{0,v}}{\sigma_v^2} v \qquad (9)
$$

### **Selection**

**Third,** applying property3 we can show that:

$$
E\left[\frac{\varepsilon_0}{\sigma_0} \Big| \frac{\nu}{\sigma_\nu} > z\right] = E\left[E\left(\frac{\varepsilon_0}{\sigma_0} \Big| \frac{\nu}{\sigma_\nu} > z\right) \Big| \frac{\nu}{\sigma_\nu} > z\right] \tag{10}
$$

**Finally,** Let  $s = \frac{v}{\sigma_v} \sim N(0,1)$ . Applying property2 and equations (8) and (9):

$$
E\left[\frac{\varepsilon_0}{\sigma_0} \Big| \frac{v}{\sigma_v}\right] = \frac{1}{\sigma_0} E[\varepsilon_0 | s] = \frac{1}{\sigma_0} \frac{\sigma_{0,s}}{\sigma_s^2} s =
$$
  

$$
\frac{1}{\sigma_0} \frac{\cot(\varepsilon_0, \frac{v}{\sigma_v})}{\sigma_s^2} \frac{v}{\sigma_v} = \frac{1}{\sigma_0} \frac{\frac{1}{\sigma_v} \cos(\varepsilon_0, v)}{1} \frac{v}{\sigma_v} =
$$
  

$$
E\left[\frac{\varepsilon_0}{\sigma_0} \Big| \frac{v}{\sigma_v}\right] = \rho_{0,v} \frac{v}{\sigma_v} \qquad (11)
$$

#### **Selection**

Returning to equation (6), now we have:  $E(\ln(w_0)|I > 0) = \mu_0 + \sigma_0 E \left[ \frac{\varepsilon_0}{\sigma_0} \Big| \frac{\nu}{\sigma_0} \right]$  $\frac{1}{\sigma_v} > z$  $= \mu_0 + \sigma_0 E \left[ E \left( \frac{\varepsilon_0}{\sigma_0} \right) \frac{\nu}{\sigma_1} \right]$  $\sigma_v$  $> z\bigg)\big|\frac{v}{\sigma_v}$ (we used equation 10)  $= \mu_0 + \sigma_0 E \left( \rho_{0,v} \frac{\partial v}{\partial x} \right)$  $\frac{v}{\sigma_v}$   $\left| \frac{v}{\sigma_v} \right|$  $\sigma_v$ (we used equation 11) =  $\mu_0 + \sigma_0 \rho_{0,\nu} E \left( \frac{\nu}{\sigma_v} \right) \frac{\nu}{\sigma_v}$  $\frac{1}{\sigma_v} > z$  $= \mu_0 + \sigma_0 \rho_{0,\nu} \left( \frac{\varphi(z)}{1 - \Phi(z)} \right)$ (we used property4)  $E(\ln(w_0)|I>0) = \mu_0 + \sigma_0 \rho_{0,\nu} \left( \frac{\varphi(z)}{1 - \Phi(z)} \right)$  (12) We can derive a similar expression for  $E(\ln(w_1) | I > 0)$  $E(\ln(w_1)|I>0) = \mu_1 + \sigma_1 \rho_{1,\nu} \left(\frac{\varphi(z)}{1-\Phi(z)}\right)$  (13)

#### **Selection** Now let us further simplify equations 12 and 13. We will start with equation 12. Using that  $\sigma_{0,v} = cov(\varepsilon_0, v) = E(\varepsilon_0[\varepsilon_1 - \varepsilon_0]) = E(\varepsilon_0 \varepsilon_1) - E(\varepsilon_0 \varepsilon_0) = \sigma_{0,1} - \sigma_0^2$ :  $E(\ln(w_0)|I>0) = \mu_0 + \sigma_0 \rho_{0,\nu} \left( \frac{\varphi(z)}{1 - \Phi(z)} \right) =$  $\mu_0 + \sigma_0 \frac{\sigma_{0,v}}{\sigma_{0,v}}$  $\sigma_0 \sigma_v$  $\frac{\varphi(z)}{1-\Phi(z)}\bigg) =$  $\mu_0 + \frac{\sigma_{0,v}}{\sigma_v}$  $\left(\frac{\varphi(z)}{1-\Phi(z)}\right) = \mu_0 + \frac{\sigma_{0,1} - \sigma_0^2}{\sigma_v}$  $\overline{c}$  $\sigma_v$  $\left(\frac{\varphi(z)}{1-\Phi(z)}\right)=$  $\mu_0 + \frac{\sigma_0 \sigma_1}{\sigma_v}$  $\sigma_{0,1}$  $\frac{\sigma_{0,1}}{\sigma_0 \sigma_1} - \frac{\sigma_0}{\sigma_1}$  $\left(\frac{\varphi(z)}{1-\Phi(z)}\right)=$  $E(\ln(w_0)|I>0) = \mu_0 + \frac{\sigma_0 \sigma_1}{\sigma_0} \left(\rho_{0,1} - \frac{\sigma_0}{\sigma_1}\right)$  $\frac{\varphi(z)}{1-\Phi(z)}$  (14) Analogously for equation 13, destination country:  $E(\ln(w_1)|I>0) = \mu_1 + \frac{\sigma_0 \sigma_1}{\sigma_v}$  $\sigma_1$  $\frac{\sigma_1}{\sigma_0} - \rho_{0,1} \bigg) \bigg( \frac{\varphi(z)}{1 - \Phi(z)} \bigg)$  (15)

### **Selection**

Define income differential Q and  $\mu_j + Q_j$  as the expected wage of migrants in country j. In order to understand the position of migrants in the distribution of workers in each country (that is, whether migrants are positively or negatively selected), we want to know the signs of  $Q_0$  and  $Q_1$ :

$$
Q_0 \equiv E(\varepsilon_0 | I > 0) = \frac{\sigma_0 \sigma_1}{\sigma_v} \left( \rho_{0,1} - \frac{\sigma_0}{\sigma_1} \right) \left( \frac{\varphi(z)}{1 - \Phi(z)} \right) \tag{16}
$$

$$
Q_1 \equiv E(\varepsilon_1 | I > 0) = \frac{\sigma_0 \sigma_1}{\sigma_v} \left( \frac{\sigma_1}{\sigma_0} - \rho_{0,1} \right) \left( \frac{\varphi(z)}{1 - \Phi(z)} \right) \tag{17}
$$

Note that the Inverse Mills Ration is always positive, as well as  $\frac{\sigma_0 \sigma_1}{\sigma_v}$ , hence the sign depends only on  $\rho_{0,1} - \frac{\sigma_0}{\sigma_1}$  and  $\frac{\sigma_1}{\sigma_0} - \rho_{0,1}$ .

### **Three Cases of Immigrant Selection**

**Positive selection:**  $Q_0 > 0$  and  $Q_1 > 0$  Migrants are positively selected relative to either country's income distribution  $\Leftrightarrow \rho_{0,1} > \frac{\sigma_0}{\sigma_1} \Rightarrow \frac{\sigma_1}{\sigma_0} > 1$  as  $\rho_{0,1} \le 1$ . This requires a high correlation between the value of skills in countries 0 and 1, and that income is more dispersed in the US than in country 0. Borjas's example is high-skilled migration from Western Europe. **Negative selection:**  $Q_0 < 0$  and  $Q_1 < 0$ . Migrants are negatively selected relative to either country's income distribution  $\Leftrightarrow \rho_{0,1} > \frac{\sigma_1}{\sigma_0} \Rightarrow \frac{\sigma_0}{\sigma_1} > 1$  as  $\rho_{0,1} \le 1$ . This requires a high correlation between the value of skills in countries 0 and 1, and that income is less dispersed in the US than in country 0. Borjas's example is the US social safety net drawing low-skilled immigrants from countries with less of a social safety net. **Refugee sorting:**  $Q_0 < 0$  and  $Q_1 > 0$ . Migrants are negatively selected relative to the home country income distribution, but fall in the top of the US income distribution  $\Leftrightarrow \rho_{0,1}$  $min\left\{\frac{\sigma_0}{\sigma_1};\frac{\sigma_1}{\sigma_0}\right\}$  $\frac{a_1}{\sigma_0}$ . This requires a low correlation between the value of skills in country 0 and in country 1. Borjas argues this may be the case for (communist) countries/countries that have recently experienced a Communist takeover. **No fourth case:**  $Q_0 > 0$  and  $Q_1 < 0$ . Mathematically, this case is ruled out because it would require  $\rho_{0,1} > 1$ . To see this note that  $Q_0 > 0$  implies  $\rho_{0,1} > \frac{\sigma_0}{\sigma_1}$  and, hence,  $\frac{\sigma_0}{\sigma_1} < 1 \Rightarrow \frac{\sigma_1}{\sigma_0} > 1$ and  $Q_1 < 0$  implies  $\rho_{0,1} > \frac{\sigma_1}{\sigma_0}$  but this is not possible as  $\frac{\sigma_1}{\sigma_0} > 1$ .

#### **WHO MIGRATES? Chiswick's model application to migrant selectivity**

*Model of why migrants can be "favourably" or "unfavorably" selected*

•The rate of return in line with HC framework can be rewritten as:

$$
r = \frac{W_b - W_a}{C_f + C_d}
$$

•Where Cf are foregone earnings and Cd direct out of pocket money, Wb represents earnings in destination, Wa in origin.

•Migration occurs if the rate of return from the investment in migration  $(r)$  >= the interest rate for investment in HC (i)

•Suppose, two groups of workers, low and high ability; lets assume wages are e.g. 100k percent higher for more able:

$$
W_{b,h} = (1 + k)W_{b,l}
$$
  

$$
W_{a,h} = (1 + k)W_{a,l}
$$

It is assumed that direct costs do not wary with ability, but ability raises the value of foregone earnings:

 $C_{f,h} = (1 + k)C_{f,l}$ 

The rate of return to high-ability person:

$$
r_{h} = \frac{(1+k)W_{b,l} - (1+k)W_{a,l}}{(1+k)C_{f,l} + C_{d}} = \frac{W_{b,l} - W_{a,l}}{C_{f,l} + (C_{d}/(1+k))}
$$

Thus  $r_h > r_l$  as long as earnings increase with ability ( $k > 0$ ) and there are positive out of pocket costs of migration => selectivity of those people to migrate.

If there were no out-of-pocket costs  $(C_d = 0)$ , then  $r_h = r_l$  and there would be no selectivity in migration on basis of ability OR if there were no labor-market premium for higher level of ability (i.e. k=0), then  $r_h = r_l$  and there is no selectivity in migration on basis of this dimension of ability.

Now, Chiswick (1999) adds further an assumption that more able are more efficient in migration,

The more able need less time *t* to accomplish the task, and greater efficiency gives greater returns in migration.

If the more able may also be more efficient in utilizing out-of-pocket expenditure, then the difference in the rate of return to migration is even greater.

If  $C_{d,h} = (1 + \gamma)C_{d,l}$ , where  $\gamma$  is a direct cost-efficiency parameter, and  $\gamma < 0$ , then:

$$
r_{h} = \frac{W_{b,l} - W_{a,l}}{C_{f,l} + \frac{C_{d,l}(1+\gamma)}{(1+k)}}
$$

Thus, there are several implications from the model: the larger out-of-pocket costs of migration, the lower is the propensity to migrate, but also the lower is the return migration rate and the **greater the propensity for favorable selectivity in migration**. This propensity for favorable selectivity is intensified if those who are more able in the labor market are also more efficient in the migration and adjustment process.



## Empirical evidence on Selection - who moves

- Earlier studies limited due to lack of data some early studies are in line with the self selection (Borjas 1987) model, e.g. they find immigrant's (in the US) wages are negatively related to income inequality in origins (Borjas, 1987; Cobb-Clark, 1993).
- More recently, when the data become available, a number of studies in odds with Borjas 1987 model predictions:
	- Belot and Hatton (2012) examine the education levels of immigrants from 70 source countries to 21 OECD countries. They find that immigrants tend to be positively selected in terms of education as the difference in wages between high- and low-educated workers, as a proxy for the relative return to skills, widens between the destination and the source country, i.e as the return to skill increases in the destination relative to the origin, selection becomes more positive. This result is present only after controlling for poverty rates in the source countries – poverty prevents low-skilled from migrating from countries with high returns to skills.
	- Grogger and Hanson (2011) study the selection of immigrants from 100 source countries to 15 highincome OECD destinations. They find that immigrants to those countries are positively selected, although for many countries they should be negatively according to Borjas (1987) model. They find that bigger differences in the relative return to skill between the destination and the origin decreases selectivity -> the opposite of the Borjas (1987) model. The as well argue that liquidity constrains/poverty constrains prevent low-skilled people from migrating.

# Empirical evidence on Selection - who moves – other mechanisms..

- Migration costs and cultural factors shape selectivity as well: research finds that the bigger the distance between the destination and origin (distance=proxy for migration costs), the more educated immigrants are (in line with Chiswicks 1999 HC model predictions).
- Linguistic proximity is usually positively related to immigrants educational levels => when skills are more transferable (or  $\rho$  is higher) immigratns are more positively selected.
- Interestingly, historical colonial relationship between source and destination is associated with more negative selectivity.
- NETWORKS immigrants selectivity tends to decrease as migrant networks grow bigger immigrant networks are associated with more negative selection of immigrants (Bertoli and Rapoport, 2013). Immigrant networks tend to be most important pull factor in migration in particular for migrants coming from poor countries and countries with low educational levels (Pedersen, Pytlikova and Smith, 2008).
- IMMIGRATION POLICY selective (Canada point system, Australia..recently Denmark, Germany)





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#### **ADJUSTMENT OF IMMIGRANTS**

•More on immigrant assimilation and integration; and performance of second generations of immigrants during the next lectures

•*Now an example of how do CEE migrants fare..*

#### **How do CEE migrants fare? Post-enlargement evidence**

- Main sending countries:
	- **UK**: Poland, Slovakia, Lithuania,
	- **Ireland**: Poland, Lithuania, Latvia
	- **Sweden**: Poland, Lithuania, Estonia
- Sectoral distribution of immigrants:
	- **UK**: hotels and restaurants, manufacturing, agriculture/construction
	- **Ireland**: construction, manufacturing, hotels and restaurants
	- **Sweden**: health care, trade, manufacturing

#### **How do CEE migrants fare? Post-enlargement evidence**

•Characteristics of post-enlargement immigrants:

- **UK**:
	- young,
	- males,
	- single,
	- rel. highly educated (with qualifications),
	- higher empl. rate than of natives and non-EU migrants.
	- Earn less than natives, later arrivals earn less than earlier arrivals.













