Understanding Interest Rates

FIN 204 Lecture 2.1.

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Outline

1. What Are the Interest Rates?
2. The Internal Rate of Return
3. The Difference Between Interest Rates and Internal Rate of Return
4. The Difference Between Interest Rates and the Rate of Return
5. Real and Nominal Interest Rates
Why are interest rates important?

- the most closely watched variables in the economy. Why?
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- the most closely watched variables in the economy. Why?
- directly affect life of citizen and firms
- buy a house or invest on the stock market or put your money in a bank?
- invest in a new project, or buy government bonds?
What are the interest rates?

Definition: Interest Rate

**Interest rate** is the amount of money that a given asset or debt instrument worth 100 units of money produces per unit of time to its owner.

However, assets live infinitely long, bonds can be very short-term. How do we compare the money that they produce for us?
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However, assets live infinitely long, bonds can be very short-term. How do we compare the money that they produce for us? *present value*
You win a lottery promising 1 mln. per year for 10 yrs. Did you win 10 mln?
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**Definition: Present Value**

*Present Value* of an asset is what the lifetime yield of an asset is worth today.

How do we calculate the present value of an asset?
Calculating the Present Value

Suppose you give a loan of 100 to your friend. You require 10% back at the end of the period. How much will you get at the end of the period?
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\[ 100 + 10 = 110 = \]

\[ 100 \times (1 + 0.1) = \]
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Suppose you give these 110 to your friend. You require 10% back at the end of the period. How much will you get at the end of the period?

\[ 110 + 11 = 121 = 100(1 + 0.1)^2 \]
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Suppose you give these 121 to your friend again. You require 10% back at the end of the period. How much will you get at the end of the period?

\[ 121 + 12 = 133 = 100(1 + 0.1)^3 \]
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Suppose you give these 121 to your friend again. You require 10% back at the end of the period. How much will you get at the end of the period?

\[ 121 + 12.1 = 131.1 = 100(1 + 0.1)^3 \]
Calculating the Present Value

How to generalize the argument for $n$ periods?

The present value is:

$$PV = \frac{FV}{(1 + i)^n}$$

Back to the lottery example. How much did you really win?
Calculating the Present Value

How to generalize the argument for \( n \) periods?

The present value is:

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Back to the lottery example. How much did you really win? 6,759,019 or something;-)
The Internal Rate of Return (IRR) is the most accurate measure of interest rates; it is what economists mean when they use the term interest rate.

**Definition: Internal Rate of Return (IRR)**

The **IRR** is an *interest rate* with a special quality: it equates the present value of payments received from a debt instrument with its face value today.

How do we calculate the IRR?
How to calculate the IRR of a 1-year simple loan with 10% interest rate?

Face value today

100

Present value

\[
\frac{110}{1+i}
\]

Now solve for \(i\).

\[i = \ldots\]
How to calculate the IRR of a 1-year simple loan with 10% interest rate?

Face value today
100

Present value
\[ \frac{110}{1+i} \]

Now solve for \( i \).

\( i = ... \)

For simple loans, the interest rate equals the IRR.
Calculating the IRR for a fixed payment loan

The interest rate and the $i$ for simple loans are equal. Is that so for other types of loans?
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**Definition: Fixed Payment Loan**

The **Fixed Payment Loan** is a loan in which the lender requires that a fixed amount of money is paid back every period. This fixed amount consists of part of the principal and an interest payment.

Examples?
Calculating the IRR for a fixed payment loan

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Examples? Mortgages, leasing of a car...
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Examples? Mortgages, leasing of a car...

Example: The price of a car today is 1000. The car company offers you a payment plan of \( 2 \times 600 \) at the end of the 1st and the 2nd years. What is the IRR for this loan?
Calculating the IRR for a fixed payment loan

Value today
1000

Let’s solve for $i$...

\[
PV = \frac{600}{1 + i} + \frac{600}{(1 + i)^2}
\]
Calculating the IRR for a fixed payment loan

Value today

1000

Let’s solve for $i$...

Answer: $i = 13.1$
Calculating the IRR for a fixed payment loan

Value today

\[ \frac{600}{1 + i} + \frac{600}{(1 + i)^2} \]

Let’s solve for \( i \)...

Answer: \( i = 13.1 \)

How much will you pay with a simple loan of 1000 for 2 yrs, and interest rate of 13.1?
Calculating the IRR for a fixed payment loan

Value today

\[ 1000 \]

Let’s solve for \( i \)...

Answer: \( i = 13.1 \)

How much will you pay with a simple loan of 1000 for 2 yrs, and interest rate of 13.1?

Answer: \( 1000(1 + 0.131)^2 = \ldots \)
Calculating the IRR for a fixed payment loan

Value today

\[
PV \quad 1000 \quad \frac{600}{1+i} + \frac{600}{(1+i)^2}
\]

Let’s solve for \( i \)...
Answer: \( i = 13.1 \)

How much will you pay with a simple loan of 1000 for 2 yrs, and interest rate of 13.1?
Answer: \( 1000(1 + 0.131)^2 = ... = 1279 \)
Calculating the IRR for a fixed payment loan

Value today

\[ 1000 \]

\[ \frac{600}{1 + i} + \frac{600}{(1 + i)^2} \]

Let’s solve for \( i \)...

Answer: \( i = 13.1 \)

How much will you pay with a simple loan of 1000 for 2 yrs, and interest rate of 13.1?

Answer: \( 1000(1 + 0.131)^2 = \ldots = 1279 \)

So, the interest rate and the IRR for any other loan besides a simple one are different.
The Difference Between Interest Rates and the Rate of Return

Is the interest rate an accurate measure of what an asset can bring you?
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Is the interest rate an accurate measure of what an asset can bring you? Does the price of the asset matter?
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**Definition: Rate of return**

*Rate of return* of an asset is the interest payments to the owner *plus* the change in its value, expressed as a fraction of its purchase price.
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Example: A bond with price $p_t = 100$. 
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Example: A bond with price $p_t = 100$. Interest rate (coupon payment, yield) at the end of the year $C = 10\% = 10$. 

Let's follow the definition:

$$\text{RET} = C + \frac{p_{t+1} - p_t}{p_t}$$
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$\text{Rate of return} = \frac{C + p_{t+1} - p_t}{p_t}$
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Disentangling the Rate of Return

What stands behind it?

**Definition: Current yield**

*Current yield* of an asset is the interest payments to the owner expressed as a fraction of its purchase price: 

\[ i_c = \frac{C}{p_t} \]

**Definition: Capital gain**

*Capital gain* of an asset is the increase of its price expressed as a fraction of its purchase price: 

\[ g = \frac{p_{t+1} - p_t}{p_t} \]
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Therefore

\[
RET = C + g
\]
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P. Stankov (AAC)
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\]

Pretty different concepts: the *interest rate*, the *rate of return*, and the *internal rate of return*. 
Real and Nominal Interest Rates

What is the effect of inflation on interest rates?

**Definition: Nominal Interest Rate**

The **Nominal Interest Rate** is the interest rate that is written down in a mortgage contract, on the face of a bond as a coupon, or on another debt instrument such as a fixed-term loan.

**Definition: Real Interest Rate**

The **Real Interest Rate** is the inflation-adjusted nominal interest rate:

\[ i_r = i - \pi_e \]

Why do we need the real interest rate?
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Example: A deposit in a bank with 5% interest rate. Expected inflation rate is 7%. How much is your money worth today?

\[ FV = 100 \times (1 + 0.05) = 105 \]

However, \[ PV = \frac{FV}{1 + 0.07} = \frac{105}{1.07} = 98.13 \]
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Real and Nominal Interest Rates

How do the real and nominal interest rates compare?

[Graph showing interest rate trends from 1955 to 2000, with labels for nominal rate and estimated real rate.]