Seasonal Unit Root Tests in a Time Series with A-priori Unknown Deterministic Components

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Abstract

The present work introduces a sequential strategy of testing for seasonal unit roots. In each step, we test jointly for seasonal unit roots at all frequencies and for the presence of a specific combination of deterministic components. For this purpose, we propose F-type statistics and provide their percentiles. The fixed regressors may include a constant, seasonal dummies, a common trend or a seasonal trend. The most general non-rejected specification is employed to test for seasonal unit roots at distinct frequencies. The procedure is applied to a number of seasonally unadjusted quarterly time series.

JEL classification: C12, C15, C22

Keywords: seasonal, unit root, F-test, deterministic components

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I. Introduction

Over the last quarter of a century there has been a growing interest in testing for unit roots, starting with the seminal paper by Dickey and Fuller (1979). The importance of deterministic components was recognized early on and Dickey and Fuller (1981) introduced likelihood ratio statistics for joint tests of unit roots and an intercept or a linear trend. Smith and Taylor (1999) generalize their results in a seasonal context. In these tests, it is implicitly assumed that a researcher *apriori* knows the structure of the deterministic component. The uncertainty about existence of a trend is explicitly considered in Perron (1988)[3] and more thoroughly in Ayat and Burridge (2000). In a seasonal time series, the deterministic component can be quite involved and may include a constant term, seasonal dummies, a common trend or a seasonal trend. The present work offers a systematic treatment of the significance of a particular combination of fixed regressors while testing for seasonal unit roots.

Dickey, Hasza, and Fuller (1984), henceforth DHF, proposed the test statistics to test for unit roots at seasonal lags and also provided the percentiles of the distributions of these test statistics. Their methodology is an extension of unit roots testing (see Dickey and Fuller 1979) to seasonal time series. DHF statistics tests the joint hypothesis of both seasonal and non-seasonal unit roots. In a quarterly series, the DHF t-statistics tests the null of roots of $e^{i\theta}$ at $\theta = 0$, $\Pi/2$, Π and $3\Pi/2$ simultaneously. Hylleberg, Engle, Granger and Yoo (1990), henceforth HEGY, offer a generalization of this approach, which enables one to distinguish among the hypothesis of a zero frequency unit root (i.e., the non-seasonal unit root), the hypothesis of unit roots at frequencies different from zero (i.e. seasonal unit roots), and the joint hypothesis of unit roots at both zero and non-zero frequencies.[4] HEGY generate critical values for their statistics and also show that the asymptotic results from Dickey and Fuller

[3]See Section 2.6 of his paper and the interpretation of his testing strategy given in Ayat and Burridge (2000), Section 3.5.

[4]Both DHF and HEGY test the null hypothesis of the presence of unit roots. An alternative is to test the null of a stationary process as in Canova and Hansen (1995). Hylleberg (1995) compares this approach with that of HEGY.

(1979) and from DHF apply in some special cases to their tests. Smith and Taylor (1998) further extend the HEGY procedure by including the possibility of differential seasonal drift under the null hypothesis of a seasonal unit root. Franses and Hobijn (1997) consider the effect of increasing seasonal variation and seasonal mean shifts. Applications of the HEGY methodology to various time series can be found for instance in Beaulieu and Miron (1993), Franses (1995), and Paap, Franses, and Hoek (1997).

Ghysels, Lee and Noh (1994), henceforth GLN, investigate performance of the DHF and HEGY test procedures for many cases encountered in practice. They consider autoregressive models with seasonal dummies and seasonal ARIMA models for data generating processes with and without seasonal dummies. To compare the DHF and HEGY tests directly, a joint HEGY-type test is introduced which tests for the presence of unit roots at the zero as well as all seasonal frequencies. GLN claim that the HEGY-type tests are somewhat superior to the DHF test; however, its size deteriorates for data generating processes with a parameter close to a unit circle.

In this study, we extend the available testing procedures by constructing the F-type statistics, to test jointly the unit roots and the deterministic components in a seasonal time series. These statistics are an extension of Dickey-Fuller (1981) statistics to seasonal time series. Specifically, the tested null hypothesis is that of a unit root and the following deterministic components: (i) seasonal dummies and a seasonal trend; (ii) seasonal dummies and a common trend; (iii) seasonal dummies; (iv) a constant and a common trend; (v) a constant; and finally, (vi) no deterministic components. Monte Carlo simulation is employed to generate the percentiles of the proposed statistics. The introduced F-statistics provide an alternative to the more complex tests proposed by Smith and Taylor (1999), which allow for testing of unit roots at distinct frequencies.[5]

[5] Critical values for their test statistics are only available in a working paper version, see Smith and Taylor (1998).

Next, a sequential procedure is suggested, based on the proposed F-type statistics in conjunction with existing tests for seasonal unit roots. In the first step, the seasonal, DF-type F-tests are applied to a seasonally non-adjusted time series. If the joint test for a unit root and a deterministic component in (i) is rejected, the series does not contain a unit root. Testing should continue using deterministic components defined in (ii), (iii), etc., until the null hypothesis is rejected. The last accepted hypothesis determines the specification with the smallest number of free parameters out of the accepted models. The only exception is the case (iv) where fixed regressors consist of a constant and a common trend. Then, the accepted null should be seasonal dummies and a common trend in (ii) rather than just seasonal dummies in (iii). In the second step, the accepted specification is tested using DHF and HEGY tests.

The proposed statistics are illustrated by using fifteen quarterly, seasonally unadjusted time series. The log transformation of the data is employed. According to the seasonal F-tests, only one series has a unit root with no deterministic regressors. In ten series, the null hypothesis of a unit root and a constant cannot be rejected. In two cases, the accepted specification includes seasonal dummies and a trend together with the unit root. Finally, there are two times series with only deterministic components and no unit roots. The HEGY tests in the second step indicate that while most of our time series contain the non-seasonal unit root, strong evidence for presence of seasonal unit roots is only available for the United Kingdom consumption of non-durables and services, the Swedish real per-capita non-durable consumption and the Swedish real per-capita disposable income.

The paper is organized as follows. Section II describes our seasonal F-tests and proposes a way of using them. Section III gives details of simulation of critical values. In Section IV, we apply the procedure to quarterly time series. Section V concludes.

II. Joint Tests for Seasonal Unit Roots and Deterministic Components

In this section, the Dickey and Fuller (1981) F-type statistics are extended to deal with deterministic components of seasonal time series processes. Using these F-tests, it is possible to formulate a general approach to test for seasonal unit roots. The generalized procedure enables us to distinguish among integrated seasonal processes with the set of fixed regressors which may include a constant, seasonal dummies, the time trend and the seasonal linear trend.

Of interest is a time-series process x_t of the type

$$\phi(B)x_t = \epsilon_t \tag{1}$$

where $\phi(B)$ is in general an n-th order polynomial with n distinct roots and ϵ is an i.i.d. stochastic process with a zero mean and a constant variance. The focus of the paper is on quarterly data frequency - consequently, the special case of (1) is the fourth-order polynomial

$$\phi(B) = (1 - B^4) = (1 - B)(1 + B)(1 - iB)(1 + iB), \tag{2}$$

with respective unit roots $e^{i\theta}$ where $\theta = 0, \Pi/2, \Pi, 3\Pi/2$, i.e. 1, -1, i and $-i$. 1 is the non-seasonal, zero-frequency root (zero cycles per year) while $-1, i$ and $-i$ are the seasonal unit roots. -1 corresponds to 2 cycles per year and i or $-i$ correspond to one cycle per year.

There are two main types of tests available. The first is the DHF test, which is based on the auxiliary regression:

$$x_t = \mu_t + \rho x_{t-4} + \epsilon_t \tag{3}$$

where μ_t may contain deterministic components such as the intercept, seasonal dummies, the regular trend and the seasonal trend. DHF use the z-type and t-type test statistics for $\rho = 1$. The DHF procedure assumes that there are unit roots at some or all of the other frequencies in (2).

The second approach is that of HEGY, which uses the model:

$$\phi^*(B)y_{4t} = \mu_t + \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \epsilon_t \quad (4)$$

where

$$\begin{aligned} y_{1t} &= (1 + B + B^2 + B^3)x_t, \\ y_{2t} &= -(1 - B + B^2 - B^3)x_t, \\ y_{3t} &= -(1 - B^2)x_t, \\ y_{4t} &= (1 - B^4)x_t, \end{aligned}$$

with $\phi^*(B)$ being an AR polynomial whose order is determined using diagnostic checks ensuring that $\hat{\epsilon}_t$ is roughly white noise. If $\pi_1 = 0$, one cannot reject the non-seasonal unit root 1. If $\pi_2 = 0$, the presence of the seasonal unit root -1 cannot be rejected. Similarly, if $\pi_3 = \pi_4 = 0$, the seasonal unit roots $i, -i$ cannot be rejected. HEGY propose t-tests for each π_i $(t(\pi_i), i = 1, \ldots, 4)$ and joint F-type statistics for π_3 and π_4 (denoted F34; see HEGY, Tables 1a and 1b), which are further extended by GLN F-type statistics for the joint significance of $\{\pi_1, \pi_2, \pi_3, \pi_4\}$ and $\{\pi_2, \pi_3, \pi_4\}$ (denoted respectively as F1234 and F234; see GLN, Tables C1 and C2).[6] Smith and Taylor (1998, Tables 1a, 1b, and 1c) are the first ones to include seasonal time trend in μ_t. The F-type statistics for the joint significance of $\{\pi_1, \pi_2, \pi_3, \pi_4\}$ tests the same hypothesis as the DHF statistics but, according to GLN, has better power properties against common alternatives. Also, the HEGY test statistics enable one to test for a particular unit root and simulated critical values are available for all combinations of deterministic elements in μ_t. Consequently, we use HEGY tests in combination with the proposed F-type statistics.

The shortcoming of both the DHF and HEGY type tests is the implicit assumption of knowing the exact composition of the deterministic component of μ_t. However, the distribution of these components is affected by the potential presence of unit roots. In a non-seasonal context, Dickey and Fuller (1981) tabulate critical values for statistics testing various hypotheses regarding the intercept, the time trend and the autoregressive coefficient in the

[6] Most of the critical values from HEGY and GLN are also available for different sample sizes in Franses and Hobijn 1997.

regression $x_t = \mu_t + \rho x_{t-1} + \epsilon_t$. Smith and Taylor (1999) extend this line of research in the seasonal context. Perron (1988) and Ayat and Burridge (2000) suggest strategies of systemic approach to uncertainty about the non-stochastic trend. We construct tests similar to ones in Dickey and Fuller (1981), which are suitable for our general auxiliary regression and provide a simple alternative to tests in Smith and Taylor (1999). We then suggest how to use them in a sequential procedure.

Consider the auxiliary regression:

$$x_t = \sum_{i=1}^{4} \alpha_i D_{it} + \sum_{i=1}^{4} \beta_i D_{it} t + \rho x_{t-4} + \epsilon_t \tag{5}$$

where x is the time series of interest, D_{it}'s are the seasonal dummies, t the time trend, and α's, β's, and ρ are the corresponding coefficients. This formulation encompasses all the possible combinations of deterministic elements in μ_t. We now formulate the null hypotheses, which aim at deciding on the deterministic part of the seasonal stochastic processes:

$$\begin{array}{lrcl}
H_{01}: & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \rho) & = & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, 1) \\
H_{02}: & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \rho) & = & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta, \beta, \beta, \beta, 1) \\
H_{03}: & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \rho) & = & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, 0, 0, 0, 0, 1) \\
H_{04}: & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \rho) & = & (\alpha, \alpha, \alpha, \alpha, \beta, \beta, \beta, \beta, 1) \\
H_{05}: & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \rho) & = & (\alpha, \alpha, \alpha, \alpha, 0, 0, 0, 0, 1) \\
H_{06}: & (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \rho) & = & (0, 0, 0, 0, 0, 0, 0, 0, 1)
\end{array}$$

The above listed null hypotheses do not include all the possible combinations of fixed regressors. Their choice is driven by availability of critical values of the HEGY type tests. However, the missing hypotheses are not very useful anyway. For instance, it is unlikely for a time series to have zero intercepts and four different seasonal time trends. Also, note that H_{0i}'s are ordered according to the increasing number of restrictions on the parameters.

The F-statistics are calculated as

$$F_i = \frac{(RSS_0^i - RSS_1)/\nu_0^i}{RSS_1/\nu_1}, \quad i = 1, \ldots, 6, \tag{6}$$

where SSE_0^i and SSE_1 denote respectively the residual sum of squares in the restricted model (under the null hypothesis i) and in the unrestricted regression (5). Similarly, ν_0^i and

ν_1 denote degrees of freedom. $\nu_0^i = 9 - k$ with k being the number of regressors in the restricted models and $\nu_1 = T - 13$ with T the number of observations. The critical values for these F statistics are calculated using simulation in the next section.[7]

The first step of the proposed testing strategy is to conduct the F-tests to identify the joint hypothesis of a unit root and a certain combination of deterministic components, which cannot be rejected. H_{01} is tested first using F_1 defined in (6). If the null hypothesis is rejected, there are no unit roots and one can use standard F tests to find the right combination of deterministic variables. If the null cannot be rejected, we continue by testing H_{02} and so on, until we reject H_{0i} for some i. Then the specification defined by $H_{0,(i-1)}$ is tested for the seasonal (and non-seasonal) unit roots using the HEGY type test with critical values from HEGY, GLN, and Smith and Taylor (1998).[8] When H_{04} is the first rejected hypothesis, the adopted specification is defined by H_{02} since H_{03} does not encompass H_{04}.

III. Percentiles of Proposed Statistics

The details of the Monte Carlo experiment to tabulate the empirical distributions of the F-statistics in (6) are discussed next. The data generating process (DGP) used is $x_t = x_{t-4} + \epsilon_t$ where ϵ follows the standard normal distribution. In the simulations, the first four values of x_t are set equal to zero, 32 observations (eight years of data) are generated, and then another T observations, with T respectively equal to 48, 100, 160, and 200. To eliminate the impact of initial values, only the last T observations are employed in the computations.

Using the generated time series, the unrestricted regression equation (5) is estimated with the trend variable $t = 1, 2, \ldots, T$. Second, the estimation of the restricted regression of

[7] The power of the proposed tests could be increased by sequential alternation of the unrestricted regression. For instance, to test H_{06}, the unrestricted regression would contain only a constant besides the lagged x variable. However, we opt for a more convenient, unifying framework with the unrestricted regression (5).

[8] For example, for the null H_{06}, one can use the Tables 1a and 1b in HEGY for the t-test of whether $\pi_1 = 0$ in the auxiliary regression (4) with $\mu_t = 0$ and listed under "no intercept, no seas. dum., no trend".

$x_t - x_{t-4}$ on μ_t is conducted where

$$\begin{aligned}
\mu_t &= \textstyle\sum_{i=1}^{4} \alpha_i D_{it} + \sum_{i=1}^{4} \beta_i D_{it} t && \text{for } H_{01}, \\
&= \textstyle\sum_{i=1}^{4} \alpha_i D_{it} + \beta t && \text{for } H_{02}, \\
&= \textstyle\sum_{i=1}^{4} \alpha_i D_{it} && \text{for } H_{03}, \\
&= \alpha + \beta t && \text{for } H_{04}, \\
&= \alpha && \text{for } H_{05}.
\end{aligned} \tag{7}$$

To compute F_i, we simply use RSS^i from these regressions for $i = 1, \ldots, 5$. For H_0^6, RSS_0^i is simply $\sum_{t=5}^{T}(x_t - x_{t-4})^2$. We repeat these steps 50,000 times and report the resulting critical values in Table 1.[9]

IV. Applications

In this section, the data sources are described in detail and tests for seasonal unit roots are conducted using the proposed F-type statistics and the HEGY tests.

Fifteen non seasonally adjusted quarterly time series either used elsewhere in the literature or available in the public domain are analyzed. We used three data sources: the Web site of Bureau of Economic Analysis, Franses (1998) and Smith and Taylor (1998). The detailed description of time sries analyzed is given in Table 2. One of the series, the UK nondurables consumption from Smith and Taylor (1998), has been widely used in the literature on seasonal integrated processes, notably in HEGY.

Typically, log transformation of variables is used to test for non-seasonal unit roots to justify the use of a linear trend variable. In the seasonal roots literature, the actual time series tested can be in levels, in logs or various differences of the two. For instance, HEGY use the logs of the UK consumption of nondurables while Smith and Taylor (1998) use the first difference in levels. To compare results across series, the log transformation is used in this study. The unrestricted regression (5) is estimated first by OLS with results reported

[9] Since the joint F-tests are by construction different from both the HEGY and DHF-type tests, no power calculations are conducted. Power comparison in this situation would be similar to comparing the Dickey and Fuller (1979) t and z tests with Dickey and Fuller (1981) F-tests.

in Table 3. The results indicate several tendencies. The seasonal dummies do not seem to differ much for any of the time series and the same observation applies to the seasonal time trend. Moreover, the time trend looks negligible as compared to the seasonal dummies. The autoregressive coefficient is close enough to 1 to warrant a closer look at the possible presence of unit roots, either seasonal or non-seasonal. Since the fixed set of deterministic regressors is included in the regression equation, the unit root remains the only potential source of seasonality.

Next, the seasonal F-type statistics proposed for the joint hypothesis regarding the fixed regressors and the autoregressive coefficient are employed. We test the null hypotheses H_{0i} using the F_i-tests, $i = 1, \ldots, 6$ and the critical values available in Table 1. All the tests are conducted even though this is not necessary in some cases according to the sequential procedure proposed in Section II. Table 4 presents the results for all time series together with the corresponding number of observations to determine the appropriate critical values in each case. The results reflect the tendencies already identified using the unrestricted OLS regression. The most parsimonious unit root model for ten out of fifteen time series contains only the intercept as a fixed regressor. The exceptions are canun, ukimp, ukinv, and swecon. There is nothing surprising in the United Kingdom imports series since the unit root hypothesis is rejected for all considered sets of fixed regressors. Interestingly, in all the remaining series, the joint hypothesis of a unit root and some combination of deterministic components is rejected only to be accepted with a smaller set of fixed regressors. All these series have a relatively small estimate of the autoregressive coefficient (under 0.75, see Table 3) and small standard errors for seasonal dummies. In addition, the F-statistics are very close to critical values for all hypotheses tested. Therefore, it is sensible to use the last accepted unit root model before the rejected one.

Having identified the right combination of deterministic components, the HEGY tests can be applied to the time series. The regression (4) is estimated with $\phi^*(B) = 1$ and μ_t

indicated in Table 5. To test for the unit roots 1 and -1, we use one-sided t-type tests for π_1 and π_2 to test $H_0: \pi_1 = 0, \ \ H_A: \pi_1 < 0$ and $H_0: \pi_2 = 0, \ \ H_A: \pi_2 < 0$. To test for the presence of the complex unit roots one can use the F34 test. Alternatively, one can conduct a two-sided test for $\pi_4 = 0$ - if the null cannot be rejected, follow up with a one-sided test of $\pi_3 = 0$ against $\pi_3 < 0$. Finally, the F1234 tests whether the $(1 - B^4)$ filter is appropriate. Results of the HEGY tests are reported in Table 5.

Note that the HEGY tests are redundant for the Canadian unemployment and United Kingdom public investment since the most general unit root model is rejected using the joint F-statistics. For illustration purposes, we conduct the HEGY tests anyway. The HEGY tests indicate that there are no seasonal unit roots in either case. While the zero-frequency unit roots are not rejected, the t-statistics are negative and not far from their critical values. Therefore, there is a possibility that presence of the non-seasonal unit root is not rejected due to the lack of power of the HEGY tests. To investigate this possibility, the standard DF t-statistics are used with a constant and a trend. The null hypothesis is rejected in both cases, thus confirming the outcome of the seasonal F-tests. If a researcher wants to use the two series in a regression, no differencing is necessary provided that the regression includes seasonal dummies and a seasonal trend.

With the exception of gergnp and sweinc, all the remaining series contain the non-seasonal unit root. Seasonal unit roots are present only in three time series - ukndc, swecon, and sweinc, respectively. Out of these series, just in the case of sweinc the complex unit roots cannot be rejected. Let us consider the series gergnp and swecon to illustrate how the results should be interpreted. The German GNP is stationary and no differencing is necessary, assuming that a regression on gergnp includes a constant. To use swecon properly in a regression, this regression should contain seasonal dummies and a linear trend and the series should be differenced using the $(1 - B)(1 + B)$ filter.

V. Conclusion

This study takes a step further the current methodology of testing seasonal unit roots by taking into account the uncertainty about the deterministic components. Specifically, we propose a set of F-type statistics to test jointly seasonal unit roots and deterministic components in quarterly series. These components may include the intercept, seasonal dummies, a common trend or a seasonal trend. The percentiles of the proposed statistics obtained by the Monte Carlo methods are reported. A series of the joint null hypotheses is proposed to be used. The last accepted null hypothesis encompassing the first rejected hypothesis identifies the appropriate combination of the deterministic regressors. Finally, DHF or HEGY type tests are conducted using this set of fixed regressors, where the HEGY procedure distinguishes among the potential non-seasonal and seasonal unit roots.

The suggested algorithm is applied to fifteen seasonally non adjusted quarterly time series. In two cases, seasonality is due to seasonal dummies and a seasonal trend. In ten time series, adding the intercept to the HEGY or DHF type regression is sufficient. In two cases, seasonal dummies and a simple trend are employed. In the remaining case, no fixed regressor is necessary. In the second step, the HEGY tests are conducted. Results indicate that while many of the series contain the non-seasonal unit root, only three of them contain a seasonal one.

The proposed testing procedure provides a simple and well defined framework on how the deterministic components should be handled when testing for seasonal unit roots. So far, a battery of diagnostic tests needed to be conducted. Moreover, inference based on many of the standard tests may have been flawed due to presence of unit roots. The F-type statistics described in this study improve the existing methodology on both of these fronts. We see the following extensions of the present results. First, one can easily accommodate the use of seasonal data at different frequency, say monthly. Somewhat more demanding would be generalization of the present testing procedure to account for potential autocorrelation in residuals.

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Table 1: Percentiles of F-type Statistics, DGP: $x_t = x_{t-4} + \epsilon_t,\ \epsilon_t \sim$ N(0,1)

T	0.90	0.95	0.975	0.99	0.90	0.95	0.975	0.99
		F_1				F_2		
48	27.04	31.40	35.40	40.79	7.86	8.99	10.04	11.41
100	25.87	29.28	32.56	36.51	7.29	8.15	8.96	10.03
160	25.47	28.66	31.66	35.63	7.12	7.91	8.65	9.65
200	25.38	28.48	31.30	34.92	7.06	7.83	8.56	9.44
		F_3				F_4		
48	6.59	7.52	8.37	9.52	5.41	6.18	6.92	7.95
100	6.04	6.74	7.39	8.25	4.84	5.38	5.93	6.57
160	5.90	6.53	7.12	7.92	4.68	5.18	5.64	6.23
200	5.85	6.47	7.06	7.79	4.63	5.11	5.54	6.11
		F_5				F_6		
48	4.93	5.61	6.28	7.14	4.63	5.25	5.85	6.65
100	4.37	4.85	5.34	5.92	4.06	4.51	4.95	5.47
160	4.22	4.65	5.07	5.58	3.91	4.31	4.69	5.15
200	4.17	4.59	4.98	5.48	3.86	4.24	4.61	5.06

T is the number of observations.

Table 2: Data

Symbol	Series	Source	Sample
usgdp	US gross domestic product, 1992 dollars	BEA	1946.1-1997.4
uspce	US personal consumption expenditures, 1992 dollars	BEA	1946.1-1997.4
usindp	US industrial production index, 1985=100	Franses (1998)	1960.1-1991.4
canun	Unemployment in Canada, in thousands	Franses(1998)	1960.1-1991.4
gergnp	Real gross national product in Germany	Franses(1998)	1960.1-1990.4
ukinv	Real total investment in the United Kingdom	Franses(1998)	1955.1-1988.4
ukgdp	UK gross domestic product, 1985 prices	Franses(1998)	1955.1-1988.4
uktcon	UK total consumption, 1985 prices	Franses(1998)	1955.1-1988.4
ukndc	UK nondurables consumption, 1985 prices	Smith and Taylor (1998)	1955.1-1992.1
ukexp	UK exports of goods and services, 1985 prices	Franses(1998)	1955.1-1988.4
ukimp	UK imports of goods and services, 1985 prices	Franses(1998)	1955.1-1988.4
ukpinv	UK public investment, 1985 prices	Franses(1998)	1962.1-1988.4
ukeu	UK employment and unemployment (labor force)	Franses(1998)	1955.1-1988.4
swecon	Real per capita non-durables consumption in Sweden	Franses(1998)	1963.1-1988.1
sweinc	Real per capita disposable income in Sweden	Franses(1998)	1963.1-1988.1

Table 3: Estimates from the Model: $x_t = \sum_{i=1}^{4} \alpha_i D_{it} + \sum_{i=1}^{4} \beta_i D_{it} t + \rho x_{t-4} + \epsilon_t$

series	α_1	α_2	α_3	α_4	β_1	β_2	β_3	β_4	ρ
usgdp	0.28	0.28	0.28	0.28	1e-3	1e-3	1e-3	1e-3	0.94
	(0.01)	(0.01)	(0.01)	(0.01)	(2e-7)	(2e-7)	(2e-7)	(2e-7)	(6e-4)
uspce	0.20	0.20	0.19	0.19	8e-4	8e-4	8e-4	8e-4	0.96
	(0.00)	(0.00)	(0.00)	(0.00)	(1e-7)	(1e-7)	(1e-7)	(9e-8)	(3e-4)
usindp	1.23	1.25	1.24	1.25	2e-3	2e-3	2e-3	2e-3	0.68
	(0.06)	(0.06)	(0.06)	(0.06)	(4e-7)	(3e-7)	(4e-7)	(3e-7)	(4e-3)
canun	1.78	1.70	1.63	1.64	5e-3	5e-3	6e-3	6e-3	0.69
	(0.13)	(0.11)	(0.10)	(0.11)	(1e-6)	(2e-6)	(2e-6)	(2e-6)	(4e-3)
gergnp	0.85	0.86	0.86	0.86	1e-3	9e-4	9e-4	9e-4	0.84
	(0.04)	(0.05)	(0.05)	(0.05)	(1e-7)	(1e-7)	(9e-8)	(1e-7)	(2e-3)
ukinv	1.24	1.24	1.24	1.24	7e-4	6e-4	7e-4	8e-4	0.87
	(0.11)	(0.11)	(0.11)	(0.12)	(1e-7)	(1e-7)	(1e-7)	(1e-7)	(1e-3)
uktcons	2.44	2.45	2.45	2.46	2e-3	1e-3	1e-3	2e-3	0.76
	(0.50)	(0.51)	(0.51)	(0.51)	(2e-7)	(2e-7)	(2e-7)	(2e-7)	(5e-3)
ukndc	1.65	1.66	1.66	1.67	1e-3	1e-3	1e-3	1e-3	0.84
	(0.23)	(0.23)	(0.23)	(0.23)	(9e-8)	(8e-8)	(9e-8)	(1e-7)	(2e-3)
ukgdp	2.31	2.32	2.32	2.32	1e-3	1e-3	1e-3	1e-3	0.78
	(0.33)	(0.33)	(0.33)	(0.33)	(1e-7)	(1e-7)	(1e-7)	(1e-7)	(3e-3)
ukexp	2.68	2.69	2.67	2.68	3e-3	3e-3	3e-3	3e-3	0.70
	(0.34)	(0.34)	(0.33)	(0.33)	(5e-7)	(6e-7)	(6e-7)	(6e-7)	(4e-3)
ukimp	4.14	4.14	4.14	4.12	5e-3	5e-3	5e-3	5e-3	0.54
	(0.49)	(0.49)	(0.49)	(0.48)	(7e-7)	(7e-7)	(7e-7)	(7e-7)	(6e-3)
ukpinv	1.29	1.24	1.26	1.27	-2e-4	-1e-4	-2e-4	-3e-4	0.85
	(0.19)	(0.19)	(0.19)	(0.19)	(4e-7)	(4e-7)	(4e-7)	(4e-7)	(3e-3)
ukeu	0.78	0.78	0.78	0.77	1e-4	1e-4	1e-4	1e-4	0.92
	(0.23)	(0.23)	(0.23)	(0.23)	(3e-9)	(3e-9)	(4e-9)	(4e-9)	(2e-3)
swecon	0.59	0.60	0.59	0.62	9e-4	7e-4	7e-4	5e-4	0.71
	(0.01)	(0.01)	(0.01)	(0.02)	(6e-8)	(5e-8)	(4e-8)	(3e-8)	(4e-3)
sweinc	0.60	0.61	0.63	0.68	7e-4	6e-4	4e-4	7e-4	0.74
	(0.02)	(0.02)	(0.02)	(0.03)	(1e-7)	(7e-8)	(6e-8)	(9e-8)	(4e-3)

Standard errors reported in parentheses

Table 4: Joint Tests of Seasonal Unit Roots and Deterministic Components

series	n	F_1	F_2	F_3	F_4	F_5	F_6
usgdp	208	5.57	1.44	1.15	0.84	0.73	98.90***
uspce	208	5.69	1.49	1.54	0.91	1.01	169.33***
usindp	128	24.85	6.26	6.43*	3.60	4.03	10.44***
canun	112	26.25 *	6.61	5.74	3.80	3.61	3.97
gergnp	124	14.44	3.69	5.06	2.16	3.21	26.44***
ukgdp	136	15.77	4.02	3.32	2.30	2.08	15.94***
ukinv	136	12.47	3.18	3.12	1.82	1.95	7.26***
uktcons	136	11.69	2.96	2.80	1.70	1.76	19.00***
ukc	149	11.64	2.94	2.43	1.69	1.53	28.02***
ukexp	136	20.85	5.29	4.26	3.04	2.67	11.40***
ukimp	136	34.54**	8.72**	7.23**	5.00*	4.54*	12.62***
ukpinv	136	8.40	2.14	1.76	1.23	1.10	1.13
ukeu	136	2.58	0.65	3.43	0.38	2.15	7.67***
swecon	104	22.70	5.72	6.09*	3.41	3.96	11.77***
sweinc	104	16.48	4.19	4.74	2.43	2.98	5.61***

Table 5: Seasonal HEGY type t-tests and F-tests

series	n	μ_t	$t(\pi_1)$	$t(\pi_2)$	$t_(\pi_3)$	$t(\pi_4)$	$F34$	$F234$	$F1234$
usgdp	208	α	0.03	-2.77 **	-5.33 **	-5.32 **	33.02 **	26.67 **	20.01 **
uspce	208	α	1.22	-1.90 *	-4.27 **	-3.31 **	15.70 **	12.04 **	9.44 **
usindp	128	$\sum_{i=1}^{4} \alpha_i D_{it} + \beta t$	-2.81	-5.08 **	-5.63**	-8.51**	85.66**	226.32**	173.42**
canun	112	$\sum_{i=1}^{4} \alpha_i D_{it} + \sum_{i=1}^{4} \beta_i D_{it} t$	-2.91	-9.12**	-3.14	-5.19**	21.54**	137.24**	109.69**
gergnp	124	α	-3.13 **	-3.13 **	-2.27 **	-1.80 *	4.33 **	6.58 **	8.43 **
ukgdp	136	α	-0.67	-2.46 **	-3.27 **	-2.72 **	9.76 **	9.16 **	7.09 **
ukinv	136	α	-1.63	-5.72 **	-3.72 **	-2.69 **	11.43 **	22.48 **	18.73 **
uktcon	136	α	1.43	-1.89 *	-2.27 **	-2.10 **	4.98 **	4.71 **	4.09 **
ukndc	149	α	0.48	-1.11	-1.65 *	-1.86 *	3.19 **	2.61 *	2.03
ukexp	136	α	-0.41	-4.09 **	-7.10 **	-2.70 **	32.24 **	30.46 **	22.94 **
ukimp	136	$\sum_{i=1}^{4} \alpha_i D_{it} + \sum_{i=1}^{4} \beta_i D_{it} t$	-2.46	-6.83**	-6.58**	-4.91**	44.95**	78.40**	59.84**
ukpinv	136	0	1.02	-2.10 **	-3.35 **	-3.88 **	14.67 **	12.07 **	9.48 **
ukeu	136	α	2.11	-7.48 **	-4.56 **	-4.19 **	22.54 **	54.66 **	49.37 **
swecon	104	$\sum_{i=1}^{4} \alpha_i D_{it} + \beta t$	-2.01	-2.28	-3.31*	-1.26	34 6.43*	6.21**	6.11*
sweinc	104	α	-2.89 **	-0.23	-0.30	-0.58	0.21	0.16	2.28

The t-statistics $t(\pi_i), i = 1, 2, 3$ are one sided while $t(\pi_4)$ is two-sided. We only test for significance at 10% and 5% levels of significance due to availability of critical values in GLN.