Asset Pricing and the US Financial & Real Estate Markets

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To my wife Oksana, for her support and the brave decision to marry an economist.

Contents

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The second part of the present habilitation work is a joint paper with Scott Gilbert from the Southern Illinois University at Carbondale. The third chapter is a reprint of "Testing for Latent Factors in Models with Autocorrelation and Heteroskedasticity of Unknown Form," also co-authored with Scott Gilbert and published in Southern Economic Journal 72(1): 236-252. This article appears here with permission of the Southern Economic Association. Part IV is my article "Mean Reversion in Asset Returns and Time Non-Separable Preferences," International Review of Economics and Finance 10, 2001, 223-245, reprinted based on the License Agreement No 1767721373745 with Elsevier Limited. I would like to thank David DeJong, John Duffy, and two anonymous referees for helpful suggestions and comments on this work. Chapter V is a reprint of my paper "An Empirical Investigation of the Consumption Based Capital Asset Pricing Model Using a Modified Variance-Ratio Test," Journal of Economics and Finance Vol. 25, No. 1, Spring 2001, 1-21, with kind permission of Springer Science and Business Media. Here I would like to thank Craig Burnside, David DeJong, John Duffy, and Steve Husted for their comments on previous versions of the paper. The research underlying the last section was supported in part by the Marie Curie International Reintegration Grant No 014864, "Consumption, Household's Welfare, and Dynamics of Property Prices," financed by the European Commission within the Sixth Framework Programme. Finally, I am grateful to Olga Aslanidi for her excellent editorial and research assistance with preparation of this book.

Part I

Introduction

1 Asset Pricing: Theory and Evidence

In this section, I provide an extensive commentary to the subsequent five papers and a conclusion, which form the present monograph. The main objective of the commentary is putting this substantive body of original research in the context of literature on asset pricing. First, I discuss empirical properties of asset returns relevant to this work, which include stocks, bonds, and property returns. The theoretical background of the presented research is then divided in two parts, beta pricing models and consumption based asset pricing models, respectively. Third, a lot of attention is paid to the econometric methodology appearing in the finance literature since improvements of the empirical toolbox of a financial economist play an important part in all the included papers. Finally, the articles are briefly described and I characterize their scientific contribution.

1.1 Empirical Properties of Asset Returns

The primary objective of the presented work is to study economic foundation of asset returns from the empirical perspective. Two main groups of assets will be considered: financial securities and real estate. Financial securities are further divided into basic securities such as equities (stocks) and fixed-income securities (bonds, deposits, etc.) and into derivatives (options, swaps, futures and forwards, etc.). Here the focus will be solely on the basic financial assets. The real estate market has been included in this analysis because of its undeniable impact on performance of economies via savings and consumption decisions of economic agents. It affects not only the size of savings but their allocation as well. Various econometric and economic models will be calibrated or estimated using data from the United States. This decision reflects the scope of the US economy, the richness of available data and the possibility to compare the results with a large body of existing research.

Stock returns have been studied extensively. up to the 1980s, they were considered unpredictable as a result of market efficiency - see Malkiel (1996) for an informal treatment and Campbell, Lo, and MacKinlay (1997, Ch.2) for a survey of early research regarding the random walk hypothesis. However, evidence started mounting in the 1980s that the asset returns were in fact predictable to some extent. In particular, both individual and portfolio returns tend to be positively serially correlated in the short run (typically in less than a year) and negatively in the long run. The former empirical observation is indicative of the so called *momentum*. The latter provides the basis for *contrarian* investment strategies when you buy (financial) assets when the market (a stock, a portfolio, etc.) is low and sell them when it is high. Some variables such as the dividend/price ratios and term premium have been found useful in forecasting returns on equities (see Cochrane 1999). Also, stock market volatility changes over time, it increases in recessions and falls in booms. Deviations from the random walk hypothesis pose a challenge for the asset pricing theory. The observed patterns can be either rationalized by generalization of existing theory (e.g. via introduction of market frictions or of a generalized class of preferences, see Campbell, Lo, and MacKinlay 1997, Ch.8) or attributed to irrational behavior of investors (see Shiller 2001 and Barberis and Thaler 2002).

Bonds are fixed-income securities, which do not entitle their owner to any type of ownership. Corporate bonds are typically less risky than stocks partly because of their preferential treatment in case a company is in trouble. Government bonds are almost risk-free as default is highly unlikely in developed countries and hence the only substantial risk is due to inflation. The interest rates (and holding returns) of government bonds differ based on their maturity. Plotting interest rates against maturities characterizes the so called term structure (the yield curve). Similarly to stocks returns, bond returns were thought to be unpredictable in the 1980s. Unpredictable bond returns correspond to the expectations model of the term structure, in which the upward sloping yield curve is interpreted as an expectation of higher short-term rates in the future rather than a term premium. However, further research showed that an unusually steep yield curve does in fact signal higher returns on long term bonds with respect to short term bonds in the next period (see Cochrane 1999). Even though the predictability of bond returns is not a main concern in this monograph, some of the methodology developed here can be used to find whether the bond returns are statistically predictable. My primary interest is the relationship between the stock returns and the (almost) risk-free government bond returns. The difference between the two is referred to as an equity premium and it has been the center of attention of a large body of financial literature. It seems (see Mehra and Prescott 1985 and numerous subsequent papers) that the observed equity premium is irrationally high. In other words, it is puzzling that investors do not invest more in the stock market, drive the stock prices up and the returns down, and reduce the empirical risk premium of stocks over government bonds.

The last class of asset returns considered are returns on real estate i.e. the first-differenced real house prices. Since the mid 1990s, real house prices have been rising around the world, in countries such as Australia, United Kingdom, Ireland, the Netherlands, Spain, Sweden, and in the United States. Many Central and Eastern European countries, including the Czech Republic, have also experienced raising property values. The bullish real estate markets may have helped to hold off the worldwide recession but there were many indicators that the high real estate prices were not entirely supported by economic fundamentals. This proved to be the case especially for the United States and Britain but potentially for other European countries as well. Collapsing real estate prices can have dire consequences, which include instability on the financial markets and a possible recession. Studying patterns of real estate returns and their relationship with financial securities is clearly of significant importance. Englund and Ioannides (1997) list some stylized facts regarding the real returns on residential real estate. Mainly, they are predictable by their past values as well as by the GDP growth and the rate of change in the real rate of interest rate. Other attributes of the housing returns are their positive correlation with financial returns, mainly stocks (e.g. see Kennedy and Andersen 1994) and with consumption (e.g. Case, Quigley, and Shiller 2001).

1.2 Beta Asset Pricing Models

The beta asset pricing models derive their name from the market beta in the Capital Asset Pricing Model (CAPM). The origins of the CAPM start with the mean-variance analysis of Markowitz (1952), in which he first characterizes the solution to a portfolio problem by minimizing a variance (as a measure of risk) for a given expected return on a portfolio. This solution has become known as the efficient frontier. Tobin (1959) adds a risk-fee asset to the analysis of Markowitz (1952), which simplifies the solution. In such a case, the efficient frontier becomes a straight line and all investors hold the same proportions of risky assets in their portfolios. This does not mean that investors' attitudes towards risk are the same; on the contrary, their risk aversion differs. The only assumptions made are concavity of the utility function (i.e. all investors are risk averse) combined with normality of asset returns.[1] Finally, it was shown (see Sharpe 1964 and Lintner 1965) that in equilibrium, all investors hold the market portfolio. Expected return on an asset in excess of the risk free rate in the CAPM is then given by a product of beta and the excess market return over the risk-less rate. The beta is the measure of risk in this model and is expressed as the ratio of the variance of the asset return with the market portfolio relative to the variance of the market portfolio.

The CAPM has been tested extensively empirically since the beginning of its existence. Early on, during the 1960s and 1970s, the CAPM seemed to capture well both the cross-section and time-series properties of asset returns. One of the first deviations from this conclusion was the size effect identified in Banz (1981). The nature of the

[1] Alternatively, the utility function can be quadratic.

size effect lies in the observation that the returns on stocks of relatively small firms tend to do better than predicted by the CAPM. In other words, the market beta is not sufficient to capture the cross-section of asset returns. Similar observations were made using other firm-specific characteristics. Bhandari (1988) finds that there is a positive relation between average returns and leverage, even after controlling for size measured by the market equity (ME). Rosenberg, Reid, and Lanstein (1985) show that the ratio of a firm's book value of equity (BE) to size is also positively related to average returns. Controlling for size and the market beta, Basu (1983) documents that earnings-price ratio (E/P) adds to the explanatory power of cross-section of returns. All such evidence was brought together in Fama and French (1992). In a univariate setting, they confirm a strong relationship between the average returns and the market beta, ME, leverage, E/P, and BE/ME, respectively. In multivariate regressions, it is ME and BE/ME, whose relation with average returns persists.

Results summarized in Fama and French (1992) led to the conclusion that rationally priced assets (mainly stocks) were related to multidimensional risks. A number of factors were considered as proxies for these risks. The standard economic factors in multivariate regressions include the market return and consumption, which are both crucial elements in equilibrium asset pricing models. Other variables characterizing the state of an economy or the default premium, term structure, industrial production and money supply. A seminal paper that uses macroeconomic risk factors is Chen, Roll, and Ross (1986). Some researchers have instead focused on statistical factors typically based on the principle-components analysis - see for example Lehman and Modest (1988) and Connor and Korajczyk (1988). Predictions based on statistical factors do not outperform predictions based on macro variables (see Ferson and Korajczyk 1995).

Widely popular are two new factors constructed in Fama and French (1993) which were inspired by the results from Fama and French (1992). The first factor captures the excess returns on stocks with high BE/ME over stocks with low BE/ME while controlling for size. The other factor is the return on stocks of small firms over the return on stocks of large firms while controlling for BE/ME. The former factor is often referred to as HML "high-minus-low") and the latter as SML ("small-minus-big"). Recently, the Fama and French factors have been related to macroeconomy. For example Vassalou (2003) emphasizes business cycle information of the Fama-French factors while Lettau and Ludvigson (2001) link the factors to consumption. The macroeconomic factors are therefore in some sense substitutes for the Fama-French factors. Also, considering the turmoil on the financial markets since August 2007, they may be more important than factors related to properties of individual stocks.

1.3 Consumption Based Asset Pricing Models

The CAPM is essentially a single-period model which obviously restricts the analysis of an individual's consumption and portfolio choices. To find the individual's optimal life-time savings and investment strategies, one needs to solve an individual's multi-period consumption and portfolio choice problem. The multi-period model of consumption and portfolio choice characterizes demand for assets in a general equilibrium-setting. The supply side can be fully determined by an explicit model of firm production technologies. Lucas (1978) introduces the so called 'tree' model where he refers to production assets as trees. A consumer can purchase shares which entitle her to ownership of a tree (trees) and the fruit it produces. This is a modelling analogy of a stock market where trees correspond to firms and fruit to dividends. The amount of fruits (production) each year is determined exogenously by a stochastic process. This model will be referred to as the Consumption based Capital Asset Pricing Model (CCAPM).

Using the exogenous law of motion for production leads to an endowment economy, which will be the basis of a subsequent analysis in the presented work. While the trees represent risky assets, it is also assumed that there is a risk-free asset. To solve for the law of motion for asset prices (and hence returns on the assets), it remains to specify preferences over the consumption process and the exact nature of the stochastic process driving the output. Benchmark examples are a power utility function for preferences and a Markov state process for the output. Once all the necessary ingredients are available, the stock price process is determined by equaling supply and demand for both assets. This characterizes the returns on both stocks and bonds and the resulting process can be compared with the real-life data.

The Lucas tree-model is a full-equilibrium model whose predictions can be easily tested against the observed asset returns. Mehra and Prescott (1985) assumed consumers had a utility function with constant relative risk aversion. This is a power utility function for which the curvature parameter is equal to the relative risk aversion coefficient, which is in turn the reciprocal of the intertemporal elasticity of substitution. They also assumed that the law of motion for the endowment process was a two-state Markov chain. The intuition behind the Markov process was fairly simple - there were two states of the world, in one there is a booming economy and hence faster growing consumption and in the other there is a recession and slowly growing consumption. Specifying the endowment process as a Markov chain also enables one to discretize the first order conditions of the consumer optimization problem. This solution method is a special case of a gaussian quadrature rule (see Tauchen and Hussey 1991 for details).

Mehra and Prescott (1985) calibrated the parameters of this process to match data on the US per consumption of non-durables and services from 1989 to 1978. The combination of the utility function parameters and the parameters of the consumption process was then used to solve for the expected value of the equity premium which was contrasted to about 6% observed on the US stock market using inflation adjusted stock and bond returns. Then Mehra and Prescott considered a wide range of values of the risk aversion parameter to see which values can generate returns with moments matching the data. This only occurred for high values of risk aversion, above 10. To put this result in perspective, let us consider risk aversion of 25, a number often found in various other empirical studies, which is needed to generate and equity premium around 6% (e.g. see Burnside 1994). With this risk aversion, a consumer would prefer an 18% reduction in consumption to a 50-50 % chance bet of gaining or loosing 20% of consumption. Clearly, this is not how investors behave when facing such a bet. This issue has been labelled the equity premium puzzle.

To summarize, the intertemporal consumption based asset pricing model with power utility can generate expected equity premium matching the data only for an unrealistically high risk aversion. To take a different perspective, it is puzzling that stocks have had such a high premium over risk-free bonds in the last 100 years. If the stocks were such a bargain, why do investors not purchase them, driving the prices up and the risk premium down? Hansen and Jagannathan (1991) offer a different perspective on the equity premium puzzle. Rather than assuming an exogenous law of motion for the endowment process, they start with moments of asset returns. They use these moments to derive restrictions on the intertemporal marginal rate of substitution, or more generally, a stochastic discount factor. Given an expected value of the stochastic discount factor, its model implied variance is lower for reasonable risk aversion coefficients than the bound reflecting sample moments of asset returns.[2]

In general, there are at least two ways in which the equity premium puzzle can be resolved. The first is introducing market frictions. The second possibility is to generalize preferences, which is the route taken in this study. Constantinides (1990) suggests considering habit formation as a part of preferences. Habit persistence reflects the notion that a consumer values the changes in her utility rather than the level. A real life example would be eating in fast food restaurants by students, later on replaced by higher quality restaurants thanks to higher income in a first job. If the consumer had to go back to the fast food places due to a negative income shock (perhaps being fired), the utility drawn from them would be much lower than in the student years. A related concept is durability of services, which tends to appear in a somewhat shorter horizon than habit persistence. If the consumer gets a haircut, she will not need it in the subsequent weeks,

[2]Cecchetti, Lam, and Mark (1994) and Burnside (1994) construct some of the first tests, which take into account the sampling error due to estimation of the various moments.

still drawing positive utility from the recently consumed service. Constantinides (1990) demonstrates that habit persistence drives a wedge between the risk aversion coefficient and the stochastic discount factor, hence resolving the the equity premium puzzle by increasing the volatility of the intertemporal marginal rate of substitution while keeping the risk aversion constant.

Cecchetti, Lam and Mark (1990) consider predictability of asset returns in the CCAPM with the standard power utility function. They assume that the endowment process is a Markov chain and have two means conditional on the current state of the world - say, an economic expansion and an economic recession/slowdown. In the CCAPM, consumption, dividend payments, and output can be all viewed as the empirical counterpart of the endowment process. Bonomo and Garcia (1994) argue that this result is solely due to a miss-specification of the endowment process. They carefully study the properties of the US consumption, dividends, and GNP, and conclude that the proper Markov model includes switching variance rather than the mean due to heteroskedasticity present in the data. They further show that the CCAPM with the two-variance Markow switching process for endowment generates returns, which are not statistically predictable i.e. they follow a random walk. One can then ask the question if a CCAPM with a more general utility function can in fact produce predicable asset returns with a proper specification of the endowment process. Empirical evidence suggests that durability prevails for data frequencies quarter of a year and smaller (see Eichenbaum and Hansen 1990) while habit persistence dominates for longer data periods (see Ferson and Constantinides 1991 and Heaton 1995). This empirical finding indicates that habit persistence and/or durability may be able to provide rationalization of predictable asset returns in the context of the CCAPM.

1.4 Econometric Methodology

Random Walk for Stock Returns

There is a direct mapping of the financial theory of efficient markets into statistical methodology. The so called weak-form market efficiency states that a current stock price summarizes all the available information and is the best forecast of a future stock price. This is essentially a definition of a martingale, a stochastic process of a variable with the same property. The martingale property captures the notion of a fair game where the expected pay-off is zero. In other words, the expected change in the stock price is zero. The famous random walk hypothesis imposes somewhat stronger restrictions by assuming in addition that the price increments are independent i.e. any linear or non-linear function of the price changes are not correlated. If an error term in the random walk process for the stock price is normal then the stock price follows an arithmetic Brownian motion. This implies that stock returns follow a geometric Brownian motion. Therefore, the returns also follow a random walk in a discrete form.

There is a large number of tests of the null hypothesis of a random walk. The tests typically fall into three groups: parametric, semi-parametric, and non-parametric. In the present work, the exclusive focus is on classical parametric tests. Autocorrelation coefficients between returns at different periods are a basis of numerous parametric tests, including the variance ratio test. The variance ratio test is based on the implication of the random walk hypothesis that variance of price changes is a linear multiple of the number of periods. This means that the variance of returns for k periods is equal k times the variance in a single period. Hence the ratio of the two should equal 1 under the random walk hypothesis. The variance ratio can also be expressed in the terms of autocorrelations of returns up to the k-th order. Lo and MacKinlay (1988) derive the asymptotic distribution of the variance ratio statistic, which is robust to presence of heteroskedasticity in returns. Variance ratios are used as means of documenting predictability of asset returns in both Cecchetti, Lam, and Mark (1990) and in Bonomo and Garcia (1994). Variance ratios are typically significantly greater than one for data frequencies shorter than one year and significantly smaller than one for longer frequencies. This is consistent with autocorrelation patterns in stock returns. The variance ratios for different time periods k can also be estimated jointly using GMM.

Empirical evidence regarding stock returns suggests that an econometric model characterizing their behavior should encompass heteroskedasticity and autocorrelation. Similarly, the same holds true for consumption and property returns. Bonomo and Garcia (1994) examine carefully aggregate per capita consumption in the United States and conclude that at annual frequency, the consumption process should be modelled using a two-variance one-mean Markov chain. Therefore, a joint process for consumption and the stock and real estate returns can be formulated as a two-mean two-variance process for individual series. Assuming there are only two states of the world for each series, there are eight states for the joint tri-variate process with the corresponding correlation matrix for the three variables.

Testing Factor Asset Pricing Models

The initial tests of the CAPM were based on a simple time series regression of individual asset (excess) returns on a constant (alpha) and the (excess) return of the market portfolio. If the CAPM is in fact a good model for asset returns then the constant should be insignificantly different from zero while the coefficient of the market portfolio (our CAPM beta) should be significant. The beta is then our measure of exposure to the systematic risk. The regression results reflect the underlying assumptions of individually independent returns. However, we typically allow for contemporaneous correlation across assets, which is likely to be non-zero. Asset returns can be organized in a panel. In such a case, the maximum likelihood estimation (MLE) yields the same results as equation-by-equation regressions assuming joint normality of returns. If we are not willing to make the assumption of joint normality, we can estimate the regression coefficients in our panel data set using the Generalized Method of Moments (GMM) from Hansen (1982). In the panel (or in a system of equations), we can test the null hypothesis of zero alphas using a Wald, Likelihood ratio, Langrange Multiplier and Hansen J tests. Gibbons, Ross, and Shenken (1989) show that the multivariate tests can be interpreted as a measure of distance from the efficient frontier.

Aside from time series considerations, the CAPM has cross-sectional implications as well. It states that differences in mean returns in a cross-section of assets depend linearly and entirely on the corresponding asset betas. This relationship is referred to as the Security Market Line (SML). However, there are several issues when we attempt to test validity of the cross-sectional CAPM hypothesis. The first problem is a high volatility of asset returns which results in our frequent inability to reject the hypothesis that average returns across different assets are the same. The solution to this problem involves sorting assets (mainly stocks) into portfolios to maximize differences in average returns. The stocks can be sorted based on characteristics, which are known empirically and/or theoretically to affect the returns, such as size, book-to-market ratios, and the asset betas.

The second problem in testing the SML stems from the fact that asset betas have to be first estimated in a time-series first-pass regression. Then, the average returns are regressed on the beta estimates. Significant coefficients of betas (lambdas) in this second-pass regression indicate that betas do explain the cross-sectional variation in mean returns. In this case, asset betas are measured with an error, implying an errors-in-variables problem. Due to this problem, lambdas are biased downward and can be wrongly deemed insignificant and thus refuting predictions of the CAPM. Fama and MacBeth (1973) offer methodology that helps to solve the errors-in-variables problem. They propose using rolling cross-sectional regressions. Every period (say a year), they estimate the asset betas using a time-series regression based on the previous five years of data. The betas are then used to estimate lambdas in the cross-sectional regression. The result is a time-series of lambda estimates. It can be tested using a simple t-statistic whether a mean of the lambda series is different from zero. If it is, beta is priced. Shanken (1992) offers a newer version of the Fama and MacBeth (1973) method that improves its finite sample properties. Alternatively, one can jointly estimate cross-sectional and time-series equations using GMM.[3]

CCAPM Estimation

The majority of empirical papers estimate the parameters of the CCAPM in two ways - using MLE or GMM. Hansen and Singleton (1983) derive the maximum likelihood function for the CCAPM under the assumption of joint normality and homoskedasticity of asset returns and consumption. The risk premium in this case is proportional to a relative risk aversion coefficient and the covariance between an asset return and an aggregate consumption growth. Also, the risk-free rate depends linearly on the expected consumption growth. The model is often not rejected in the original Hansen and Singleton (1983) set-up since the risk premium is viewed as a constant. Hansen and Singleton (1982) derive the moment restrictions on asset returns implied by the CCAPM and estimate its parameters using GMM. The moment restrictions consist of first and second order moments. GMM is based on minimizing the objective function of optimally weighted deviations of the model implied moments from the sample moments. The minimized objective function is chi-square distributed with the degrees of freedom equal to the number of over-identifying restrictions i.e. the number of restrictions minus the number of parameters. This is the Hansen J test. The results of the estimation when both a risky and risk-free assets are used are reflective of the equity premium puzzle. The Hansen J test of over-identifying restrictions indicates a strong rejection of the CCAPM. Since the GMM estimation is robust with respect to both heteroskedasticity and autocorrelation, the model rejection provides powerful evidence against the CCAPM. In principle, GMM can be used using other first and second order moments.

[3] Wang (2005) offers a comprehensive summary of tests of asset pricing models.

1.5 Research Outcome

The main body of this habilitation work consists of five papers and a conclusion. The papers roughly follow the previously outlined structure of the asset pricing literature. The first article focuses on the time series regression of excess returns on a number of explanatory variables, which proxy for the underlying economic risk. The center of attention is the uncertainty in the beta estimation. This uncertainty causes the errors-in-variables problem in the cross-sectional regression of expected returns on betas, which is relevant for the Fama and MacBeth method. Interestingly, this issue has not been addressed in detail in financial literature in spite of its importance. The beta estimates are mostly treated as the true betas and hence exact measures of the underlying risk. However, we for example cannot conclude that two stocks with betas say 1 and 1.5 have different levels of risk (as it would appear) if the betas are not different from each other *statistically*.

We study the importance of this consideration using the size sorted portfolios of US stocks. The realized returns on portfolios of small and large firms should be sufficiently different even after accounting for the variation in returns. In reality, they are different only prior to 1982 but not since. This indicates disappearance of the size effect. The size effect refers to empirical observation that expected returns for small firms tend to be greater then predicted by the CAPM. The empirical evidence and potential theoretical explanations regarding the size effect are extensively and comprehensively summarized in Fama and French (1992). Joint inter-asset beta equality tests are used to find a potential cause of this empirical observation. While the market sensitivities in the time series regression are significantly different in the two sub-samples, this is not the case for a default premium and consumption growth. Sensitivities of these two variables became statistically different only after 1982. An offered conjecture is that the different loadings of the two variables influence the expected returns in such a way that they eliminate the differences due to market betas. Therefore, the expected returns on portfolios of small and large firms are statistically indistinguishable but the sensitivities are different.

The second article also enhances the empirical methodology and extends evidence regarding the factor models for expected returns, of which the CAPM is a special case. The considered empirical model includes four asset returns. The first two are (excess) returns on portfolios of the top and bottom 50% firms traded on the US stock market based on capitalization. The choice of the size related stock returns enables one to comment on existence of the often discussed size effect. The other two considered returns in the factor model from the first article are the interest rates for long-term and short-term US government bonds. Their presence in the model makes it possible to study two phenomena appearing in the financial literature, the existence of a term premium and the risk free rate puzzle. Combining them with the stock returns addresses the issue of the equity premium and attempts to find the factors that drive its behavior. Explanatory factors included in the model are based on the variables used in Chen, Roll, and Ross (1986) i.e. the stock market return, consumption, money supply, industrial production, and the unexpected inflation. These factors proxy for underlying economic risk, which has proved to be important during the financial crisis starting in August 2007. Economic risk may matter more than the firm specific factors prevailing in explanation of the time series and cross-sectional properties of asset returns during the 1990s and the early 2000s.

The main objective in this case is to find the number of factors, which drive asset returns. It is of interest to know if the stock and bond returns are driven by the same underlying latent factors. The underlying factors are not observable and we only have information regarding variables such as the stock market return, consumption, etc. To find the number of latent factors a new test is constructed. It is based on the distance between an OLS estimate of the unrestricted model of returns with either two or four dependent variables (excess asset returns) and up to five explanatory variables. The test is robust to both autocorrelation and heteroskedasticity and outperforms the Hansen J test used in this context. The results indicate that at least two factors are needed to explain the cross-sectional and time series behavior of bond returns and hence confirm the existence of a term premium. Also, at least two factors are necessary in the case of returns for portfolios consisting of large and small firms, respectively. This outcome is consistent with a size effect. Finally, at least four factors are needed in a joint model of stock and bond returns. In other words, different factors drive the bond and stock markets. This result is representative of the equity premium puzzle.

The third paper analyzes implications of the CCAPM with habit formation for predictability of stock returns. There are two main elements of the CCAPM, which are combined together. The first is the appropriate process for endowment, which in this setting can be real per capita consumption, GDP, and dividends. The second is a time non-separability parameter. If positive, it illustrates durability of services, such as drawing positive utility from a vacation taken not long ago or a recent haircut so a fresh one is not necessary. If the parameter is negative, the utility contains habit persistence, in which a representative values positive changes in the utility rather then its level. The results demonstrate that the time non separability parameter is related to predictability of model implied asset returns. The CCAPM calibrated to monthly data generates positively autocorrelated returns for durable utility. The CCAPM based on the annual data produces negatively autocorrelated stock returns for habit persistence.

The fourth paper formulates a new statistic based on a joint estimation of variance ratios and simple mean returns for several periods by GMM. The statistic is therefore formulated in such a way that it simultaneously captures both the autocorrelation structure and the levels of returns for several holding periods. Analogically to the previous paper, the CCAPM is calibrated using the two-mean two-variance Markov switching process for endowment. Also, the CCAPM contains the time non-separability parameter. The results confirm findings regarding the non-separability parameter, favoring habit persistence for annual data, time non-separability for quarterly data, and durability for monthly data. Power of the newly designed test is examined as well. It is found that the power is greater for alternative hypotheses formulated by a varying risk aversion as compared with a varying time non-separability parameter. Relative lack of sensitivity to the changes in this parameter suggests its smaller importance relative to the relative risk aversion, which plays a crucial role in the equity premium puzzle.

The last paper focuses on interaction of stock returns, real estate returns, and consumption. Clearly, modelling such interaction is a very useful exercise, especially considering the recent development on the financial markets with collapsing housing prices in the US quickly followed by a severe global decline on the stock markets. It seems that a global recession is inevitable and the reduced output will be accompanied by reduced consumption. In the early 2000s, a bursting dot com bubble did not affect consumption mainly due to a wealth effect of ever higher real estate prices. The econometric methodology generalizes the univariate two-mean two-variance process for endowment to a tri-variate Markov chain with eight states of the world. The eight states correspond to various combinations of two states for each of the three variables: a recession (or a mere slowdown) and an expansion. For example, stock returns tend to be more volatile during a period with falling stock prices. The tri-variate model is estimated using data up to May 2004. The state with the declining stock prices, rising property prices and slowly increasing consumption resembling the beginning of the 21st century does have a positive probability though the state corresponding to the current situation does not. This is perhaps not surprising since the real estate prices have not collapsed as much and as fast as they have in the last year or say at any point in time prior to 2004.

The conclusion then discusses the presented results from the perspective of the current global economic and financial crisis. The time separable power utility CCAPM is estimated using GMM to find if there is still an equity premium puzzle with depressed stock prices in recent months. The issue of future research is addressed in this light as well, especially the role of housing in financial models.

Part II

Inter-Asset Comparisons of Betas and Returns to Small and Large Firms' Stocks

In finance, differences among assets' mean returns are often accounted for by differences in betas in asset return regressions. We test whether betas and groups of betas differ *statistically* across assets in a multifactor asset pricing model. We use the individual and joint inter-assets beta equality tests to analyze why expected returns on portfolios of small and large firms differed prior to 1982 but not since. The disappearance of the difference is due to differing sensitivities to variables other than the market e.g. default premium or consumption growth.

2 Introduction

The main purpose of asset pricing models is to explain differences in expected returns among stocks and other risky assets. In the tradition of the Sharpe (1964) and Lintner (1965) capital asset pricing model (CAPM) and its generalizations via the Merton (1973) and Breeden (1979) intertemporal equilibrium models and the Ross (1976) arbitrage pricing theory (APT), such performance differences arise due to differing sensitivities ('betas') to some economic variables, either some explicit source(s) of risk or some underlying state variable(s). We illustrate how formal tests of equality of betas across assets can be used to interpret time series and cross sectional behavior of expected returns. We focus on the model of returns in the spirit of Chen, Roll and Ross (1986) and combine beta equality tests with point beta estimates and tests for zero intercepts (see Gibbons, Ross, and Shanken 1989) to account for differences between large and small firms' stock returns.

There is a vast number of studies involving beta estimation and some of the studies do take into account standard errors of beta estimates. To our surprise though, we have not been able to find a study, which compares *formally* betas across assets. The present work attempts to fill this gap in the literature. While informal comparison of point estimates maybe sufficient in some applications, it is not when we attempt to evaluate the effect of a group of factors rather then just that of the market beta (market vs. non-market factors, economic vs. statistical, etc.). Rather then merely stating that a multifactor model is needed to explain time series and cross-sectional behavior of expected returns,[4] we would like to uncover more about the nature of these differences. Hence we propose various joint tests for equality of factor sensitivities across assets. Applying these tests to portfolios of stocks of large and small firms yields new insights. Namely, we are able to explain the somewhat puzzling recent disappearance of a measurable difference in mean returns on stocks of small and large firms.

To test formally for differences among particular betas, we make use of panel (time-series + cross-section) regression models of excess returns. This is a standard modelling framework for defining and discussing sensitivities of excess returns on covariates/regressors, e.g. risk factors and/or state variables. For example, estimation of such a model using panel data is the main focus of the influential papers by Fama and French (1993, 1996). With sensitivities defined via the slope parameters in the model, differences in sensitivity are clearly in the domain of formal, testable hypotheses. The financial data typically contain some form of heteroskedasticity and autocorrelation (see for example French, Schwert, and Stambaugh 1987 and Campbell, Lo, and MacKinlay, 1997, Ch. 2), which calls for the use of robust tests. The heteroskedasiticity and autocorrelation consistent methods (HAC) built on earlier work by White (1980) are developed in Newey and West (1987, 1994), Andrews (1991) and Andrews and Monahan (1992), and are further studied by den Haan and Levin (1996, 1997).

[4] Cochrane (1999) lists this observation as one of the consensual results in recent financial research.

Robust tests often suffer from distortions arising due to the test rule, which relies on the asymptotic distribution of test statistics, and which generally differs from rules based on the exact (but unknown) finite-sample distribution.[5] To find limitations of these methods in our testing framework, we examine performance of Wald and Hansen (1982) tests with HAC estimates of covariance matrices of residuals and compare it with that of classical F tests. In simulation the HAC Hansen tests distort less than the F test and HAC Wald tests, and simple pre-whitening is as good or better than other methods of handling serial correlation. Consequently, we report results of estimation conducted using the Hansen method with simple pre-whitening (see den Haan and Levin 1996, 1997).

We use the afore-mentioned methods to test for differences in the economic sensitivity of small and large firm excess returns measured by the top and bottom deciles of the CRSP Capitalization Indices since 1959. The potential for such differences has long been recognized,[6] and as covariates we include standard economic risk factors (market return and consumption growth) as well as other standard economic variables (default premium, term structure, industrial production, inflation, and money growth) related to the economy. Some researchers use instead covariates consisting of size and book-to-market related portfolios (see Fama and French, 1993, and Chan and Chen, 1991), or other statistical factors (Lehmann and Modest, 1988, and Connor and Korajczyk, 1988), but since statistical and economic covariates appear to have similar predictive power with respect to stock returns (see Ferson and Korajczyk, 1995), we use just the latter,

[5] MacKinnon and White (1985) acknowledge the distortions problem for heteroscedasticity-robust tests, proposing corrective methods, and Ferson and Foerster (1994) examine the importance of distortions for heteroskedasticity-robust tests of some financial models. den Haan and Levin (1997) report on test distortions for a variety of HAC tests in a single equation context (see also Cushing and McGarvey, 1999) and Cochrane (2001, Ch. 15) studies the zero intercepts hypothesis in the multi-equation context, using HAC methods of Newey and West.

[6] See Schwert (1983) for a review of early theories, and Fama and French (1992, 1993) and Cochrane (1999) for further discussion.

similarly to Chen, Roll and Ross (1986).

We first focus on the standard CAPM. Prior to 1982, the sample mean returns for small and large firms are statistically different, indicative of the size effect. We then test for equality of market betas and find that this difference in performance can be in part accounted for by the difference in the level of risk of the two considered portfolios measured by the market beta. The remaining part is due to the positive intercept for small firms' stock returns and corresponds to rejection of the CAPM by the test for zero intercepts, confirming the small size effect. Since 1982, the mean excess returns for the small and large returns are formally indistinguishable with returns on small firms' stocks being smaller. However, the market betas are statistically different and the market beta for stocks of small firm is actually the smaller one. With the CAPM not rejected, this suggests that investment in small firms is less risky and has the same (or greater if one considers only the point estimates) return, making it clearly the better investment opportunity.

In the next step, we investigate bivariate models with the market as one factor and one of the above mentioned macroeconomic variables as the other. We also consider a model with all seven risk factors. Implications of the bivariate and multivariate models are similar. The point estimates of the market betas in both sub-periods do not change much as compared with the CAPM. The formal test of their equality also shows that they are statistically different at both sub-samples. Before 1982, test results lead to similar conclusions as in the CAPM. Namely, the small firms' stocks are somewhat riskier mainly due to differing market betas and the risk premium is greater than it would be accounted for by the multifactor asset pricing model. Since 1982, implications of our tests are very different from those of the CAPM. Consistent with findings of Horowitz, Loughran and Savin (2000) and Fama and French (1993), we find that the size effect has either disappeared or has been reversed in favor of the large firms. Results of the joint test of beta equality for all variables but the market return signal that there are other sources of risk differences than market beta. This impression is confirmed by individual tests of beta equality; more specifically, all our variables with the exception of industrial production have statistically different sensitivities for the two portfolios. A brief look at point estimates indicates higher sensitivity of large firms to the market but smaller to the other economic variables, which explains why mean returns are similar since 1982.

Our analysis indicates that formal testing of betas, jointly and individually, can be a useful source of information for a financial economist in addition to tests for zero intercepts and point beta estimates. A possible strategy for comparison of groups of assets would be: (i) Get point beta estimates and test for zero intercepts; (ii) Formally compare market betas; (iii) Formally compare sensitivities of other available factors, jointly and individually. Using this approach, we can draw conclusions regarding the disappearance of the statistical difference in expected returns on stocks of small and large firms since 1982. The disappearance is due to previously minor but lately significant differences in sensitivities of expected returns to variables other than the market. Since the early 1980's, the risk exposure to those variables works in the opposite direction than market risk. Consequently, the mean returns on portfolios of small and large firms are similar.

The rest of the paper is organized as follows. Section 3 presents the formal test of equality of betas in a time-series regression model and Section 4 discusses the asymptotic properties of the HAC robust Wald and Hansen tests. Section 5 addresses the data selection and lists data sources. Section 6 studies the finite sample properties of the tests in a simulation exercise. Section 7 applies the tests to the data and Section 8 concludes.

3 Model

For a collection of n risky assets, each earning a return during periods $t = 1, 2, ..., T$, let r_{it} denote the excess return to the i-th asset. We recall that a unifying implication of the finance theories listed above is the following restriction (see Cochrane, 2001):

$$Er_{it} = \beta_i \lambda, \;\; i = 1, ..., n, \tag{1}$$

where β_i is a $1 \times K$ vector of betas (sensitivities) for asset i with respect to risk factors, and λ a $K \times 1$ vector of risk premia. It is obvious here that if the mean values $Er_{it}, i = 1, ..., n$ are not all the same then neither are the betas $\beta_i, i = 1, ..., n$.

To estimate the risk premia in (1), one needs to estimate betas first. This first step is common for both the two-pass method and for the Fama and MacBeth (1973) empirical method[7]. Both methods use the linear regression model of asset returns of the form:

$$r_{it} = \alpha_i + \beta_i x_t + \varepsilon_{it}, \;\; i = 1, ..., n, \;\; t = 1, ..., T, \tag{2}$$

where x_t is a $K \times 1$ vector of covariates (risk factors and/or state variables), β_i is the same beta as in (1), α_i is the i-th intercept, and the errors ε_{it} have conditional expectation $E[\varepsilon_{it}|x_t] = 0$. In this model, β_{ik} is the expected increase in the excess return r_{it}, given a one unit increase in the covariate x_{tk}, while $\alpha_i = E[r_{it}|x_t = (0, ..., 0)']$, e.g. α_i is the expected excess return when each covariate equals 0. The model is linear in the parameters α and β, but x_t itself may be non-linear in some underlying variables which themselves may be non-contemporaneous with r_t, hence the model may be both non-linear and dynamic in some underlying variables (see Ferson and Harvey, 1999, for a recent example).

In the Sharpe-Lintner CAPM version of the model, x is the excess return on the market portfolio, and the betas measure sensitivity to market risk. Other candidates for x include consumption growth, as in the Breeden (1979) consumption-based CAPM, and other variables, possibly instruments for some latent factors (see Section 6 for a detailed discussion). Such models offer an explanation of differences among average returns for various assets, provided that sensitivities differ among assets. Informal comparisons

[7]Shanken (1992) relates the time-series and cross-sectional regressions (2) and (1) with respect to these two methods.

of betas across assets are widespread in the industry, facilitated by point and interval estimates, but formal comparison via hypothesis tests has not received attention in the literature.

The hypotheses of present interest take the form of linear restrictions on β. To concisely express such hypotheses for the purpose of testing, for each equation i we denote by $\theta_{[i]}$ the $(K+1)\times 1$ vector $(\alpha_i, \beta_{i1}, ..., \beta_{iK})'$, and let θ be the $n(K+1)\times 1$ vector $(\theta'_{[1]}, \theta'_{[2]}, ..., \theta'_{[n]})'$. The intercept α_i will be unrestricted with the exception of our simulation exercise. With 0_p the column vector consisting of p entries each equal to 0, and with A some user-specified $p \times n(K+1)$ matrix, each linear restriction on the model parameters takes the form:

$$H_0 : A\,\theta = 0_p.$$

For testing differences in slopes across equations, and the relevant restriction is of the form:

$$D\,\theta_{[i]} = D\,\theta_{[j]},\ \ i,j = 1, ..., n, \tag{3}$$

for some $r \times (K+1)$ matrix D, some number r of restrictions, and all assets i, j. The appropriate form of the matrix A in H_0 is then:

$$A = J_n \otimes D, \tag{4}$$

where J_n is the $(n-1)\times n$ matrix with entries $J_{ni1} = 1$, $J_{n,i,i+1} = -1$, and $J_{nij} = 0$ otherwise, and $\otimes$ is the Kronecker product operator. For example, for the test of equality of slopes across equations for $n = 2$ and $K = 1$, we have $p = 1$, $A = \begin{bmatrix} 0, & 1, & 0, & -1 \end{bmatrix}$, $D = [0, : 1]$, and $J_2 = [1, : -1]$.

4 Tests

In this section we describe methods of hypothesis testing based on HAC Wald and Hansen tests which we later study as alternatives to the F test. The HAC robust tests are prone to small sample distortion and we can only use them in our models since we focus on just two portfolios. In the standard modelling framework where a large number of assets is studied, these robust methods would be impractical. Typically, a researcher attempting HAC estimation would have to assume a certain form of heteroskedasticity and autocorrelation.

4.1 HAC Test Statistics

To conduct generalized Wald tests we let $\hat{\theta}$ denote the ordinary least squares (OLS) estimator, and we let $\hat{V}_{\hat{\theta}}$ denote an estimator, further described below, of the variance-covariance matrix for $\hat{\theta}$. For each given choice of $\hat{V}_{\hat{\theta}}$, the test statistic is:

$$W = \hat{\theta}' A' \left(A \hat{V}_{\hat{\theta}} A' \right)^{-1} A \hat{\theta}. \tag{5}$$

The statistic W measures the distance (in R^p, with norm $||v|| = v'(A\hat{V}_{\hat{\theta}}A')^{-1} v$) between the vector $A\hat{\theta}$ and the value 0_p hypothesized under H_0, hence larger values of W suggest larger departures of the data from H_0. Under the null hypothesis, W is distributed as chi square asymptotically, with p degrees of freedom.

To conduct generalized Hansen tests, for any parameter values α_i and β_i define the regression residuals for the model (2):

$$e_{it} = r_{it} - \alpha_i - \beta_i x_t, \quad i = 1, ..., n, \quad t = 1, \ldots, T.$$

The relevant sample moments comprise the $n(K+1) \times 1$ vector $m(\theta)$, given by:

$$m(\theta) = \frac{1}{T} \sum_{t=1}^{T} z_t \otimes e_t,$$

where z_t is the $(K+1) \times 1$ vector $(1, x_t')'$. Denoting by $\hat{V}_m$ an estimator (specified below) of the variance-covariance matrix of $m(\hat{\theta})$, the Hansen test statistic is:

$$S = \min_{\theta \in H_0} m(\theta)' \hat{V}_m^{-1} m(\theta). \tag{6}$$

The Hansen test measures the distance (in $R^{n(K+1)}$, with the norm $||v|| = v'\hat{V}_m^{-1} v$) between the vector $m(\theta)$ of sample moments and the value $0_{n(K+1)}$ hypothesized under

H_0, hence larger values of S suggest larger departures from H_0. Like the Wald statistic, S is distributed chi square (asymptotically) under the null hypothesis, with p degrees of freedom.

For econometric testing of linear restrictions H_0 on linear regression systems, Hansen tests are seldom used while F and Wald tests are popular, whereas for nonlinear problems the Hansen test is common, as in Hansen (1982) and Ferson and Foerster (1994). Yet our simulations (reported later) suggest a useful role for HAC Hansen tests of parameter equality across equations in linear systems.

4.2 Computation

To compute the test statistics we apply formulas (4), (5) and (6), with various specifications for the covariance matrix estimators $\hat{V}_{\hat{\theta}}$ and $\hat{V}_m$. For the HAC Wald and Hansen tests, we use a variety of HAC covariance estimators. Among these are the Bartlett kernel and the data-dependent Newey and West (1994) bandwidth, with and without pre-whitening (denoted NW and NW-P, respectively), the quadratic spectral kernel with the Andrews (1991) data-dependent bandwidth (without prewhitening, denoted A), and the Andrews and Monahan (1992) method (denoted AM) with pre-whitening. Further, we include the simple pre-whitening method (denoted VARHAC) with parametric, vector autoregressive, adjustment for serial correlation, studied by den Haan and Levin (1996, 1997). Finally, for comparison purposes we include the White covariance estimator (WH) which is robust to heteroskedasticity but not serial correlation. Since the technical details of covariance estimators are neatly summarized in Campbell, et al. (1997) and Cushing and McGarvey (1999), we omit them for brevity.

To carry out the minimization (6) required for the Hansen statistic S, we use the GMM (simultaneous-iteration) routine, which at each iteration stage simultaneously solves for updated parameter and covariance matrix estimates, as in Hansen, Heaton and Yaron (1996).[8]

5 Data

We examine excess returns on stocks of firms ranked by capitalization. We use the industry standard CRSP Stock File Capitalization Decile Indices, monthly time series based on portfolios rebalanced annually. To limit the number of dependent variables (and the potential for test distortions, reported later), we use one return for Decile 1 portfolio and one for Decile 10 portfolio, respectively corresponding to the largest and smallest companies. In all cases, we calculate excess returns using the 30-Day Treasury Bill return, also provided by CRSP. We denote the excess returns as r_{LARGE} and r_{SMALL}, respectively.

Summary statistics, for monthly excess returns in the period 1959:02 - 2003:12, are in Table 1. The starting period of the data series is determined by availability of the consumption series (defined below). We further split the sample in two sub-samples, 1959:02-1982:10 and 1982:11-2003:12, enabling us to examine stability of regression parameters. October 1982 marks the approximate ending of Paul Volcker's war on inflation in the early eighties. In all considered sample periods, the excess return on small caps tends to be more volatile, in accord with Malkiel and Xu (1997). A comparison of the sample means for excess returns reveals that the excess return on the large capitalization portfolio is greater than the excess return on the small-cap portfolio (by 9.93% annually) but the gap is much smaller in the second sub-sample (2.07% annually), consistent with Fama and French (1993) and Horowitz, at al. (2000) (in fact, for the Cap based portfolios, the large firms have actually outperformed the small ones in the second sub-sample). To confirm the impression based on a simple comparison we also conduct formal t-tests for differences in means, which account for the covariance between the two

[8] We use the econometrics software Eviews 3.1 for all our calculations. The relevant code is available upon request.

portfolios. The t-statistics and p-values are respectively 2.29/0.02 overall, 2.43/0.02 for the first sub-sample, and 0.59/0.55 for the second sub-sample, which substantiates our conclusions.

As covariates in the model (see Table 1 for summary statistics of covariates, and Table 2 for correlations with dependent variables), we choose ones likely to affect the stochastic discount rate and/or the expected stream of cash flows. We follow Chen, Roll and Ross (1986) and use data on the stock market, bond market, the business cycle and inflation, and we augment the dataset by the growth of monetary base to address the issue of asymmetric reaction of firms of different capitalization to restrictive monetary policy (see Gertler and Gilchrist, 1994, Li and Hu, 1998, and Perez-Quiros and Timmermann, 2000).

To describe the stock market we use the CRSP NYSE value-weighted index. Again, we use returns in excess of the 30-Day Treasury Bill, denoting the results by r_{VW}. The correlation with the large-cap return is close to one (see Table 2), and since the large-cap firms account for most of the market value, this is not surprising.[9]

We consider two bond market variables. The effect of unanticipated changes in bond risk premia is measured by the difference (denoted r_{DEF}) between interest rates on the low grade bonds and long-term government securities. The low grade bond interest rate is measured by the Seasoned Baa Corporate Bond Yield, collected by Moody's Investors Service. The long-term government bond return-to-maturity is from the 5-year Treasury Bonds, obtained from the web site of the Board of Governors of the Federal Reserve System (BGFRS). To describe the term structure we use the difference between the one-period holding return on the 5-year Treasury Bond, collected by CRSP, and the first lag of the return on a 30-Day Treasury Bill. This term premium (r_{TERM}) proxies for the influence of changes in the term structure on equity returns.

As measures of real economic activity, we include the growth rates of industrial production (g_{IP}) and real per capita consumption (g_{CONS}). We obtain industrial production

[9] Fama and French (1996) report a similar correlation.

data (series INDPRO, seasonally adjusted) from the BGFRS web site, and we obtain consumption data (series PCEND, non-durables, series PCES, services, POP, population, series CPIAUCSL, Consumer Price Index For All Urban Consumers, All Items 1982-84=100, all series are seasonally adjusted), from the Bureau of Economic Analysis.

The consumer price index is used as an inflation variable. Since the null hypothesis of the unit root cannot be rejected in some sub-samples of this series, we use the first difference in our analysis. For money growth, we use the growth rate of the seasonally adjusted monetary base (g_{MON}), obtained from the St. Louis Fed's web site (series AMBSL, seasonally adjusted).

6 Simulation

In this section, we attempt to find the limit of a sensible employment of the HAC methods to see exactly what level of model complexity they can handle. We use computer simulation, based on a calibrated model of asset returns, to assess test performance. Of interest are rejection rates under the null hypothesis and under the alternative. If the nominal distribution (F distribution for the F test, chi square distribution for the HAC tests) is an accurate approximation then the tests should reject under H_0 at a rate near the theoretical test size; otherwise, the tests will exhibit noticeable distortions.

To set up the simulation, we define a first-order vector autoregressive (VAR) process for covariates x_t:

$$x_t = c + \Phi\, x_{t-1} + u_t, \tag{7}$$

where c is a $K \times 1$ vector of constants, Φ is an $K \times K$ matrix of coefficients, and u_t is a $K \times 1$ vector of random variables which are independent over time and normally distributed with zero mean and cross-sectional variance-covariance matrix Λ.

To see what range of values might be realistic for the parameters of the x_t process, we estimate (7) for $K = 4$ by OLS using $x_t = (r_{VWNY}, r_{TERM}, g_{CONS}, g_{MON})'$. Estimates of elements the matrix Φ range from -0.30 to 0.43. We also try several other combinations of explanatory variables and while estimates differ to a large extent, the diagonal elements tend to be greater than offdiagonal ones, which are often close to 0. Therefore, for our simulation we set $\Phi_{ij} = 0.10$ for $i = j$ and $\Phi_{ij} = 0$ for $i \neq j$. Estimates of the constant term tend to be small relative to elements of Φ, and we set $c = 0.002$ in our simulation exercise. The diagonal elements of the estimated residual covariance matrix $\hat{\Lambda}$ are typically of order 0.0001, and the off-diagonal elements are typically much smaller, hence we let Λ be a diagonal matrix with each diagonal entry equal to 0.0001.

For the regression errors ε_{it} in (2), we posit a dynamic model with serial correlation and generalized autoregressive conditional heteroskedasticity (GARCH), as follows:

$$\varepsilon_{it} = \psi_1 \varepsilon_{i,t-1} + \psi_2 \sqrt{1 + \psi_3\, \varepsilon_{i,t-1}^2}\, \eta_{it}, \;\; i = 1, \ldots, n,$$

with η standard normal noise. Parameter ψ_1 specifies the autocorrelation, and parameters ψ_2 and ψ_3 specify the conditional heteroskedasticity. We choose ψ so that the autocorrelation of the error term ε_{it}, as well as its variance relative to that of x's, corresponds to what we observe in historical data series, with r_{1t} and r_{2t} excess returns on portfolios of small and large firms, respectively. In this case, we set $\psi_1 = .1$, $\psi_2 = .003$ and $\psi_3 = .2$. The cross-sectional empirical covariance of η_{it} is sometimes positive and sometimes negative, and we specify the population covariance between η_{1t} and η_{2t} to be 0.

To get a sense for the behavior of the F test and 'robust' HAC tests, we first generate results for the case $n = 2$, with $K = 2$ and, alternatively, $K = 4$, using 500 simulated time series for r_{it}, $i = 1, 2$, with 250 and 500 observations, roughly corresponding to one half of our sample and the whole sample of our historical monthly data, respectively. We conduct a Monte-Carlo experiment based on a calibrated model, rather than a bootstrap method as in Ferson and Foerster (1994), for two reasons: First, the calibrated model allows us to identify the source of test success or failure; second, the regression errors have posited dynamics which would not be replicated by standard bootstrap sampling.[10]

[10] Alternatively, one could employ a block-bootstrap method, as in Cochrane (2001, Ch. 15).

We record the number of rejections of the null hypothesis using the chi square critical values at the 5% level of significance.

Table 3 reports rejection rates under the null hypothesis of cross-equation equality for all coefficients, e.g. the case where the restriction defining matrix D in Section II equals the $p \times p$ identity matrix. We calibrate all β values to equal to 1, and all α values to equal 0. Our simulations show a serious tendency for distortion in most but not all tests. Specifically, the F test and the HAC Wald tests over-reject[11], and two of the Hansen tests (Newey-West and Newey-West with pre-whitening) under-reject the null hypothesis. On the other hand, three of the Hansen tests (VARHAC, Andrews and Andrews-Monahan) show minimal distortion, and of these three the VARHAC test is by far the simplest to compute and interpret. We have examined the Hansen VARHAC test in numerous other simulations exercises: For $n = 2$, we gradually increase the number of covariates K by two up to $K = 8$, and rejection rates fall toward 0.03 and 0.04 for sample sizes 250 and 500. Since many studies consider decile indices, we also look at $n = 10$ and increase the number of covariates from two to eight, in which case the rejection rates for the VARHAC Hansen tests are respectively 0.01 and 0.02 for the two sample sizes. For no other test method do we find less distortion than for the VARHAC Hansen test, and our results suggest that a researcher attempting to investigate the relationship between various variables and asset returns is 'safer' when the number of assets is smaller since the asymptotic and finite sample distributions of the test statistic are closer.

To describe performance under the alternative hypothesis, we generate simulated time series for excess returns via:

$$\begin{aligned} r_{1t} &= x_{1t} + x_{2t} + \ldots + x_{Kt} + \varepsilon_{1t}, \\ r_{2t} &= x_{1t} + x_{2t} + \ldots + x_{\frac{K}{2},t} + (1 + \tfrac{0.2}{K})\big(x_{(\frac{K}{2}+1),t} + x_{(\frac{K}{2}+2),t} + \ldots + x_{Kt}\big) + \varepsilon_{2t}. \end{aligned}$$

Table 4 reports rejection rates under the alternative hypothesis for $K = 2$ and $K = 4$, with relatively high rejection rates for the F test, and with higher rejection rates for the

[11] For similar results see Cushing and McGarvey (1999) and Cochrane (2001, Ch. 15).

HAC Wald test than for the corresponding HAC Hansen test. Among the HAC Hansen tests, the Andrews, Andrews-Monahan and VARHAC methods reject more frequently than the others. These results describe the frequency with which an economist would correctly reject the null hypothesis, using the nominal (F or chi square) distribution of the relevant statistic. A related, but different, issue is the frequency of correct rejection for an economist who knows and uses the exact test distribution. The latter power calculations are not interesting here because the economist does not know the exact distribution, and it is impossible to concisely report on this distribution in a way that would be broadly useful for asset return regression. We have nevertheless done such power calculations, with the same rankings described above, for the various tests.

Overall, the simulations reveal some serious problems with the F test and with the 'robust' HAC Wald tests, in terms of over-rejection under the null hypothesis, whereas three of the HAC Hansen tests avoided serious distortions and were also best among the Hansen tests under the alternative hypothesis. Among these favored three we recommend the VARHAC Hansen test, with it's simple, parametric pre-whitening approach to serial correlation adjustment. For the range of sample sizes under study, the VARHAC Hansen test performance under null and alternative hypotheses suggests that for a small number of assets, $n = 2$, we can have as many as 8 covariates and still avoid major test distortions. In cases of $n = 10$ asset returns, the number of covariates in a restricted econometric model should be kept small, perhaps no more than 4 or 6. In cases where larger models and a greater number of restrictions are desired, larger sample sizes (weekly rather than monthly data, for example) may be necessary for satisfactory results.

7 Empirical results

Having scrutinized a variety of test methods, we turn now to the problem of testing for differences in sensitivity among firms of different size. As our simulations warn against the use of overly large models, we only use up to seven explanatory variables in our two-asset model. To save space we report only the Hansen-type tests with parametric VARHAC adjustment for residual serial correlation and heteroskedasticity, as these tests showed relatively little distortion in simulation, and are generally in agreement with the other tests for the models we analyze. The tests are formulated by defining the matrix D in Section II accordingly.

We first examine the CAPM. Table 5 gives results for the full monthly sample (1959-2003) and two sub-samples. The test of equality of market betas suggests significant difference in risk exposure, for large and small firms, in both sub-samples but not overall. This is a result of changing beta for small firms, which is 1.28 in the first but only 0.81 in the second sub-sample. We also test for whether the intercepts are each 0; since the only factor in the CAPM is the market excess return, the test of zero intercepts is essentially a HAC robust version of the standard F-test commonly applied in testing the CAPM.[12] The CAPM is rejected overall and in the first sub-sample but not in the second sub-sample.

In our discussion of the results, we will focus on the two sub-samples since the results for the whole time period are (more) likely to reflect changing betas over time. The theory, on which CAPM is based, offers the following interpretation. Higher betas in the first sub-sample are not sufficient to explain higher mean returns for the same period. The CAPM is rejected and the intercept for small firms is significantly positive, suggesting that investment in small firms delivers a premium higher than accounted for by the CAPM. This is in fact the so called firm size effect. In this case, statistically different betas do not provide us with information that makes a difference. The recommendation here is clear: invest in small firms. The theory of CAPM gives no such recommendation for the second sub-sample, where any differences in expected returns are simply due to differing market betas. However, statistically different betas (at 10% level of significance)

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[12] As pointed out in Gibbons, Ross, and Shanken (1989), the test of the CAPM is equivalent to the test of *ex-ante* mean-variance efficiency of a particular portfolio and the test statistic (either F, S or W) can then be interpreted as a measure of distance from the mean-variance frontier.

are in contrast with mean returns, which do not differ for small and large firms from a statistical point of view. In other words, the mean returns are approximately the same but the large firms are riskier (recall that small firms' beta is now 0.81). Using this interpretation, the recommendation is again clear: buy stocks of small firms.

We next examine bivariate models, with covariates given by the market return and one of the remaining five economic variables, with results reported in Table 6. The properties of the market betas are not changed with addition of another covariate, i.e. the market slopes differ in the first sub-sample as well as in the second one. Sensitivity to the second covariate shows in each case no significant differences in the first sub-sample. In the second sub-sample, there are now significant differences in slopes for the default premium and consumption. We also conduct the joint test for zero intercepts, which can be loosely interpreted as a test of our asset pricing model[13] In the first period, three out of six models have non-zero intercepts and all have either an insignificant or positive intercept for small firms. In the second period, only one out of six indicates non-zero intercepts (the default premium being the second variable), in this case with a positive intercept for large firms. The recommendation for the period from 1959:02 till 1982:10 stands: statistical differences in mean returns are driven by differences in market betas and there is extra premium on investment in small firms, which dominates investment in large ones. For the second period, the situation has become more complex. We have statistically undistinguishable mean returns and statistically differing market betas. The smaller market beta for small firms indicates a better investment opportunity. However, we also have other variables whose beta differ, namely the default premium and consumption. The results with default premium even indicate that it might be the large firms, which have become a bargain. So, while performance of the two portfolios is similar in the second sub-sample there are significant differences in risk exposure between the large and small firms in addition to previously identified differences in market betas. These differences indicate that returns on small firms' stock is more sensitive (i.e. riskier)

[13] Loosely because not all the variables can be interpreted as asset returns.

to variables other than the market excess return.

Results from bivariate models call for a model with more explanatory variables. We examine the model in which all seven covariates are included at once. Table 7 reports parameter estimates and their standard errors, computed via the VARHAC method.[14] To further describe the model we report in Table 8 residual diagnostic tests. As indicated, there is strong evidence of both residual heteroskedasticity and autocorrelation, in which case our use of HAC test methods is highly appropriate.[15] Finally, Table 9 reports results of tests of cross-section restrictions.

Table 7 indicates that market betas still differ in both sub-periods and the point estimates are similar to those in the uni- and bivariate regressions. For other sensitivities, with the exception of the monetary slope, the differences in the point estimates have grown in the second sub-sample. The test for zero intercepts interestingly indicates non-zero intercepts in the second rather then the first sub-period. Table 9 shows slopes for the default premium and consumption betas statistically different in the period from 1982:11 to 2003:12 and they are joined, for this sub-sample, by sensitivities to the term premium, inflation and money supply. Consumption betas differ also for the first period. In addition, a joint test of beta equality for all but the market variables results in rejection of the null in the second sub-sample.

We will now attempt to interpret our results. Let us start with the period from 1959:02 to 1982:10. Risk measured by betas is different mainly due to differences in the market beta, which partly explains differing mean returns. Stocks of small firms appear underpriced, and hence are a better investment opportunity. The situation is more complex in the period from 1982:11 to 2003:12 where the recommendation based solely on

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[14] The reported estimates are also in accord with other studies on firm-size effects, which use multi-factor models - see Fama and French (1993, market beta, betas for the default and term premia), Chan, Chen, and Hsieh (1985, all but the money supply beta), and Li and Hu (1998, industrial production and money supply betas).

[15] We found similarly strong evidence of conditional heteroskedasticity and serial correlation in a majority of the univariate and bivariate models we studied.

market betas is essentially reversed when one uses additional factors. While investment in small firms appears less risky based on market betas, it is riskier as measured by other betas. Being underpriced, the large firms seem to be the bargain here.

8 Conclusion

As finance theory suggests, differing sensitivities (betas) of economic factors translate in differing asset performance (unless they affect expected returns in opposite directions). While this connection is widely recognized, almost no attention has been paid to *formal* differences in betas. The present work explicitly acknowledges this link by testing for statistical differences of betas across assets and by considering implications of these differences for mean excess returns.

We first provide a general regression framework, which can be easily used to test for equality of betas across equations. A number of methods can be used to conduct such tests. We consider the standard F, Wald, and Hansen tests. The advantage of the Wald and Hansen methods is relatively simple accommodation of robustness to general forms of autocorrelation and heteroskedasticity, often present in the financial data. The price for generality in this case is potential distortion of the tests in larger regression systems. In a simulation exercise tailored to our data application, we find that finite sample distortions are relatively minor and that the Hansen method with parametric pre-whitening outperforms the other methods.

We illustrate the usefulness of formal comparison of betas in application to stocks sorted by firm capitalization. Namely, a simple t-test indicates that small firms outperformed the larger firms prior to 1982:10 but not since then. We attempt to shed some light on this empirical observation by carefully analyzing betas of several economic factors. We find that the market beta difference is the main source of differing mean returns before 1982:10. However, while the market beta differ also since 1982:10, the mean returns do not. Moreover, it is the small firms, which appear safer. Testing of beta equality of factors other than the market reveals the reason behind this seeming inconsistency. The other sensitivities (especially to the default premium) also differ but are higher (in absolute value) for the small firms, making them riskier from this perspective.

Overall, our empirical analysis suggests that formally comparing, individually and jointly, market betas and betas of other macroeconomic variables can be helpful in explaining behavior of expected returns and can lead to investment recommendations. While individual statistical comparison of betas may be redundant at times, joint comparisons are useful in any case as they summarize information contained in a number of point beta estimates. They can be combined with mean returns, regression intercepts and point beta estimates to form a clearer basis for judging investment opportunities. A by-product of our calculations is confirmation of the now widely accepted need for multi-factor models. Default premium and consumption growth seem to be two important sources of risk differences other than the market.

There are several directions for future research. As our results indicate, any two assets or a group of assets can be compared by formally testing equality of betas across equations. For instance, one could revisit the influential Fama and French (1993) paper to evaluate whether the stocks sorted by size and book-to-market ratios really differ as the point beta estimates suggest. The importance of formal comparison of betas across assets also leads to the possibility of uncertainty in betas being priced by the market. This possibility could be investigated by the Fama and MacBeth (1973) method using some measure of uncertainty in the market beta as one of the factors in the time-series regression. One such measure could be the difference in publicly available beta estimates. The cross-sectional regression would then reveal whether this factor is priced or not.

Table 1

Summary Statistics

	r_{SMALL}	r_{LARGE}	r_{VW}	r_{DEF}	r_{TERM}	g_{IP}	g_{CONS}	π	g_{MON}
				1959:02-2003:12					
Mean	11.22	4.98	5.67	23.76	3.15	3.16	2.02	4.12	6.60
Median	8.25	6.83	9.32	22.68	2.39	3.52	2.16	3.57	6.48
Max	694.93	210.29	195.37	57.36	113.86	71.98	21.63	21.53	47.03
Min	-367.96	-241.97	-266.66	1.68	-86.59	-43.36	-21.58	-6.58	-32.16
St.Dev.	78.56	49.92	50.98	10.87	19.48	9.81	5.24	3.65	5.49
Skewness	-0.29	1.20	-0.39	0.38	0.22	0.03	-0.18	0.97	0.67
Kurtosis	4.73	15.61	4.98	2.62	6.58	9.67	4.54	4.64	18.38
				1959:02-1982:10					
Mean	11.84	1.91	3.21	18.29	0.74	3.22	1.95	5.13	5.98
Median	6.86	4.63	4.69	15.84	0.57	3.60	2.29	3.99	6.08
Max	694.93	210.29	195.37	47.16	113.86	71.98	21.63	21.53	23.48
Min	-321.65	-157.32	-145.90	1.68	-86.59	-43.36	-21.58	-3.90	-5.56
St.Dev.	89.24	49.51	51.84	9.50	21.11	12.07	6.01	4.26	3.96
Skewness	1.61	0.06	0.00	0.74	0.47	0.00	-0.17	0.69	0.01
Kurtosis	15.40	4.25	3.90	2.70	7.98	7.77	3.95	3.16	4.04
				1983:11-2003:12					
Mean	10.51	8.44	8.43	29.90	5.85	3.09	2.10	2.99	7.30
Median	9.77	10.93	12.20	28.56	5.81	3.51	2.00	2.94	7.09
Max	291.21	156.09	149.07	57.36	56.35	23.83	16.03	11.35	47.03
Min	-367.96	-241.97	-266.66	13.92	-40.34	-14.49	-14.78	-6.58	-32.16
St.Dev.	64.68	50.24	49.95	8.84	17.12	6.42	4.22	2.34	6.74
Skewness	-0.19	-0.67	-0.87	0.72	-0.06	0.15	-0.10	-0.20	0.56
Kurtosis	9.53	5.48	6.58	2.97	3.06	3.31	4.71	5.14	16.29

Notes: r_{SMALL} and r_{LARGE} denote respectively the excess returns on the small-cap and large-cap portfolios, r_{VW} is the excess return on the market portfolio, r_{DEF} and r_{TERM} are the default and risk premium, respectively, g_{IP} and g_{CONS} are growth rates of industrial production and per capita consumption, respectively, π measures the inflation rate and g_{MON} the growth rate of the money supply, respectively. All reported numbers are annualized, in percentages.

Table 2

Correlations

		r_{SMALL}	r_{LARGE}
1959:02-2003:12	r_{VW}	0.69	0.99
	r_{DEF}	0.16	0.12
	r_{TERM}	0.11	0.19
	g_{IP}	-0.01	0.00
	g_{CONS}	0.20	0.16
	π	-0.12	-0.18
	g_{MON}	-0.07	-0.01
1959:02-1982:10	r_{VW}	0.74	0.98
	r_{DEF}	0.21	0.21
	r_{TERM}	0.18	0.22
	g_{IP}	0.05	0.07
	g_{CONS}	0.20	0.18
	π	-0.13	-0.19
	g_{MON}	-0.05	-0.01
1982:11-2003:12	r_{VW}	0.63	0.99
	r_{DEF}	0.19	-0.03
	r_{TERM}	-0.01	0.13
	g_{IP}	-0.18	-0.14
	g_{CONS}	0.22	0.14
	π	-0.12	-0.13
	g_{MON}	-0.10	-0.03

Notes: See notes in Table 1 for variable definitions.

Table 3

Rejection Rates Under the Null Hypothesis

				Covariance Matrix Estimator					
K	Sample Size	Test		WH	NW	NW-P	A	AM	VARHAC
2	250	F	0.08						
		Hansen		0.07	0.03	0.02	0.05	0.05	0.05
		Wald		0.08	0.08	0.07	0.07	0.07	0.07
2	500	F	0.07						
		Hansen		0.07	0.05	0.05	0.06	0.05	0.05
		Wald		0.08	0.08	0.07	0.07	0.06	0.06
4	250	F	0.07						
		Hansen		0.05	0.02	0.01	0.04	0.04	0.04
		Wald		0.10	0.13	0.13	0.10	0.08	0.08
4	500	F	0.06						
		Hansen		0.05	0.03	0.03	0.04	0.04	0.04
		Wald		0.06	0.08	0.08	0.06	0.05	0.05

Notes: We simulate 500 times the series $r_{it} = x_{1t} + x_{2t} + \ldots + x_{Kt} + \varepsilon_{it}, i = 1, 2, t = 1, ..., T, T = 250$ or 500, $K = 2$ or $K = 4$. $x_{jt} = 0.002 + 0.10\, x_{j,t-1} + u_{jt}$, $j = 1, 2, \ldots, K$, where $u_{1t}, ..., u_{Kt}$ are mutually independent and i.i.d. normally distributed with zero mean and variance 0.0001. $\varepsilon_{it} = 0.1\varepsilon_{i,t-1} + 0.003 \times \sqrt{1 + 0.2\,\varepsilon^2_{i,t-1}}\eta_{it}$, $i = 1, 2$, with η standard normal noise. We test the null hypothesis of equality of all parameters across the two assets.

Table 4

Rejection Rates Under the Alternative Hypothesis

				Covariance Matrix Estimator					K
K	Sample Size	Test		WH	NW	NW-P	A	AM	VARHAC
2	250	F	0.89						
		Hansen		0.87	0.78	0.77	0.85	0.84	0.84
		Wald		0.90	0.89	0.89	0.88	0.87	0.87
2	500	F	1.00						
		Hansen		0.99	0.99	0.99	0.99	0.99	0.99
		Wald		1.00	1.00	1.00	1.00	1.00	1.00
4	250	F	0.58						
		Hansen		0.51	0.26	0.26	0.43	0.48	0.48
		Wald		0.61	0.62	0.61	0.61	0.58	0.58
4	500	F	0.89						
4	500	Hansen		0.86	0.76	0.74	0.82	0.84	0.84
		Wald		0.89	0.87	0.87	0.88	0.88	0.88

Notes: We simulate 500 times series

$$r_{1t} = x_{1t} + x_{2t} + \ldots + x_{Kt} + \varepsilon_{1t},$$

$$r_{2t} = x_{1t} + x_{2t} + \ldots + x_{\frac{K}{2},t} + (1 + \tfrac{0.2}{K})(x_{(\frac{K}{2}+1),t} + x_{(\frac{K}{2}+2),t} + \ldots + x_{Kt}) + \varepsilon_{2t},$$

$t = 1, 2, \ldots, T$, $T = 250$ or 500, $K = 2$ or $K = 4$. $x_{jt} = 0.002 + 0.10\, x_{j,t-1} + u_{jt}$, $j = 1, 2, \ldots, K$, where $u_{1t}, ..., u_{Kt}$ are mutually independent and i.i.d. normally distributed with zero mean and variance 0.0001. $\varepsilon_{it} = 0.1\, \varepsilon_{i,t-1} + 0.003\sqrt{1 + 0.2\, \varepsilon^2_{i,t-1}}\, \eta_{it}, i = 1, 2$, with η standard normal noise. We test the null hypothesis of equality of all parameters across the two assets.

Table 5

Tests of the CAPM

Hypothesis	Years	r_{VW}
equal slopes	59:02-03:12	1.16
		(0.28)
	59:02-82:10	5.32
		(0.02)
	82:11-03:12	3.25
		(0.07)
zero intercepts	59:02-03:12	5.07
		(0.08)
	59:02-82:10	5.73
		(0.06)
	82:11-03:12	1.94
		(0.38)

Notes: The model is $r_{it} = \alpha_i + \beta_i x_t + \varepsilon_{it},\ i = 1, 2$, where α_i is the i-th intercept, β_i is the i-th slope, $r_{1t} = r_{SMALL}$, $r_{2t} = r_{LARGE}$, x_t is r_{VW} (see notes for Table 1 for variables' definitions) and ε_{it} is the regression error. Reported are HAC Hansen statistics (VARHAC method) for testing equality of slopes and zero values for intercepts. p-values are in parentheses.

Table 6

Tests of Assorted Bivariate Models

Hypothesis	Years	independent variable (in addition to market return)					
		r_{DEF}	r_{TERM}	g_{IP}	g_{CONS}	$\Delta\pi$	g_{MON}
equal slopes	59:02-03:12	0.83	1.23	1.16	0.62	1.17	1.15
(market)		(0.36)	(0.27)	(0.28)	(0.43)	(0.28)	0.28
	59:02-82:10	5.28	4.90	5.09	4.63	5.15	5.04
		(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)
	82:11-03:12	3.34	2.71	4.02	3.90	3.04	3.59
		(0.07)	(0.10)	(0.04)	(0.05)	(0.08)	(0.06)
equal slopes	59:02-03:12	1.82	3.13	0.06	7.67	0.00	0.68
(other)		(0.18)	(0.08)	(0.81)	(0.01)	(0.98)	(0.41)
	59:02-82:10	0.64	0.01	0.04	0.80	0.19	0.02
		(0.42)	(0.92)	(0.85)	(0.37)	(0.66)	(0.88)
	82:11-03:12	13.42	2.61	1.52	5.15	0.04	0.27
		(0.00)	(0.11)	(0.22)	(0.02)	(0.84)	(0.60)

Notes: The model is $r_{it} = \alpha_i + \beta_i x_t + \varepsilon_{it},\ i = 1, 2,$ where α_i is the i-th intercept, β_i is the i-th vector of slopes, $r_{1t} = r_{SMALL}$, $r_{2t} = r_{LARGE}$, elements of x_t are r_{VW} and of one the following variables: r_{VW}, r_{DEF}, r_{TERM}, g_{IP}, g_{CONS}, $\Delta\pi$, g_{MON} (see notes for Table 1 for variables' definitions) and ε_{it} is the regression error. Reported are HAC Hansen statistics (VARHAC method) for testing equality of slopes. p-values are in parentheses.

Table 7

Estimated Model with Seven Covariates

		independent variable							
Years	Size	Intcpt.	r_{VW}	r_{DEF}	r_{TERM}	g_{IP}	g_{CONS}	$\Delta\pi$	g_{MON}
59:02-03:12	small	-0.005	1.036	0.660	-0.185	-0.097	1.442	1.139	-1.058
		(0.005)	(0.079)	(0.248)	(0.138)	(0.269)	(0.499)	(0.713)	(0.608)
	large	0.000	0.973	-0.081	-0.005	0.047	-0.156	0.123	0.171
		(0.001)	(0.011)	(0.036)	(0.022)	(0.035)	(0.071)	(0.107)	(0.067)
59:02-82:10	small	0.003	1.247	0.584	-0.125	0.014	0.985	0.643	-1.342
		(0.007)	(0.126)	(0.495)	(0.209)	(0.304)	(0.548)	(0.874)	(1.870)
	large	0.000	0.950	-0.048	-0.058	0.036	-0.109	0.071	0.007
		(0.001)	(0.018)	(0.063)	(0.030)	(0.041)	(0.088)	(0.141)	(0.206)
82:11-03:12	small	-0.022	0.778	1.260	-0.387	-1.042	2.306	2.554	-0.909
		(0.010)	(0.099)	(0.405)	(0.232)	(0.759)	(1.071)	(1.031)	(0.484)
	large	0.004	1.000	-0.219	0.067	0.096	-0.218	0.116	0.201
		(0.001)	(0.011)	(0.059)	(0.027)	(0.064)	(0.119)	(0.154)	(0.058)

Notes: The estimated model is: $r_{it} = \alpha_i + \beta_i x_t + \varepsilon_{it},\ i = 1, 2$, where α_i is the i-th intercept, β_i is the i-th vector of slopes, $r_{1t} = r_{SMALL}$, $r_{2t} = r_{LARGE}$, $x_t = (r_{VW}, r_{VW}, r_{DEF}, r_{TERM}, g_{IP}, g_{CONS}, \Delta\pi, g_{MON})'$ (see notes for Table 1 for variables' definitions) and ε_{it} is the regression error. Reported are OLS estimates of the model parameters with VARHAC standard errors in parentheses.

Table 8

Tests for Residual Heteroskedasticity and Correlation

Residual Property		Size	Test	from 59:02 to 03:12	59:02 82:10	82:11 03:12
correlation	across equations		Pearson	–0.70	-0.70	-0.57
				(0.14)	(0.02)	(0.03)
	across time	small	Q	98.11	66.42	45.47
				(0.00)	(0.00)	(0.00)
		large	Q	41.47	25.14	14.80
				(0.000)	(0.01)	(0.25)
heteroskedasticity		small	White	4.61	5.88	3.35
				(0.00)	(0.00)	(0.00)
		large	White	4.77	6.17	2.17
				(0.00)	(0.00)	(0.01)

Notes: The estimated model is: $r_{it} = \alpha_i + \beta_i x_t + \varepsilon_{it}$, $i = 1, 2$, where α_i is the i-th intercept, β_i is the i-th vector of slopes, $r_{1t} = r_{SMALL}$, $r_{2t} = r_{LARGE}$, $x_t = (r_{VW}, r_{VW}, r_{DEF}, r_{TERM}, g_{IP}, g_{CONS}, \Delta\pi, g_{MON})'$ (see notes for Table 1 for variables' definitions) and ε_{it} is the regression error. Residuals are calculated using OLS estimates, equation by equation; Pearson = chi-square test for correlation; White test = F test with no cross terms; Q = Q statistic for testing 12 lags of autocorrelation; p-values in parentheses.

Table 9

Tests of Seven Covariate Model

	period		
Hypothesis	1959:02 2003:12	1959:02 1982:10	1982:11 2003:12
equal slopes:			
r_{VW}	0.51	3.84	4.78
	(0.48)	(0.05)	(0.03)
r_{DEF}	6.84	1.16	10.57
	(0.01)	(0.28)	(0.00)
r_{TERM}	1.36	0.08	2.81
	(0.24)	(0.78)	(0.09)
g_{IP}	0.24	0.00	1.88
	(0.62)	(0.95)	(0.17)
g_{CONS}	7.77	3.01	3.59
	(0.01)	(0.08)	(0.06)
$\Delta\pi$	1.86	0.37	4.39
	(0.17)	(0.54)	(0.04)
g_{MON}	2.99	0.40	3.52
	(0.08)	(0.53)	(0.06)
equal slopes:	14.99	5.60	20.89
(all but mkt.)	(0.02)	(0.47)	(0.00)

Notes: The model is $r_{it} = \alpha_i + \beta_i x_t + \varepsilon_{it},\ i = 1, 2$, where α_i is the i-th intercept, β_i is the i-th vector of slopes, $r_{1t} = r_{LARGE}$, $r_{2t} = r_{SMALL}$, $x_t = (r_{VW}, r_{DEF}, r_{TERM}, g_{IP}, g_{CONS}, \pi_{UI}, g_{MON})'$ (see notes for Table 1 for variables' definitions) and ε_{it} is the regression error. Reported are respectively statistics for the Hansen tests (VARHAC method) of equality of slopes across equations for a given covariate and of equality of all slopes with the exception of the market. p-values are in parentheses.

Part III

Testing for Latent Factors in Models with Autocorrelation and Heteroskedasticity of Unknown Form

This paper considers the problem of testing for "latent factors" or "reduced rank" in a broad class of (multivariate linear stationary) time series models, wherein model errors have autocorrelation and heteroskedasticity of unknown form. It is easy to motivate these models and methods in the context of finance models, and we illustrate with a familiar macro model of asset returns, proposed by Chen, Roll and Ross (1986). Unfortunately, previously used tests for reduced rank are not sufficiently robust, so we examine two heteroskedasticity and autocorrelation consistent (HAC) methods: A HAC version of Hansen's (1982) GMM test, and a lesser known but more user-friendly "minimum distance" or "ratio of asymptotic densities" (RAD) test. We recommend the RAD test, for which we supply computer code. In application the tests lend more (HAC) robust support to the hypothesis that multiple factors drive the link between the macroeconomy and the returns on bonds and stocks.

9 Introduction

Empirical research in financial economics frequently suggests the existence of few latent factors driving the systematic component of asset returns. Existence of such latent factors makes it easier to understand the effect on asset returns of the many variables that comprise the systematic component. Results depend on the number and type of assets used and the number and types of instruments, which themselves serve as proxies for the latent factors (for examples see Campbell 1987, Zhou 1995, and Costa, Gardini, and Paruolo 1997). In econometric terms, the existence of latent factors translates into a reduced rank restriction on the (array of) coefficients in an asset return regression system.

The present work considers the problem of testing for latent factors in a broad class of (multivariate linear stationary) time series models, wherein model errors have autocorrelation and heteroskedasticity of unknown form. The generality of error dynamics is well suited to financial models of bond and stock returns, as in the macro model of Chen, Roll and Ross (1986). To deal with such generality we consider heteroskedasticity and autocorrelation consistent (HAC) methods, a HAC version of Hansen's (1982) GMM test, and a lesser known but more user-friendly minimum distance or ratio of asymptotic densities (RAD) test.

The primary benefit of using a HAC-type test of economic hypotheses, in time series models, is a certain kind of increased robustness relative to tests that rely on parametric assumptions about error dynamics. This robustness implies that stated significance levels of HAC tests are frequently closer to their true values, at least in sufficiently large samples. So, for the financial economist who wants to know: "Are there *really* multiple factors driving the link between the macroeconomy and the returns on bonds and stocks?", HAC tests (and the underlying mental exercise regarding error dynamics) give added insight. HAC tests may or may not agree with less robust tests. In our application, the HAC test results agree with the results of simpler (and more popular) implementation of Hansen's (1982) test for reduced rank, but the crucial point is that stated significance levels in the popular version of Hansen's test are not correct, in statistical terms, when the data have dynamics that cause residual serial correlation. Hence, to say that the HAC tests "agree" with the popular version of Hansen's test is really too liberal an interpretation; more accurate is to say that the nominal (but likely invalid) conclusions from the popular test coincide with that of the autocorrelation-robust tests.

The HAC Hansen test and the RAD test each require some special calculation, and for this we do some programming. The computational complexity is due partly to the presence of corrections for residual autocorrelation and heteroskedasticity, and if instead we assumed that model errors were independent and identically distributed then we could test for reduced rank via Anderson's (1951) convenient likelihood ratio (LR) test (see Reinsel and Velu [1998] for discussion and Zhou [1994] for a related test couched in GMM terms). It is of course possible to extend Anderson's LR test to (parametric) probability models with autocorrelated errors (see Reinsel and Velu 1998), but this approach relies on a correctly specified error dynamic. We instead take the non-parametric HAC approach, allowing a wider variety of error dynamics.

Is special calculation really necessary for our testing purposes? In application to asset pricing models, Zhou (1994) gives an interesting modification of Hansen's (1982) testing approach, with an analytic (hence computationally convenient) test for latent factors in asset returns. However, to implement his analytical test Zhou relies on parametric assumptions about the model's error dynamics. Specifically, he considers the case of white noise errors and also the case of errors which are uncorrelated but have a known (parametric) form of conditional heteroskedasticity. The method can, in principle, be extended to models with a parametric form of error autocorrelation, and while this is also the case with Anderson's analytical LR test, both methods are necessarily parametric. Since we pursue instead a nonparametric HAC objective, we undertake a computationally harder problem.

Comparing the HAC robust Hansen-type and RAD tests for reduced rank, the latter is much easier to implement with full flexibility regarding the reduced rank form, e.g. the selection of linearly independent matrix rows in the reduced-rank coefficient matrix. For this reason, the RAD test is more user-friendly than the robust Hansen test, and with it we obtain conclusions robust to the form of reduced rank, as well as the form of error dynamics. Both are calculated by minimizing a quadratic form with the optimal weighting matrix given by the inverse of a relevant covariance matrix. Covariance matrix estimation is made robust to both heteroskedasticity and autocorrelation via various non-parametric and parametric methods. We consider several kernel-based, heteroskedasticity and autocorrelation consistent (HAC) procedures, with various combinations of kernel, bandwidth selection, and pre-whitening filter (see Newey and West 1987, 1994, Andrews 1991, Andrews and Monahan 1992). We also implement a simple parametric procedure with pre-whitening, advocated by den Haan and Levin (1997). For the Hansen test we follow Hansen, Heaton and Yaron (1996), simultaneously iterating over both the weighting matrix and model parameters. For the RAD test such simultaneous iteration is unnecessary.

Our empirical study, like the influential work of Chen, Roll and Ross (1986), looks at the link between asset markets and macroeconomic fundamentals. As dependent variables in our regression system, we choose a set of excess returns broadly characterizing the U.S. bond and stock markets over the last four decades. Specifically, we use monthly excess returns on the Treasury securities of maturities of 90 days and 5 years. To capture the main features of the U.S. stock markets, we sort returns by firm size and use excess returns on the CRSP small capitalization and large capitalization portfolios. There are many candidates for our explanatory variables. Given our dependent variables (a small number of both stock and bond returns), we do not consider size and book-to-market related portfolios of Fama and French 1993 (henceforth FF) and term and default premia (FF, Chen, Ross and Ross 1986). The size related variables are mostly used in studies focusing on stocks; also, in those studies, a large number of assets is involved. The problem with bond related factors is that we might run into econometric problems with term structure related dependent and independent variables.[16] Moreover, while the FF factors do a good job in explaining cross-sectional and time-series variation of stock returns, they are somewhat ad hoc. Hence, we focus on macroeconomic variables which can be theoretically linked (at least loosely) to expected returns. We are left with a subset

[16] This is to some extent true for the size related factors as well; see FF for a detailed discussion.

of risk factors similar to Chen, Roll, and Ross (1986), consisting of the stock market, consumption, industrial production, money supply, and the unexpected inflation rate.

The models of asset returns with macroeconomic explanatory variables are well specified statistically, provided that the used time series are stationary. From the theoretical perspective, the models used can be considered to be examples of the inter-temporal Capital Asset Pricing Model (CAPM), in which case the test for reduced rank is the test for the number of latent risk factors inherent in this model.[17] If we wanted to interpret the test of our model as a test of the Arbitrage Pricing Theory, all the explanatory variables would have to be excess returns on assets. Interpreting the growth rate of consumption and the expected inflation rate literally as asset returns is problematic but the return on the stock market is obviously an asset return, the industrial production growth rate may be viewed as a return on physical assets, and the growth rate of money as a (negative) return on cash holdings (due to inflation).

We analyze the reduced rank structure in the bond and stock markets, both separately and jointly. We first document the presence of autocorrelation and heteroskedasticity in residuals of all unrestricted regression systems, using a battery of tests. Then we apply the RAD and the Hansen tests (as a benchmark) with several HAC robust covariance matrix estimators. We find that the one-factor hypothesis is rejected both in the bond market and the stock market. A joint estimation and tests of the stock and bond markets does not alter these results - statistically, at least four factors are needed for an accurate description of both markets. The sources of differences between bonds of different maturity can be traced to significantly different market and industrial production betas, suggesting (consistently with existing theory, see Campbell [1999] for a survey on stylized facts regarding various premia) that there is a term premium mainly due to a higher sensitivity to the market risk and to business cycles. For stocks of different firm size such differences can be traced to consumption and monetary betas. This confirms

[17] A special case of this model uses the stock market return as the only explanatory variable is the standard CAPM.

that there are differences between stocks of different sizes but in a sense diverging from the literature since the returns in our sample actually do not statistically differ,[18] only the betas do. Presence of consumption in our data is motivated by the consumption based CAPM and the higher consumption beta suggests that small firms are riskier from this perspective. Differing monetary betas may be due to greater sensitivity to tighter monetary policy of small firms (see Gertler and Gilchrist 1994). The differences between bonds and stocks confirm the equity premium puzzle. However, the prospective is rather novel in this case as we combine reduced rank tests with cross-sectional Wald tests.

The paper is organized as follows. Section 10 introduces the unrestricted and restricted asset pricing models, Section 11 describes the Hansen tests and the generalized Wald tests, Section 12 discusses the data selection and data sources, Section 13 reports our results and Section 14 concludes.

10 Model

We are interested in testing for latent factors in a broad class of multivariate linear models. However, to make the exposition more readable for the general economist we will couch our discussion in the specific context of asset pricing models. The formal setup in Equation 8 is still quite general, representing as it does a multivariate linear relationship between some (dependendent) variable y and other variables (x), so the same formal model and methods can be applied to other kinds of economic data.

For a collection of assets, let $y_{1t}, \ldots, y_{nt}$ denote the (excess) returns to holding each asset from time $t-1$ to time t. The regression model of interest is:

$$y_t = \beta\, x_t + \varepsilon_t, \quad t = 1, \ldots, T, \tag{8}$$

where β is an $n \times K$ coefficient matrix, with $n < K$, and ε_t is an $n \times 1$ vector of regression

[18] Similar findings of the diminishing firm size effect are reported by FF and by Horowitz, Loughran and Savin 2000.

errors for which $E[\varepsilon_t|x_t] = 0$. x_t is a $K \times 1$ vector of observables which may include a constant and such factors as the market excess return, consumption growth, etc. (see section 3 for details). The series y_t, x_t, ε_t are presumed stationary and conformable to standard central limit theory, and the errors ε_t can exhibit conditional heteroskedasticity and/or autocorrelation, in the usual manner described in White (1984) and Davidson (1994, 2000), for example.

Consider, in addition to the regression model (8), the following latent factor specification of the conditional mean of asset returns:

$$E[y_{it}|x_{it}] = \gamma_i E[z_t|x_{it}], \quad i = 1, \ldots, n, \ t = 1, 2, \ldots, T, \tag{9}$$

where z_t is a $q \times 1$ vector of unobserved latent factors, for some $q < K$, and γ_i, $i = 1, \ldots, n$, are their $1 \times q$ coefficient vectors. One can likewise specify a linear relationship between z and x:

$$E[z_{jt}|x_t] = \lambda_j \, x_t, \quad j = 1, ..., q, \quad t = 1, \ldots, T \tag{10}$$

for some $1 \times K$ vectors $\lambda_1, ..., \lambda_q$. In this case, the observable variables in vector x_t serve as *proxies* of the underlying latent factors z_t. The general null hypothesis is reduced rank $q < K$ for the matrix β. This reduced rank hypothesis is, equivalently, expressible as:

$$H_0 : \beta = \gamma \, \lambda \text{ for some } n \times q \text{ matrix } \gamma \text{ and some } q \times K \text{ matrix } \lambda. \tag{11}$$

To test H_0 via the Hansen (1982) approach, it is common to impose further parameter identification, as in Campbell (1987) and Ferson and Foerster (1994). We will follow this approach to implementing the Hansen test, due to the tremendous computational simplification it affords. At the same time, we will avoid these additional identifications when using the alternative RAD testing approach.

To further identify parameters under H_0, we can optionally specify $\gamma = [I \; \delta]' \lambda$, with I the $q \times q$ identity matrix and δ some $(n - q) \times q$ matrix, in which case the specialized null hypothesis is:

$H_0^*: \beta = [I\ \delta]'\lambda$ for some $q \times q$ identity matrix I, $(n-q) \times q$ matrix δ and $q \times K$ matrix λ.

While hypothesis H_0^* is clearly stronger than H_0, it is actually a very common normalization to impose in reduced-rank regression models (of which our financial models are special cases), and is typically invoked when reporting estimates of such models in economics (as in the cointegration output of Eviews software). If we are worried that H_0^* is too strong, we may prefer to put more weight on the RAD test, rather than the version of Hansen's (1982) test under study.

11 Tests

In this section, we describe two tests for reduced rank. We first define the test statistics, then describe their computation which in each case can be carried out using GMM. We first define the minimum-distance or RAD test of interest. The RAD test statistic is:

$$W = \min_{\beta \in H_0} (vec(\hat{\beta}) - vec(\beta))' \, \hat{\Omega}^{-1}_{vec(\hat{\beta})} \, (vec(\hat{\beta}) - vec(\beta)), \tag{12}$$

where $vec(\hat{\beta})$ is the unconstrained OLS estimator and $\hat{\Omega}_{vec(\hat{\beta})}$ is the associated GMM covariance matrix estimate, both implemented via heteroskedasticity and autocorrelation robust covariance estimators. The RAD test can be viewed as a special case of a very interesting (but little known) general econometric test proposed by Szroeter (1983).

The RAD test is a "minimum distance" test, in that it is based on the minimum squared "distance" between $\hat{\beta}$ and the reduced-rank approximations to $\hat{\beta}$. This minimum squared distance can also be interpreted as (proportional to the log of) the ratio of constrained and unconstrained approximate (normal) densities for the parameter estimator $\hat{\beta}$, with constraint given by the reduced-rank restriction (see Gilbert and Zemcik 2004). Hence, the label RAD (ratio of asymptotic densities) is fitting and also intuitive in its similarity to the likelihood ratio (LR) test statistic. An advantage of RAD over LR is that we can make RAD robust to error autocorrelation and heteroskedasticity of unknown form, via a HAC form for $\hat{\Omega}_{vec(\hat{\beta})}$, whereas consistency of LR requires a known form of error dynamics.

Next we define the relevant Hansen (1982) tests. For each given value of β in Equation 8 define:

$$e_{it} = y_{it} - \beta_i x_t, \quad i = 1, \ldots, n, \quad t = 1, \ldots, T. \tag{13}$$

Relevant sample moments take the form:

$$m(\beta)_{ij} = \frac{1}{T} \sum_{t=1}^{T} e_{it} u_{jt}, \quad i = 1, ..., n, \ j = 1, ..., L, \tag{14}$$

with $u_{jt}, j = 1, ..., L$ a set of instruments.

The general Hansen (1982) GMM statistic is defined as:

$$S = \min_{\beta \in H_0^*} vec(m(\beta))' \, \hat{\Omega}^{-1}_{vec(m(\beta))} \, vec(m(\beta)), \tag{15}$$

where $\hat{\Omega}_{vec(m(\beta)))}$ is the GMM covariance-matrix estimator, which we later specify in ways robust to heteroskedasticity and serial correlation. To arrive at the solution S to Equation 15, we iterate over repeated trials, at each stage simultaneously solving for updated parameter and covariance-matrix estimates, as in Hansen et al. (1996).

Under the null hypothesis (H_0 or H_0^*) and suitable regularity conditions (stationarity, finite moments, mixing, etc., as in White [1984] and Davidson [1994, 2000], for example), both the Hansen test S and RAD test W can be shown to be distributed asymptotically as chi-square variables with $(n-q)(K-1)$ degrees of freedom. Hansen (1982) shows this result for S under general conditions (and for recent discussion see Harris and Mátyás [1999]). For the RAD test, Szroeter (1983) provides some general (but quite abstract) theory, and Gouriéroux and Monfort (1989) give a somewhat more streamlined and intuitive version of this broad theory. In specific application to reduced-rank linear models, Gilbert and Zemcik (2004) give an extensive theoretical description of the RAD test.

To summarize briefly the logic of proving the asymptotic chi-square distribution of W: (a) By construction, the statistic W is obtained as the minimum (the Malhanobis) distance between an unconstrained parameter vector (consisting of $\hat{\beta}$ elements) and a set of candidate values $\beta \in H_0$, (b) the estimator $\hat{\beta}$ is itself assumed to be consistent and asymptotically normal, with a covariance matrix $\Omega_{vec(\hat{\beta})}$, (c) the consistent estimator $\hat{\Omega}_{vec(\hat{\beta})}$ of $\Omega_{vec(\hat{\beta})}$ is also the distance-defining matrix appearing in the Malanobis distance function. We can then view the constrained estimator $\tilde{\beta}$ of β as a function of $\hat{\beta}$ and $\hat{\Omega}_{vec(\hat{\beta})}$, and applying the Delta (asymptotic expansion) method (see, for example, van der Vaart [1998]) to $\tilde{\beta}$, we can readily obtain a first-order normal approximation to the difference $\tilde{\beta} - \hat{\beta}$, and from this conclude that W is asymptotically chi square. For other, more exhaustive proofs of the asymptotic normality of the minimized (square) distance objective function, see Dahm and Fuller (1986), Cragg and Donald (1995), and Gilbert and Zemcik (2004).

To compute the Hansen (simultaneous-iterated) J-type test S, we use the GMM routine in EViews 3.1, with a variety of choices for the covariance estimation method. To compute the RAD test W, we use a simple and convenient iterative method to get an alternating sequence of γ and λ estimates, which are needed to estimate β under H_0. Appendix 1 contains a derivation of the required mathematical formulae. Alternatively, we could use the Newton-Raphson method of Ahn and Reinsel (1988, 1990). At each stage of the iteration, we hold fixed the current estimate (γ or λ) and solve the quadratic optimization problem (12) for the remaining parameters. We start with the initial values

$$\hat{\lambda}_i = \hat{\beta}_i, \; i = 1, ..., q. \tag{16}$$

Holding fixed the initial choice of $\hat{\lambda}$, we solve the quadratic problem in Equation 12 in terms of γ:

$$vec(\hat{\gamma}') = \left[(I_n \otimes \hat{\lambda}')' \hat{\Omega}^{-1}_{vec(\hat{\beta}')} (I_n \otimes \hat{\lambda}') \right]^{-1} (I_n \otimes \hat{\lambda}')' \hat{\Omega}^{-1}_{vec(\hat{\beta}')} vec(\hat{\beta}'). \tag{17}$$

We express the solution in terms of transposes γ', β', and λ' due to the fact that the covariance matrix $\hat{\Omega}_{vec(\hat{\beta}')}$ is readily obtained from standard regression packages, whereas $\hat{\Omega}_{vec(\hat{\beta})}$ is equivalent but requires extra computation.

Next, having computed the first-round choice $\hat{\gamma}$, we hold it fixed and solve the quadratic problem in Equation 12 in terms of λ,

$$vec(\hat{\lambda}') = \left[(\hat{\gamma} \otimes I_K)'\hat{\Omega}^{-1}_{vec(\hat{\beta}')}(\hat{\gamma} \otimes I_K)\right]^{-1} (\hat{\gamma} \otimes I_K)'\hat{\Omega}^{-1}_{vec(\hat{\beta}')} vec(\hat{\beta}'). \tag{18}$$

We repeat the alternating updates of $\hat{\gamma}$ and $\hat{\lambda}$ numerous times (at least 10). To ensure convergence, at each round of the sequence, we normalize the matrix γ by diving each of its elements by its upper left element. We carry out these computations using a program we wrote in the EViews 3.1 environment. This program is available from the authors upon request, and is able to handle more general reduced-rank structures where only some of the regressors are restricted.

We estimate the covariance matrices needed to calculate S and W in a number of different ways. While we use mainly HAC-based estimators, for comparison purposes, we also examine the White heteroskedasticity-consistent estimator (denoted White). Ferson and Foerster (1994) study the finite sample properties of the Hansen/White test for reduced rank equal to 1 and 2, as implied by various versions of the CAPM. As for HAC methods, we include ones based on the Bartlett kernel and the data-dependent Newey and West (1994) bandwidth, with and without prewhitening (denoted NW and NW-P, respectively). We also include the quadratic spectral kernel with the Andrews (1991) data-dependent bandwidth (without prewhitening, denoted A), and the Andrews-Monahan (1992) method (denoted AM) with prewhitening. We have also examined the simple prewhitening method studied by den Haan and Levin (1997) (VARHAC). We have spot checked some of our EViews-based computations using a Gauss code written by Hansen, Heaton and Okagi, and we have noticed that EViews 3.1 versions prior to June 2000 appear to have an error in computing some of the J tests, but versions June 2000 and later do not have this problem, as confirmed by the EViews technical staff.

Den Haan and Levin (1997) study finite sample properties of kernel-based and parametric covariance-matrix estimators in a single equation context with complex serial correlation structures. Their Monte Carlo experiments favor a simple parametric method with prewhitening - VARHAC. In computations, which we omit here for brevity, we extend their simulations to systems of equations in which we study the small sample rejection rates of the S and W statistics under the hypotheses of reduced and full rank. Our results also support the use of the VARHAC method in most cases. In addition, we find that some finite-sample properties (including empirical size) worsen as we increase the number of equations, increase the number of explanatory variables, increase rank and decrease the sample size. We conclude that, for a sample size T of about 500 (our number of observations), four equations and six explanatory variables (including a constant term), the S and W statistics are reasonably close to the chi-square distribution under the null.

12 Data

Most studies focusing on factor models of expected asset returns either assume or ultimately conclude that the number of factors equals 1, 2 or 3 (see Campbell 1987; Ferson and Foerster 1994; Zhou 1995; Backus, Foresi, and Telmer 1998; de Jong 1998; Dai and Singleton 2000). To test for up to three factors, we need at least four asset returns. Therefore, we choose two bond returns and two stock returns to characterize the bond and stock markets, respectively.

Treasury 90-day Bills and 5-year Bonds seem a natural choice as representatives for the bond market. While there is no difference in the default risk, there is a difference in levels of risk due to differing maturities. The data source is CRSP (indno 1000707 for the 90-day T-bill and 1000704 for the 5-year T-bond) and we subtract the 30-day T-bill rate (indno 1000708) to get excess returns r_{T90} and r_{T5}, respectively. The data frequency is monthly, the sample period 1959:02-2000:11 is given by the availability of per capita consumption series (see below) and the summary statistics are in Table 10. As expected, the rate of return on the longer maturity bond is higher and so is the corresponding risk level as measured by the standard deviation.

To capture the basic features of the stock market, we need stock returns covering a wide range of stocks but with different risk characteristics. We use CRSP NYSE Portfolio Indices ranked by capitalization, combining deciles 1-5 for the large firms (indno 1000314) and 6-10 for the small firms (indno 1000315). These monthly time series are based on quarterly rebalanced portfolios. Excess returns are again calculated using the 30-day Treasury bill return and we denote them as r_{LARGE} and r_{SMALL}, respectively. The summary statistics in Table 10 indicate an overall higher level of both return and risk for small firms. The excess return on small caps is more volatile (consistent with Malkiel and Xu [1997], for example) and this feature is independent of the chosen time period. On the other hand, the mean excess return is actually greater for large firms since 1980, a trend noticed by Fama and French (1993) and carefully documented by Horowitz, Loughran, and Savin (2000).

As covariates, we opt for variables for which there is an established theoretical link to expected excess returns. This excludes the size-related stock market factors (see Fama and French 1993; Chan and Chen 1991) and term structure and default risk related bond factors (see Chen et al. 1986; Fama and French 1993). Inclusion of these variables could also lead to econometric problems, with firm size and-term-structure-related dependent and independent variables.

Both the static CAPM and the intertemporal CAPM suggest the use of the market excess return as an explanatory variable. We use the CRSP value-weighted index of the S&P 500 Universe (indno 1000502) in excess of the 30-day T-bill (r_{SP}). The time series characteristics in Table 10 are similar to those of r_{LARGE} due to the fact that the value weighted index is dominated by large firms.

The consumption CAPM and business-cycle models specify the relationship between expected returns, consumption, and production. Hence, we include the growth rates of industrial production (g_{IP}) and real per capita consumption of nondurables and services (g_{CONS}) as measures of real economic activity. We obtain both series from the St. Louis Fed's website. Specifically, we use the variable INDPRO to calculate g_I and variables PCEND, PCES, and POP to calculate g_C. The per-capita consumption is adjusted to inflation using the CRSP price index (indno 1000709). The FED series are seasonally adjusted. Interestingly, the correlation of consumption with stock-market variables (see Table 10) is greater than with bond-market variables, and the opposite is true for industrial production.

We also include the monetary growth as one of the explanatory variables. The link between expected returns and money can be motivated by overlapping generations models (see Brock [1990] for a survey), models with money in the utility function (Brock 1975), and models with the cash-in-advance constraint (Svensson 1985)[19]. We use the seasonally adjusted monetary base series from the St. Louis Fed's website (series AMBSL) to calculate the monetary growth rate, g_M, which seems to move more closely with the bond market than the stock market (see Table 10).

Finally, we control for the effect of inflation. Because inflation happens to be the only nonstationary series (the augmented Dickey Fuller test does not reject the unit root), we use its first difference, which is the unexpected inflation.

13 Empirical Results

In this section, we investigate the latent variable structure of the bond market, the stock market, and the market for both bonds and stocks. In each case, we first estimate the unrestricted model of the form (8), test corresponding residuals for heteroskedasticity and correlation, and then conduct tests for reduced rank. In this application, the afore-mentioned GMM setup will reduce to a simple method of moments setup, with u consisting of regressors x and with $L = K$. We report reduced-rank test results for both Hansen and RAD tests with VARHAC covariance matrix. However, our results are robust with respect to the choice of covariance matrix estimators described in section 11

[19] As Feenstra (1986) shows, the cash-in-advance models can be interpreted as a special case of the money in utility function models.

(i.e., besides VARHAC, we also use White, NW, NW-P, A, and AM).

13.1 Bond Market

The estimates of the unrestricted model for expected bond excess returns are reported in panel A of Table 11. The intercepts are small but significantly different from zero, indicating the presence of a term premium unexplained by the simplistic asset pricing model. A Wald test for zero intercepts in both equations at once can be loosely interpreted as a test of an asset pricing model - see Gibbons, Ross, and Shanken (1989); Fama and French (1993, 1996); and Cochrane (2001). The Wald test statistic is, in this case, 17.10 and the corresponding p-value is 0, so our explanatory variables themselves do not entirely explain the time-series behavior of bond returns, a result consistent with the notion of the riskfree rate puzzle (see Campbell 1999, for example). Because we can think of our risk factors as proxies for the underlying latent factors, this should not undermine our reduced-rank analysis.

According to CAPM and the intertemporal CAPM, the beta of excess market return is expected to be close to zero because the default risk is presumably very small for U.S. government bonds. The beta is likely to be higher for excess returns on bonds with higher maturity where the differences in the overall risk level increase. Panel A of Table 11 confirms this prediction with the market beta being insignificant for r_{T90} and somewhat larger and significant for r_{T5}. The consumption growth rate is insignificant in both equations, and this is consistent with first-order conditions of the consumer optimization problem (in the power utility consumption CAPM) only for large risk-aversion coefficients (see Eqn. 1.16 in Cochrane [2001]). The beta for industrial production is significantly negative for both types of bonds, reflecting the fact that industrial production is a leading indicator for output. Because we consider a multiple regression that includes industrial production in addition to the market excess return, the industrial production beta characterizes reaction of returns to output fluctuations unusually large for a given level of market return. Bond prices are typically higher earlier in contractions, which pushes down the next period's interest rates and returns. Bonds with higher maturity seem to be more sensitive to business cycles. The sign of the monetary beta is in accord with a simple intuition of lower interest rates as a result of increasing the money supply, but this beta is insignificantly different from zero. The estimate of the expected inflation coefficient is also insignificantly different from zero, thus suggesting that, while the expected inflation affects the returns according the Fisher equation, it does not influence excess returns.

Preliminary to testing for reduced rank, we first document the need for the HAC robust methodology by testing for heteroskedasticity and various forms of correlation in the regression residuals of our unrestricted model. Panel B of Table 11 indicates the residuals are correlated across equations and time and heteroskedastic, thus justifying our HAC robust estimation methods. The Hansen and RAD tests of the null hypothesis of rank=1 are then conducted using the VARHAC covariance-matrix estimator.

For the Hansen test (but not the RAD test), for ease of computation, we apply the more specialized version H_0^* of reduced-rank hypothesis H_0, via the specification

$$\begin{aligned} y_{1t} &= \lambda x_t + \epsilon_{1t}, \\ y_{2t} &= \delta y_{1t} + \epsilon_{2t}. \end{aligned} \tag{19}$$

Here, the first asset serves as the reference asset, through which we can describe (up to a multiple δ) the dependence of y_2 on x.

For data, we use the sample 1959:02-2000:11, where $y_{1t} = r_{T90}$, $y_{2t} = r_{T5}$, λ is a (6×1) vector of coefficients, $x_t = (1,\, r_{SP},\, g_I,\, g_C,\, g_M,\, \pi_{UI})'$, ϵ_{it} is the regression error, and δ is the multiple coefficient characterizing the different sensitivity of the second asset. We also use the vector x_t as instruments in the J-test.

Both tests strongly reject the (respective) null hypothesis - see panel C of Table 11. These results suggest that behavior is very different for the two government bonds even though the only source of difference in risk is the maturity term. More than one macroeconomic factor is needed to explain the cross-section of expected bond returns. Implicitly, the term premium is thus characterized by at least two underlying factors. To identify potential sources for differences between bonds of different maturities, we run simple Wald tests of equality of individual coefficients across equations. The equality of coefficients is only rejected for the market and industrial production betas, which were also the only sensitivities statistically different from zero in our unrestricted model.

While we focus on bond excess returns, our study is complementary to research concentrated on the term structure of interest rates (Cochrane [1999] relates bond returns and interest rates with respect to the yield curve and Campbell, Lo, and MacKinlay [1997, ch. 10] provide basic formulae tying returns and yields together). For example, Backus et al., (1998) give a survey of (multi) factor models of the term structure and Ang and Piazzesi (2001) use a VAR model with macroeconomic and latent variables. The latent variables are often referred to as slope, curvature, and level factors and correspond to the shape of the yield curve. Ang and Piazzesi (2001) treat macroeconomic variables characterizing inflation and the business cycle as observable and argue that the slope and curvature factors can be related to macro factors. Consistent with our results, this leaves them with three factors needed to explain the term structure - the inflation, business cycle, and level factors.

13.2 Stock Market

Panel A of Table 12 reports the OLS estimates of betas in the unrestricted model.[20] Neither the Wald test (with the statistic equal 0.16 and corresponding p-value 0.92) nor individual t-tests can reject the hypothesis of zero intercepts for this stock return model. This is rather different than the case of bonds returns mentioned earlier.

As implied by the CAPM and the intertemporal CAPM, the market beta is positive and significant. It is higher than 1 for small firms, reflecting a higher level of risk associated with their returns. For large firms, the market beta is close to 1 due to the

[20]The estimation of the unrestricted model is often used as the first step of the Fama and MacBeth (1973) method. For example, see Chen, Roll, and Ross (1986), who also use macroeconomic variables.

fact that the time series characteristics of the stock market portfolio are dominated by firms of a greater market value. The consumption beta is significant for small firms but still too small as compared with predictions of the consumption CAPM with power utility function (see Eqn. 1.16 in Cochrane [2001]), confirming the equity premium puzzle. Industrial production is again negative but insignificant. The money betas are both negative but only the small-firm money beta significantly so. The expected inflation coefficient is positive for both portfolios and significant for small firms. A positive coefficient suggests that expected inflation affects stock returns more that it affects the riskfree rate.[21]

Turning to tests for reduced rank, we first test for residual heteroskedasticity and correlation. Results in panel B of Table 12 indicate that both are present, further justifying the use of HAC robust methods. The restricted model for the Hansen test is specified by the system (19) with $y_1 = r_{SMALL}$ and $y_2 = r_{LARGE}$. As shown in panel C of Table 12, the null hypothesis of rank one is strongly rejected by both the Hansen and RAD test. This results is robust to exclusion of intercepts and to the choice of the covariance-matrix estimator. Interestingly, because the signs of all coefficients are the same, this outcome is due to disproportionately greater sensitivities for small firms. Specifically, Wald tests for equality of individual coefficients across the two equations show that only consumption, monetary, and expected inflation betas differ at 5% level of significance. The market betas are statistically indistinguishable, consistent with recent evidence suggesting that returns on stocks sorted by size may not differ as much as previously thought.[22] Our results suggest that, while differences between small and large firms are more subtle, they do exist, mainly due to quantitative variation of sensitivities to variables other than the market excess return. Therefore, at least two factors are

[21] Our results are consistent with studies on firm-size effects - see Fama and French (1993, market beta), Chan, Chen, and Hsieh (1985, market and industrial production betas), and Li and Hu (1998, industrial production and money- supply betas).

[22] Among others, Horowitz, Loughran, and Savin (2000) report that no consistent relationship can be found between size and realized returns since the 1980s.

needed to explain both the mean excess returns on small and large firms. Similarly, Costa, Gardini, and Paruolo (1997) consider monthly returns on common stocks listed on the Milan stock of Exchange and use maximum likelihood inference in reduced-rank regression models to conclude that the number of (latent) factors appears to be greater than four.

13.3 Bond and Stock Markets Jointly

In this subsection, we consider simultaneously the bond and stock markets. Panel A of Table 13 reports residual correlation across equations - the correlations across markets are small and insignificant (the rest of the correlation matrix can be found in Tables 11 and 12). Campbell and Ammer (1993), for example, argue that the low correlation can be explained by the real interest rate and by news about future excess stock returns and inflation. Because the residual heteroskedasticity and autocorrelation tests are conducted equation by equation, they are identical to ones reported in panel B of Tables 2 and 3.

When applying the HAC-robust Hansen test (but not the RAD test), for ease of computation, we again apply a restricted version H_0^* of the latent factor structure H_0. The restricted model uses the first q assets as reference assets, as follows:

$$\begin{aligned} y_{it} = & \quad \lambda_i x_t + \epsilon_{it}, \quad i = 1, 2, \ldots, q, \\ y_{jt} = & \quad \delta_1 y_{1t} + \ldots + \delta_q y_{qt} + \epsilon_{jt}, \quad , j = q+1, \ldots, n, \end{aligned} \tag{20}$$

where the rank q=1,. . . ,3, and $y_{1t} = r_{T90}$, $y_{2t} = r_{T5}$, $y_{3t} = r_{SMALL}$, $y_{4t} = r_{LARGE}$. λ's and x_t are defined above.

For both the Hansen test and RAD test, panel B of Table 13 indicates a strong rejection of ranks 1, 2, and 3, that is, at least four latent factors are necessary to characterize jointly the cross-sectional and time-series behavior of expected excess returns. The rejection of the factor models with a small number of factors is representative of problems connected with accounting for the high risk-free rate and the term, bond equity, and equity premia (see Campbell [1999] for a survey).

The question arises whether this result could be anticipated given the fact that at least two factors were needed to explain expected returns for both the bond and stock markets. The answer is no because there is a possibility that both bond and stock excess returns are driven by the same two latent factors. Such a possibility is rejected by the reduced-rank analysis. The source of differences lies in different patterns of sensitivities in the two markets. While the variability in the bond returns is mostly due to differing market and industrial production betas, the variation in the stock excess returns can be traced to betas for the growth rates of consumption and money supply, and expected inflation.

In a quite different modelling context, Campbell (1987) tests for reduced rank in a VAR model with bond excess returns of several maturities and the excess return on the market portfolio. As instruments he uses lagged yield spreads. The residuals in his VAR model are heteroskedastic but not serially correlated and he uses the White/score method to find that there are at least three latent factors. We have confirmed his results in a similar setup, using both the White and HAC robust score and RAD methods, the RAD test being robust to the choice and ordering of the reference assets.[23] Moreover, we repeated his analysis for the bond market separately, later adding the only stock market variable. In the case of the bond market, two-factor null hypothesis could not be rejected and, in the case of the joint model, three factors could not be rejected.[24] This result is consistent with our tests in the four-variable model, that is, several (more than three) factors are necessary to account for the cross-sectional and time-series patterns of stocks and bonds.

Our empirical work has been directed at counting the number of latent (macro) factors in asset returns, in the spirit of Chen, Roll, and Ross (1986). Of course, for a given number of latent factors, it is important to understand the nature of these factors

[23] Of course, the autocorrelation robustness is not really needed in this case.

[24] For the purpose of replicating Campbell's (1987) analysis, we extend our dataset by adding several Treasury securities. Details regarding this data and our results are available upon request.

and the plausible ways that they might cause events in financial markets. For this, we would need to extend considerably the empirical investigation, by reporting fitted values of coefficients in the latent factor models. We leave this important work to future research.

14 Summary

In this article, we propose tests for latent factors, or reduced rank, in multivariate linear models, in the case where model errors exhibit error serial correlation and heteroskedasticity of unknown form. We considered two types of tests, a version of Hansen's (1982) GMM test and a different, more user-friendly test called the RAD test. It would be interesting to extend the analysis to include other tests robust to error dynamics, including a general test of matrix rank proposed by Gill and Lewbel (1992). Their test, while perhaps less intuitive than the RAD test, may offer some computational advantages, although in simulations, we have not yet been able to show that either test is faster to compute than the other. We provide (Eviews) programs/macros for computing the RAD test, and a convenient stand-alone Windows program will soon be available from the first author. We encourage economists to apply the RAD test for latent factors, to many kinds of economic data (finance, macro, micro, international, etc.).

While we have tried hard to achieve extra robustness in our tests, in terms of error dynamics, we rely on asymptotic theory for our test significance levels and decision rules. In small samples, asymptotic significance levels may be poor approximations, and in that case bootstrap/simulation methods may be a useful substitute. Gilbert and Zemčík (2004) report some such simulations and, while we have not encountered serious test distortions in simulations fit to the sample sizes and models in the present work, we can produce big distortions by drastically reducing the sample size, or by drastically increasing the number of (y) variables in the model.

Appendix 1

The proposed algorithm for computing RAD test relies on two mathematical formulas, Equations 17 and 18 in the text. To derive these formulas, first note that, under the reduced-rank restriction H_0 in the text we have $\hat{\beta}' = \hat{\lambda}'\hat{\gamma}'$, and applying standard rules of linear algebra (Ruud 2000, p. 925),

$$vec(\hat{\lambda}'\hat{\gamma}') = (I_n \otimes \hat{\lambda}')\, vec(\hat{\gamma}') = (\hat{\gamma} \otimes I_K)\, vec(\hat{\lambda}').$$

To derive Equation 10, note that the desired $\hat{\gamma}$ is such that, given the fixed initial estimate 9 of λ, the first-order conditions for the quadratic problem (12) reduce to:

$$\frac{\partial\, (vec(\hat{\beta}') - vec(\hat{\lambda}'\hat{\gamma}'))'}{\partial\, vec(\gamma')} \hat{\Omega}^{-1}_{vec(\hat{\beta}')} (vec(\hat{\beta}') - vec(\hat{\lambda}'\hat{\gamma}')) \;=\; 0.$$

Using the first of the standard algebra rules stated above, we obtain:

$$\frac{\partial\, (vec(\hat{\beta}') - vec(\hat{\lambda}'\hat{\gamma}'))'}{\partial\, vec(\gamma')} = -(I_n \otimes \hat{\lambda}')',$$

and from these last two equations we obtain the Formula (17) in the text.

To derive (18), note that the desired $\hat{\lambda}$ is such that, given the fixed estimate $\hat{\gamma}$, the first order conditions for the quadratic problem (12) reduce to:

$$\frac{\partial\, (vec(\hat{\beta}') - vec(\hat{\lambda}'\hat{\gamma}'))'}{\partial\, vec(\lambda')} \hat{\Omega}^{-1}_{vec(\hat{\beta}')} (vec(\hat{\beta}') - vec(\hat{\lambda}'\hat{\gamma}')) \;=\; 0.$$

Applying the second of the standard algebra rules stated above, we obtain:

$$\frac{\partial\, (vec(\hat{\beta}') - vec(\hat{\lambda}'\hat{\gamma}'))'}{\partial\, vec(\lambda')} = -(\hat{\gamma} \otimes I_K)',$$

and from these last two equations we obtain the Formula (18) in the text.

Table 10

Summary Statistics

	r_{T90}	r_{T5}	r_{SMALL}	r_{LARGE}	r_{SP}	g_C	g_I	g_M	π
Mean	0.58	1.44	8.13	6.21	6.50	2.09	3.45	6.55	4.30
St. Dev.	1.21	18.69	66.54	50.49	50.85	5.56	10.47	5.56	3.92
Skewness	2.36	0.20	-0.17	-0.37	-0.36	-0.25	-0.10	0.17	0.92
Kurtosis	18.63	7.19	7.38	5.15	5.06	4.00	9.08	12.71	4.28
				Correlation					
r_{T90}	1.00	0.62	0.13	0.11	0.11	-0.02	-0.20	-0.05	-0.02
r_{T5}		1.00	0.16	0.25	0.24	0.01	-0.17	-0.06	-0.11
r_{SMALL}			1.00	0.85	0.82	0.21	-0.03	-0.04	-0.17
r_{LARGE}				1.00	0.99	0.15	-0.02	0.00	-0.18
r_{SP}					1.00	0.15	-0.02	0.01	-0.17
g_C						1.00	0.15	0.10	-0.45
g_I							1.00	0.02	-0.12
g_M								1.00	0.02
π									1.00

Sample 1959:02 2000:11, r_{T90} and r_{T5} are excess returns on 90-Day T-Bills and 5-Year T-Bonds, r_{SMALL} and r_{LARGE} denote the excess returns on the small-cap and large-cap portfolios, r_{SP} is the excess return on the market portfolio, g_I, g_C and g_M are growth rates of real per capita consumption, industrial production and money supply, respectively. π denotes the inflation rate. Reported numbers for means and standard deviations are annualized, in percentages.

Table 11

Bond Market

Panel A: Unrestricted Model

	int.	r_{SP}	g_C	g_I	g_M	$\Delta\pi$	R^2
r_{T90}	0.00060	0.00242	-0.00473	-0.02242	-0.01058	-0.02166	0.05807
	(0.00012)	(0.00182)	(0.01572)	(0.01069)	(0.01167)	(0.02428)	
r_{T5}	0.00276	0.08835	-0.00533	-0.29870	-0.21372	-0.07272	0.09216
	(0.00138)	(0.02322)	(0.21187)	(0.09078)	(0.16409)	(0.26758)	

The estimated model is: $y_{it} = \beta_i x_t + \varepsilon_{it}$, $i = 1, 2$, sample 1959:02-2000:11 where $y_{1t} = r_{T90}$, $y_{2t} = r_{T5}$, β_i is a (6×1) vector of coefficients, $x_t = (\text{int.}, r_{SP}, g_I, g_C, g_M, \Delta\pi)'$ and ε_{it} is the regression error. r_{T90} and r_{T5} are excess returns on 90-Day T-Bills and 5-Year T-Bonds, r_{SP} is the excess return on the market portfolio, g_I, g_C and g_M are growth rates of real per capita consumption, industrial production and money supply, respectively. $\Delta\pi$ is the first difference in the inflation rate. We report OLS coefficient estimates and p-values in parentheses are calculated using the VARHAC standard errors.

Table 11

Bond Market

Panel B: Tests for Residual Heteroscedasticity and Correlation

correlation	across equations		Pearson	0.60
				(0.02)
	across time	r_{T90}	Q	35.11
				(0.00)
		r_{T5}	Q	21.36
				(0.05)
heteroskedasticity		r_{T90}	White	8.15
				(0.00)
		r_{T5}	White	5.33
				(0.00)

Residuals are calculated using OLS estimates, equation by equation ; Pearson = chi-square test for correlation; White test = F test with no cross terms; Q = Q statistic for testing 12 lags of autocorrelation; p-values in parentheses.

Table 11

Bond Market

Panel C: Reduced Rank Tests

test	rank 1
Hansen	30.23
	(0.00)
generalized Wald	21.00
	(0.00)

Calculation conducted using the VARHAC covariance matrix estimator. p-values in parentheses.

Table 12

Stock Market

Panel A: Unrestricted Model

	int.	r_{SP}	g_C	g_I	g_M	$\Delta\pi$	R^2
r_{SMALL}	0.00342	1.05880	1.11352	-0.16953	-0.704334	-0.12976	0.68988
	(0.00222)	(0.05081)	(0.30487)	(0.17948)	(0.31882)	(0.46281)	
r_{LARGE}	0.00045	0.98534	-0.02430	-0.00470	-0.10313	-0.19138	0.98550
	(0.00088)	(0.00746)	(0.04867)	(0.01644)	(0.15291)	(0.09300)	

The estimated model is: $y_{it} = \beta_i x_t + \varepsilon_{it},\ i = 1, 2$, sample 1959:02-2000:11 where $y_{1t} = r_{SMALL}$, $y_{2t} = r_{LARGE}$, β_i is a (6×1) vector of coefficients, $x_t = ($int., r_{SP}, g_I, g_C, g_M, $\Delta\pi)'$ and ε_{it} is the regression error. r_{SMALL} and r_{LARGE} denote the excess returns on the small-cap and large-cap portfolios, r_{SP} is the excess return on the market portfolio, g_I, g_C and g_M are growth rates of real per capita consumption, industrial production and money supply, respectively. $\Delta\pi$ is the first difference in the inflation rate. We report OLS coefficient estimates and p-values in parentheses are calculated using the VARHAC standard errors.

Table 12

Stock Market

Panel B: Tests for Residual Heteroscedasticity and Correlation

correlation	across equations		Pearson	0.42
				(0.03)
	across time	r_{SMALL}	Q	70.17
				(0.00)
		r_{LARGE}	Q	109.45
				(0.00)
heteroskedasticity		r_{SMALL}	White	6.59
				(0.00)
		r_{LARGE}	White	12.11
				(0.00)

Residuals are calculated using OLS estimates, equation by equation ; Pearson = chi-square test for correlation; White test = F test with no cross terms; Q = Q statistic for testing 12 lags of autocorrelation; p-values in parentheses.

Table 12

Stock Market

Panel C: Reduced Rank Tests

test	rank 1
Hansen	15.77
	(0.01)
generalized Wald	18.24
	(0.00)

Calculation conducted using the VARHAC covariance matrix estimator. p-values in parentheses.

Table 13

Bond and Stock Markets Jointly

Panel A: Tests for Residual Correlation Across Equations

	r_{SMALL}	r_{LARGE}
r_{T90}	0.07	0.07
	(0.17)	(0.18)
r_{T5}	-0.08	0.06
	(0.16)	(0.20)

Residuals are calculated using OLS estimates from the unrestricted model (equation by equation): $y_{it} = \beta_i x_t + \varepsilon_{it},\ i = 1, \ldots, 4$, sample 1959:02-2000:11 where $y_{1t} = r_{T90}$, $y_{2t} = r_{T5}$, $y_{3t} = r_{SMALL}$, $y_{4t} = r_{LARGE}$, β_i is a (6×1) vector of coefficients, $x_t = (\text{int.}, r_{SP}, g_I, g_C, g_M, \Delta\pi)'$ and ε_{it} is the regression error. r_{T90} and r_{T5} are excess returns on 90-Day T-Bills and 5-Year T-Bonds, r_{SMALL} and r_{LARGE} denote the excess returns on the small-cap and large-cap portfolios, r_{SP} is the excess return on the market portfolio, g_I, g_C and g_M are growth rates of real per capita consumption, industrial production and money supply, respectively. $\Delta\pi$ is the first difference in the inflation rate. p-values in parentheses are calculated using the Pearson chi-square test for correlation.

Table 13

Bond and Stock Markets Jointly

Panel B: Reduced Rank Tests

test	rank 1	rank 2	rank 3
Hansen	87.68	31.85	15.92
	(0.00)	(0.00)	(0.00)
generalized Wald	166.60	27.59	16.84
	(0.00)	(0.00)	(0.00)

Calculation conducted using the VARHAC covariance matrix estimator. p-values in parentheses.

Part IV

Mean Reversion in Asset Returns and Time Non-separable Preferences

Time non-separable preferences are used in combination with various specifications of the endowment process to calibrate the Capital Asset Pricing Model (CAPM). Time non-separability is caused either by habit persistence or durability. It is demonstrated that the model can indeed produce the amount of mean reversion detected in historical returns. Specifically, habit persistence is required to match negative autocorrelation of annual asset returns and durability is needed to replicate positive autocorrelation detected in monthly asset returns. In addition, the CAPM with habit persistence can predict negative expected returns when calibrated to monthly data.

15 Introduction

Various studies of the U.S. stock market report evidence that equity returns display positive serial correlation at horizons shorter than one year and negative serial correlation at longer horizons (see Campbell, Lo, and MacKinlay 1997, Chapter 2 for a survey). Though autocorrelation of asset returns does not imply a violation of market efficiency, it does raise the question of whether the behavior of security markets can be explained by a rational expectations asset-pricing model. This paper argues that it can, provided that consumer preferences are time non-separable.

The hypothesis of serially independent returns (the random walk hypothesis) is often tested using the variance ratio test which, according to Poterba and Summers (1988), has a higher power than alternatives such as the likelihood-ratio test or the regression of current returns on lagged returns. Since the volatility of asset returns changes over time, it would be of no interest to reject the random walk model due to heteroskedasticity. Lo and MacKinlay (1988) thus derive the asymptotic distribution of variance ratios under the null hypothesis of random walk allowing for changing variances. Their specification test is applied here to both annual and monthly stock market returns and the random walk hypothesis is strongly rejected for all considered time horizons. As expected, returns with holding periods less than one year are positively autocorrelated and returns with longer holding periods are negatively autocorrelated.

Several studies make successful attempts to rationalize deviations of asset returns from random walk. Typically, they employ the Lucas (1978) Capital Asset Pricing Model (CAPM) with time separable, constant relative risk aversion utility function. Kandel and Stambaugh (1990) use the CAPM to replicate autocorrelations of equity returns as well as other unconditional moments. Their model is calibrated to the quarterly consumption growth rate, which is assumed to follow a four-state Markov switching process. Parameters of the Markov process are selected jointly with preference parameters to reflect various characteristics of the consumption and returns data. A similar approach is adopted in Cecchetti, Lam, and Mark (1990) who model the endowment process by a two-mean, one-variance Markov chain whose parameters are estimated using the consumption data only. Cecchetti *et al.* (1990) generate the distribution of variance ratios implied by the CAPM and then test the null hypothesis of the model being true using point estimates of variance ratios from historical returns. Comparison of variance ratios of model returns with variance ratios of historical returns demonstrates that historical returns could have in fact been generated by this type of a model. However, Bonomo and Garcia (1994) use variance ratios to show that the degree of mean reversion in Kandel and Stambaugh (1990) and in Cecchetti *et al.* (1990) is sensitive to the choice of the Markov switching model for the endowment process. They conclude that the CAPM cannot account for the magnitude of mean reversion observed in the data once the proper Markov specification for the endowment process is chosen, the proper specification being the two-state, one-mean, and two-variance Markov switching model for yearly data. Based on likelihood-ratio tests, this specification is considered superior to both the four-state process in Kandel and Stambaugh (1990) and to the two-state process with two means and one variance in Cecchetti *et al.* (1990). Bonomo and Garcia (1994) also show that the CAPM with time additive preferences is unable to generate negative excess returns.

In this paper, I employ the Markov switching model of Bonomo and Garcia (1994) to model endowment at annual frequency. For monthly data, a more general two-state, two-mean, and two-variance specification is used. In addition - following Cecchetti, Lam and Mark (1994) - a first-order autoregressive process is considered to investigate the robustness of results at both yearly and monthly frequencies. Since the Lucas CAPM does not provide any guidelines to distinguish among consumption, dividends and output, historical series on real consumption, dividends, and GNP growth rates are all respectively used to estimate parameters of the endowment process[25].

In an attempt to account for autocorrelation patterns found in the U.S. stock market, time non-separable preferences are introduced. Time non-separability can be brought in the CAPM by making current utility dependent on past consumption in two ways: either utility depends on aggregate consumption or on an individual's own consumption. We speak about *external habit* in the case of the former and about *internal habit* in the case of the latter. Campbell and Cochrane (1999) use a slow-moving external habit to replicate (among other things) the long-horizon forecastability of stock returns. In their study, dynamics in asset returns is produced by interaction between the surplus consumption ratio, which evolves as a heteroskedastic AR(1) and endowment, which is assumed to follow a random walk. The current study on the other hand focuses on the relatively more popular internal habit formulation where there is a basis for comparison to results based on estimation (see Ferson and Constantinides 1991, Heaton 1995, and Eichenbaum and Hansen 1990) and on volatility bounds (see Cecchetti *et al.* 1994 and Balduzzi and Kallal 1997). This formulation also allows one to investigate durability in addition to habit persistence.

[25] GNP data are not available at monthly frequency.

The exact specification of the preference structure is taken from Ferson and Constantinides (1991). The consumer values how much more she can consume today in comparison with how much she consumed yesterday. So, what matters is not the level of current consumption but the difference between current consumption and lagged consumption. If yesterday's consumption increases the agent's utility - one can think of a vacation or of a haircut - preferences display durability. If the lagged consumption lowers utility there is habit persistence. For instance, it is hard to go back to junk food after one has become accustomed to eating in good restaurants. There is an ongoing dispute of which effect dominates. The evidence from testing the overidentifying restrictions is somewhat ambiguous but seems to suggest that habit prevails in the long run and durability in the short run. Ferson and Constantinides (1991) use Generalized Method of Moments (GMM) to test the CAPM and conclude that the complementarity effect is strong for quarterly and annual data even if time averaging is accounted for. Heaton (1995) exploits a more complicated form of the utility function by adding more lags of consumption. He finds the first few coefficients on consumption are positive, and then the sign switches. Eichenbaum and Hansen (1990) use monthly data and GMM to show that local substitutability dominates. Testing the CAPM based on volatility bounds tends to favor habit persistence to durability (see Cecchetti *et al.* 1994 and Balduzzi and Kallal 1997). To investigate whether either of the two effects can generate mean reversion, several versions of the model are examined: strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability.

The CAPM with a time non-separability parameter is calibrated using estimated parameters of the corresponding endowment process. Then, the equilibrium returns are solved for. The solution method is based on discretization of the first order conditions using the Gaussian quadrature rule (see Tauchen and Hussey 1991) and enables one to calculate model variance ratios without the small sample bias characteristic to Monte-Carlo simulations. The results demonstrate that the amount of mean reversion in historical returns can be matched by the CAPM with time non-separable preferences for all considered endowment models, time series and data frequencies. Specifically, habit persistence generates negative and durability positive autocorrelation of model returns. Therefore, habit persistence is necessary to replicate negative serial correlation in yearly historical returns and durability is needed to reproduce positive serial correlation observed at monthly frequency. As established by Bonomo and Garcia (1994), time separable preferences do not imply mean reversion in model returns for a two-variance Markov switching model. Finally, the CAPM calibrated to monthly data can predict negative excepted returns when consumption is complementary over time.

The paper is organized as follows. Section 16 uses the asymptotic distribution of variance ratios derived in Lo and MacKinlay (1988) to show that asset returns do not follow random walk. In Section 17, parameters of the endowment processes are estimated. The estimates are used to calibrate the CAPM. Section 18 describes the CAPM with time non-separable preferences and indicates how equilibrium price-dividend ratios can be used to calculate model variance ratios and expected excess returns. Section 19 presents results and Section 20 concludes. Appendix 2 gives a detailed account of data sources and Appendix 3 demonstrates how the Gaussian quadrature method is employed to solve the CAPM with time non-separable utility function.

16 Mean Reversion in Historical Returns

The random walk hypothesis of asset returns has been tested extensively in the financial literature. The consensus is that asset returns tend to be positively serially correlated for horizons shorter than one year and negatively serially correlated for longer horizons. To test the random walk hypothesis for equity returns, I adopt the framework of Lo and MacKinlay (1988), who develop a specification test based on the asymptotic distribution of variance ratios that is robust to the presence of heteroskedasticity.

The variance ratio test exploits the fact that if the stock return follows a random walk, the return variance should be proportional to the return horizon. The variance ratio statistic is defined as

$$VR(q) = \frac{Var(R_t^q)}{qVar(R_t^1)} = 1 + \frac{2}{q}\sum_{j=1}^{q-1}(q-j)^{\rho_j}, q = 1, 2, ... \tag{21}$$

where R_t^q is the simple q-period return and ρ_j is the j-th serial correlation coefficient of returns. Alternatively, the variance ratio statistic for monthly data can be defined as $VR(q) = \frac{Var(R_t^q)}{q} / \frac{Var(R_t^{12})}{12}$ i.e. variances of simple returns are compared to the variation over a one-year period (e.g. see Poterba and Summers 1988). Campbell *et al.* (Chapter 2, 1997) argue that this approach might be problematic if the time horizon is large relative to the time period covered by the available data. Therefore, the formulation in (21) is employed.

The specification test in Lo and MacKinlay (1988)(see Section 1.2. of their paper) is designed to test the random walk hypothesis allowing for dependent but uncorrelated increments. The asymptotic distribution of the variance ratio statistic based on (21) is derived under a compound null hypothesis that imposes rather general restrictions on the type and degree of heteroskedasticity present. Under this null hypothesis, the variance ratio estimator still approaches unity asymptotically and importantly, estimators of autocorrelation coefficients in (21) are asymptotically uncorrelated. Consequently, estimates of their variances can be summed up using squared weights in (21) to estimate the variance of the variance ratio statistic. The variance then can be used for statistical inference.

Let $\widehat{VR(q)}$ and $\widehat{\vartheta(q)}$ denote the variance ratio estimator and the heteroskedasticity-consistent estimator of its variance, respectively[26] . The statistic $z(q) = \sqrt{Tq}(\widehat{VR(q)} - 1)/\sqrt{\widehat{\vartheta(q)}}$ is asymptotically standard normal. The estimates of variance ratios and the $z(q)$ statistics are computed using both annual and monthly data on real returns for the S&P Composite Index (see Appendix 2 for details).

Results for the yearly frequency are reported in Table 14. The variance ratios are

[26] See Lo and MacKinlay (1988), the second expression in (13) for the variance ratio estimator and equation (20) for its variance.

greater than one for the second period and they are lower than one from the third period on. Since the variance ratios can be expressed as a function of the autocorrelations, this means that real returns display the pattern of at first positive and then negative serial correlation. A variance ratio lower than one for the time periods beyond two years indicates very strong negative autocorrelation at long horizons. The autocorrelation has to be large in absolute terms to make up for the first period when the returns are positively serially correlated. So, long term returns are to some extent predictable. The variance ratios are slightly higher than those reported in Bonomo and Garcia (1994) since the sample bias is taken into account. Nevertheless, they are significantly different from one in all cases and the random walk hypothesis is strongly rejected.

Variance ratios calculated using monthly returns are displayed in Table 15. Since they are all significantly greater than one, the random walk hypothesis is again rejected in all cases. Variance ratios greater than one indicate positive serial correlation in monthly returns. Tables 14 and 15 confirm stylized facts regarding equity returns. The identified autocorrelation pattern stands as a challenge for the CAPM.

17 The Endowment Process

In equilibrium of a typical representative agent economy, the consumption stream equals the dividend stream. In addition, the output (perishable 'fruits') is equivalent to the dividend payment. Therefore, the following time series are considered for the empirical analysis: the real per capita consumption of non-durables and services[27], the dividend growth rate, and the real per capita GNP growth rate. Appendix 2 describes both annual

[27] Most studies based on monthly data employ consumption of non-durables and services to analyze the performance of the CAPM. Non-durable and services consumption can be used under the assumption that preferences over durables, and non-durables and services, are separable. Among the cited studies using lower frequency data, Cecchetti *et al.* (1990) and Bonomo and Garcia (1994) use total consumption, while Kandel and Stambaugh (1990) use consumption of non-durables and services. Hence, both types of consumption data are used in the case of yearly frequency.

and monthly series. Table 16 and 17 provide corresponding summary statistics. Since the data for output are not collected monthly, only series for consumption and dividends are used in the case of monthly frequency.

Bonomo and Garcia (1994) consider the following L-state Markov switching model for the endowment process:

$$x_t = \alpha_0 + \alpha_1 S_{1,t-1} + \cdots + \alpha_{L-1} S_{L-1,t-1} + (\omega_0 + \omega_1 S_{1,t-1} + \cdots + \omega_{L-1} S_{L-1,t-1})\epsilon_t, \quad (22)$$

where x_t is the natural logarithm of the endowment process and $S_{i,t} = 1$ if whenever the state of the economy is i and 0 otherwise. ϵ_t is an i.i.d. $N(0,1)$ error term.

Specification given by (22) encompasses the two-state Markov switching model with two means and one variance (2SMS2M1V) used in Cecchetti *et al.* (1990) as well as the four-state Markov switching model with two means and two variances (4SMS2M2V) employed in Kandel and Stambaugh (1990). Bonomo and Garcia (1994) use the likelihood-ratio test to reject the 2SMS2M1V model when the two-state Markov switching model with two means and two variances (2SMS2M2V) is used as an alternative. However, the 2SMS1M2V model cannot be rejected against the same alternative. In addition, the 2SMS1M2V model cannot be rejected neither against the alternative of the three-state, three-mean, and three-variance Markov switching model nor against the alternative of the 4SMS2M2V model. Therefore, for reasons of parsimony, Bonomo and Garcia (1994) adopt the 2SMS1M2V model as the model according to which the endowment growth rate evolves.

For the 2SMS1M2V model, $L = 2$ and $\alpha_1 = 0$. α_0 is both the conditional and unconditional mean of x_t. If $S_t = 0$, the conditional variance of x_t is ω_0^2 and $(\omega_0 + \omega_1)^2$ otherwise. The transpose of the transition matrix for the Markov process S is defined as follows: $P = \begin{pmatrix} p_{00} & (1-p_{00}) \\ (1-p_{11}) & p_{11} \end{pmatrix}$. As the notation suggests, p_{00} is the probability of remaining at the state 0 while p_{11} is the probability of remaining at the state 1. I replicate the maximum likelihood estimation[28] undertaken in Bonomo and Garcia (1994)

[28] The likelihood function for autoregressive processes in which the parameters of the autoregression

(see their Table I, p. 23) and report the results in Table 18[29].

For the monthly data, a more general 2SMS2M2V process is used to characterize the endowment processes for consumption and dividends. $\alpha_1 \neq 0$ allows for autocorrelation in the endowment process. Estimates of the parameters of the 2SMS2M2V process are summarized in Table 19. α_1 is significantly different from zero only for dividends.

Sometimes, an AR(1) model is also used to characterize the endowment process, especially for the monthly data frequency (see Cecchetti *et al.* 1994 for instance). Even though the AR(1) model does not capture heteroskedasticity implied by findings of Bonomo and Garcia (1994), I use it to evaluate the performance of the CAPM as well. At annual frequency, estimates of the autocorrelation coefficient by GMM are spurious and differ greatly depending on what instruments are used. Thus, OLS estimates are used instead. For monthly data, the GMM estimation is robust and GMM estimates are used to calibrate the CAPM. Even so, the AR(1) model is rejected for consumption using the Hansen (1982) J-statistic. The AR(1) process does not seem to capture the time series properties of endowment series very well and consequently, its implications will be only briefly mentioned in the text where they are different from results based on the Markov switching models[30]. An alternative to AR(1) could be a higher order autoregressive process. However, this is prohibitive due to restrictions imposed by the used solution method. Moreover, Cecchetti, *et al.* (1994) use monthly consumption data to show that coefficients on the second through twelfth lags of consumption growth are not significantly different from zero.

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can change as the result of a regime-shift variable is derived for example in Hamilton (1994), Chapter 22.

[29] Estimates are not identical to those of Bonomo and Garcia (1994) because their dataset is updated by two observations.

[30] A detailed description of results for the AR(1) model is available upon request.

18 The Asset Pricing Model with Time Non-Separability

In this section, a version of the Lucas (1978) tree model is presented where the utility at time t depends on the utility at time t-1. The consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, C_{t-1}),$$

subject to the budget constraint

$$C_t + P_t^E A_{t+1}^E + P_t^F A_{t+1}^F \leq (P_t^E + D_t) A_t^E + A_t^F,$$

where A_t^E, P_t^E, and D_t are the amount of risky assets (equity or 'trees') held, the market price of the risky asset, and the dividend, respectively. A_t^F and P_t^F are the investment in the risk-less asset and its price, respectively. C_t is consumption. The value of the utility function depends on both consumption at time t and t-1 and is assumed to have standard properties. β is the discount factor. Let Λ_t and M_t denote the Lagrange multiplier of the maximization problem connected with the budget constraint, and the Intertemporal Marginal Rate of Substitution ($IMRS$), respectively. Then it follows from the first order conditions that

$$M_{t+1} = \frac{\Lambda_{t+1}}{\Lambda_t} = \frac{\beta[U_1(C_{t+1}, C_t) + \beta E_{t+1} U_2(C_{t+2}, C_{t+1})]}{U_1(C_t, C_{t-1}) + \beta E_t U_2(C_{t+1}, C_t)}.$$

To make sure that the $IMRS$ is stationary and that there exists a representative consumer with the same preference specification over aggregate consumption, the following class of utility functions is adopted:

$$U(C_t, C_{t-1}) = \frac{(C_t + \delta C_{t-1})^{1-\gamma}}{(1-\gamma)}.$$

One can think of utility being derived from a good called *services* where services are linear in both current and past consumption. δC_{t-1} is the internal habit. The sign of δ determines whether the consumption is substitutable or complementary over time. If the consumption is substitutable then the utility from the flow of services from dominates the effect of habit persistence. On the other hand, if δ is negative the habit developed by the consumer is stronger than durability. The coefficient was estimated with different results. Eichenbaum and Hansen (1990) report a positive sign for monthly data while Ferson and Constantinides (1991) find evidence in monthly, quarterly, and annual data that habit persistence prevails. Tests based on volatility bounds also support negative δ (see Balduzzi and Kallal 1997 for a monthly frequency and Cecchetti *et al.* 1994 for monthly and annual frequencies). To investigate implications of both durability and habit persistence, performance of the model is evaluated for positive as well as negative values of δ. γ is approximately equal to the expected value of the Relative Risk Aversion (RRA) coefficient and if $\delta = 0$ then γ is exactly equal to the RRA coefficient.

Using the utility function specification, the $IMRS$ can be expressed as

$$M_{t+1} = \frac{\beta[(1+\delta X_{t+1}^{-1})^{-\gamma} + \beta\delta E_{t+1}(X_{t+2}+\delta)^{-\gamma}]}{(1+\delta X_t^{-1})^{-\gamma} + \beta\delta E_t(X_{t+1}+\delta)^{-\gamma}} X_{t+1}^{-\gamma}, \tag{23}$$

where $X_{t+1} = \frac{C_{t+1}}{C_t}$. The Euler equation for the risky asset

$$P_t^E = E_t M_{t+1}(P_{t+1}^E + D_{t+1})$$

can be written as

$$V_t = E_t M_{t+1} H_{t+1}(1 + V_{t+1}), \tag{24}$$

V_t is the price-dividend ratio and H_t is the gross growth rate of the dividend. The gross return on the risky asset is defined as

$$R_{t+1}^E = \frac{P_{t+1}^E + D_{t+1}}{P_t^E} = \frac{V_{t+1}+1}{V_t} H_{t+1}.$$

If one solves for autocorrelations of the model implied equity returns, variance ratios can be calculated.

The Euler equation for the risk-free asset is $P_t^F = E_t M_{t+1}$. The return on the risk-less asset can be written as

$$R_{t+1}^F = \frac{1}{P_t^F} = \frac{1}{E_t M_{t+1}}.$$

Bonomo and Garcia (1994) argue that the CAPM with time separable preferences cannot generate negative excess returns. To address the issue, conditional expected excess returns can be expressed as

$$E_t(R^E_{t+1} - R^F_{t+1}).$$

The presence of time non-separability makes the model more difficult to solve. To solve for the value function of the model, a version of the method described in Tauchen and Hussey (1991) is used. They develop a discrete space approximation to solutions of nonlinear asset pricing models which is based on the quadrature method (also known as Nystrom's method).

The solution method is described thoroughly in Appendix 3. Briefly, the conditional normal distribution from the continuous part of the 2SMS2M2V process x_t is approximated using the Gaussian N-point quadrature rule. The difference equation is discretized accordingly and solved for price-dividend ratios. Price-dividend ratios are used to calculate equity returns and their variance ratios. Finally, conditional expected excess returns are computed. The solution algorithm is easily modified for the AR(1) process.

19 Empirical Results

The CAPM can be calibrated using consumption (both total and of non-durables and services), dividends, and GNP as the endowment process. For the calibration to be complete, preference parameters have to be set as well. The considered parameter sets have the following structural interpretation: strong habit persistence, modest habit persistence, time separable preferences, modest durability, and strong durability. It is demonstrated that time non-separable preferences can indeed generate mean reversion of the degree observed in the data for all endowment processes. The negative autocorrelation detected in yearly frequency is matched when preferences exhibit modest habit persistence. The positive serial correlation in monthly returns is replicated by the CAPM with durability in utility function. The CAPM calibrated to monthly data can also produce negative expected returns but only for preferences displaying strong habit persistence.

The endowment parameters are α_0, α_1, p_{11}, p_{00}, ω_0, and ω_1. Their maximum likelihood estimates are given in Tables 18 and 19. By definition, $\alpha_1 = 0$ for the 2SMS1M2V process. Chosen values of the utility function parameters are in accordance with both Cecchetti, *et al.* (1990) and Bonomo and Garcia (1994) i.e. $\beta = 0.97$ and $\gamma = 1.70$[31]. In addition to the discount factor and the RRA coefficient, a time non-separability parameter δ is introduced to evaluate the impact of time non-separability in preferences on the (potential) autocorrelation of model returns. It can take values from -1 to 1. Negative δ generates negative values at some states of the discretized $IMRS$ process. The appearance of negative values of the $IMRS$ depends on the process used as the endowment as well as on the particular combination of parameters. δ for the large degree of habit persistence is the lowest value that implies non-negative values of the $IMRS$ given $\beta = 0.97$, $\gamma = 1.70$, and corresponding estimates of the parameters of the endowment process. δ is set to -0.07 for a small degree of habit persistence. For $\delta = 0$, preferences are time separable and results from this paper should be directly comparable to those of Bonomo and Garcia (1994). The small degree of durability is represented by $\delta = 0.07$. Finally, $\delta = 0.60$ for the large degree of durability. Model variance ratios are calculated using (21) and (30). Means and standard deviations are calculated using equations (27) and (28), respectively. Equity premiums are computed by taking the unconditional expectation of (31)[32].

Bonomo and Garcia (1994) argue that the CAPM cannot produce expected excess returns that are negative. Since they only consider a model with time separable preference, introduction of the time non-separability parameter δ can potentially render a model with expected returns being negative at some states for a favorable combination of parameters. The expected excess returns are calculated according to equation (31).

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[31] The value for the RRA coefficient corresponds to results of various empirical studies that report estimated values of γ between 1 and 2 for stocks (see for instance a maximum likelihood estimation of the RRA coefficient in Neely, Roy and Whiteman 1996).

[32] Equations (27)-(31) can be found in Appendix B.

19.1 Annual Data

Table 20 reports variance ratios, means, standard deviations, and equity premiums of the model returns for annual consumption data. The $IMRS$ becomes negative at the 16-th state for $\delta = -0.66$ in the case of total consumption and for $\delta = -0.67$ in the case of consumption of non-durables and services. Thus, values -0.65 and -0.66 are used, respectively. The degree of mean reversion as measured by variance ratios is low compared to that of the actual returns, especially for the consumption of non-durables and services. This is perhaps surprising, given the magnitude of habit persistence. Nonetheless, variance ratios for $\delta = -0.07$, which represents modest habit persistence, can in fact generate the mean reversion observed in the data for both consumption processes. For $\delta = 0$, the model has a structural interpretation of being time separable. The variance ratios are equal to unity in all the cases, which confirms results of Bonomo and Garcia (1994) who report variance ratios in the range from 0.9987 to 0.9852 for the time separable model with identical parameter values. Their variance ratios are slightly lower than one due to small sample bias since they are calculated for a sample size 1160 whereas the variance ratios reported here can be thought of as asymptotic values. For δ higher than one, the model displays durability and the variance ratios are greater than one.

For strong habit persistence, the model mean return is higher than the actual mean return and so is the standard deviation. The equity premium for the model returns is also relatively high, mainly due to negative risk-less returns at some states. A gradual increase of the time non-separability parameter implies a lower variation in the $IMRS$ and results in lower mean returns, standard deviations of returns and equity premiums, respectively.

For yearly dividends and GNP, the results are given in Table 21. The negative values of the $IMRS$ first appear at the 16-the state for $\delta = -0.47$ and $\delta = -0.55$, respectively. Therefore, $\delta = -0.46$ and $\delta = -0.54$ are used in calculation of the model returns for the CAPM displaying strong habit persistence. Mean reversion for the model with strong habit persistence is more pronounced compared to the case where consumption is used as the endowment process. Modest habit persistence again implies variance ratios with a pattern resembling that of the data i.e. variance ratios are closer to one for the first few periods and then they drop. The variance ratios again tend to increase with the increasing non-separability parameter δ and are greater than one for $\delta > 0$. Note that means and standard deviations of returns are slightly lower for dividends compared to means and standard deviations of returns resulting from using either consumption or GNP as the endowment process. Otherwise, the model mean returns, standard deviations, and equity premiums are again decreasing as δ increases.

Equilibrium variance ratios of returns for the same parameter combinations as in the case of the 2SMS1M2V model are calculated for the AR(1) model. Variance ratios lower than one appear already for the power utility model for both total consumption and consumption of non-durables and services. For GNP, strong habit persistence is necessary to match historical variance ratios. For dividends, equilibrium variance ratios are close to one or greater than one for all parameter combinations considered.

Table 22 reports the expected excess returns for all four endowment processes and the values of utility function parameters $\beta = 0.97$ and $\alpha = 1.70$. δ takes the lowest values admissible i.e. $\delta = -0.65$ for total consumption, $\delta = -0.66$ for consumption of non-durables and services, $\delta = -0.46$ for dividends, and $\delta = -0.54$ for GNP. There are 16 possible states of the economy: the first eight correspond to the lower and the other eight to the higher conditional standard deviation of the endowment process. As seen in Table 22, none of the expected excess returns is negative though the expected excess return in the 16-th state is close to zero in all the cases. The negative excess returns appear when $\delta = -0.66$ for consumption, $\delta = -0.67$ for consumption of non-durables and services, $\delta = -0.47$ for dividends, and $\delta = -0.55$ for GNP; that is only when the $IMRS$ is negative. The model calibrated using the AR(1) process is also unable of generating negative excess returns.

19.2 Monthly Data

Table 23 compares historical and model variance ratios for the calibration based on monthly data. The lowest acceptable time-nonseparability coefficient δ is -0.84 for consumption of non-durables and services and -0.77 for the dividends, respectively. The pattern of equilibrium variance ratios is similar to the one found in annual data i.e. they are lower than one for habit persistence, equal to unity for time separable preferences, and greater than one for durability. Contrary to findings in annual data, monthly returns are positively serially correlated, with variance ratios significantly greater than one. Consequently, one needs $\delta > 0$ to match model variance ratios with historical ones. Again, habit persistence in necessary to generate sufficiently large equity premium.

The AR(1) model for the consumption process implies variance ratios lower than one for both strong and modest degrees of habit persistence, time separability, and modest durability. $\delta = 0.60$ results in variance ratios greater than one. For dividends, variance ratios are lower than one only for $\delta = -0.77$ and greater than one otherwise.

Table 24 provides expected returns for strong habit persistence in both consumption and dividends. Interestingly, there are negative expected rates of returns at some states. The negative expected rates of return only appear for $\delta << 0$. When the endowment processes are modeled by AR(1), the CAPM does not generate negative expected rates of return for any parameter combination.

20 Summary

In this paper, I examine an equilibrium asset pricing model with time non-separable preferences from the prospective of its ability to match the magnitude of mean reversion detected in the data on asset returns.

The mean reversion in asset returns is documented using the variance ratio test. The null hypothesis is that of the random walk and is rejected for all holding periods considered. The variance ratios of long horizon returns (with the exception of the two-year variance ratio) imply negative autocorrelation and returns with holding periods between one and ten months are positively autocorrelated.

Two types of models for the endowment process are considered: the Markov switching model allowing for heteroskedasticity and the AR(1) model. At the annual frequency, parameters of the models are estimated using data on total consumption, consumption of non-durables and services, dividends, and GNP, respectively. Consumption of non-durables and services are utilized at the monthly frequency. Parameter estimates of the endowment process are employed together with utility function parameters to calibrate the CAPM. The model variance ratios and expected excess returns are then solved for.

Evidence regarding time separability is inconclusive since implications of the CAPM are sensitive to the choice of the endowment process. On the other hand, the results clearly indicate that there is a connection between time non-separability in preferences and mean reversion. A sufficient degree of habit persistence can produce negatively autocorrelated asset returns. Similarly, strong enough durability implies positively serially correlated returns. This result is robust across all endowment models, times series and frequencies considered. To match the pattern of at first positive and then negative serial correlation in historical returns, one needs a combination of local substitution and long run habit persistence. Heaton (1995) finds evidence that such a combination is also consistent with Hansen and Jagannathan (1991) bounds. So, the endowment process could be approximated by a higher order autoregressive model with the first few autoregressive coefficients positive and the others negative. This approach posits two problems however. First, it is difficult to solve the CAPM given the current framework and second, the autoregressive model might not be acceptable from the statistical point of view. For example, Cecchetti *et al.* (1994) rule out higher order autoregressive processes in favor of the AR(1) model using monthly consumption data.

Finally, the CAPM with consumption complementary over time is showed to generate negative conditional expected returns when calibrated to monthly data.

Appendix 2

Appendix 2.1. Annual Data

The annual data considered here are those used by Cecchetti at al. (1993) and by Bonomo and Garcia (1994). A detailed description of the data sources is given in Cecchetti at al. (1990). The data consist of the following series:

1. Consumption: The real per capita total consumption and consumption of non-durables and services, 1889-1987.

2. GNP: The real per capita GNP, 1869-1987.

3. CPI: Both the annual average and end of year observations from 1870 to 1987.

4. Dividends (D): The nominal dividends, 1871-1987, deflated by the annual average CPI.

5. Standard and Poor's Composite Stock Price Index (P): January observations, 1871-1988, adjusted to inflation by the end of period CPI.

6. Risk-free yield (R^F): The nominally risk-less yields on Treasury securities, 1871-1987. Adjusted to inflation by the end of period CPI.

The summary statistics for growth rates of consumption, dividends, and GNP are reported in Table 16. Real annual returns on equity are constructed using the series P and D as $R^E_{t+1} = \frac{P_{t+1}+D_t}{P_t}$. The mean equity premium is computed as $E[R^E_t - R^F_t]$.

Appendix 2.2. Monthly Data

The monthly data include the following series:

1. Consumption: The real per capita consumption of non-durables and services in 1987 dollars - CITIBASE series $(GMCSQ + GMCNQ)/POP$, 1959:02 1993:03.

2. Price Index: Calculated as $(GMCS + GMCN)/(GMCSQ + GMCNQ)$, where $GMCS$, $GMCN$, $GMCSQ$, $GMCNQ$ are respectively nominal consumption expenditures on services, nominal consumption expenditures on non-durables, real

consumption expenditures in 1987 dollars on services, and real consumption expenditures in 1987 dollars on non-durables, 1947:02 1993:03.

3. Standard and Poor's Composite Common Stock Price Index: CITIBASE series FSPCOM adjusted for inflation by the above price index, 1947:02 1993:03.

4. Risk-Free Rate: Monthly collected interest rate on the three-months Treasury Bills (CITIBASE series FYGM3) adjusted for inflation by the above price index, 1947:02 1993:03.

5. Dividends: Calculated using the dividend yield on Standard and Poor's Composite Common Stock (CITIBASE series FSDXP), Standard and Poor's Composite Common Stock Price Index, and the price index, both defined above, 1947:02 1993:03.

Table 17 provides summary statistics for monthly consumption and dividends. Real returns and mean equity premium are calculated in a manner similar to annual data.

Appendix 3

Appendix 3.1. Price-dividend Ratios

The first part of Appendix 3 derives price-dividend ratios implied by the joint hypothesis of the CAPM and the forcing process driving endowment.

Let us construct a Markov process for x_t with the number of states given by $2N$ and let x be a $(2N \times 1)$ vector of values corresponding to the $2N$ states i.e.,

$x = \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}.$

x^0 is an $(N \times 1)$ vector with elements:

$x_i^0 = \alpha_0 + \omega_0 \alpha_i, i = 1, 2, ...N,$

where α_i is the abscissa for an N-point quadrature rule for the standard normal density.[33] Similarly, x^1 is an $(N \times 1)$ vector with elements:

$x_i^1 = \alpha_0 + \alpha_1 + (\omega_0 + \omega_1)\alpha_1, i = 1, 2, ...N.$

The transpose of the transition matrix for x is:

$$T = \begin{pmatrix} p_{00}\Pi_{00} & (1 - p_{00})\Pi_{01} \\ (1 - p_{11})\Pi_{10} & p_{11}\Pi_{11} \end{pmatrix}. \tag{25}$$

x_t is normally distributed with the conditional mean v_t and the conditional variance σ_t^2. $v_t = \alpha_0$ for $S_{t-1} = 0$ and $v_t = \alpha_0 + \alpha_1$, otherwise. $\sigma_t^2 = \omega_0^2$ for $S_{t-1} = 0$ and $\sigma_t^2 = (\omega_0 + \omega_1)^2$, otherwise. Let us define $z = (x_t - v_t)/\sigma_t$. Since z is a random variable with the standard normal density, we can write the conditional probability density function $f(x_t \mid x_{t-1})$ as $\phi(z)/\sigma_t$, where $\phi()$ denotes the standard normal density function. Also, the cumulative density function $F(x_t = y \mid x_{t-1}) = \int_{-\infty}^{y} \frac{f(x_t \mid x_{t-1})}{\sigma_t} dx_t = \int_{-\infty}^{\frac{y-v_t}{\sigma_t}} \phi(z)dz = \Phi(\frac{y - v_t}{\sigma_t})$, where $\Phi()$ denotes the standard normal cumulative density function. So, the conditional mean of x_t does not depend on x_{t-1} and $\Pi_{00} = \Pi_{01} = \Pi_{10} = \Pi_{11} = \Pi$, where

[33] As N increases, the approximate solution converges to the exact solution uniformly. In most applications, accuracy does not increase much beyond $N = 5$. I use $N = 8$, which is a compromise between desired precision and computational tractability.

$\Pi_{ij} = \omega_j, i, j = 1, 2, ...N.$

ω_j are the weights of an N-point quadrature rule for the standard normal density. Note that the IMRS (see Eq. (23)) can be written as:

$M_{t+1} = \frac{\beta + \beta^2 \delta E_{t+1} B_{t+2}}{1 + \beta \delta E_t B_{t+1}} B_{t+1},$

where $B_{t+1} = (\frac{\delta + X_{t+1}}{\delta + X_t} X_{t+1})^{-\gamma}.$

Let us define elements of a $(2N \times 2N)$ matrix B as

$B_{ij} = (\frac{\delta + e^{x_j}}{\delta + e^{x_i}} e^{x_i})^{-\gamma}, i, j = 1, 2, ..., 2N.$

B can be used to discretize the IMRS by defining a $(2N \times 2N)$ matrix M with elements:

$M_{ij} = \frac{\beta + \beta^2 \delta E[B_{ij} \mid j]}{1 + \beta \delta E[B_{ij} \mid i]} B_{ij}, i, j = 1, 2, ..., 2N.$

Using Eq. (25), $E[B_{ij} \mid j] = \sum_{j=1}^{2N} B_{ij} T_{ij}$. Finally, the Euler equation (Eq. (24)) can be discretized as well:

$v = K\iota + Kv,$

where v is a $(2N \times 1)$ vector of price-dividend ratios and ι is a $(2N \times 1)$ vector of ones. Elements of the $(2N \times 2N)$ matrix K are defined as:

$K_{ij} = M_{ij} e^{x_j} T_{ij}, i, j = 1, 2, ..., 2N.$

Solving for v,one gets:

$v = (I - K)^{-1} K\iota,$

where I is the $(2N \times 2N)$ identity matrix.

Appendix 3.2. Model Returns

The tomorrow's return to the equity conditioned on today's state is

$$R_{ij}^E = \frac{P_j^E + D_j}{P_i^E} = \frac{v_j + 1}{v_i} e^{x_j}, \quad i, j = 1, ..., 2N^{34}. \tag{26}$$

The return is implied by the model calibrated to the process of the growth rate of endowment will be used for the derivation of the model variance ratios.

In Appendix 3.1, the endowment growth rate is approximated by a Markov chain with $2N$ states where the transition probabilities are given by T. The equilibrium real

[34] Note that when $N = 8$, there are 256 values for the rate of return.

return at time t depends on the endowment growth rates at times t and t-1 and is given by (26). Thus, a Markov chain for the returns can be constructed where the number of states is $4N^2$. Using the transition matrix of the equilibrium returns one is able to compute autocorrelations of those returns, and consequently, the variance ratios implied by the model.

The transpose of the transition matrix for the model returns is

$$
\boldsymbol{Q} = \begin{pmatrix}
T_{1,1}, & T_{1,2}, & ... & T_{1,2N}, & 0, & 0, & ... & 0, & 0, & 0, & ... & 0 \\
0, & 0, & ... & 0, & T_{2,1}, & T_{2,2}, & ... & T_{2,2N}, & 0, & 0, & ... & 0 \\
 & & ... & & & & ... & & & & ... & \\
0, & 0, & ... & 0, & 0, & 0, & ... & 0, & T_{2N,1}, & T_{2N,2}, & ... & T_{2N,2N} \\
T_{1,1}, & T_{1,2}, & ... & T_{1,2N}, & 0, & 0, & ... & 0, & 0, & 0, & ... & 0 \\
0, & 0, & ... & 0, & T_{2,1}, & T_{2,2}, & ... & T_{2,2N}, & 0, & 0, & ... & 0 \\
 & & ... & & & & ... & & & & ... & \\
0, & 0, & ... & 0, & 0, & 0, & ... & 0, & T_{2N,1}, & T_{2N,2}, & ... & T_{2N,2N} \\
 & & ... & & & & ... & & & & ... & \\
T_{1,1}, & T_{1,2}, & ... & T_{1,2N}, & 0, & 0, & ... & 0, & 0, & 0, & ... & 0 \\
0, & 0, & ... & 0, & T_{2,1}, & T_{2,2}, & ... & T_{2,2N}, & 0, & 0, & ... & 0 \\
 & & ... & & & & ... & & & & ... & \\
0, & 0, & ... & 0, & 0, & 0, & ... & 0, & T_{2N,1}, & T_{2N,2}, & ... & T_{2N,2N}
\end{pmatrix}
$$

Let $\boldsymbol{\psi}$ denote the $(4N^2 \times 1)$ vector of unconditional probabilities of the returns. The following procedure delivers the unconditional expected value of the product of today's and lagged returns:

(i) Compute the unconditional expected value of returns by

$$E[R_t] = \boldsymbol{\psi}' R = \kappa, \tag{27}$$

where R is the $(4N^2 \times 1)$ vector of possible values of the returns and κ is the expected value;

(ii) compute the variance of returns by

$$Var[R_t] = (R.R) - \kappa^2 = \eta^2; \tag{28}$$

(iii) get the unconditional expected value of the product of the today's and lagged return:

$$E[R_{t+s}R_t] = (R.\boldsymbol{\psi})' \boldsymbol{Q}^s R. \tag{29}$$

Equilibrium values of the variance ratios are then computed using (21) and

$$\rho_s = \frac{E[R_{t+s}R_t] - \kappa^2}{\eta^2}. \tag{30}$$

The expected excess returns can be computed using the transition matrix T (Eq. (26)) and the risk-free returns. The risk-free return is simply one over the price of the risk-free asset and can be expressed as

$$R_i^F = \frac{1}{\sum_{j=1}^{2N} T_{ij} M_{ij}}, i = 1, 2, \ldots, 2N.$$

The expected excess returns then are:

$$E[R_i^E - R_i^F | i] = \sum_{j=1}^{2N} T_{ij}(R_{ij}^E - R_i^F). \tag{31}$$

Table 14

Variance Ratios for Historical Returns; Yearly Data 1870-1987

q	VR(q)	z(q)
2	1.0275	2.9952
3	0.8891	-7.9440
4	0.8923	-6.0742
5	0.8760	-5.9204
6	0.8205	-7.5561
7	0.7918	-7.9245
8	0.8013	-6.9658
9	0.7928	-6.7778
10	0.7705	-7.0959

The random walk hypothesis allowing for heteroskedasticity is rejected in all cases at 1% level.

Table 15

Variance Ratios for Historical Returns; Monthly Data 1947:02 1994:03

q	VR(q)	z(q)
2	1.2652	111.4259
3	1.3629	106.7755
4	1.4248	103.2105
5	1.4902	104.2021
6	1.5669	108.5213
7	1.6150	107.9693
8	1.6339	103.3748
9	1.6491	99.4246
10	1.6636	96.0809

The random walk hypothesis allowing for heteroskedasticity is rejected in all cases at 1% level.

Table 16

Summary Statistics for Growth Rates in Sample; Yearly data

	Total Consumption	Consumption of Non-durables and Services	Dividends	GNP
Time Period	1890-1987	1890-1987	1872-1987	1870-1987
Obs.	98	98	116	118
Mean	0.0182	0.0172	0.0112	0.0178
St.Dev.	0.0374	0.0342	0.1262	0.0514
Skewness	-0.4097	-0.4045	-0.8228	-0.7574
Kurtosis	3.8750	3.9773	6.3321	7.6627
Maximum	0.0990	0.0994	0.4168	0.1613
Minimum	-0.0987	-0.0874	-0.4314	-0.2216
First Autocor.	-0.0679	-0.1343	0.2089	0.3908

Table 17

Summary Statistics for Growth Rates in Sample; Monthly Data

	Consumption	Dividends
Time Period	1959:02 1993:03	1947:02 1993:03
Obs.	410	554
Mean	0.00159	0.000768
St.Dev.	0.00394	0.005666
Skewness	0.0195	1.73730
Kurtosis	3.5174	16.72803
Maximum	0.01598	0.03945
Minimum	-0.010795	-0.0341
First Autocor.	-0.2442	0.1992

Table 18

Maximum Likelihood Estimates of the 2SMS1M2V Process, Yearly Data

	Total Consumption	Consumption of Non-durables and Services	Dividends	GNP
α_0	0.0197	0.0187	0.0144	0.0179
	(8.087)	(10.416)	(2.304)	(5.701)
p_{11}	0.9897	0.9885	0.8193	0.9281
	(3.742)	(3.500)	(1.746)	(2.707)
p_{00}	0.9874	0.9854	0.8165	0.9834
	(3.338)	(3.086)	(2.228)	(3.966)
ω_0	0.0165	0.0113	0.0381	0.0303
	(8.714)	(8.436)	(7.569)	(10.913)
ω_1	0.0299	0.0315	0.1350	0.0698
	(6.328)	(7.523)	(6.922)	(4.161)

Asymptotic t-ratios in parentheses. For $p_{ii}, i = 0, 1$, the reported t-ratios are those of the transformation $ln(p_{ii}/(1 - p_{ii})), i = 0, 1$, respectively. The transformation was employed to restrict probability estimates to the interval $(0, 1)$.

Table 19

Maximum Likelihood Estimates of the 2SMS2M2V Process; Monthly Data

	Consumption	Dividends
α_0	0.0015	0
	(5.940)	(0.180)
α_1	0.0003	0.007
	(0.331)	(3.237)
p_{11}	0.5377	0.6037
	(0.139)	(0.898)
p_{00}	0.8483	0.9516
	(1.216)	(7.712)
ω_0	0.0034	0.0033
	(8.588)	(19.030)
ω_1	0.0020	0.0095
	(2.085)	(6.858)

Asymptotic t-ratios in parentheses. For $p_{ii}, i = 0, 1$, the reported t-ratios are those of the transformation $ln(p_{ii}/(1 - p_{ii})), i = 0, 1$, respectively. The transformation was employed to restrict probability estimates to the interval $(0, 1)$.

Table 20

Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Total Consumption and to Consumption of Non-durables and Services, the 2SMS1M2V Process, Yearly Data

Total Consumption

	Actual	$\delta = -0.65$	$\delta = -0.07$	$\delta = 0$	$\delta = 0.07$	$\delta = 0.60$
VR(2)	1.0275	0.9100	0.8831	1.0001	1.1120	1.4576
VR(3)	0.8891	0.8835	0.8442	1.0001	1.1493	1.6101
VR(4)	0.8923	0.8729	0.8248	1.0002	1.1680	1.6864
VR(5)	0.8760	0.8685	0.8132	1.0003	1.1792	1.7322
VR(6)	0.8205	0.8672	0.8055	1.0003	1.1867	1.7627
VR(7)	0.7918	0.8677	0.8000	1.0004	1.1921	1.7845
VR(8)	0.8013	0.8692	0.7959	1.0005	1.1961	1.8009
VR(9)	0.7928	0.8715	0.7928	1.0005	1.1993	1.8136
VR(10)	0.7705	0.8741	0.7903	1.0006	1.2018	1.8238
mean	0.0818	0.1912	0.0666	0.0664	0.0663	0.0661
st.dev.	0.1871	1.2891	0.0439	0.0386	0.0350	0.0284
eq. premium	0.0529	0.1459	0.0029	0.0024	0.0020	0.0011

Table 20

Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Total Consumption and to Consumption of Non-durables and Services, the 2SMS1M2V Process, Yearly Data

Consumption of Non-durables and Services

	Actual	$\delta = -0.66$	$\delta = -0.07$	$\delta = 0$	$\delta = 0.07$	$\delta = 0.60$
VR(2)	1.0275	0.9651	0.8830	1.0001	1.1121	1.4577
VR(3)	0.8891	0.9550	0.8440	1.0001	1.1495	1.6103
VR(4)	0.8923	0.9511	0.8246	1.0002	1.1682	1.6866
VR(5)	0.8760	0.9496	0.8130	1.0003	1.1794	1.7324
VR(6)	0.8205	0.9494	0.8053	1.0003	1.1869	1.7629
VR(7)	0.7918	0.9498	0.7998	1.0004	1.1923	1.7847
VR(8)	0.8013	0.9506	0.7957	1.0005	1.1964	1.8011
VR(9)	0.7928	0.9517	0.7926	1.0005	1.1995	1.8138
VR(10)	0.7705	0.9530	0.7901	1.0006	1.2020	1.8240
mean	0.0818	0.1904	0.0647	0.0645	0.0644	0.0643
st.dev.	0.1871	2.0772	0.0399	0.0351	0.0318	0.0257
eq. premium	0.0529	0.1444	0.0024	0.0020	0.0017	0.0009

$\beta = 0.97$ and $\gamma = 1.70$; values of δ represent respectively strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

Table 21

Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Dividends and to GNP, the 2SMS1M2V Process, Yearly Data

	Dividends					
	Actual	$\delta=-0.46$	$\delta=-0.07$	$\delta=0$	$\delta=0.07$	$\delta=0.60$
VR(2)	1.0275	0.8611	0.8866	1.0013	1.1100	1.4484
VR(3)	0.8891	0.8219	0.8496	1.0022	1.1471	1.5980
VR(4)	0.8923	0.8057	0.8314	1.0030	1.1658	1.6729
VR(5)	0.8760	0.7977	0.8208	1.0035	1.1771	1.7179
VR(6)	0.8205	0.7933	0.8137	1.0040	1.1847	1.7479
VR(7)	0.7918	0.7906	0.8088	1.0043	1.1902	1.7694
VR(8)	0.8013	0.7889	0.8051	1.0046	1.1943	1.7855
VR(9)	0.7928	0.7878	0.8023	1.0049	1.1975	1.7980
VR(10)	0.7705	0.7869	0.8000	1.0051	1.2000	1.8080
mean	0.0818	0.3255	0.0632	0.0608	0.0593	0.0570
st.dev.	0.1871	1.5981	0.1552	0.1359	0.1231	0.0987
eq. premium	0.0529	0.3886	0.0346	0.0282	0.0238	0.0133

Table 21

Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Dividends and to GNP, the 2SMS1M2V Process, Yearly Data

	GNP					
	Actual	$\delta = -0.54$	$\delta = -0.07$	$\delta = 0$	$\delta = 0.07$	$\delta = 0.60$
VR(2)	1.0275	0.7406	0.8845	1.0006	1.1115	1.4541
VR(3)	0.8891	0.6755	0.8466	1.0013	1.1489	1.6055
VR(4)	0.8923	0.6576	0.8280	1.0018	1.1679	1.6813
VR(5)	0.8760	0.6576	0.8172	1.0024	1.1793	1.7268
VR(6)	0.8205	0.6657	0.8102	1.0029	1.1871	1.7571
VR(7)	0.7918	0.6778	0.8054	1.0034	1.1927	1.7789
VR(8)	0.8013	0.6920	0.8019	1.0038	1.1970	1.7952
VR(9)	0.7928	0.7071	0.7993	1.0042	1.2004	1.8079
VR(10)	0.7705	0.7225	0.7973	1.0046	1.2031	1.8180
mean	0.0818	0.1335	0.0639	0.0635	0.0633	0.0629
st.dev.	0.1871	0.5558	0.0624	0.0548	0.0498	0.0402
eq. premium	0.0529	0.0946	0.0058	0.0047	0.0040	0.0022

$\beta = 0.97$ and $\gamma = 1.70$; values of δ represent respectively strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

Table 22

Equilibrium Expected Excess Returns, the 2SMS1M2V Process, Yearly Data

State	Total Consumption	Consumption of Non-durables and Services	Dividends	GNP
	$\delta = -0.65$	$\delta = -0.66$	$\delta = -0.46$	$\delta = -0.54$
1	0.0429	0.0226	0.2465	0.0540
2	0.0366	0.0201	0.2125	0.0457
3	0.0318	0.0181	0.1878	0.0396
4	0.0277	0.0164	0.1679	0.0347
5	0.0242	0.0148	0.1508	0.0305
6	0.0209	0.0134	0.1356	0.0268
7	0.0178	0.0119	0.1214	0.0233
8	0.0145	0.0103	0.1071	0.0198
9	0.9887	0.9584	34.5342	2.8414
10	0.6129	0.6061	3.1920	1.1976
11	0.4124	0.4124	1.2929	0.6751
12	0.2819	0.2840	0.6835	0.4166
13	0.1878	0.1901	0.3906	0.2611
14	0.1151	0.1167	0.2193	0.1560
15	0.0554	0.0559	0.1060	0.0785
16	0.0023	0.0013	0.0223	0.0156

$\beta = 0.97$ and $\gamma = 1.70$; values of δ represent strong habit persistence.

Table 23

Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Consumption and to Dividends, the 2SMS2M2V Process, Monthly Data

Consumption

	Actual	$\delta=-0.84$	$\delta=-0.07$	$\delta=0$	$\delta=0.07$	$\delta=0.60$
VR(2)	1.2652	0.6113	0.8808	1.0000	1.1141	1.4600
VR(3)	1.3629	0.4820	0.8411	1.0000	1.1522	1.6134
VR(4)	1.4248	0.4174	0.8212	1.0000	1.1712	1.6900
VR(5)	1.4902	0.3787	0.8093	1.0000	1.1826	1.7360
VR(6)	1.5669	0.3529	0.8013	1.0000	1.1902	1.7667
VR(7)	1.6150	0.3344	0.7957	1.0000	1.1956	1.7886
VR(8)	1.6339	0.3206	0.7914	1.0000	1.1997	1.8050
VR(9)	1.6491	0.3099	0.7881	1.0000	1.2029	1.8178
VR(10)	1.6636	0.3013	0.7854	1.0000	1.2054	1.8280
mean	0.006759	0.1073	0.0339	0.0339	0.0339	0.0339
st.dev.	0.03431	0.4575	0.0047	0.0041	0.0037	0.0030
eq. premium	0.002612	0.0751	0.0000	0.0000	0.0000	0.0000

Table 23

Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Consumption and to Dividends, the 2SMS2M2V Process, Monthly Data

Dividends

	Actual	$\delta = -0.77$	$\delta = -0.07$	$\delta = 0$	$\delta = 0.07$	$\delta = 0.60$
VR(2)	1.2652	0.6916	0.8807	1.0000	1.1143	1.4601
VR(3)	1.3629	0.5928	0.8409	1.0000	1.1524	1.6135
VR(4)	1.4248	0.5450	0.8210	1.0000	1.1714	1.6901
VR(5)	1.4902	0.5170	0.8091	1.0000	1.1828	1.7361
VR(6)	1.5669	0.4986	0.8011	1.0000	1.1905	1.7668
VR(7)	1.6150	0.4856	0.7954	1.0000	1.1959	1.7887
VR(8)	1.6339	0.4759	0.7912	1.0000	1.2000	1.8052
VR(9)	1.6491	0.4684	0.7879	1.0000	1.2032	1.8179
VR(10)	1.6636	0.4624	0.7852	1.0000	1.2057	1.8282
mean	0.006759	0.0483	0.0313	0.0313	0.0313	0.0313
st.dev.	0.03431	0.2407	0.0067	0.0059	0.0053	0.0043
eq. premium	0.002612	0.0182	0.0001	0.0001	0.0000	0.0000

$\beta = 0.97$ and $\gamma = 1.70$; values of δ represent respectively strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

Table 24

Equilibrium Expected Excess Returns, the 2SMS1M2V Process, Monthly Data

State	Consumption	Dividends
	$\delta = -0.84$	$\delta = -0.77$
1	0.8598	0.3498
2	0.5826	0.2443
3	0.3577	0.1563
4	0.1583	0.0764
5	-0.0269	0.0006
6	-0.2054	-0.0740
7	-0.3846	-0.1505
8	-0.5787	-0.2353
9	1.4583	1.6300
10	0.9560	1.0240
11	0.5659	0.5834
12	0.2327	0.2267
13	-0.0666	-0.0790
14	-0.3458	-0.3523
15	-0.6176	-0.6078
16	-0.9026	-0.8647

$\beta = 0.97$ and $\gamma = 1.70$; values of δ represent strong habit persistence

Part V

An Empirical Investigation of the Consumption Based Capital Asset Pricing Model Using a Modified Variance-Ratio Test

A chi-square statistic is constructed that compares variance ratios and mean simple returns from data with those implied by an asset pricing model. The statistic is applied to the Consumption based Capital Asset Pricing Model with time non-separable preferences. It favors habit persistence for annual data, time-separability for quarterly data, and durability for monthly data, respectively. Introduction of time non-separability yields only a marginal improvement. The power of the test is high when alternative hypotheses are formed by varying the relative risk aversion coefficient. It is lower for alternative hypotheses generated by varying the time non-separability parameter, especially for durability.

21 Introduction

The statistical test often used to investigate whether asset returns follow a random walk is based on variance ratios of returns (see Campbell. Lo, and MacKinlay 1997, Chapter 2, for a survey). Variance ratios capture the autocorrelation structure of asset returns and though their statistical power to detect mean reversion is rather low, it is still higher than for alternatives such as the likelihood-ratio test or simple regressions of current returns on lagged returns (see Poterba and Summers 1988). The test developed in this paper is based on variance ratios and mean simple returns. The presence of mean simple returns ensures that the level of returns is captured together with their autocorrelation structure, I avoid the necessity of choosing the variance ratio with the highest power (see Faust 1992) by constructing a chi-square statistic, which is a weighted sum of squared deviations from estimated and hypothesized variance ratios and mean simple returns for several time periods. The chi-square test statistic can be used to test any model of asset prices including the Consumption based Capital Asset Pricing Model (CCAPM).

In the standard CCAPM, the representative agent maximizes expected utility where preferences are defined by time-separable iso-elastic functions of the flow of nondurable goods and services. Cecchetti, Lam, and Mark (1990) compare variance ratios of asset returns implied by the CCAPM with historical variance ratios. They find that the model can match the pattern observed in the U.S. data. Bonomo and Garcia (1994), on the other hand, show that the CCAPM can produce serial correlation in equilibrium returns only if its endowment process is misspecified. Zemcik (2001) uses a proper specification of the endowment process together with time non-separable preferences to demonstrate that the CCAPM can in fact generate autocorrelation in asset returns. Time non-separability in Zemcik (2000) is introduced by adopting the internal habit formation where the current utility depends on an individual's past consumption. Preferences are thus defined by three parameters: the discount factor, the time non-separability parameter, and the relative risk aversion (RRA) coefficient. An identical preference specification is utilized in this paper.

Following Bonomo and Garcia (1994), the consumption in the model is assumed to follow a two-state, one-mean, and two-variance Markov switching process. The parameters of the consumption process are estimated by the method of maximum likelihood using the U.S. consumption of nondurables and services at annual, quarterly, and monthly frequencies. The CCAPM is calibrated to various combinations of the parameters and solved for. The equilibrium returns are then compared to the U.S. equity returns using the proposed chi-square statistic. The highest p-values are recorded with negative time non-separability parameter (habit persistence) for annual data, a zero time non-separability parameter (time separability) for quarterly data, and a positive time non-separability parameter (durability) for monthly data. However, the modified variance-ratio test cannot reject time-separability at any data frequency, though p-values are lower compared to time non-separable preferences. This result is in accord with recent evidence based on Bayesian comparison of various preference specifications in Gordon and Samson (1999). They conclude that extensions of the standard power utility function do not yield a demonstrable improvement.

A standard approach to test asset pricing models is to estimate parameters of a model by minimizing the distance between various moments of the model returns with their sample analogs. The empirical adequacy of the model then can be tested using the fact that the objective function evaluated in its minimum is chi-square distributed. See Hansen and Singleton (1982) and Ferson and Constantinides (1991) for examples of this methodology. More recently, Lee and lngram (1991), Duffie and Singleton (1993), and Heaton (1995) discuss and/or apply an estimation procedure which accounts for cases when the moments of model returns or the stochastic discount factor are simulated. Since Ferson and Constantinides (1991) and Heaton (1995) both estimate the CCAPM with time non-separable preferences, their results are directly comparable to those of the calibration exercise presented here. Ferson and Constantinides (1991) estimate the CCAPM at yearly, quarterly, and monthly frequencies, and Heaton (1995) uses monthly data. While findings based on annual and monthly data frequencies are roughly consistent with results presented here, there are differences at the quarterly frequency. Estimates of the time non-separability parameter in Ferson and Constantinides (1991) are strongly negative, whereas the modified variance-ratio test used in this paper favors time-separability.

The chi-square test statistic constructed here is similar in spirit to other tests based on distance from sample moments of asset returns. Hung (1994) offers a statistic which takes into account uncertainty from point estimates of means of the risk-free rate and the equity premium as well as their mutual correlation. Burnside (1994) and Cecchetti, Lam, and Mark (1994) devise tests based on volatility bounds. Specifically, even though the restriction imposed by Hansen and Jagannathan (1991) may be violated, the distance between the minimum variance bound and the variance of a pricing kernel does not have to be statistically significant.[35] Volatility bound tests tend to favor habit persistence for all data frequencies, which is due to the increased severity of the equity premium puzzle when durability is present. Hung (1994), Burnside (1994), and Cecchetti et al. (1994) all focus on the cross-sectional characteristics of asset returns; on the other hand, the test presented in this paper concentrates on the autocorrelation pattern of a single asset. In this context, durability is needed for the CCAPM to match positive serial correlation in historical monthly returns.

To evaluate information in accepting or rejecting the CCAPM, the power of the modified variance-ratio test is examined given the null hypothesis of the CCAPM being the true underlying model. The endowment process parameters are set equal to their maximum-likelihood estimates for each data frequency. The preference parameters of the CCAPM are set equal to parameter combinations with highest p-values. The asymptotic distribution of the above mentioned test statistic is chi-square. To avoid potential issues with a small-sample bias, I also compute the empirical distribution of the statistic. Critical values from both the empirical and asymptotic distributions are then used for computing the power against alternative hypotheses. Various versions of the alternative hypothesis are formed by varying the time non-separability parameter and relative risk aversion coefficient, respectively. Power is calculated using a sequence of time series of model asset returns that is generated under a given alternative hypothesis. The power is close to one for most alternative parameter combinations, which are far enough from the combinations with highest p-values. The only exception is a region with a positive non-separability parameter. This may be attributed to the fact that according to the statistic used, these parameter combinations cannot be rejected using the data on the

[35] For recent development regarding volatility bounds, see Hansen, Heaton, and Luttmer (1995); Hansen and Jagannathan (1997), and Balduzzi and Kallal (1997).

U.S. stock returns and consumption growth.

The paper is organized as follows. Section 22 derives the chi-square statistic based on variance ratios and mean simple returns. Section 23 discusses the CCAPM with time dependent preferences; Section 24 solves for the equilibrium returns implied by the model; the implied process for returns is used to evaluate the chi-square statistic for various parameter combinations in Section 25 The power of the test is investigated in Section 26 Section 27 concludes. The used data set is described in Appendix 4.

22 The Modified Variance-Ratio Test

This section derives a test of asset pricing models based on estimates of variance ratios and mean simple returns. The random walk hypothesis for asset returns is frequently tested using the standard variance-ratio test (see Campbell et al. 1997, Chapter 2, for a survey). A typical approach is to derive an asymptotic distribution of variance ratios under the null hypothesis and then test the hypothesis. Variance ratios used may be chosen according to their power as suggested in Faust (1992). Rather than choosing variance ratios based on their power, a joint estimation of variance ratios for several time horizons is conducted here. In addition, variance ratios are estimated together with mean simple returns. The estimation is carried out by the Generalized Method of Moments (GMM) and the asymptotic distribution of estimates is normal regardless of the distribution of returns and a form of potential heteroskedasticity. Finally, a Wald statistic is constructed that compares mean simple returns and variance ratios implied by an asset-pricing model with their estimates. The variance-ratio statistic can be written as

$$VR(k) = \frac{Var(R_t^k)}{kVar(R_t)} = 1 + \frac{2}{k}\sum_{s=1}^{k-1}(k-s)\rho_s, k = 2, 3, ..., \tag{32}$$

where R_t^k, is the simple k-period gross return, R_t is the simple one-period gross return, and ρ_s, is the s-th serial correlation coefficient of returns.[36] $VR(k) = 1$ if the k-period

[36]Poterba and Summers (1988) offer an alternative definition of she variance-ratio

return follows a random walk. For $k = 2$, the variance ratio is simply unity plus the first-order autocorrelation. Thus, $VR(2) < 1$ for a negatively serially correlated two-period return, and $VR\{2) > 1$ for a positively serially correlated two-period return. For $k > 2$, $VR(k)$ is a linear combination of autocorrelation coefficients of returns with declining weights.

Let μ_i, denote the mean of the k-period simple gross return. Let us also assume that both μ_k and $VR(k)$can be obtained by solving a given model of asset pricing. Define $z_t = (R_t^1, R_t^2, ..., R_t^L)'$, and

$\theta = (\mu_{1,}\mu_{2,...,}\mu_{L,}VR(2), VR(3), ..., VR(L))'$, where L is a positive integer. The moment restrictions used for the GMM estimation follow from the definition of the variance ratio statistic (32):

$E[h(z_t, \theta)] = 0,$

where

$$h(z_t, \theta) = \begin{pmatrix} R_t^1 - \mu_1 \\ R_t^2 - \mu_2 \\ \dots \\ R_t^L - \mu_L \\ (R_t^2 - \mu_2)^2 - 2VR(2)(R_t^1 - \mu_1)^2 \\ (R_t^3 - \mu_3)^2 - 3VR(3)(R_t^1 - \mu_1)^2 \\ \dots \\ (R_t^L - \mu_L)^2 - LVR(L)(R_t^1 - \mu_1)^2 \end{pmatrix}$$

Let $g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} h(z_t, \theta)$and W_T is some positive definite symmetric matrix. The GMM estimator of θ maximizes the quadratic form $J_T(\theta) = g_T(\theta)'W_T g_T(\theta)$. In our case, $J_T(\widehat{\theta}) = 0$ because the estimator is exactly identified. It can be proved that $\sqrt{T}\ (\widehat{\theta} - \theta_0) \xrightarrow{d} N(0, V(\widehat{\theta}))$. $V(\widehat{\theta})$ is the asymptotic covariance matrix and is given by

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statistic for monthly data: $VR(k) = \frac{Var(R_t^k)}{k} / \frac{Var(R_t^{12})}{12}$, i.e., they compare variances of simple returns in relation to (he variation over a one-year period. For quarterly data, 12 would be replaced by 4. This approach is not adopted here because of potential problems with variance ratios, which arise when the time horizon is large relative to the total time span of the data (see Campbell et al. 1997, Chapter 2).

$(D_0 S_0^{-1} D_0)^{-1}$, where $D_0 = E[\frac{\partial h(z_t, \theta)}{\partial \theta}]$, and $S_0 = \sum_{i=-\infty}^{\infty} E[h(z_t, \theta_0)] E[h(z_t, \theta_0)]'$. $V(\widehat{\theta})$ is estimated by $\widehat{V}(\widehat{\theta}) = (D_T S_T^{-1} D_T)^{-1}$where

$$D_T = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial h(z_t, \widehat{\theta})}{\partial \theta},$$

and S_T is estimated using the Newey and West (1987) method, i.e.,

$$\begin{aligned} S_T &= \frac{1}{T} \sum_{t=1}^{T} h(z_t, \widehat{\theta}) h(z_t, \widehat{\theta})' + \sum_{i=1}^{n} [1 - \frac{i}{n+1}] \times \\ & [\frac{1}{T} \sum_{t=1+i}^{T} h(z_t, \widehat{\theta}) h(z_{t-1}, \widehat{\theta})' + \frac{1}{T} \sum_{t=1}^{T-i} h(z_t, \widehat{\theta}) h(z_{t+i}, \widehat{\theta})'], \end{aligned}$$

where the number of lags n is equal to 15.[37] The estimates of $\widehat{\theta}$ are reported in Table 25 for the S&P Index at yearly, quarterly, and monthly frequencies for L=10.[38] The data sources are described in Appendix 4. When annual data are used, the variance ratio for a two-year investment horizon equals one. However, $VR(2)$ can be greater than one if a different data span is used. For investment horizons greater than two years, variance ratios are lower than one, which corresponds to findings of Cecchetti et al. (1990) and Poterba and Summers (1988). For quarterly returns, positive autocorrelation coefficients overweight the negative ones up to the investment horizon of seven quarters (1.75 years). Finally, variance ratios for monthly returns are greater than one for all periods. Estimates of θ are consistent with the following stylized facts: equity returns display positive serial correlation at horizons shorter than one year and negative serial correlation for longer horizons.

Using the fact that $\widehat{\theta}$ is asymptotically normally distributed with covariance matrix $V(\widehat{\theta})$, one can write

$$Q = T(\widehat{\theta} - \theta_0)' V(\widehat{\theta})^{-1} (\widehat{\theta} - \theta_0) \sim \chi^2_{2L-1}. \qquad (33)$$

[37]The Gauss code for GMM written by Hansen, Heaton, and Okagi is available upon request.

[38]L = 10 is chosen since Cecchetti et al. (1990), Bonomo and Garcia (1994), and Zemcik (2001) examine VR(k) for k = 2, 3,..., 10.

The above relationship also holds when $V(\widehat{\theta})$ is replaced by its consistent estimate, $\widehat{V}(\widehat{\theta})$. The vector θ can be thought of as a function of a vector of underlying parameters of an asset pricing model. In this case, one might falsely conclude that the number of degrees of freedom in the Q-statistic is the difference between the number of elements in θ (which equals the number of restrictions) and the number of model parameters. However, since no estimation with respect to model parameters is conducted, the number of degrees of freedom is simply the number of elements in θ. The Q statistic will be employed to test the CCAPM with time non-separable preferences.

23 The CCAPM with Time Non-Separable Preferences

The framework adopted here is that of Lucas (1978). The model is solved for the case with time-dependent preferences. The lifetime utility function of the representative consumer takes the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t + \alpha c_{t-1})^{1-\gamma}}{1-\gamma}, \tag{34}$$

where c_t, is the consumption of services at time t and β and α denote the discount factor and the time non-separability parameter, respectively, α is the coefficient of interest since it affects the autocorrelation structure of the model returns (Zemcik 2001) as well as the size of the equity premium implied by the model (Constantinides 1990). γ is approximately equal to the expected value of the RRA coefficient (Ferson and Constantinides 1991) for $\alpha \neq 0$ and equals the RRA for $\alpha = 0$.

The present version of the CCAPM has been estimated and tested extensively. Ferson and Constantinides (1991) estimate the CCAPM by GMM at annual, quarterly, and monthly frequencies to find negative time non-separability parameters for annual and quarterly data and a positive one for monthly data. Estimation of alternative versions of the CCAPM by Eichenbaum and Hansen (1990) and Heaton (1995) also supports durability in monthly data. Cecchetti et al. (1994) test the CCAPM using tests based on volatility bounds. Their results favor habit persistence for annual and monthly data frequencies. A certain degree of habit persistence is necessary to generate a more volatile pricing kernel, which in turn increases the equity premium implied by the model.

Let $s_{t,}p_{t,}d_t$ be the amount of assets (trees) held, the market price of the asset, and the dividend, respectively. The representative agent then maximizes (34) with respect to the following budget constraint:

$$c_t + p_t s_{t+1} \leq (p_t + d_t)s_t...$$

The first order necessary conditions for the optimization problem imply

$$p_t = E_t m_{t+1}(p_{t+1} + d_{t+1}), \tag{35}$$

where m_{t+1} is the Intertemporal Marginal Rate of Substitution (IMRS) and is given by

$$m_{t+1} = \frac{\beta[(1+\alpha x_{t+1}^{-1})^{-\gamma} + \beta\alpha E_{t+1}(x_{t+2}+\alpha)^{-\gamma}]}{(1+\alpha x_t^{-1})^{-\gamma} + \beta\alpha E_t(x_{t+1}+\alpha)^{-\gamma}} x_{t+1}^{-\gamma},$$

where $x_{t+1} = \dfrac{c_{t+1}}{c_t}$. Using the expression for the IMRS, the Euler equation (35) can be written as

$$v_t = E_t m_{t+1} h_{t+1}(1 + v_{t+1}), \tag{36}$$

where v_t denotes the price-dividend ratio and h_t the gross growth rate of the dividend, respectively.

24 Shocks and Solution Method

This section derives variance ratios implied by the CCAPM. At first, the state dependent continuous distribution of the consumption growth rate is discretized by the Gaussian Appoint quadrature rule for N=6.[39] Then, the IMRS is discretized as well and so is the

[39]The resulting number of corresponding rates of returns is $(\#\text{states}\times\text{N})^2$, i.e., 144.

Euler equation, which can be written in the terms of price-dividend ratios. Next, the Markov process forcing equilibrium returns is derived using the price-dividend ratios and the transition matrix of the consumption process. Finally, variance ratios are calculated from the moments of model returns.[40]

Bonomo and Garcia (1994) argue that the two-state Markov switching model with one mean and two variances (2SMSIM2V) is the most parsimonious, statistically acceptable model for consumption growth rate. I follow Bonomo and Garcia (1994) and estimate parameters of this process using the method of maximum likelihood.[41]

The 2SMS1M2V model is defined as:

$$\ln(x_{t+1}) = \delta_0 + (w_0 + w_1 u_t)\epsilon_{t+1},$$

where $u_t = 1$ or 0, depending on the state of the economy. $\epsilon_{t+1} \sim N(0,1)$. The transpose of the transition matrix between the states 0 and 1 is:

$$G = \begin{pmatrix} g_{00} & 1 - g_{00} \\ 1 - g_{11} & g_{11} \end{pmatrix},$$

where g_{00} is the probability of remaining at state 0, while g_{11} is the probability of remaining at state 1. To conduct the estimation, consumption data on nondurables and services in the U.S. are used—see Appendix 4 for description of the data and Table 26 for summary statistics. The maximum likelihood estimates of $\delta_0, g_{00}, g_{11}, w_0$,and w_1 are reported in Table 27.

[40]For the sake of brevity, some details of the solution method are omitted. The details are available upon request from the author.

[41]Cecchetti et al. (1994) use a random walk model for annual data and an AR(1) model for monthly data. The random walk model is explicitly ruled out by Bonomo and Garcia (1994). For the sake of comparison, I also solve the model using the AR(1) process at all data frequencies and conduct the calibration exercise described in testing the CCAPM below. The CCAPM then generates autocorrelated returns even for time separable preferences, contrary to calibration of the model by the 2SMS1M2V process. However, differences in results seem minor when the Q-statistic is used and uncertainty regarding point estimates of historical variance ratios is accounted for.

Let $\ln(x^0)$ be an (N $\times$ 1) vector with elements

$$\ln(x_i^0) = \delta_0 + w_0 a_{i,} \quad i = 1, 2, ..., N,$$

where $a_{i,}$, is the abscissa for an N-point quadrature rule for the standard normal density. Also, let $\ln(x^1)$ denote an (N $\times$ 1) vector with elements

$$\ln(x_i^1) = \delta_0 + (w_0 + w_1) a_{i,} \quad i = 1, 2, ..., N,$$

Then, the consumption growth rate $\ln(x_{t+1})$ is approximated by a Markov chain, where the vector of possible values of the consumption process is defined as

$$\ln(x_t) = \begin{pmatrix} \ln(x^0) \\ \ln(x^1) \end{pmatrix},$$

with the transpose of the transition matrix

$$T = \left(\begin{array}{cc} g_{00}\Pi & (1 - g_{00})\Pi \\ (1 - g_{11})\Pi & g_{11}\Pi \end{array} \right), \tag{37}$$

where $\Pi_{ij} = w_j, i, j = 1, 2, ..., N$, and w_j are the weights of an N-point quadrature rule for the standard normal density.

The IMRS is written in the terms of x_{t+1}, and discretized. Let M denote the (2 N $\times$ 2N) matrix of values of the IMRS. Thus, the Euler equation (36) can be expressed as:

$$v = K\iota + Kv, \tag{38}$$

where v is a {2N $\times$ 1) vector of price-dividend ratios and ι is a (2N $\times$ 1) vector of ones. The (2N $\times$2N) matrix K is defined as

$$\{K_{ij}\} = M_{ij} x_j T_{ij}, \quad i, j = 1, 2, ..., 2N.$$

(38) is a system of 2N equations in 2N unknowns and can be easily solved for v . The equity returns then are

$$R_{ij} = \frac{p_j + d_j}{p_i} = \frac{v_j + 1}{v_i} x_j, \quad i, j = 1, ..., 2N. \tag{39}$$

Using T and (39), the equilibrium path for returns is characterized by another Markov chain with (4N^2 $\times$ 1) vector of possible values of returns, R, the (4N^2$\times$4N^2) transition matrix, P_R, and the $(4\text{N}^2 \times 1)$ vector of unconditional probabilities, Π_R (see Hamilton 1994, Section 22.4. for an example of a similar approach). The implied moments of returns are

$$\begin{aligned} E(R_t) &= \Pi_{R'} R = \mu, \\ Var[R_t] &= \Pi_{R'}(R.R) - \mu^2, \end{aligned} \tag{40}$$

and

$$E[R_{t+s}R_t] = (R.\Pi_R)' P_R^s R.$$

Thus, the s-th autocorrelation coefficient of the equilibrium returns can be calculated using

$$\rho_s = \frac{E[R_{t+s}R_t] - \mu^2}{Var[R_t]}. \tag{41}$$

The variance ratios implied by the CCAPM are calculated using (41) and (32). θ_0 is defined as $(\mu, 2\mu, ..., L\mu, VR(2), VR(3), ..., VR(L))$, where L = 10. The Q-statistic then can be computed for a given set of parameter values by substituting for θ_0 in (33).

25 Testing the CCAPM

In this section, contour maps of p-values associated with the Q-statistic are constructed. Annual, quarterly, and monthly data are used. At each data frequency, the CCAPM is fully characterized by the endowment process parameters $\delta, g_{00}, g_{11}, w_0$, $w_1$and by the utility function parameters α, β, γ. The consumption process parameters are set equal to their maximum-likelihood estimates given in Table 27. Preference parameters are chosen to be within a range of "reasonable" values. Values for β being examined for annual data are 1.03, 1.00, 0.97, and 0.90, respectively, Kocherlakota (1990) provides a theoretical justification for $\beta > 1$. β's for the other two data frequencies are adjusted to account for a different time-period. α varies from -1 to 1 and Y from 0 to 25. Based on the outcome, a narrower parameter space is then examined in finer intervals. Finally, parameter combinations with the highest p-values are presented for each data frequency. The described calibration exercise can be thought of as an informal estimation procedure.

A rigorous estimation of the identical version of the CCAPM is conducted in Ferson and Constantinides (1991) and is beyond the scope of this paper.

Results for annual data are summarized in Figure 1. Contour diagrams A, B, and C in Figure 1 differ only by the value of β. Admissible values for the model are the ones for which the null hypothesis of the equilibrium model being valid cannot be rejected. For instance when the p-value taken from one of the contour diagrams is higher than 10 percent it is not possible to reject the model at 10 percent level of significance. Since there are no p-values higher than 1 percent for β= 0.90, no contour diagram is drawn. Interestingly, in all the other cases, both durability and habit persistence are plausible. In addition, time separability is not rejected either, though corresponding p-values are somewhat lower. The highest p-value across all evaluated preference parameters is 0.99999 for β= 1.00, α = -0.04, and γ= 4.30.

The outcome of testing the CCAPM using quarterly data is depicted in Figure 2. βs for quarterly data are 1.0074 in part A of the picture, 1.0000 in B, 0.9924 in C. and 0.9740 in D, respectively. Contours are similar to their counterparts in Figure 1 in admitting durability, habit persistence, and time-separability for all βs considered, even for β= 0.9740, which is equivalent to β = 0.90 at a yearly frequency. The parameter combination with the highest p-value of 0.99995757 is β= 1.00742, α= 0, and γ=6.4.

Results for monthly data are illustrated in Figure 3. Contrary to Figures 1 and 2, Figure 3 favors durability, though time-separable preferences and habit persistence mostly cannot be rejected. When β = 0.99126, no p-value is greater than 1 percent and no picture is drawn. The highest p-value of 1 is obtained for β = 1.00000, α = 0.10, and γ = 2.80.

26 Power of the Modified Variance-Ratio Test

The null hypothesis is defined by the maximum-likelihood estimates of the consumption process parameters (Table 27) in combination with preference parameters with the highest p-value according to the Q-statistic (see Section "Testing the CCAPM"). The null hypothesis is used to parameterize the Data Generating Process, DGP_0. The DGP_0 is employed to produce a sequence of 20,000 repetitions of the Monte-Carlo experiment where a series of returns is generated under the null hypothesis of the CCAPM model being true. The number of observations for each data frequency is taken from Table 26. Simple means of returns and variance ratios are estimated for each series of returns generated by the DGP_0. The estimates are used to calculate the Q-statistic. An empirical distribution of Q is derived. The empirical distribution provides the size corrected critical value of the test. Then an alternative hypothesis is formulated by varying utility function parameters. The DGP_1 where the preference parameters are supplied by the alternative hypothesis, is formed. The DGP_1 is used to generate 1,000 time series of returns. Mean simple returns and variance ratios are estimated so that the Q-statistic can be calculated. The Q-statistic is compared to its critical value. Both the size corrected and asymptotic critical values are considered. The size corrected critical value is obtained from the distribution of Q under the null hypothesis. The alternative hypothesis is either accepted or rejected using the Q-statistic. Finally, the ratio of the number of rejections to the number of repetitions determines the power of the modified variance-ratio test.

A more formal discussion follows closely Spanos (1993). Let

$b = (\beta, \alpha, \gamma, \delta, g_{00}, g_{11}, w_0, w_1)' \in B$,

where B is the set of plausible values for parameters of the CCAPM. Suppose we have two competing hypotheses, $H_0 : b \in B_0$ and $H_1 : b \in B_1$, where $B_0 \cup B_1 = B$ and $B_0 \cap B_1 = \varnothing$. Let us also define the acceptance region $C_0 = \{b : Q(b) < \chi_{s,2L-1}\}$ and the rejection region $C_1 = \{b : Q(b) > \chi_{s,2L-1}\}$, where s is the level of significance of the test. Thus, if $b \in C_0$ we accept H_0 at the s level of significance and if $b \in C_1$ we reject H_0 at the s level of significance. Q is defined by equation (33). Then the power of the test is the probability of rejecting H_0 when false i.e. $\Pr(b \in C_1 \mid b = b_1)$ for $b_1 \in B_1$.

To compute the empirical distribution of Q, an artificial dataset has to be constructed.

We know from Section "Shocks and Solution Method" that returns satisfying restrictions given by the CCAPM are driven by a Markov chain with vector of possible values R and the transition matrix P_R. The Markov chain is fully characterized by the parameters of the CCAPM, combined in a vector b. Let $d = (\delta, g_{00}, g_{11}, w_0, w_1)'$ denote a vector of consumption process parameters. Table 27 defines d for annual data (d^a), quarterly data (d^q), and monthly data (d^m), respectively. Values of preference parameters are taken from the calibration exercise in Section "Testing the CCAPM". Thus, we can formulate the null hypotheses as follows:

Annual Data H_0 : $b_0^a = (1.00, -0.04, 4.30, d^a{\prime})\prime$,

Quarterly Data H_0 :$b_0^q = (1.00742, 0, 6.4, d^q{\prime})\prime$,

Monthly Data H_0 : $b_0^m = (1.00, 0.1, 2.8, d^m{\prime})\prime$.

To see the difference between the asymptotic and empirical distributions of Q I calculate the Q-statistic for each series produced by the DGP_0. The model implied means of simple returns and variance ratios are defined using b_0 or in other words we can express θ in (33) as a function of the underlying parameters of the CCAPM represented by b. Under H_0, $\theta(b) = \theta(b_0^a)$ for annual data and similarly for quarterly and monthly data. Graph 4 compares the two distributions for each data frequency. The theoretical relative frequency for 10-unit intervals is computed as $F(Q_2) - F(Q_1)$, where $Q_2 - Q_1 = 10$ and F(.) is the cumulative distribution function of χ_{19}. To account for the difference I conduct two kinds of tests to assess the power of the modified variance-ratio test: the asymptotic test and the size corrected test. The 10% critical value for χ_{19} is 27.2036. So, the rejection region for the asymptotic test is $C_1 = \{b : Q(b) > 27.2036\}$. Using size corrected critical values, rejection regions are

Annual Data: $C_1 = \{b : Q(b) > 89.108211\}$,

Quarterly Data: $C_1 = \{b : Q(b) > 50.610095\}$,

Monthly Data: $C_1 = \{b : Q(b) > 40.614630\}$.

Not surprisingly, as the number of observations increases, the empirical distribution of Q gets closer to the asymptotic distribution. However, the empirical distribution remains fat-tailed for all frequencies.

Finally, the power is calculated. The alternatives are intentionally constructed so that only one element in the parameter vector b differs from the null hypothesis. The alternative hypothesis is formulated by varying either the RRA coefficient γ, or the time non-separability parameter α. The hypothesis is either rejected or accepted based on the Q-statistic and the corresponding rejection region. The power of the test is estimated by

$\widehat{\Pr}(b \in C_1 \mid b = b_1) = \frac{number\ of\ \ rejections}{number\ of\ \ repetitions}.$

The results of power calculations are depicted in Figure 5. For annual data, see figures A and B where the alternative hypotheses are respectively:

Annual Data, γ varies $H_1^{\gamma} : b_1^a = (1.00, -0.04, \gamma, d^{a}\prime)\prime, \gamma \in [0, 5]$,

Annual Data, α varies $H_1^{\alpha} : b_1^a = (1.00, \alpha, 2.5, d^{a}\prime)\prime, \alpha \in [-0.5, 0.5]$.

The pattern in both figures clearly corresponds to what can be seen in Figure 1. As the null hypothesis is approached, the power decreases in both cases. However, aside from the close proximity to b_0^a , the power is unity or close to unity. The power of the size corrected test is somewhat lower by construction (size corrected critical value is greater than the asymptotic critical value) but it reaches unity as well for $\gamma < 1.0$ and $\gamma > 3.8$ in Figure 5A and for $\alpha = -0.6$ in Figure 5B.

Alternatives for quarterly data are given as:

Quarterly Data, γ varies $H_1^{\gamma} : b_1^q = (1.00742, 0, \gamma, d^{q}\prime)\prime, \gamma \in [6, 7]$,

Quarterly Data, α varies $H_1^{\alpha} : b_1^q = (1.00742, \alpha, 6.4, d^{q}\prime)\prime, \gamma \in [-0.5, 1.0]$.

Results are summarized in Figures 5C and 5D and correspond to Figure 2. There are two peaks with high p-values in Figure 2 and a through for $\alpha = 0.1$. The power is relatively low for the peaks and high for the through. Alternative hypotheses for monthly data are as follows:

Monthly Data, γ varies $H_1^{\gamma} : b_1^m = (1.00, 0.1, \gamma, d^{m}\prime)\prime, \gamma \in [0, 7]$,

Monthly Data, α varies $H_1^{\alpha} : b_1^m = (1.00, \alpha, 2.8, d^{m}\prime)\prime, \alpha \in [-0.5, 0.5]$.

The power for monthly data is shown in Graphs 5E and 5F. Again, results are compatible with testing the CCAPM using the Q-statistic in Figure 3.

The power of the test is high when γ deviates from it's null hypothesis' value. However, the power for $\alpha > -0.1$ is rather low since the data cannot clearly distinguish between habit persistence and durability.

27 Summary

Various forms of the variance-ratio test are utilized to document predictability and to test asset pricing models. This paper introduces the modified variance-ratio test where a joint distribution of variance ratios is derived together with mean simple returns to construct a test statistic, which may be employed to test a variety of models generating asset returns. The intuition behind the metric originates from the autocorrelation pattern in the U.S. equity data i.e. a positive autocorrelation in shorter investment horizons and negative autocorrelation in longer investment horizons. The statistic evaluates whether the difference between the model implied variance ratios and the model implied mean simple returns and their historical counterparts is statistically significant.

The CCAPM with time non-separable preferences is tested using the modified variance-ratio test. The results reflect what we observe in the U.S. stock market data. Variance ratios lower than one translate into habit persistence for annual data though neither durability nor the time-separable model can be rejected. For quarterly data frequency, the variance ratios are at first higher and then lower than one and consequently, the highest p-value is found for the time separable model. However, there are parameter combinations representing respectively habit persistence and durability, which cannot be rejected either. Finally, variance ratios greater than one in monthly data imply test results favoring durability.

Finally, the power of the modified variance-ratio test is examined to evaluate the extent of information contained in test results. The null hypothesis is formed using parameter combinations with highest p-values according to the modified variance-ratio test. Both the asymptotic and size corrected critical values are calculated from empirical distribution of the test statistic under the null hypothesis of the CCAPM being the true model. An alternative hypothesis is either accepted or rejected using both the asymptotic and size corrected tests. The ratio of rejections to the number of repetitions determines the power. Alternative hypotheses are formed by varying one parameter of the model at a time. The parameters changing their values are the RRA coefficient and the time non-separability parameter, respectively. When the RRA coefficient varies, the power of the test is mostly one across all data frequencies and is lower only in the neighborhood of the parameter combination defined by the null hypothesis. While the test rules out successfully values of the RRA that are different from the null hypothesis, it does not distinguish among habit persistence, time-separability, and durability very well. The low power in this case is caused by data supporting a wide range of values of the time non-separability coefficient.

Appendix 4

Annual Data

The yearly data considered here were used in Cecchetti et al. (1993). Cecchetti et al. (1990) provides a detailed account of the data sources. There are four series:

1.Consumption: The real per capita consumption of non-durables and services, 1889-1987.

2.CPI: Both the annual average and end of year observations from 1890 to 1987.

3.Dividends: The nominal dividends, 1890-1987, deflated by the annual average CPI.

4.Standard and Poor's Composite Stock Price Index: January observations, 1890-1988, adjusted to inflation by the end of period CPI.

Quarterly Data

The quarterly data I use contain observations form 1947 II to 1993 I and consist of two series:

1.Consumption: Real per capita consumption of non-durables and services - CITIBASE series (GCNQ + GCSQ)/GPOP).

2.Stock Return: Quarterly value weighted Standard and Poor's 500 returns taken from the CRSP tape, adjusted for inflation.

Monthly Data

The monthly data considered start from February 1959 and end in March 1993. They include the following series:

1.Consumption: The real per capita consumption of non-durables and services in 1987 dollars - CITIBASE series (GMCSQ + GMCNQ)/POP.

2.Price Index: Computed as (GMCS + GMCN)/(GMCSQ + GMCNQ), where GMCS, GMCN, GMCSQ, GMCNQ are respectively nominal consumption expenditures on services, nominal consumption expenditures on non-durables, real consumption expenditures in 1987 dollars on services, and real consumption expenditures in 1987 dollars on non-durables

3.Standard and Poor's Composite Common Stock Price Index: CITIBASE series FSPCOM adjusted for inflation by the above price index.

4.Dividends: Constructed using the dividend yield on Standard and Poor's Composite Common Stock (CITIBASE series FSDXP), Standard and Poor's Composite Common Stock Price Index, and the price index, both defined above.

Table 25

Summary Statistics for Real Historical Returns

	Yearly		1890-	1987	Quarterly		1947:II-	1993:I	Monthly		1959:2-	1993:3
Observations	98				184				410			
Mean	0.07978				0.02195				0.00495			
St.dev.	0.19500				0.07670				0.03488			
First Autocorr.	−0.00998				0.13309				0.28204			
k	$\hat{\mu}_k$	s.e.	$\widehat{VR(k)}$	s.e.	$\hat{\mu}_k$	s.e.	$\widehat{VR(k)}$	s.e.	$\hat{\mu}_k$	s.e.	$\widehat{VR(k)}$	s.e.
1	1.078	0.021			1.021	0.006			1.005	0.002		
2	2.156	0.030	1.000	0.153	2.043	0.009	1.147	0.154	2.010	0.003	1.283	0.092
3	3.234	0.034	0.849	0.147	3.065	0.011	1.137	0.187	3.015	0.004	1.375	0.132
4	4.310	0.039	0.859	0.140	4.087	0.012	1.119	0.193	4.019	0.004	1.426	0.146
5	5.390	0.043	0.846	0.152	5.110	0.014	1.105	0.196	5.024	0.005	1.473	0.152
6	6.471	0.046	0.798	0.148	6.132	0.015	1.095	0.202	6.029	0.005	1.525	0.159
7	7.552	0.049	0.763	0.157	7.155	0.016	1.059	0.204	7.034	0.006	1.545	0.164
8	8.635	0.052	0.757	0.161	8.177	0.017	0.997	0.195	8.038	0.006	1.534	0.167
9	9.719	0.054	0.739	0.154	9.200	0.017	0.938	0.184	9.043	0.006	1.520	0.169
10	10.799	0.056	0.708	0.145	10.223	0.018	0.904	0.176	10.048	0.007	1.502	0.167

Notes
The mean simple gross returns, variance ratios and their standard errors are based on the joint GMM estimation.

Table 26

Summary Statistics for Per Capita Consumption Growth Rate

	Annual Data 1890-1987	Quarterly Data 1947:II-1993:I	Monthly Data 1959:02-1993:03
Observations	98	184	410
Mean	0.0172	0.00444	0.00160
St. Dev.	0.0342	0.00563	0.00395
Skewness	-0.4045	-0.44717	0.003436
Kurtosis	3.9773	3.6211	3.5229
First Autocorrelation	-0.1331	0.1986	-0.2424

Table 27

Maximum Likelihood Estimates of the 2SMS1M2V Process

	Annual Data	Quarterly Data	Monthly Data
δ	0.0187	0.0046	0.0017
	(10.416)	(11.813)	(8.258)
g_{11}	0.9885	0.9832	0.4510
	(3.500)	(2.396)	(-0.231)
g_{00}	0.9854	0.9942	0.9247
	(3.086)	(4.316)	(2.878)
ω_0	0.0113	0.0050	0.0042
	(8.436)	(17.091)	(19.226)
ω_1	0.0315	0.0037	-0.0028
	(7.523)	(2.557)	(-4.482)
LF	2.20788	3.78981	4.12347

Notes

(1) Asymptotic t-ratios in parentheses. For $g_{ii}, i = 0, 1$, the reported t-ratios are those of the transformation $ln(g_{ii}/(1 - g_{ii})), i = 0, 1$, respectively. The transformation was employed to restrict probability estimates to the interval $(0, 1)$. (2) LF refers to the mean log-likelihood function as calculated by the GAUSS Constrained Maximum Like-lihood Estimation module.

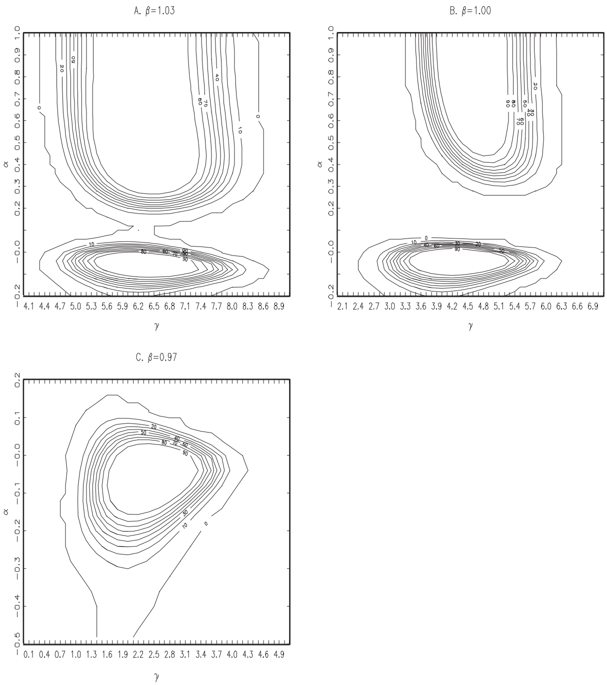

Figure 1. P-values; Annual Data

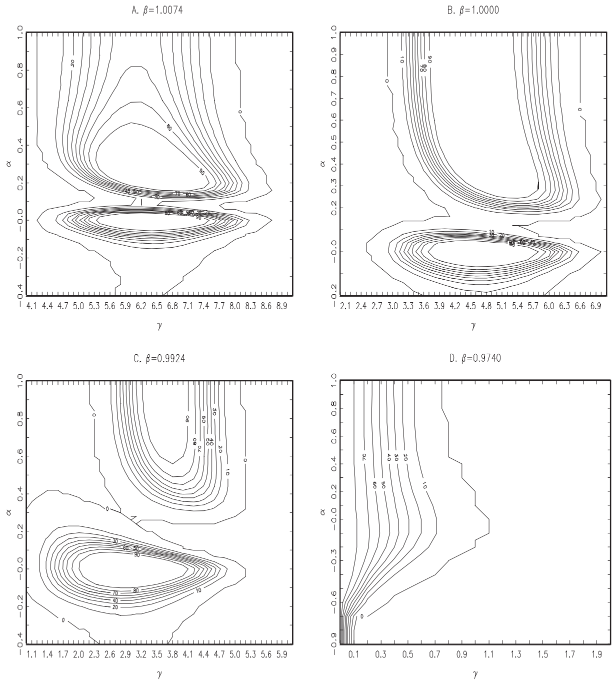

Figure 2. P-values; Quarterly Data

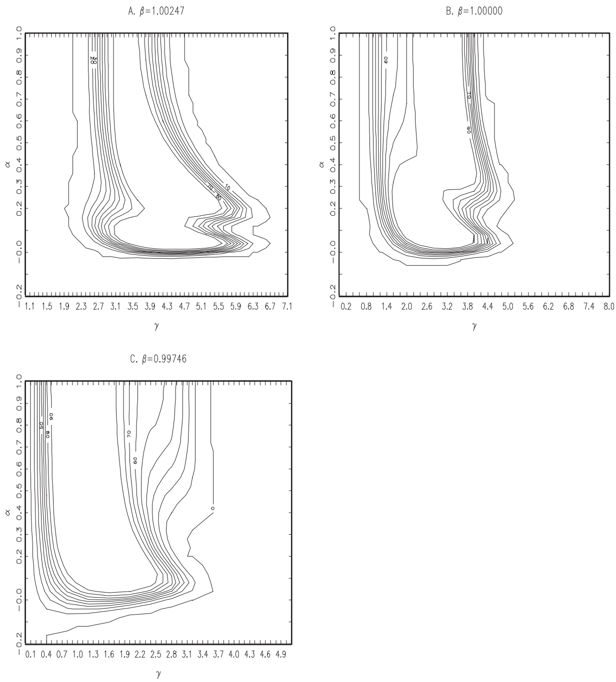

Figure 3. P-values; Monthly Data

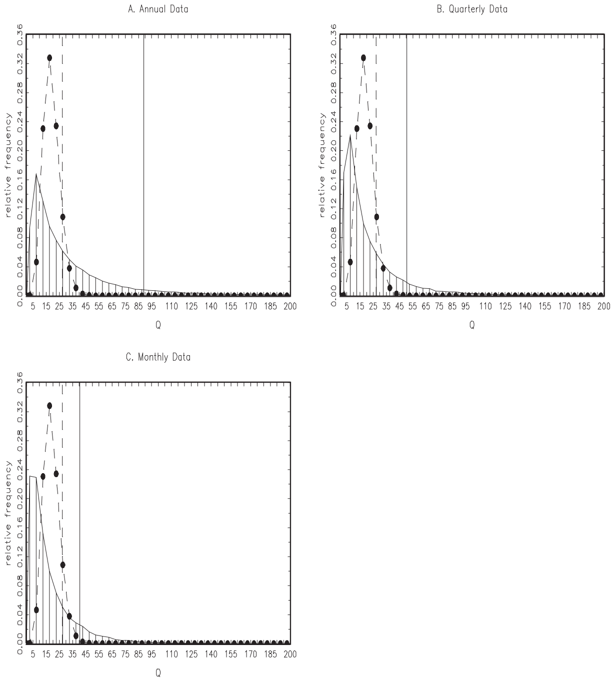

Figure 4. Empirical Distribution of Q Compared to χ_{19}

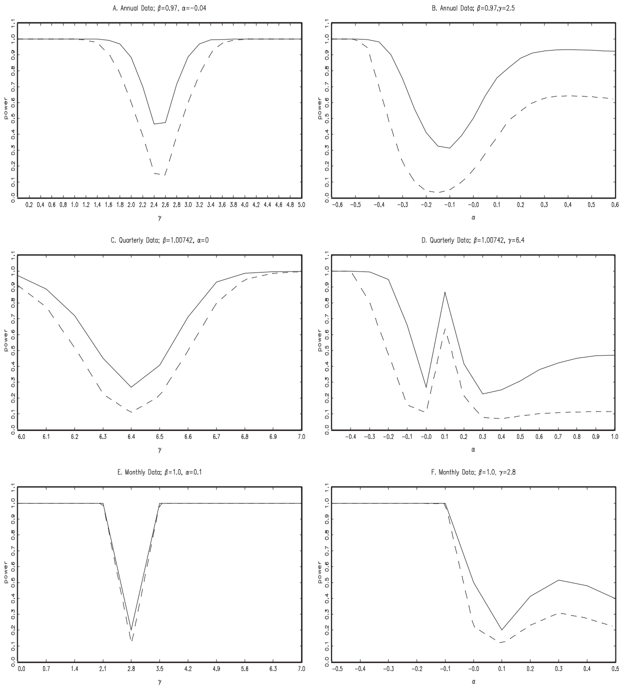

Figure 5. Power of the Modified Variance-Ratio Test

Part VI

Housing, Consumption, and Stock Returns: A Joint Econometric Model

Various theoretical models and empirical studies suggest a dynamic relationship among property returns, consumption, and stock returns. The present paper proposes a joint Markov switching model for the three variables, which allows for differing means and variances of the individual series. Combining two states for each series results in an eight-state Markov model with a discrete covariance matrix. The model parameters include unconditional state probabilities and are estimated by constrained MLE. One of the states with non-zero probability is characterized by high-mean-high-variance property returns with low-mean-high-variance stock returns and high-mean-low-variance consumption, similar to the state of the US economy in the beginning of the 21st century. Findings of the constrained MLE are also supplemented by Granger causality tests and impulse response functions.

28 Introduction

The recent worldwide inflation of share prices followed by a sharp decline has spurred an interest in the impact of the household wealth on consumption. Surprisingly high level of consumption kept the U.S. (and with it the world's) economy from going in a deeper recession and it is often claimed that it were the high real estate prices, which positively affected consumption (see Benjamin, Chinloy, and Jud 2004). On the other hand, there is a possibility of a reverse process due to potentially declining real estate prices,[42] which

[42] Gary Shilling, "Housing Bust Ahead," Forbes, 10/3/2005, Vol. 176 Issue 6, p110.

raises the question of an accurate representation of the empirical relationship between housing prices, stock returns and consumption.

Rational expectations models with focus on pricing of assets often include explicit treatment of housing. Flavin and Nakagawa (2004) model housing with adjustment costs and compare the housing stock with habit persistence. They use a partial equilibrium approach with an exogenously given joint process for property, stock, and bond returns. Implications of their model depend on the assumption of zero-correlation between property returns and financial returns. This assumption is relaxed in Yao and Zhang (2003) who allow for positive correlation between real returns on housing assets and stock returns. Campbell and Cocco (2002) investigate the impact of fixed- and floating-rate mortgage contracts on a life-time utility of home owners. Housing prices are also exogenously given in their model and are presumed to follow a random walk process. Bajari, Benkard, and Krainer (2005) study how fluctuations in property prices affect aggregate consumer welfare in a closed economy and Reis (2005) uses a similar framework to construct a dynamic price index, which includes prices of financial assets as well as real estate prices.

The price of the housing asset can also be endogenized. Ortalo-Magné and Rady (2003) explicitly consider the role of the young credit-constrained households to generate over-reaction of housing prices to income shocks and positive correlation of house prices and the number of housing transactions. Lustig and Van Niewerburgh (2003) abstract from life-cycle considerations and express the dependance of household exposure to idiosyncratic risk in the terms of housing collateral, the ratio of housing wealth to human wealth. All of these studies suggest presence of dynamic interactions among property returns, stock returns, and consumption, and importance of mutual correlations.

Proper calibration of rational asset pricing models and/or formulation of a multivariate econometric model requires detailed knowledge of features of the individual series as well as their mutual interaction. Dynamics of property prices in the OECD countries are explicitly studied by Englund and Ioannides (1997) who find a significant first-order autocorrelation in the first-differenced real annual house prices. Autocorrelation in property returns is also discussed in Cho (1996), a summary article on house price dynamics. Consumption processes are modelled in Bonomo and Garcia (1994), who argue that a Markov process for consumption should account for heteroskedasticity - a failure to do so results in seemingly mean reverting equilibrium equity returns in an asset pricing model with a power utility function.[43] Autocorrelation and heteroskedasticity of stock returns have been documented in numerous studies; for a thorough survey see Campbell, Lo, and MacKinlay (1997).

The interaction between house prices and stock prices is studied for example in Kennedy and Andersen (1994) who report positive correlation between the two variables,[44] contradicting the main assumption in Flavin and Nakagawa (2004). Lustig and Van Niewerburgh (2003) find that an increase in the US stock returns can be predicted by decline in the ratio of housing wealth to human wealth. Many empirical studies also estimate marginal propensity to consume out of housing or stock market wealth, rather than from returns. Case, Quigley, and Shiller (2001) use panel data from twelve developed European countries, Canada, and United States and Tan and Voss (2000) use Australian regional panel data to demonstrate that consumer spending is more sensitive to changes in housing wealth than to changes in financial wealth. These findings suggest ambiguity with respect to the correlation between housing and financial returns, strong causality from property prices to consumption, and weak causality from stock prices to consumption.

The present paper formulates an econometric model for the joint process of housing returns, consumption, and stock returns. Following Bonomo and Garcia (1994) and Zemčík (2001), it allows for two means and two variances for each series. For example, a bearish stock market can be associated with a higher volatility while the volatility is

[43] Zemčík (2001) shows that mean reverting equity returns can in fact be generated when the heteroskedastic consumption process is combined with time non-separability in preferences .

[44] See their Table III.2. They do not include statistical significance.

reduced on the bull market. Similar analogies can be drawn for housing or consumption: For instance, the model specification allows for two states of the economy, one with quickly growing house prices and a slow decline - similarly to Japan by the late 1980's- with respective smaller and larger fluctuations. The paper then generalizes some results for univariate mixture distributions processes in Hamilton (1994, Ch. 22.3) in the spirit of a bi-variate Markov switching model for stock returns and industrial production in Hamilton and Li (1996). In the resulting tri-variate model, two states for every time series translate into a joint eight-state Markov model. Each state corresponds to a particular combination of regimes of the three time series. In one of the states, consumption growth rate is positive, the stock market is declining and the housing market is on the rise, a situation reminiscent of the state of the US economy since 2000. Parameters of the process include probabilities of occurrence of each of the eight states.

Estimation of the tri-variate Markov chain poses a challenge due to the discreetness of the covariance matrix of parameters, which in turn reflects presence of differing volatilities of the time series processes. The challenge is addressed by explicit formulation of the Jacobian matrix of the non-linear inequality constraints with respect to restricted parameters and estimation by the method of restricted maximum likelihood. Estimates of covariances among the three series are all zero with binding constraints to keep the covariance matrix positive definite. This result supports validity of the assumption of zero covariances between house and financial returns adopted in Flavin and Nakagawa (2004). In mean-variance combinations of individual series, housing returns are more volatile when they rise, contrary to consumption and stock returns. The most likely overall state of the economy is when all the series are at their higher levels of means. Other non-zero states include growing house prices with declining stock market and either faster or slower growth of consumption. The former corresponds to the state of the US economy shortly after the recent stock market decline. Finally, declining stock market can be observed together with high state of consumption and low state of housing returns. It is also possible for the stock market to be bullish but accompanied by low consumption and house prices.

Causality tests and VAR impulse response functions complete the picture of the mutual relationship among housing, stock markets and consumption. The stock market returns Granger-cause both consumption and housing returns. Housing returns come close to being statistically significant in predicting stock returns, which is consistent with conclusions in Lustig and Van Niewerburgh (2003). Consumption does not have any statistically significant predictive information content for either stock or housing returns; on the other hand, housing does Granger-cause consumption. These observations are roughly confirmed by impulse response functions where consumption is sensitive to shocks in either stock or housing market and the stock market reacts (almost significantly) to shocks on the housing market and viceversa. Presented findings indicate causality (both in its standard meaning and in the sense of Granger) from housing and property returns to consumption. However, contrary to evidence from panel data studies using housing and stock market wealth, stock returns seem to have a stronger impact on consumption spending than property returns.

The paper is organized as follows. Section 29 describes the tri-variate Markov model and estimation of its parameters, Section 30 characterizes data, Section 31 estimation output, Section 32 discusses Granger causality tests and impulse response functions, and Section 33 concludes.

29 Estimation Methodology

Processes of the stock market prices and macroeconomic variables (e.g. consumption, industrial production, and GDP) have been studied extensively, including their (realistic) simplified versions meant for calibration of life-cycle models. The process for real estate prices has not yet been the center of attention from this prospective, which will be remedied in this study. A potential specification can be found in Hamilton and Lin (1996), who focus on the stock market prices and adopt a Markov-switching process with ARCH features, which allows for asymmetric effects on volatility. However, this model may be over-parameterized for the purposes of this study and does not allow for differing growth rates. Therefore, a better alternative is the two-state two-mean two-variance Markov Switching Model (2S2M2VMS) used in Zemčík (2001) for consumption, which can be extended to accommodate all the three variables.[45]

To estimate a multivariate Markov process, this section generalizes discussion of Markov chains for univariate processes in Hamilton (1994). Let us consider a multivariate Markov-switching process $y_t,\ t = 1, \ldots, T$ with mean μ_t and variance Ω_t. y_t is a (3×1) vector representing the housing, consumption and stock return processes, respectively. Each of the processes $j = 1, \ldots, 3$ has two regimes denoted $S_j = 0$ and $S_j = 1$. The 8×3 matrix S then characterizes all 8 states of the economy:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ . & . & . \\ 1 & 1 & 1 \\ \end{pmatrix}. \tag{42}$$

Let us define a variable $S_t^* = i$ whenever the state of the economy is i and 0 otherwise. Each state of the economy corresponds to the i-th row of the matrix S. The unconditional

[45] This 2S2M2VMS generalizes the random walk model in Campbell and Cocco (2002) by allowing for different growth rates and volatility. The structure of the Markov process makes it relatively easy to incorporate in life-cycle rational asset pricing models, which then can be solved.

probability that S_t^* equals i is

$$P\{S_t^* = i\} = \pi_i. \tag{43}$$

Conditional mean and covariance matrix are respectively given by:

$$\mu_i = (\mu_t | S_t^* = i) = \begin{pmatrix} \mu_1 + \mu_1^\dagger S_{i1} \\ \mu_2 + \mu_2^\dagger S_{i2} \\ \mu_3 + \mu_3^\dagger S_{i3} \end{pmatrix} \tag{44}$$

and

$$\Omega_i = (\Omega_t | S_t^* = i) = \begin{pmatrix} \omega_1 + \omega_1^\dagger S_{i1} & \omega_4 & \omega_5 \\ \omega_4 & \omega_2 + \omega_2^\dagger S_{i2} & \omega_6 \\ \omega_5 & \omega_6 & \omega_3 + \omega_3^\dagger S_{i3} \\ \end{pmatrix}, \tag{45}$$

where Ω_i is a positive definite symmetric matrix. Further assume that y_t is conditionally normally distributed i.e.

$$f(y_t | S_t^* = i) = \frac{1}{(2\Pi|\Omega_i|)^{1/2}} \exp\left[-\frac{1}{2}(y_t - \mu_i)' \Omega_i^{-1} (y_t - \mu_i)\right]. \tag{46}$$

Then the joint density distribution of y_t and S_t^* is

$$f(y_t, S_t^* = i) = \pi_i f(y_t | S_t^* = i). \tag{47}$$

Summing up (47) over the states results in the unconditional density of y_t:

$$f(y_t) = \sum_{i=1}^{8} f(y_t | S_t^* = i). \tag{48}$$

Let us stack elements of μ_i and Ω_i together with all π_i's except the last one in a (21×1) vector θ. The log-likelihood for the observed data is

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t). \tag{49}$$

Maximum likelihood estimate $\hat{\theta}$ is calculated by maximizing $L(\theta)$ with respect to θ and subject to $\sum_{i=1}^{8} \pi_i = 1$, $\omega_k + \omega_k^\dagger S_{ik} \geq 0$ for all i, k, and Ω_i being positive definite for all i. Typically, positive definiteness is ensured by restricting eigenvalues of a covariance matrix to be positive. This approach cannot be applied here since there are in fact eight covariance matrices. This problem can be circumvented by following an alternative definition of positive definiteness where a matrix is positive definite if and only if its leading principal minors are positive (e.g. definition 21.30 in Sydsæter, Strøm, and Berck 2000). Details are laid out in Appendix 5. The optimization is conducted using the Constrained Maximum Likelihood module in Gauss (see Gauss Applications 1995).

30 Data

The used data series are listed in Table 28, including data sources and a brief description. Housing prices are characterized by median sales of existing homes in the United States, which are seasonally adjusted by the difference from moving average method. The resulting series is adjusted for inflation by a consumer price index and a growth rate of the adjusted series is calculated (HPIEX). Stock returns are real monthly returns on the S&P index and include both capital gains and dividends (SPRET). The per capita consumption growth rate is computed using the seasonally adjusted US personal consumption expenditures and population data (CRATE). The summary statistics of the constructed series are given in Table 29. Stock returns have the highest mean and are also the most volatile of the three series. The distribution of HPIEX has a long right tail and distributions of SPRET and CRATE have both a long left tail. All the series are peaked relative to the normal distribution. The null hypothesis of normality cannot be rejected at 5% level of significance only for the house index series. Covariances provide basis of comparison with the estimates of the covariance matrices Ω_i's. Correlations among the series are around 0.10. Standard augmented Dickey-Fuller unit root tests with an intercept (not reported) reject unit roots in all the series at 1% level of significance.

31 Results

Results of the maximum likelihood estimation are reported in Table 30 where parameter estimates are accompanied by their standard errors and t-ratios, which can be used for illustrative statistical inference. The estimates of means conditional on the respective states being zero are significantly positive for stock returns and consumption growth rate. The mean estimates for state variables equal to unity are insignificant but in two cases negative, namely for stock returns and consumption. The only positive element of the covariance matrix for $S_{i1} = S_{i2} = S_{i3} = 0$ is the variance of consumption. Covariances restricted to be the same in all states are zero (compare with Table 29). While all the variances for $S_{i1} = S_{i2} = S_{i3} = 1$ are positive, only the one for stock returns is significant. There are four non-zero values for unconditional probabilities, with π_2 and π_5 statistically significant.

To interpret the estimates, they are organized in Table 31 so that one can see how the three processes move together. For example, for the state $S_i^* = 1$, the state variables for house prices, stock returns and consumption are respectively $S_{i1} = S_{i2} = S_{i3} = 0$. The mean of the growth rate of housing index is $\mu_1 = 0.0010$ from Table 30. The variance is equal to $\omega_1 = 0$. Similarly for stocks and consumption. The estimated unconditional probability of occurrence of the first state is 0 ($\pi_1 = 0$).

A close look at the table provides some additional information on the mutual dynamics of the considered processes. The housing process has two states, low mean with low variance and high mean with high variance. The estimates of the stock returns process conform stylized facts i.e. the variance is low for high returns and vice versa. In other words, there is heteroskedasticity with higher volatility associated with low returns. The consumption process is similar in that aspect though the difference in volatilities is rather small compared to the difference in means. A combination of means and variances corresponding to a particular state determines the joint law of motion for the three-variate process.

Let us consider the most likely state $S_i^* = 5$. In this case, probability of its occurrence is $\pi_t = 0.86$[46]. All processes are in the higher mean state with the corresponding higher volaitility for housing and lower volatilities for stocks and consumption, respectively. Clearly, most of the time since 1968, all three processes were growing at faster rates. State eight has the second largest probability - here housing is on the rise while stock returns and consumption growth are low. State two corresponds to low growth rates of housing and consumption with high returns on the stock market. Of special interest is $S_i^* = 7$, which can be related to the situation in the United States in the beginning of the 21st century. The stock market was bearish but high house prices fuelled consumption growth. In the laste state with non-zero probability, $S_i^* = 3$, consumption is growing fast in spite of low housing prices and stock returns.

32 Granger Causality Tests and Impulse Responses

To further investigate relations among the three series, Granger causality tests are conducted (see Granger 1969). The concept of Granger causality differs from common understanding of the word; rather then cause and effect it reflects precedence and information content. It provides a useful way of characterization of how relevant a variable (say, y) is for predicting another variable (x). y is said to Granger-cause x if x and its past values improve prediction of y when used in addition to past y's.

The following bivariate regressions are run:

$$\begin{aligned} y_t &= \alpha_0 + \alpha_1 y_{t-1} + \ldots + \alpha_l y_{t-l} + \beta_1 x_{t-1} + \ldots + \beta_l x_{t-l} + \epsilon_t, \\ x_t &= \alpha_0 + \alpha_1 x_{t-1} + \ldots + \alpha_l x_{t-l} + \beta_1 y_{t-1} + \ldots + \beta_l y_{t-l} + u_t \ , \end{aligned} \qquad (50)$$

for all three pairs of housing, stock returns and consumption series. The null hypothesis is

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_l = 0 \qquad (51)$$

[46] In the following discussion, the focus will be on point estimates since t-ratios are only approximate and not available for the state eight and zero π's.

and if it is rejected, then x Granger causes y in the first regression and viceversa in the second regression.

Table 32 reports F-statistics, which are the Wald statistics for the joint hypothesis (51). The null hypothesis that SPRET does not Granger-cause HPIEX is rejected at 10% level of significance, which indicates that the stock market helps to forecast the housing market. The same is not true in the opposite direction though the p-value only slightly exceeds 10%. The hypothesis that consumption Granger-causes housing and viceversa is accepted at 5% level of significance. The fact that both variables improve the prediction of the other series illustrate close mutual relationship of housing and consumption. Finally, CRATE does not Granger-cause SPRET but SPRET does Granger-cause CRATE i.e. the stock market contains information useful to anticipate consumption spending of households.

To complement Granger causality tests, impulse responses to exogenous shocks are calculated as well. A VAR process for the three series is estimated with two lags for endogenous variables. Impulse responses are depicted in Figure 6 together with 95% confidence intervals. The observed patternes are in accord with Granger causality tests. HPIEX is Granger-caused by CRATE and it also reacts strongly to positive shocks in consumption. Similar reaction of HPIEX can be seen to changes in SPRET and the p-value of the null hypothesis of Granger non-causality was close to rejection in this case. The stock market reacts just to the housing market. Consumption on the other hand reacts strongly to shocks on both the housing and stock markets.

33 Summary

To capture the dynamic relationship among the three series, the present study proposes an econometric model, which is a generalized version of the bi-variate Markov switching model in Hamilton and Lin (1996) and of the two means-two variances process in Bonomo and Garcia (1994) and Zemčík (2001). In spite of a high degree of parsimony, the model allows for interesting dynamics among the three variables and roughly incorporates some stylized facts for the three series, namely autocorrelation and heteroskedasticity.

The tri-variate Markov process is estimated using US data on stock returns, prices of existing homes and real per capita consumption. Estimation is conducted by the method of restricted maximum likelihood and takes into account discrete variances of the three series. Results show that there are four states with positive probability: (i) low-mean-low-variance property returns with high-mean-low-variance stock returns and high-mean-low-variance consumption; (ii) low-mean-low-variance property returns with low-mean-high-variance stock returns and low-mean-high-variance consumption; (iii) high-mean-high-variance property returns with high-mean-low-variance stock returns and high-mean-low-variance consumption; and (iv) high-mean-high-variance property returns with low-mean-high-variance stock returns and low-mean-high-variance consumption. The state described in (iii) is reminiscent of the situation in the US economy since 2000 when consumption growth rate was positive, the stock market was declining and the housing market was on the rise. The estimation results are completed by pairwise Granger causality tests and impulse response functions. Both property and stock returns Granger-cause consumption and consumption helps in prediction of real estate returns. These findings are supported by graphed impulse response functions.

Appendix 5. Positive Definiteness of Ω_i's

The matrix Ω_i is positive definite for all i if and only if:

(i) $\omega_1 + \omega_1^\dagger S_{i1} > 0$ for all i. This is a restriction on the first leading principal minor of Ω_i.

(ii) $(\omega_1 + \omega_1^\dagger S_{i1})(\omega_2 + \omega_2^\dagger S_{i2}) - \omega_4^2 > 0$ for all i. This is a restriction on the second leading principal minor of Ω_i.

(iii) $(\omega_1 + \omega_1^\dagger S_{i1})(\omega_2 + \omega_2^\dagger S_{i2})(\omega_3 + \omega_3^\dagger S_{i3}) + 2\omega_4\omega_5\omega_6 - \omega_6^2(\omega_1 + \omega_1^\dagger S_{i1}) - \omega_4^2(\omega_3 + \omega_3^\dagger S_{i3}) - \omega_5^2(\omega_2 + \omega_2^\dagger S_{i2}) > 0$ for all i. This is simply $|\Omega_i| > 0$.

The restriction (i) is linear in parameters and is easy to enforce in Gauss. Restrictions (ii) and (iii) are non-linear and performance of the optimization algorithm is improved if the Jacobian $J_{[i]}$ of the non-linear inequality constraints with respect to restricted parameters is provided (see Gauss Applications 1995 for details). $J_{[i]}$ is a (12×9) matrix. Let the vector of parameters included in non-linear restrictions be denoted as $\theta_N = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_1^\dagger, \omega_2^\dagger, \omega_3^\dagger)'$. It follows from the restriction (ii) that for $j = 1$ to 4 (two states for two state variables) the j-th row of $J_{[i]}$ is given by :

$$J_{[i]j.} = (\omega_2 + \omega_2^\dagger S_{i2}, \omega_1 + \omega_1^\dagger S_{i1}, 0, -2\omega_4, 0, 0, S_{i1}(\omega_2 + \omega_2^\dagger S_{i2}), S_{i2}(\omega_1 + \omega_1^\dagger S_{i1}), 0).'$$

Similarly, restriction (iii) implies that for $j = 5, ..., 12$:

$$J'_{[i]j.} = \begin{pmatrix} (\omega_2 + \omega_2^\dagger S_{i2})(\omega_3 + \omega_3^\dagger S_{i3}) - \omega_2^2 \\ (\omega_1 + \omega_1^\dagger S_{i1})(\omega_3 + \omega_3^\dagger S_{i3}) - \omega_5^2 \\ (\omega_2 + \omega_2^\dagger S_{i2})(\omega_1 + \omega_1^\dagger S_{i1}) - \omega_4^2 \\ 2\omega_5\omega_6 - 2\omega_4(\omega_3 + \omega_3^\dagger S_{i3}) \\ 2\omega_4\omega_6 - 2\omega_5(\omega_2 + \omega_2^\dagger S_{i2}) \\ 2\omega_4\omega_5 - 2\omega_4 6(\omega_1 + \omega_1^\dagger S_{i1}) \\ S_{i1}(\omega_2 + \omega_2^\dagger S_{i2})(\omega_3 + \omega_3^\dagger S_{i3}) - S_{i1}\omega_2^2 \\ S_{i2}(\omega_1 + \omega_1^\dagger S_{i1})(\omega_3 + \omega_3^\dagger S_{i3}) - S_{i2}\omega_5^2 \\ S_{i3}(\omega_2 + \omega_2^\dagger S_{i2})(\omega_1 + \omega_1^\dagger S_{i1}) - S_{i3}\omega_4^2 \end{pmatrix},$$

where there are eight rows which correspond to the eight rows of matrix S.

Table 28

Data Description

Series ID	Seas. Adj.	Source	Notes
CPIAUCNS	No	Bureau of Labor Statistics	Index 1982-84-100
HP	No	National Association of REALTORS	USD
PCE	Yes	Bureau of Economic Analysis	Bil. of USD
POP	N/A	Census Bureau	
SP500	No	Compustat	USD
SPDIV	No	Compustat	USD

The common sample for the data series (growth rates and returns) is 1968:02-2004:05. CPIAUCNS is Consumer Price Index for All Urban Consumers. HP is a series with median sales price of existing family homes in the United States. PCE are Personal Consumption Expenditures. POP denotes the United States population. SP500 and SPDIV are respectively the Standard & Poor's 500 index and the corresponding dividend series.

Table 29

Data Summary Statistics (1968:02-2004:05)

	HPIEX	SPRET	CRATE
Mean	0.0011	0.0056	0.0015
Median	0.0018	0.0078	0.0017
Maximum	0.0426	0.1589	0.0313
Minimum	-0.0369	-0.2174	-0.0261
Std. Dev.	0.0123	0.0452	0.0061
Skewness	0.0076	-0.3539	-0.0911
Kurtosis	3.5194	4.5499	6.0416
Jarque-Bera	4.9044	52.7446	168.6748
(Probability)	(0.0861)	(0.0000)	(0.0000)
Observations	436	436	436
Covariances			
HPIEX	0.0001514	5.44E-05	8.20E-06
SPRET	5.44E-05	0.002035138	2.90E-05
CRATE	8.20E-06	2.90E-05	3.66E-05
Correlations			
HPIEX	1.0000	0.0980	0.1102
SPRET	0.0980	1.0000	0.1062
CRATE	0.1102	0.1062	1.0000

HPIEX is the growth rate of the house pricing index of exiting homes, adjusted to inflation using a consumer priced index; SPRET is the rate of return on the SP 500 market index; and CRATE is the growth rate of real per capita consumption.

Table 30

Maximum Likelihood Estimates of the Tri-variate Markov Process

Parameters	Estimates	Std. Error	t-ratio	μ_1
μ_1	0.001	0.0008	1.25	
μ_2	0.0082	0.0025	3.28	
μ_3	0.0016	0.0003	5.33	
$\mu_1^\dagger$	0.0002	0.001	0.20	
$\mu_2^\dagger$	-0.0255	0.0187	-1.36	
$\mu_3^\dagger$	-0.0014	0.0022	-0.64	
ω_1	0	.	.	
ω_2	0	.	.	
ω_3	0.0015	0.0002	7.50	
ω_4	0	.	.	
ω_5	0	.	.	
ω_6	0	.	.	
$\omega_1^\dagger$	0.0002	.	.	
$\omega_2^\dagger$	0.0043	0.0021	2.05	
$\omega_3^\dagger$	0.0001	.	.	
π_1	0	.	.	
π_2	0.0384	0.0174	2.21	
π_3	0.0095	0.0123	0.77	
π_4	0	.	.	
π_5	0.8595	0.0807	10.65	
π_6	0	.	.	
π_7	0.0199	0.0608	0.33	

Table 31

Conditional Means and Variances in a Given State

S_i^*	S_{i1}	S_{i2}	S_{i3}	Housing		Stocks		Consumption		Prob.
				mean	var	mean	var	mean	var	1
1	0	0	0	0.0010	0	0.0082	0	0.0016	0.0015	0
2	0	0	1	0.0010	0	0.0082	0	0.0002	0.0016	0.0384
3	0	1	0	0.0010	0	-0.0173	0.0043	0.0016	0.0015	0.0095
4	0	1	1	0.0010	0	-0.0173	0.0043	0.0002	0.0016	0
5	1	0	0	0.0012	0.0002	0.0082	0	0.0016	0.0015	0.8595
6	1	0	1	0.0012	0.0002	0.0082	0	0.0002	0.0016	0
7	1	1	0	0.0012	0.0002	-0.0173	0.0043	0.0016	0.0015	0.0199
8	1	1	1	0.0012	0.0002	-0.0173	0.0043	0.0002	0.0016	0.0727

Pairwise Granger Causality Tests

Null Hypothesis:	Obs	F-Statistic	Probability
SPRET does not Granger Cause HPIEX	424	1.5673	0.0985
HPIEX does not Granger Cause SPRET		1.4500	0.1407
CRATE does not Granger Cause HPIEX	424	2.0095	0.0223
HPIEX does not Granger Cause CRATE		1.9750	0.0252
CRATE does not Granger Cause SPRET	424	0.8353	0.6140
SPRET does not Granger Cause CRATE		2.8203	0.0010

Sample: 1969:02 2004:05, Lags: 12.

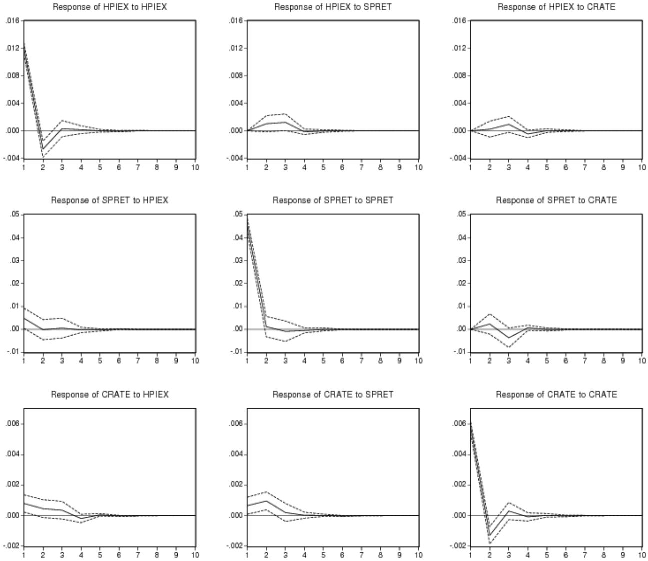

Figure 6: Response to Cholesky One S.D. Innovations ± 2 S.E.

Part VII

Conclusion

The conclusion of the present habilitation work focuses on two aspects of the asset pricing literature, the equity premium puzzle and the role of housing. This perspective is motivated by events that have taken place since August 2007. The problems originated in the market for the sub-prime mortgages. The unusual frequency of defaults put many financial institutions in a problematic position. US mortgages were highly securitized mainly in the form of collateral debt obligations and a number of other related off-balance financial vehicles, which has hidden the exposure to risk and made it difficult to estimate the impact of the collapsed housing market triggered by the defaulting mortgages. The sub-prime mortgage crisis became first a crisis of liquidity and then a crisis of solvency. The access to credit has tightened and the global economy is heading towards a recession. One implication was a bearish global stock market with elimination of 20-30% of wealth in a matter of a month in October 2008. This raises the questions if there is still an equity premium puzzle since the equity premium has become much smaller. Also, the probability of a big collapse in stock prices may have not been considered since such a collapse has not been yet observed.[47] Another question is the exact role of housing both in the crisis and in asset pricing models. Explicit modelling of housing is needed to describe accurately the causes of the global financial and economic crisis and to estimate the impact of a bursting housing bubble in many markets on household consumption and performance of economies.

33.1 Equity Premium Puzzle Revisited

Here I intend to confirm existence of the equity premium puzzle by estimating the standard power utility model by Generalized Methods of Moments (GMM), using an updated

[47] This is often referred to as "a black swan event" or a "peso problem".

dataset. To describe the estimation methodology in some detail, I follow Hamilton (1994). Let us define an $(h \times 1)$ vector of observables w_t, an $(a \times 1)$ vector of coefficients θ with the true value θ_0, and an $(r \times 1)$ vector valued function $h(\theta, w_t)$. $h(.)$ can be viewed as a residual from a model. Orthogonality conditions are defined as:

$$E[h(\theta, w_t)] = 0. \tag{52}$$

The sample equivalent of the orthogonality conditions (52) is given by

$$g(\theta, Y_T) \equiv \frac{1}{T} \sum_{t=1}^{T} h(\theta, w_t) \tag{53}$$

where Y_T is an $(Th \times 1)$ vector $[w_1, ..., w_T]'$. The idea behind GMM is to choose θ so as to make the sample momemnt $g(\theta, Y_T)$ as close as possible to the population moment of 0. The GMM estimator $\hat{\theta}_T$ is the value of θ that minimizes the scalar

$$Q(\theta, Y_T) = [g(\theta, Y_T)]' S^{-1} [g(\theta, Y_T)] \tag{54}$$

where $S = \lim_{T \to \infty} TE[g(\theta_0, Y_T) g(\theta_0, Y_T)]'$. Hansen (1982) shows that this choice of weighting matrix in the optimization problem minimizes the variance of the GMM estimator. He also shows, that the GMM estimator is normally distributed and that $J = TQ(\hat{\theta}, Y_T) \to \chi(r - a)$. This is the so called Hasen J test of overidentifying restrictions.

Hansen and Singleton (1982) apply the GMM methodology to test restrictions implied by the Lucas (1978) Consumption based Capital Asset Pricing Model (CCAPM) with the power utility function characterizing consumer preferences. The model implied residuals come from the first order conditions of the optimization problem, i.e.

$$h(\theta, w_t) = \left[\begin{array}{c} \{1 - \beta(1 + r_{e,t+1})(c_{t+1}/c_t)^{-\gamma}\} x_t^1 \\ \{1 - \beta(1 + r_{f,t+1})(c_{t+1}/c_t)^{-\gamma}\} x_t^2 \end{array} \right]. \tag{55}$$

The vector of parameters $\theta = [\beta, \gamma]$ where β is the discount factor and γ is the coefficient of relative risk aversion. r_e is the equity rate of return and r_f is the risk-free rate of return, respectively. c is the per-capita consumption of non-durables and services. $x_t^i, i = 1, 2$ are a 5×1 vectors of instruments $[1, c_t/c_{t-1}, c_{t-1}/c_{t-2}, r_{et}, r_{e,t-1}]'$ for the first equation and $[1, c_t/c_{t-1}, c_{t-1}/c_{t-2}, r_{ft}, r_{f,t-1}]'$ for the second equation, respectively. $w_t \equiv [r_{e,t+1}, r_{f,t+1}, c_{t+1}/c_t, x_t']]$.

To test restrictions (55), I use monthly US data from March 1967 to September 2008. For consumption, the data is taken from the St. Louis FED web page and for returns from the European Central Bank. The weighting matrix estimate S is robust to heteroskedasticty (White correction) and autocorrelation (Barlett kernel). The parameter estimates are as follows:

Parameter	Estimate	Error	t-statistic	P-value
β	.87	.262E-02	331.30	[.00]
γ	-1.38	1.65	-.83	[.40]

We can see that the risk aversion is negative though theory suggests it is positive. The sign is not a problem however since it is insignificant. This seems to suggest that a highly significant risk aversion coefficient is not needed anymore to match the data, especially after a big drop in stock prices in October 2008. On the other hand, the Hansen J statistic is 153.82.It has a chi-square distribution with 10-2=8 degrees of freedom. The corresponding p-value for the test of over-identifying restrictions is then 0, which means that the model is still rejected. Therefore the equity premium puzzle is weaker than before but it has not quite yet disappeared.

We have seen the impact of the current global financial crisis on the empirical validity of the equity premium puzzle. We can only speculate if there will be changes in the behavior of asset prices characterized in the present study. Reversion to the mean is likely to be strengthened for stocks, bonds, and real estate returns, with a probable downward shift of the mean in all cases. The relationship among all three asset classes has proved to be stronger then expected. In other words, the mutual correlations are much higher than previously thought. There are limits to diversification since the systemic risk is a big component of the asset prices. Theoretically, there is going to be a bigger focus on macroeconomic factors and on joint modelling of stocks, bonds, and housing returns, which are discussed in detail in the next subsection.

33.2 Housing in asset pricing models

Modelling housing is not trivial due to the two-sided character of housing. First, housing is a durable consumption good. Second, housing is an asset where households can invest their wealth. In addition, there are large transaction costs related to the changing of amount of housing consumption. Modelling becomes even more complex if we analyze renting vs. owning and take into account the presence of credit constraints. A complex life-cycle model, which addresses all the above mentioned issues is offered in Li and Yao (2004). This model is designed to study the impact of changes in housing prices on consumption and welfare of households and is ideally suited to analyze the impact of bursting real estate prices. However, such models are very complex and to some extent become black boxes. Alternatives are concentration of particular features of housing and usage of reduced form econometric models. Flavin and Nakagawa (2004) focus on the implications of a particular joint process driving asset prices including housing returns in combination with habit persistence. Piazzessi, Schneider, and Tuzel (2007) use the CCAPM to study implications of fluctuations of the relative share of housing in a household's composition of its consumption basket (i.e. composition risk) for stock prices.

The closest to the present work is Flavin and Nakagawa (2004). It also considers time non-separable preferences, similarly to Chapters IV and V. In addition, it discusses exogenous process for asset prices. The tri-variate Markov process in Chapter VI is estimated for stock returns, property returns, and consumption. However, it can be estimated using stock returns in excess of the risk free rate, bond excess returns, and property excess returns. The parameters of the estimated tri-variate process can be used to calibriate an asset pricing model similar to the model in Flavin and Nakagawa

(2004). The asset returns would be therefore modelled more realistically and off-diagonal elements of the covariance matrix of the three processes would not be zero. This is extremely important because the rapidly declining housing prices were quickly followed by bearish stock markets and by a credit crunch affecting the bond market. An outcome of this exercise would be a realistic model, which could be also used as a vehicle to analyze the impact of housing returns on consumption and various other variables. The predictions of the model could be tested using a microeconometric approach in the style of Campbell and Cocco (2007) who use a pseudo-panel of UK households to study a housing wealth effect. The asset pricing theory is likely to develop in a direction, which considers and/or combines both types of analysis.

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