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Testing for Latent Factors in Models with Autocorrelation and Heteroskedasticity of Unknown Form

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This article considers the problem of testing for latent factors or reduced rank in a broad class of (multivariate linear stationary) time-series models, wherein model errors have autocorrelation and heteroskedasticity of unknown form. It is easy to motivate these models and methods in the context of finance models, and we illustrate with a familiar macromodel of asset returns, proposed previously by Chen, Roll, and Ross. Unfortunately, previously used tests for reduced rank are not sufficiently robust, so we examine two heteroskedasticity and autocorrelation-consistent (HAC) methods, a HAC version of Hansen's GMM test and a lesser known but more user-friendly minimum-distance or ratio of asymptotic densities (RAD) test. We recommend the RAD test, for which we supply computer code. In application, the tests lend more HAC-robust support to the hypothesis that multiple factors drive the link between the macroeconomy and the returns on bonds and stocks.

JEL Classification: G12, C3

1. Introduction

Empirical research in financial economics frequently suggests the existence of few latent factors driving the systematic component of asset returns. Existence of such latent factors makes it easier to understand the effect on asset returns of the many variables that comprise the systematic component. Results depend on the number and type of assets used and the number and types of instruments, which themselves serve as proxies for the latent factors (for examples, see Campbell 1987; Zhou 1995; and Costa, Gardini, and Paruolo 1997). In econometric terms, the existence of latent factors translates into a reduced rank restriction on the (array of) coefficients in an asset return regression system.

The present work considers the problem of testing for latent factors in a broad class of (multivariate linear stationary) time-series models, wherein model errors have autocorrelation and heteroskedasticity of unknown form. The generality of error dynamics is well suited to financial models of bond and stock returns, as in the macro model of Chen, Roll, and Ross (1986). To deal with such generality, we consider heteroskedasticity and autocorrelation consistent (HAC) methods, a HAC version of Hansen's (1982) Generalized Method of Moments (GMM) test and a lesser-known but more user-friendly minimum distance or ratio of asymptotic densities (RAD) test.

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The primary benefit of using a HAC-type test of economic hypotheses, in time-series models, is a certain kind of increased robustness relative to tests that rely on parametric assumptions about error dynamics. This robustness implies that stated significance levels of HAC tests are frequently closer to their true values, at least in sufficiently large samples. So, for the financial economist who wants to know "Are there really multiple factors driving the link between the macroeconomy and the returns on bonds and stocks?" HAC tests (and the underlying mental exercise regarding error dynamics) give added insight. HAC tests may or may not agree with less robust tests. In our application, the HAC test results agree with the results of simpler (and more popular) implementation of Hansen's (1982) test for reduced rank, but the crucial point is that stated significance levels in the popular version of Hansen's test are not correct, in statistical terms, when the data have dynamics that cause residual serial correlation. Hence, to say that the HAC tests agree with the popular version of Hansen's test is really too liberal an interpretation; more accurate is to say that the nominal (but likely invalid) conclusions from the popular test coincide with that of the autocorrelation-robust tests.

The HAC Hansen test and the RAD test each require some special calculation, and for this we do some programming. The computational complexity is due partly to the presence of corrections for residual autocorrelation and heteroskedasticity, and if instead we assumed that model errors were independent and identically distributed, then we could test for reduced rank via Anderson's (1951) convenient likelihood ratio (LR) test (see Reinsel and Velu [1998] for discussion and Zhou [1994] for a related test couched in GMM terms). It is, of course, possible to extend Anderson's LR test to (parametric) probability models with autocorrelated errors (see Reinsel and Velu 1998), but this approach relies on a correctly specified error dynamic. We instead take the nonparametric HAC approach, allowing a wider variety of error dynamics.

Is special calculation really necessary for our testing purposes? In application to asset pricing models, Zhou (1994) gives an interesting modification of Hansen's (1982) testing approach, with an analytical (hence computationally convenient) test for latent factors in asset returns. However, to implement his analytical test, Zhou relies on parametric assumptions about the model's error dynamics. Specifically, he considers the case of white noise errors and also the case of errors that are uncorrelated but have a known (parametric) form of conditional heteroskedasticity. The method can, in principle, be extended to models with a parametric form of error autocorrelation, and while this is also the case with Anderson's analytical LR test, both methods are necessarily parametric. Because we pursue instead a nonparametric HAC objective, we undertake a computationally harder problem.

Comparing the HAC robust Hansen-type and RAD tests for reduced rank, the latter is much easier to implement with full flexibility regarding the reduced rank form, that is, selection of linearly independent matrix rows in the reduced-rank coefficient matrix. For this reason, the RAD test is more user friendly than the robust Hansen test and, with it, we obtain conclusions robust to the form of reduced rank as well as the form of error dynamics. Both are calculated by minimizing a quadratic form with the optimal weighting matrix given by the inverse of a relevant covariance matrix. Covariance-matrix estimation is made robust to both heteroskedasticity and autocorrelation via various nonparametric and parametric methods. We consider several kernel-based heteroskedasticity and autocorrelation-consistent (HAC) procedures, with various combinations of kernel, bandwidth selection, and prewhitening filter (see Newey and West 1987, 1994; Andrews 1991; Andrews and Monahan 1992). We also implement a simple parametric procedure with prewhitening advocated by den Haan and Levin (1997). For the Hansen test, we follow Hansen, Heaton, and Yaron (1996), simultaneously iterating over both the weighting matrix and model parameters. For the RAD test, such simultaneous iteration is unnecessary.

Our empirical study, like the influential work of Chen, Roll, and Ross (1986), looks at the link between asset markets and macroeconomic fundamentals. As dependent variables in our regression system, we choose a set of excess returns broadly characterizing the U.S. bond and stock markets over the last four decades. Specifically, we use monthly excess returns on the Treasury securities of maturities of 90 days and 5 years. To capture main features of the U.S. stock markets, we sort returns by firm size and use excess returns on the Center for Research in Security Prices (CRSP) small capitalization and large capitalization portfolios. There are many candidates for our explanatory variables. Given our dependent variables (a small number of both stock and bond returns), we do not consider size and book-to-market-related portfolios of Fama and French 1993 (henceforth FF) and term and default premia (FF; Chen, Roll, and Ross 1986). The size-related variables are mostly used in studies focusing on stocks; also, in those studies, a large number of assets is involved. The problem with bond-related factors is that we might run into econometric problems with term structure-related dependent and independent variables.[1] Moreover, while the FF factors do a good job in explaining cross-sectional and time-series variation of stock returns, they are somewhat *ad hoc*. Hence, we focus on macroeconomic variables that can be theoretically linked (at least loosely) to expected returns. We are left with a subset of risk factors similar to Chen, Roll, and Ross (1986), consisting of the stock market, consumption, industrial production, money supply, and the unexpected inflation rate.

The models of asset returns with macroeconomic explanatory variables are well specified statistically, provided that the used time series are stationary. From the theoretical perspective, the models used can be considered to be examples of the intertemporal capital asset pricing model (CAPM), in which case the test for reduced rank is the test for the number of latent risk factors inherent in this model.[2] If we wanted to interpret the test of our model as a test of the arbitrage pricing theory, all the explanatory variables would have to be excess returns on assets. Interpreting the growth rate of consumption and the expected inflation rate literally as asset returns is problematic but the return on the stock market is obviously an asset return, the industrial production growth rate may be viewed as a return on physical assets, and the growth rate of money as a (negative) return on cash holdings (due to inflation).

We analyze the rank structure in the bond and stock markets, both separately and jointly. We first document the presence of autocorrelation and heteroskedasticity in residuals of all unrestricted regression systems, using a battery of tests. Then we apply the RAD and the Hansen tests (as a benchmark) with several HAC robust covariance-matrix estimators. We find that the one-factor hypothesis is rejected both in the bond market and the stock market. A joint estimation and tests of the stock and bond markets do not alter these results—statistically, at least four factors are needed for an accurate description of both markets. The sources of differences between bonds of different maturity can be traced to significantly different market and industrial production betas, suggesting (consistently with existing theory; see Campbell [1999] for a survey on stylized facts regarding various premia) that there is a term premium mainly due to a higher sensitivity to the market risk and to business cycles. For stocks of different firm size, such differences can be traced to consumption and monetary betas. This confirms that there are differences between stocks of different sizes but in a sense diverging from the literature because the returns in our sample actually do not statistically differ,[3] only the betas do. Presence of consumption in our data is motivated by the consumption-based CAPM and the higher consumption beta suggests that small firms are riskier from this perspective.

[1] This is to some extent true for the size-related factors as well; see FF for a detailed discussion.

[2] A special case of this model uses the stock market return as the only explanatory variable is the standard CAPM.

[3] Similar findings of the diminishing firm size effect are reported by FF and by Horowitz, Loughran, and Savin (2000).

Differing monetary betas may be due to greater sensitivity to tighter monetary policy of small firms (see Gertler and Gilchrist 1994). The differences between bonds and stocks confirm the equity premium puzzle. However, the perspective is rather novel in this case, as we combine reduced-rank tests with cross-asset Wald tests.

The paper is organized as follows. Section 2 introduces the unrestricted and restricted asset pricing models, section 3 describes the Hansen tests and the generalized Wald tests, section 4 discusses the data selection and data sources, and section 5 reports our results. Section 6 concludes.

2. Model

We are interested in testing for latent factors in a broad class of multivariate linear models. However, to make the exposition more readable for the general economist, we will couch our discussion in the specific context of asset-pricing models. The formal setup in Equation 1 is still quite general, representing as it does a multivariate linear relationship between some (dependent) variable y and other variables (x), so the same formal model and methods can be applied to other kinds of economic data.

For a collection of assets, let $y_{1t}, \ldots, y_{nt}$ denote the (excess) returns to holding each asset from time $t-1$ to time t. The regression model of interest is

$$y_t = \beta x_t + \varepsilon_t, \qquad t = 1, \ldots, T, \tag{1}$$

where β is an $n \times K$ coefficient matrix, with $n < K$, and ε_t is an $n \times 1$ vector of regression errors for which $E[\varepsilon_t \,|\, x_t] = 0$. x_t is a $K \times 1$ vector of observables that may include a constant and such factors as the market excess return, consumption growth, etc. (see section 3 for details). The series y_t, x_t, ε_t are presumed stationary and conformable to standard central limit theory, and the errors ε_t can exhibit conditional heteroskedasticity and/or autocorrelation, in the usual manner described in White (1984) and Davidson (1994, 2000), for example.

Consider, in addition to the regression model (1), the following latent factor specification of the conditional mean of asset returns:

$$E[y_{it} \,|\, x_{it}] = \gamma_i E[z_t \,|\, x_{it}], \qquad i = 1, \ldots, n, \quad t = 1, 2, \ldots, T, \tag{2}$$

where z_t is a $q \times 1$ vector of unobserved latent factors, for some $q < K$, and γ_i, $i = 1, \ldots, n$, are their $1 \times q$ coefficient vectors. One can likewise specify a linear relationship between z and x,

$$E[z_{jt} \,|\, x_t] = \lambda_j x_t, \qquad j = 1, \ldots, q, \quad t = 1, \ldots, T \tag{3}$$

for some $1 \times K$ vectors $\lambda_1, \ldots, \lambda_q$. In this case, the observable variables in vector x_t serve as proxies of the underlying latent factors z_t. The general null hypothesis is reduced rank $q < K$ for the matrix β. This reduced rank hypothesis is, equivalently, expressible as

$$H_0\text{: } \beta = \gamma\lambda \text{ for some } n \times q \text{ matrix } \gamma \text{ and some } q \times K \text{ matrix } \lambda. \tag{4}$$

To test H_0 via the Hansen (1982) approach, it is common to impose further parameter identification, as in Campbell (1987) and Ferson and Foerster (1994). We will follow this approach to implementing the Hansen test due to the tremendous computational simplification it affords. At the same time, we will avoid these additional identifications when using the alternative RAD testing approach.

To further identify parameters under H_0, we can optionally specify $\gamma = [I\delta]'\lambda$, with I the $q \times q$ identity matrix and δ some $q \times (n - q)$ matrix, in which case the specialized null hypothesis is

H_0^*: $\beta = [I\delta]'\lambda$ for some $q \times q$ identity matrix I, $q \times (n - q)$ matrix δ, and $q \times K$ matrix λ.

While hypothesis H_0^* is clearly stronger than H_0, it is actually a very common normalization to impose in reduced-rank regression models (of which our financial models are special cases) and is typically invoked when reporting estimates of such models in economics (as in the cointegration output of Eviews software). If we are worried that H_0^* is too strong, we may prefer to put more weight on the RAD test rather than the version of Hansen's (1982) test under study.

3. Tests

In this section, we describe two tests for reduced rank. We first define the test statistics, then describe their computation, which, in each case, can be carried out using GMM. We first define the minimum-distance or RAD test of interest. The RAD test statistic is

$$W = \min_{\beta \in H_0} (\mathrm{vec}(\hat{\beta}) - \mathrm{vec}(\beta))' \hat{\Omega}^{-1}_{\mathrm{vec}(\hat{\beta})} (\mathrm{vec}(\hat{\beta}) - \mathrm{vec}(\beta)), \tag{5}$$

where $\mathrm{vec}(\hat{\beta})$ is the unconstrained ordinary least square (OLS) estimator and $\hat{\Omega}_{\mathrm{vec}(\hat{\beta})}$ is the associated GMM covariance matrix estimate, both implemented via heteroskedasticity and autocorrelation-robust covariance estimators. The RAD test can be viewed as a special case of a very interesting (but little known) general econometric test proposed by Szroeter (1983).

The RAD test is a minimum-distance test, in that it is based on the minimum squared distance between $\hat{\beta}$ and the reduced-rank approximations to $\hat{\beta}$. This minimum-squared distance can also be interpreted as (proportional to the log of) the ratio of constrained and unconstrained approximate (normal) densities for the parameter estimator $\hat{\beta}$, with constraint given by the reduced-rank restriction (see Gilbert and Zemčík 2004). Hence, the label RAD (ratio of asymptotic densities) is fitting and also intuitive in its similarity to the likelihood LR test statistic. An advantage of RAD over LR is that we can make RAD robust to error autocorrelation and heteroskedasticity of unknown form via a HAC form for $\hat{\Omega}_{\mathrm{vec}(\hat{\beta})}$, whereas consistency of LR requires a known form of error dynamics.

Next we define the relevant Hansen (1982) tests. For each given value of β in Equation 1, define

$$e_{it} = y_{it} - \beta_i x_t, \qquad i = 1, \ldots, n, \quad t = 1, \ldots, T. \tag{6}$$

Relevant sample moments take the form

$$m(\beta)_{ij} = \frac{1}{T} \sum_{t=1}^{T} e_{it}\, u_{jt}, \qquad i = 1, \ldots, n, \quad j = 1, \ldots, L, \tag{7}$$

with u_{jt}, $j = 1, \ldots, L$, is a set of instruments.

The general Hansen (1982) GMM statistic is defined as

$$S = \min_{\beta \in H_0^*} \operatorname{vec}(m(\beta))' \hat{\Omega}^{-1}_{\operatorname{vec}(m(\beta))} \operatorname{vec}(m(\beta)), \tag{8}$$

where $\hat{\Omega}_{\operatorname{vec}(m(\beta))}$ is the GMM covariance-matrix estimator, which we later specify in ways robust to heteroskedasticity and serial correlation. To arrive at the solution S to Equation 8, we iterate over repeated trials, at each stage, simultaneously solving for updated parameter and covariance-matrix estimates, as in Hansen et al. (1996).

Under the null hypothesis (H_0 or H_0^*) and suitable regularity conditions (stationarity, finite moments, mixing, etc., as in White [1984] and Davidson [1994, 2000], for example), both the Hansen test S and RAD test W can be shown to be distributed asymptotically as chi-square variables with $(n - q)(K - 1)$ degrees of freedom. Hansen (1982) shows this result for S under general conditions (and for recent discussion, see Harris and Mátyás [1999]). For the RAD test, Szroeter (1983) provides some general (but quite abstract) theory, and Gouriéroux and Monfort (1989) give a somewhat more streamlined and intuitive version of this broad theory. In specific application to reduced-rank linear models, Gilbert and Zemčík (2004) give an extensive theoretical description of the RAD test.

To summarize briefly the logic of proving the asymptotic chi-square distribution of W: (a) By construction, the statistic W is obtained as the minimum (the Malhanobis) distance between an unconstrained parameter vector (consisting of $\hat{\beta}$ elements) and a set of candidate values $\beta \in H_0$, (b) the estimator $\hat{\beta}$ is itself assumed to be consistent and asymptotically normal, with a covariance matrix $\boldsymbol{\Omega}_{\operatorname{vec}(\hat{\beta})}$, (c) the consistent estimator $\hat{\Omega}_{\operatorname{vec}(\hat{\beta})}$ of $\boldsymbol{\Omega}_{\operatorname{vec}(\hat{\beta})}$ is also the distance-defining matrix appearing in the Malanobis distance function. We can then view the constrained estimator $\tilde{\beta}$ of β as a function of $\hat{\beta}$ and $\hat{\Omega}_{\operatorname{vec}(\hat{\beta})}$, and applying the Delta (asymptotic expansion) method (see, for example, van der Vaart [1998]) to $\tilde{\beta}$, we can readily obtain a first-order normal approximation to the difference $\tilde{\beta} - \hat{\beta}$, and from this conclude that W is asymptotically chi square. For other, more exhaustive proofs of the asymptotic normality of the minimized (square) distance objective function, see Dahm and Fuller (1986), Cragg and Donald (1995), and Gilbert and Zemčík (2004).

To compute the Hansen (simultaneous-iterated) J-type test S, we use the GMM routine in EViews 3.1, with a variety of choices for the covariance estimation method. To compute the RAD test W, we use a simple and convenient iterative method to get an alternating sequence of γ and λ estimates, which are needed to estimate β under H_0. The Appendix contains a derivation of the required mathematical formulae. Alternatively, we could use the Newton–Raphson method of Ahn and Reinsel (1988, 1990). At each stage of the iteration, we hold fixed the current estimate (γ or λ) and solve the quadratic optimization problem (5) for the remaining parameters. We start with the initial values

$$\hat{\lambda}_i = \hat{\beta}_i, \qquad i = 1, \ldots, q. \tag{9}$$

Holding fixed the initial choice of $\hat{\lambda}$, we solve the quadratic problem in Equation 5 in terms of γ:

$$\operatorname{vec}(\hat{\gamma}') = [(I_n \otimes \hat{\lambda}')' \hat{\Omega}^{-1}_{\operatorname{vec}(\hat{\beta}')} (I_n \otimes \hat{\lambda}')]^{-1} (I_n \otimes \hat{\lambda}')' \hat{\Omega}^{-1}_{\operatorname{vec}(\hat{\beta}')} \operatorname{vec}(\hat{\beta}'). \tag{10}$$

We express the solution in terms of transposes γ', β', and λ' due to the fact that the covariance matrix $\hat{\Omega}_{\operatorname{vec}(\hat{\beta}')}$ is readily obtained from standard regression packages, whereas $\hat{\Omega}_{\operatorname{vec}(\hat{\beta})}$ is equivalent but requires extra computation.

Next, having computed the first-round choice $\hat{\gamma}$, we hold it fixed and solve the quadratic problem in Equation 5 in terms of λ,

$$\text{vec}(\hat{\lambda}') = [(\hat{\gamma} \otimes I_K)'\hat{\Omega}^{-1}_{\text{vec}(\hat{\beta}')}(\hat{\gamma} \otimes I_K)]^{-1}(\hat{\gamma} \otimes I_K)'\hat{\Omega}^{-1}_{\text{vec}(\hat{\beta}')}\text{vec}(\hat{\beta}'). \tag{11}$$

We repeat the alternating updates of $\hat{\gamma}$ and $\hat{\lambda}$ numerous times (at least 10). To ensure convergence, at each round of the sequence, we normalize the matrix γ by dividing each of its elements by its upper left element. We carry out these computations using a program we wrote in the EViews 3.1 environment. This program is available from the authors upon request and is able to handle more general reduced-rank structures where only some of the regressors are restricted.

We estimate the covariance matrices needed to calculate S and W in a number of different ways. While we use mainly HAC-based estimators, for comparison purposes, we also examine the White heteroskedasticity-consistent estimator (denoted White). Ferson and Foerster (1994) study the finite sample properties of the Hansen/White test for reduced rank equal to 1 and 2, as implied by various versions of the CAPM. As for HAC methods, we include ones based on the Bartlett kernel and the data-dependent Newey and West (1994) bandwidth, with and without prewhitening (denoted NW and NW-P, respectively). We also include the quadratic spectral kernel with the Andrews (1991) data-dependent bandwidth (without prewhitening, denoted A), and the Andrews and Monahan (1992) method (denoted AM) with prewhitening. We have also examined the simple prewhitening method studied by den Haan and Levin (1997) (VARHAC). We have spot checked some of our EViews-based computations using a Gauss code written by Hansen, Heaton, and Okagi, and we have noticed that EViews 3.1 versions prior to June 2000 appear to have an error in computing some of the J-tests, but versions June 2000 and later do not have this problem, as confirmed by the EViews technical staff.

Den Haan and Levin (1997) study finite sample properties of kernel-based and parametric covariance-matrix estimators in a single-equation context with complex serial correlation structures. Their Monte Carlo experiments favor a simple parametric method with prewhitening—VARHAC. In computations, which we omit here for brevity, we extend their simulations to systems of equations in which we study the small sample rejection rates of the S and W statistics under the hypotheses of reduced and full rank. Our results also support the use of the VARHAC method in most cases. In addition, we find that some finite-sample properties (including empirical size) worsen as we increase the number of equations, increase the number of explanatory variables, increase rank and decrease the sample size. We conclude that, for a sample size T of about 500 (our number of observations), four equations and six explanatory variables (including a constant term), the S and W statistics are reasonably close to the chi-square distribution under the null.

4. Data

Most studies focusing on factor models of expected asset returns either assume or ultimately conclude that the number of factors equals 1, 2, or 3 (see Campbell 1987; Ferson and Foerster 1994; Zhou 1995; Backus, Foresi, and Telmer 1998; de Jong 1998; Dai and Singleton 2000). To test for up to three factors, we need at least four asset returns. Therefore, we choose two bond returns and two stock returns to characterize the bond and stock markets, respectively.

Treasury 90-day Bills and 5-year Bonds seem a natural choice as representatives for the bond market. While there is no difference in the default risk, there is a difference in levels of risk due to differing maturities. The data source is CRSP (indno 1000707 for the 90-day T-bill and 1000704 for the 5-year T-bond) and we subtract the 30-day T-bill rate (indno 1000708) to get excess returns r_{T90}

Table 1. Summary Statistics

	r_{T90}	r_{T5}	r_{SMALL}	r_{LARGE}	r_{SP}	g_C	g_I	g_M	π
Mean	0.58	1.44	8.13	6.21	6.50	2.09	3.45	6.55	4.30
Standard deviation	1.21	18.69	66.54	50.49	50.85	5.56	10.47	5.56	3.92
Skewness	2.36	0.20	−0.17	−0.37	−0.36	−0.25	−0.10	0.17	0.92
Kurtosis	18.63	7.19	7.38	5.15	5.06	4.00	9.08	12.71	4.28
Correlation									
r_{T90}	1.00	0.62	0.13	0.11	0.11	−0.02	−0.20	−0.05	−0.02
r_{T5}		1.00	0.16	0.25	0.24	0.01	−0.17	−0.06	−0.11
r_{SMALL}			1.00	0.85	0.82	0.21	−0.03	−0.04	−0.17
r_{LARGE}				1.00	0.99	0.15	−0.02	0.00	−0.18
r_{SP}					1.00	0.15	−0.02	0.01	−0.17
g_C						1.00	0.15	0.10	−0.45
g_I							1.00	0.02	−0.12
g_M								1.00	0.02
π									1.00

Sample 1959:02–2000:11, r_{T90} and r_{T5} are excess returns on 90-day T-bills and 5-year T-bonds; r_{SMALL} and r_{LARGE} denote the excess returns on the small-cap and large-cap portfolios; r_{SP} is the excess return on the market portfolio; g_I, g_C, and g_M are growth rates of real per-capita consumption, industrial production, and money supply, respectively. π denotes the inflation rate. Reported numbers for means and standard deviations are annualized, in percentages.

and r_{T5}, respectively. The data frequency is monthly, the sample period 1959:02–2000:11 is given by the availability of per capita consumption series (see below) and the summary statistics are in Table 1. As expected, the rate of return on the longer maturity bond is higher and so is the corresponding risk level as measured by the standard deviation.

To capture the basic features of the stock market, we need stock returns covering a wide range of stocks but with different risk characteristics. We use CRSP NYSE Portfolio Indices ranked by capitalization, combining deciles 1–5 for the large firms (indno 1000314) and 6–10 for the small firms (indno 1000315). These monthly time series are based on quarterly rebalanced portfolios. Excess returns are again calculated using the 30-day Treasury bill return and we denote them as r_{LARGE} and r_{SMALL}, respectively. The summary statistics in Table 1 indicate an overall higher level of both return and risk for small firms. The excess return on small caps is more volatile (consistent with Malkiel and Xu [1997], for example) and this feature is independent of the chosen time period. On the other hand, the mean excess return is actually greater for large firms since 1980, a trend noticed by Fama and French (1993) and carefully documented by Horowitz, Loughran, and Savin (2000).

As covariates, we opt for variables for which there is an established theoretical link to expected excess returns. This excludes the size-related stock market factors (see Fama and French 1993; Chan and Chen 1991) and term structure and default risk-related bond factors (see Chen et al. 1986; Fama and French 1993). Inclusion of these variables could also lead to econometric problems, with firm size and term-structure-related dependent and independent variables.

Both the static CAPM and the intertemporal CAPM suggest the use of the market excess return as an explanatory variable. We use the CRSP value-weighted index of the S&P 500 Universe (indno 1000502) in excess of the 30-day T-bill (r_{SP}). The time series characteristics in Table 1 are similar to those of r_{LARGE} due to the fact that the value weighted index is dominated by large firms.

The consumption CAPM and business-cycle models specify the relationship between expected returns, consumption, and production. Hence, we include the growth rates of industrial production (g_{IP}) and real per capita consumption of nondurables and services (g_{CONS}) as measures of real economic activity. We obtain both series from the St. Louis Fed's website. Specifically, we use the variable

INDPRO to calculate g_{I} and variables PCEND, PCES, and POP to calculate g_{C}. The per-capita consumption is adjusted to inflation using the CRSP price index (indno 1000709). The FED series are seasonally adjusted. Interestingly, the correlation of consumption with stock-market variables (see Table 1) is greater than with bond-market variables, and the opposite is true for industrial production.

We also include the monetary growth as one of the explanatory variables. The link between expected returns and money can be motivated by overlapping generations models (see Brock [1990] for a survey), models with money in the utility function (Brock 1975), and models with the cash-in-advance constraint (Svensson 1985).[4] We use the seasonally adjusted monetary base series from the St. Louis Fed's website (series AMBSL) to calculate the monetary growth rate, g_{M}, which seems to move more closely with the bond market than the stock market (see Table 1).

Finally, we control for the effect of inflation. Because inflation happens to be the only nonstationary series (the augmented Dickey Fuller test does nor reject the unit root), we use its first difference, which is the unexpected inflation.

5. Empirical Results

In this section, we investigate the latent variable structure of the bond market, the stock market, and the market for both bonds and stocks. In each case, we first estimate the unrestricted model of the form (1), test corresponding residuals for heteroskedasticity and correlation, and then conduct tests for reduced rank. In this application, the afore-mentioned GMM setup will reduce to a simple method of moments setup, with u consisting of regressors x and with $L = K$. We report reduced-rank test results for both Hansen and RAD tests with VARHAC covariance matrix. However, our results are robust with respect to the choice of covariance matrix estimators described in section 3 (i.e., besides VARHAC, we also use White, NW, NW-P, A, and AM).

Bond Market

The estimates of the unrestricted model for expected bond excess returns are reported in panel A of Table 2. The intercepts are small but significantly different from zero, indicating presence of a term premium unexplained by the simplistic asset pricing model. A Wald test for zero intercepts in both equations at once can be loosely interpreted as a test of an asset pricing model—see Gibbons, Ross, and Shanken (1989); Fama and French (1993, 1996); and Cochrane (2001). The Wald test statistic is, in this case, 17.10 and the corresponding p-value is 0, so our explanatory variables themselves do not entirely explain the time-series behavior of bond returns, a result consistent with the notion of the risk-free rate puzzle (see Campbell 1999, for example). Because we can think of our risk factors as proxies for the underlying latent factors, this should not undermine our reduced-rank analysis.

According to CAPM and the intertemporal CAPM, the beta of excess market return is expected to be close to zero because the default risk is presumably very small for U.S. government bonds. The beta is likely to be higher for excess returns on bonds with higher maturity where the differences in the overall risk level increase. Panel A of Table 2 confirms this prediction with the market beta being insignificant for r_{T90} and somewhat larger and significant for r_{T5}. The

[4] As Feenstra (1986) shows, the cash-in-advance models can be interpreted as a special case of the money in utility function models.

Table 2. Bond Market

Panel A: Unrestricted Model

	Interest	r_{SP}	g_C	g_I	g_M	$\Delta\pi$	R^2
r_{T90}	0.00060	0.00242	−0.00473	−0.02242	−0.01058	−0.02166	0.05807
	(0.00012)	(0.00182)	(0.01572)	(0.01069)	(0.01167)	(0.02428)	
r_{T5}	0.00276	0.08835	−0.00533	−0.29870	−0.21372	−0.07272	0.09216
	(0.00138)	(0.02322)	(0.21187)	(0.09078)	(0.16409)	(0.26758)	

Panel B: Tests for Residual Heteroscedasticity and Correlation

Correlation	Across equations		Pearson	0.60 (0.02)
	Across time	r_{T90}	Q	35.11 (0.00)
		r_{T5}	Q	21.36 (0.05)
Heteroskedasticity		r_{T90}	White	8.15 (0.00)
		r_{T5}	White	5.33 (0.00)

Panel C: Reduced Rank Tests

Test	Rank 1
Hansen	30.23 (0.00)
RAD	21.00 (0.00)

The estimated model for panel A is $y_{it} = \beta_i x_t + \varepsilon_{it}$, $i = 1, 2$, sample 1959:02–2000:11, where $y_{1t} = r_{T90}$, $y_{2t} = r_{T5}$, β_i is a (6×1) vector of coefficients, $x_t = (\text{intercept}, r_{SP}, g_I, g_C, g_M, \Delta\pi)'$ and ε_{it} is the regression error. r_{T90} and r_{T5} are excess returns on 90-day T-bills and 5-year T-bonds, r_{SP} is the excess return on the market portfolio, g_I, g_C, and g_M are growth rates of real per capita consumption, industrial production, and money supply, respectively. $\Delta\pi$ is the first difference in the inflation rate. We report OLS coefficient estimates, and p-values in parentheses are calculated using the VARHAC standard errors.

Residuals in panel B are calculated using OLS estimates, equation by equation; Pearson = chi-square test for correlation; White test = F-test with no cross terms; Q = Q-statistic for testing 12 lags of autocorrelation; p-values in parentheses.

Calculation for panel C conducted using the VARHAC covariance matrix estimator. p-values in parentheses.

consumption growth rate is insignificant in both equations, and this is consistent with first-order conditions of the consumer optimization problem (in the power utility consumption CAPM) only for large risk-aversion coefficients (see Eqn. 1.16 in Cochrane [2001]). The beta for industrial production is significantly negative for both types of bonds, reflecting the fact that industrial production is a leading indicator for output. Because we consider a multiple regression that includes industrial production in addition to the market excess return, the industrial production beta characterizes reaction of returns to output fluctuations unusually large for a given level of market return. Bond prices are typically higher earlier in contractions, which pushes down the next period's interest rates and returns. Bonds with higher maturity seem to be more sensitive to business cycles. The sign of the monetary beta is in accord with a simple intuition of lower interest rates as a result of increasing the money supply, but this beta is insignificantly different from zero. The estimate of the expected inflation coefficient is also insignificantly different from zero, thus suggesting that, while the expected inflation affects the returns according the Fisher equation, it does not influence excess returns.

Preliminary to testing for reduced rank, we first document the need for the HAC robust methodology by testing for heteroskedasticity and various forms of correlation in the regression residuals of our unrestricted model. Panel B of Table 2 indicates the residuals are correlated across equations and time and heteroskedastic, thus justifying our HAC robust estimation methods. The Hansen and RAD tests of the null hypothesis of rank = 1 are then conducted using the VARHAC covariance-matrix estimator.

For the Hansen test (but not the RAD test), for ease of computation, we apply the more specialized version H_0^* of reduced-rank hypothesis H_0, via the specification

$$\begin{aligned} y_{1t} &= \lambda x_t + \varepsilon_{1t}, \\ y_{2t} &= \delta y_{1t} + \varepsilon_{2t}. \end{aligned} \tag{12}$$

Here, the first asset serves as the reference asset, through which we can describe (up to a multiple δ) the dependence of y_2 on x.

For data, we use the sample 1959:02–2000:11, where $y_{1t} = r_{T90}$, $y_{2t} = r_{T5}$, λ is a (6×1) vector of coefficients, $x_t = (1, r_{SP}, g_I, g_C, g_M, \pi_{UI})'$, ε_{it} is the regression error, and δ is the multiple coefficient characterizing the different sensitivity of the second asset. We also use the vector x_t as instruments in the J-test.

Both tests strongly reject the (respective) null hypothesis—see panel C of Table 2. These results suggest that behavior is very different for the two government bonds even though the only source of difference in risk is the maturity term. More than one macroeconomic factor is needed to explain the cross-section of expected bond returns. Implicitly, the term premium is thus characterized by at least two underlying factors. To identify potential sources for differences between bonds of different maturities, we run simple Wald tests of equality of individual coefficients across equations. The equality of coefficients is only rejected for the market and industrial production betas, which were also the only sensitivities statistically different from zero in our unrestricted model.

While we focus on bond excess returns, our study is complementary to research concentrated on the term structure of interest rates (Cochrane [1999] relates bond returns and interest rates with respect to the yield curve and Campbell, Lo, and MacKinlay [1997, ch. 10] provide basic formulae tying returns and yields together). For example, Backus et al. (1998) give a survey of (multi) factor models of the term structure and Ang and Piazzesi (2001) use a VAR model with macroeconomic and latent variables. The latent variables are often referred to as slope, curvature, and level factors and correspond to the shape of the yield curve. Ang and Piazzesi (2001) treat macroeconomic variables characterizing inflation and the business cycle as observable and argue that the slope and curvature factors can be related to macro factors. Consistent with our results, this leaves them with three factors needed to explain the term structure—the inflation, business cycle, and level factors.

Stock Market

Panel A of Table 3 reports the OLS estimates of betas in the unrestricted model.[5] Neither the Wald test (with the statistic equal 0.16 and corresponding p-value 0.92) nor individual t-tests can reject the hypothesis of zero intercepts for this stock return model. This is rather different than the case of bonds returns mentioned earlier.

As implied by the CAPM and the intertemporal CAPM, the market beta is positive and significant. It is higher than 1 for small firms, reflecting a higher level of risk associated with their returns. For large firms, the market beta is close to 1 due to the fact that the time series characteristics of the stock market portfolio are dominated by firms of a greater market value. The consumption beta is significant for small firms but still too small as compared with predictions of the consumption CAPM with power utility function (see Eqn. 1.16 in Cochrane [2001]), confirming the equity

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[5] The estimation of the unrestricted model is often used as the first step of the Fama and MacBeth (1973) method. For example, see Chen, Roll, and Ross (1986), who also use macroeconomic variables.

Table 3. Stock Market

Panel A: Unrestricted Model

	Interest	r_{SP}	g_{C}	g_{I}	g_{M}	$\Delta\pi$	R^2
r_{SMALL}	0.00342	1.05880	1.11352	−0.16953	−0.704334	−0.12976	0.68988
	(0.00222)	(0.05081)	(0.30487)	(0.17948)	(0.31882)	(0.46281)	
r_{LARGE}	0.00045	0.98534	−0.02430	−0.00470	−0.10313	−0.19138	0.98550
	(0.00088)	(0.00746)	(0.04867)	(0.01644)	(0.15291)	(0.09300)	

Panel B: Tests for Residual Heteroscedasticity and Correlation

Correlation	Across equations		Pearson	0.42 (0.03)
	Across time	r_{SMALL}	Q	70.17 (0.00)
		r_{LARGE}	Q	109.45 (0.00)
Heteroskedasticity		r_{SMALL}	White	6.59 (0.00)
		r_{LARGE}	White	12.11(0.00)

Panel C: Reduced Rank Tests

Test	Rank 1
Hansen	15.77 (0.01)
RAD	18.24 (0.00)

The estimated model for panel A is $y_{it} = \beta_i x_t + \varepsilon_{it}$, $i = 1, 2$, sample 1959:02–2000:11, where $y_{1t} = r_{\mathrm{SMALL}}$, $y_{2t} = r_{\mathrm{LARGE}}$, β_i is a (6×1) vector of coefficients, $x_t =$ (intercept, r_{SP}, g_{I}, g_{C}, g_{M}, $\Delta\pi)'$ and ε_{it} is the regression error. r_{SMALL} and r_{LARGE} denote the excess returns on the small-cap and large-cap portfolios r_{SP} is the excess return on the market portfolio, g_{I} , g_{C}, and g_{M} are growth rates of real per capita consumption, industrial production, and money supply, respectively. $\Delta\pi$ is the first difference in the inflation rate. We report OLS coefficient estimates, and p-values in parentheses are calculated using the VARHAC standard errors.

Residuals for panel B are calculated using OLS estimates, equation by equation; Pearson = chi-square test for correlation; White test = F-test with no cross terms; $Q = Q$ statistic for testing 12 lags of autocorrelation; p-values in parentheses.

Calculation conducted for panel C using the VARHAC covariance matrix estimator. p-values in parentheses.

premium puzzle. Industrial production is again negative but insignificant. The money betas are both negative but only the small-firm money beta significantly so. The expected inflation coefficient is positive for both portfolios and significant for small firms. A positive coefficient suggests that expected inflation affects stock returns more that it affects the riskfree rate.[6]

Turning to tests for reduced rank, we first test for residual heteroskedasticity and correlation. Results in panel B of Table 3 indicate that both are present, further justifying the use of HAC robust methods. The restricted model for the Hansen test is specified by the system (12) with $y_1 = r_{\mathrm{SMALL}}$ and $y_2 = r_{\mathrm{LARGE}}$. As shown in panel C of Table 3, the null hypothesis of rank one is strongly rejected by both the Hansen and RAD test. This result is robust to exclusion of intercepts and to the choice of the covariance-matrix estimator. Interestingly, because the signs of all coefficients are the same, this outcome is due to disproportionately greater sensitivities for small firms. Specifically, Wald tests for equality of individual coefficients across the two equations show that only consumption, monetary, and expected inflation betas differ at a 5% level of significance. The market betas are statistically indistinguishable, consistent with recent evidence suggesting that returns on stocks sorted by size may not differ as much as previously thought.[7] Our results suggest that, while differences between small and large firms are more subtle, they do exist, mainly due to quantitative variation of sensitivities to

[6] Our results are consistent with studies on firm-size effects—see Fama and French (1993, market beta), Chan, Chen, and Hsieh (1985, market and industrial production betas), and Li and Hu (1998, industrial production and money-supply betas).

[7] Among others, Horowitz, Loughran, and Savin (2000) report that no consistent relationship can be found between size and realized returns since the 1980s.

Table 4. Bond and Stock Markets Jointly

	Panel A: Tests for Residual Correlation Across Equations	
	r_{SMALL}	r_{LARGE}
r_{T90}	0.07 (0.17)	0.07 (0.18)
r_{T5}	−0.08 (0.16)	0.06 (0.20)

Panel B: Reduced Rank Test

Test	Rank 1	Rank 2	Rank 3
Hansen	87.68 (0.00)	31.85 (0.00)	15.92 (0.00)
RAD	166.60 (0.00)	27.59 (0.00)	16.84 (0.00)

Residuals for panel A are calculated using OLS estimates from hte unrestricted model (equation by equation): $y_{it} = \beta_i x_t + \varepsilon_{it}$, $i = 1, \ldots, 4$, sample 1959:02–2000:11, where $y_{1t} = r_{\text{T90}}$, $y_{2t} = r_{\text{T5}}$, $y_{3t} = r_{\text{SMALL}}$, $y_{4t} = r_{\text{LARGE}}$, β_i is a (6×1) vector of coefficients, $x_t = (\text{intercept}, r_{\text{SP}}, g_{\text{I}}, g_{\text{C}}, g_{\text{M}}, \Delta\pi)'$ and ε_{it} is the regression error. r_{T90} and r_{T5} are excess returns on 90-day T-bills and 5-year T-bonds, r_{SMALL} and r_{LARGE} denote the excess returns on the small-cap and large-cap portfolios, r_{SP} is the excess return on the market portfolio, g_{I} , g_{C}, and g_{M} are growth rates of real per capita consumption, industrial production, and money supply, respectively. $\Delta\pi$ is the first difference in the inflation rate. p-values in parentheses are calculated using the Pearson chi-square test for correlation.

Calculation conducted for panel B using the VARHAC covariance matrix estimator. p-values in parentheses.

variables other than the market excess return. Therefore, at least two factors are needed to explain both the mean excess returns on small and large firms. Similarly, Costa, Gardini and Paruolo (1997) consider monthly returns on common stocks listed on the Milan stock of Exchange and use maximum-likelihood inference in reduced-rank regression models to conclude that the number of (latent) factors appears to be greater than four.

Bond and Stock Markets Jointly

In this subsection, we consider simultaneously the bond and stock markets. Panel A of Table 4 reports residual correlation across equations—the correlations across markets are small and insignificant (the rest of the correlation matrix can be found in Tables 2 and 3). Campbell and Ammer (1993), for example, argue that the low correlation can be explained by the real interest rate and by news about future excess stock returns and inflation. Because the residual heteroskedasticity and autocorrelation tests are conducted equation by equation, they are identical to ones reported in panel B of Tables 2 and 3.

When applying the HAC-robust Hansen test (but not the RAD test), for ease of computation, we again apply a restricted version H_0^* of the latent factor structure H_0. The restricted model uses the first q assets as reference assets, as follows:

$$\begin{aligned} y_{it} &= \lambda_i x_t + \varepsilon_{it}, \qquad i = 1, 2, \ldots, q, \\ y_{jt} &= \delta_1 y_{1t} + \cdots + \delta_q y_{qt} + \varepsilon_{jt}, \qquad j = q+1, \ldots, n, \end{aligned} \tag{13}$$

where the rank $q = 1, \ldots, 3$, and $y_{1t} = r_{\text{T90}}$, $y_{2t} = r_{\text{T5}}$, $y_{3t} = r_{\text{SMALL}}$, $y_{4t} = r_{\text{LARGE}}$. λ's and x_t are defined above.

For both the Hansen test and RAD test, panel B of Table 4 indicates a strong rejection of ranks 1, 2, and, 3, that is, at least four latent factors are necessary to characterize jointly the cross-sectional and time-series behavior of expected excess returns. The rejection of the factor models with a small number of factors is representative of problems connected with accounting for the high risk-free rate and the term, bond equity, and equity premia (see Campbell [1999] for a survey).

The question arises whether this result could be anticipated given the fact that at least two factors were needed to explain expected returns for both the bond and stock markets. The answer is no because there is a possibility that both bond and stock excess returns are driven by the same two latent factors. Such a possibility is rejected by the reduced-rank analysis. The source of differences lies in different patterns of sensitivities in the two markets. While the variability in the bond returns is mostly due to differing market and industrial production betas, the variation in the stock excess returns can be traced to betas for the growth rates of consumption and money supply and expected inflation.

In a quite different modeling context, Campbell (1987) tests for reduced rank in a VAR model with bond excess returns of several maturities and the excess return on the market portfolio. As instruments, he uses lagged yield spreads. The residuals in his VAR model are heteroskedastic but not serially correlated and he uses the White/score method to find that there are at least three latent factors. We have confirmed his results in a similar setup, using both the White and HAC robust score and RAD methods, the RAD test being robust to the choice and ordering of the reference assets.[8] Moreover, we repeated his analysis for the bond market separately, later adding the only stock market variable. In the case of the bond market, two-factor null hypothesis could not be rejected and, in the case of the joint model, three factors could not be rejected.[9] This result is consistent with our tests in the four-variable model, that is, several (more than three) factors are necessary to account for the cross-sectional and time-series patterns of stocks and bonds.

Our empirical work has been directed at counting the number of latent (macro) factors in asset returns, in the spirit of Chen, Roll, and Ross (1986). Of course, for a given number of latent factors, it is important to understand the nature of these factors and the plausible ways that they might cause events in financial markets. For this, we would need to extend considerably the empirical investigation, by reporting fitted values of coefficients in the latent factor models. We leave this important work to future research.

6. Summary

In this article, we propose tests for latent factors, or reduced rank, in multivariate linear models, in the case where model errors exhibit error serial correlation and heteroskedasticity of unknown form. We considered two types of tests, a version of Hansen's (1982) GMM test and a different, more user-friendly test called the RAD test. It would be interesting to extend the analysis to include other tests robust to error dynamics, including a general test of matrix rank proposed by Gill and Lewbel (1992). Their test, while perhaps less intuitive than the RAD test, may offer some computational advantages, although in simulations, we have not yet been able to show that either test is faster to compute than the other. We provide (Eviews) programs/macros for computing the RAD test, and a convenient stand-alone Windows program will soon be available from the first author. We encourage economists to apply the RAD test for latent factors, to many kinds of economic data (finance, macro, micro, international, etc.).

While we have tried hard to achieve extra robustness in our tests, in terms of error dynamics, we rely on asymptotic theory for our test significance levels and decision rules. In small samples, asymptotic significance levels may be poor approximations, and in that case, bootstrap/simulation methods may be a useful substitute. Gilbert and Zemčík (2004) report some such simulations and, while we have not encountered serious test distortions in simulations fit to the sample sizes and

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[8] Of course, the autocorrelation robustness is not really needed in this case.

[9] For the purpose of replicating Campbell's (1987) analysis, we extend our dataset by adding several Treasury securities. Details regarding this data and our results are available upon request.

models in the present work, we can produce big distortions by drastically reducing the sample size or by drastically increasing the number of (y) variables in the model.

Appendix

The proposed algorithm for computing RAD test relies on two mathematical formulas, Equations 10 and 11 in the text. To derive these formulas, first note that, under the reduced-rank restriction H_0 in the text, we have $\hat{\beta}' = \hat{\lambda}'\hat{\gamma}'$, and applying standard rules of linear algebra (Ruud 2000, p. 925),

$$\mathrm{vec}(\hat{\lambda}'\hat{\gamma}') = (I_n \otimes \hat{\lambda}')\mathrm{vec}(\hat{\gamma}') = (\hat{\gamma} \otimes I_K)\mathrm{vec}(\hat{\lambda}').$$

To derive Equation 10, note that the desired $\hat{\gamma}$ is such that, given the fixed initial estimate 9 of λ, the first-order conditions for the quadratic problem 5 reduce to

$$\frac{\partial(\mathrm{vec}(\hat{\beta}') - \mathrm{vec}(\hat{\lambda}'\hat{\gamma}'))'}{\partial\ \mathrm{vec}(\gamma')}\hat{\Omega}^{-1}_{\mathrm{vec}(\hat{\beta}')}(\mathrm{vec}(\hat{\beta}') - \mathrm{vec}(\hat{\lambda}'\hat{\gamma}')) = 0.$$

Using the first of the standard algebra rules stated above, we obtain

$$\frac{\partial(\mathrm{vec}(\hat{\beta}') - \mathrm{vec}(\hat{\lambda}'\hat{\gamma}'))'}{\partial\ \mathrm{vec}(\gamma')} = -(I_n \otimes \hat{\lambda}')',$$

and from these last two equations, we obtain the Formula 10 in the text.

To derive Equation 11, note that the desired $\hat{\lambda}$ is such that, given the fixed estimate $\hat{\gamma}$, the first-order conditions for the quadratic problem 5 reduce to

$$\frac{\partial(\mathrm{vec}(\hat{\beta}') - \mathrm{vec}(\hat{\lambda}'\hat{\gamma}'))'}{\partial\ \mathrm{vec}(\lambda')}\hat{\Omega}^{-1}_{\mathrm{vec}(\hat{\beta}')}(\mathrm{vec}(\hat{\beta}') - \mathrm{vec}(\hat{\lambda}'\hat{\gamma}')) = 0.$$

Applying the second of the standard algebra rules stated above, we obtain

$$\frac{\partial(\mathrm{vec}(\hat{\beta}') - \mathrm{vec}(\hat{\lambda}'\hat{\gamma}'))'}{\partial\ \mathrm{vec}(\lambda')} = -(\hat{\gamma} \otimes I_K)',$$

and from these last two equations, we obtain the Formula 11 in the text.

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