An Empirical Investigation of the Consumption Based Capital Asset Pricing Model Using a Modified Variance-Ratio Test

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Abstract

A chi-square statistic is constructed that compares variance ratios and mean simple returns from data with those implied by an asset pricing model. The statistic is applied to the Consumption based Capital Asset Pricing Model with time non-separable preferences. It favors habit persistence for annual data, time-separability for quarterly data, and durability for monthly data, respectively. Introduction of time non-separability yields only a marginal improvement. The power of the test is high when alternative hypotheses are formed by varying the relative risk aversion coefficient. It is lower for alternative hypotheses generated by varying the time non-separability parameter, especially for durability. (*JEL* G12)

Introduction

The statistical test often used to investigate whether asset returns follow a random walk is based on variance ratios of returns (see Campbell, Lo and MacKinlay 1997, Chapter 2 for a survey). Variance ratios capture the autocorrelation structure of asset returns and though their statistical power to detect mean reversion is rather low, it is still higher than for alternatives such as the likelihood-ratio test or simple regressions of current returns on lagged returns (see Poterba and Summers 1988). The test developed in this paper is based on variance ratios and mean simple returns. The presence of mean simple returns ensures that the level of returns is captured together with their autocorrelation structure. I avoid the necessity of choosing the variance ratio with the highest power (see Faust 1992) by constructing a chi-square statistic, which is a weighted sum of squared deviations from estimated and hypothesized variance ratios and mean simple returns for several time periods. The chi-square test statistic can be used to test any model of asset prices including the Consumption based Capital Asset Pricing Model (CCAPM).

In the standard CCAPM, the representative agent maximizes expected utility where preferences are defined by time-separable iso-elastic functions of the flow of non-durable goods and services. Cecchetti, Lam, and Mark (1990) compare variance ratios of asset returns implied by the CCAPM with historical variance ratios. They find that the model can match the pattern observed in the U.S. data. Bonomo and Garcia (1994) on the other hand show that the CCAPM can produce serial correlation in equilibrium returns only if its endowment process is misspecified. Zemčík (2000) uses

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a proper specification of the endowment process together with time non-separable preferences to demonstrate that the CCAPM can in fact generate autocorrelation in asset returns. Time non-separability in Zemčík (2000) is introduced by adopting the internal habit formation where the current utility depends on an individual's past consumption. Preferences are thus defined by three parameters: the discount factor, the time non-separability parameter, and the relative risk aversion (RRA) coefficient. An identical preference specification is utilized in this paper.

Following Bonomo and Garcia (1994), the consumption in the model is assumed to follow a two-state, one-mean, and two-variance Markov switching process. The parameters of the consumption process are estimated by the method of maximum likelihood using the U.S. consumption of non-durables and services at annual, quarterly and monthly frequencies. The CCAPM is calibrated to various combinations of the parameters and solved for. The equilibrium returns are then compared to the U.S. equity returns using the proposed chi-square statistic. The highest p-values are recorded with negative time non-separability parameter (habit persistence) for annual data, a zero time non-separability parameter (time separability) for quarterly data, and a positive time non-separability parameter (durability) for monthly data. However, the modified variance-ratio test cannot reject time-separability at any data frequency, though p-values are lower compared to time non-separable preferences. This result is in accord with recent evidence based on Bayesian comparison of various preference specifications in Gordon and Samson (1999). They conclude that extensions of the standard power utility function do not yield a demonstrable improvement.

A standard approach to test asset pricing models is to estimate parameters of a model by minimizing the distance between various moments of the model returns with their sample analogs. The empirical adequacy of the model then can be tested using the fact that the objective function evaluated in its minimum is chi-square distributed. See Hansen and Singleton (1982) and Ferson and Constantinides (1991) for examples of this methodology. More recently, Lee and Ingram (1991), Duffie and Singleton (1993) and Heaton (1995) discuss and/or apply an estimation procedure which accounts for cases when the moments of model returns or the stochastic discount factor are simulated. Since Ferson and Constantinides (1991) and Heaton (1995) both estimate the CCAPM with time non-separable preferences, their results are directly comparable to those of the calibration exercise presented here. Ferson and Constantinides (1991) estimate the CCAPM at yearly, quarterly and monthly frequencies and Heaton (1995) uses monthly data. While findings based on annual and monthly data frequencies are roughly consistent with results presented here, there are differences at the quarterly frequency. Estimates of the time non-separability parameter in Ferson and Constantinides (1991) are strongly negative whereas the modified variance-ratio test used in this paper favors time-separability.

The chi-square test statistic constructed here is similar in spirit to other tests based on distance from sample moments of asset returns. Hung (1994) offers a statistic which takes into account uncertainty from point estimates of means of the risk-free rate and the equity premium as well as their mutual correlation. Burnside (1994) and Cecchetti, Lam and Mark (1994) devise tests based on volatility bounds. Specifically, even though the restriction imposed by Hansen and Jagannathan (1991) may be violated, the distance between the minimum variance bound and the variance of a pricing kernel does not have to be statistically significant[1]. Volatility bound tests tend to favor habit persistence for all data frequencies which is due to the increased severity of the equity premium puzzle when durability is present. Hung (1994), Burnside (1994), and Cecchetti et al. (1994) all focus on the cross-sectional characteristics of asset returns; on the other hand, the test presented in this paper concentrates on the autocorrelation pattern of a single asset. In this context, durability is needed for the CCAPM to match positive serial correlation in historical monthly returns.

To evaluate information in accepting or rejecting the CCAPM, the power of the modified variance-ratio test is examined given the null hypothesis of the CCAPM being the true underlying model. The endowment process parameters are set equal to their maximum-likelihood estimates for each data frequency. The preference parameters of the CCAPM are set equal to parameter combinations with highest p-values. The asymptotic distribution of the above mentioned test statistic is chi-square. To avoid potential issues with a small-sample bias I also compute the empirical distribution of the statistic. Critical values from both the empirical and asymptotic distributions are then used for computing the power against alternative hypotheses. Various versions of the alternative hypothesis are formed by varying the time non-separability parameter and relative risk aversion coefficient, respectively. Power is calculated using a sequence of time series of model asset returns that is generated under a given alternative hypothesis. The power is close to one for most alternative parameter combinations which are far enough from the combinations with highest p-values. The only exception is a region with a positive non-separability parameter. This may be attributed to the fact that according to the statistic used, these parameter combinations cannot be rejected using the data on the U.S. stock returns and consumption growth.

The paper is organized as follows. Section "The Modified Variance-Ratio Test" derives the chi-square statistic based on variance ratios and mean simple returns. Section "The CCAPM with Time Non-Separable Preferences" discusses the CCAPM with time dependent preferences: Section "Shocks and Solution Method" solves for the equilibrium returns implied by the model; the implied process for returns is used to evaluate the chi-square statistic for various parameter combinations in Section "Testing the CCAPM" . The power of the test is investigated in Section "Power of the Modified Variance-Ratio Test". Section "Summary" concludes. The used dataset is described in the Data Appendix.

The Modified Variance-Ratio Test

This section derives a test of asset pricing models based on estimates of variance ratios and mean simple returns. The random walk hypothesis for asset returns is frequently tested using the standard variance-ratio test (see Campbell et al. 1997, Chapter 2 for a survey). A typical approach is to derive an asymptotic distribution of variance ratios under the null hypothesis and then test the hypothesis. Variance ratios used may be chosen according to their power as suggested in Faust (1992). Rather than choosing variance ratios based on their power, a joint estimation of variance ratios for several time horizons is conducted here. In addition, variance ratios are estimated together with mean simple returns. The estimation is carried out by the Generalized Method of Moments (GMM) and the asymptotic distribution of estimates is normal regardless of the distribution of returns and a form of potential heteroskedasticity. Finally, a Wald statistic is constructed that compares mean simple returns and variance ratios implied by an asset pricing model with their estimates.

The variance-ratio statistic can be written as

$$VR(k) = \frac{\text{Var}(R_t^k)}{k\text{Var}(R_t)} = 1 + \frac{2}{k}\sum_{s=1}^{k-1}(k-s)\rho_s, \quad k = 2, 3, ..., \tag{1}$$

where R_t^k is the simple k-period gross return, R_t is the simple one-period gross return, and ρ_s is the s-th serial correlation coefficient of returns[2]. $VR(k) = 1$ if the k-period return follows a random walk. For $k = 2$, the variance ratio is simply unity plus the first-order autocorrelation. Thus, $VR(2) < 1$ for negatively serially correlated two-period return, and $VR(2) > 1$ for positively serially correlated two-period return. For $k > 2$, $VR(k)$ is a linear combination of autocorrelation coefficients of returns with declining weights.

Let μ_k denote the mean of the k-period simple gross return. Let us also assume that both μ_k and $VR(k)$ can be obtained by solving a given model of asset pricing. Define $\boldsymbol{z_t} = (R_t^1, R_t^2, ..., R_t^L)'$, and $\boldsymbol{\theta} = (\mu_1, \mu_2, ..., \mu_L, VR(2), VR(3), ..., VR(L))'$, where L is a positive integer. The moment restrictions used for the GMM estimation follow from the definition of the variance ratio statistic (1):

$$E[\boldsymbol{h}(\boldsymbol{z_t}, \boldsymbol{\theta})] = 0,$$

where

$$\boldsymbol{h}(\boldsymbol{z_t},\boldsymbol{\theta}) = \begin{pmatrix} R_t^1-\mu_1 \\ R_t^2-\mu_2 \\ \vdots \\ R_t^L-\mu_L \\ (R_t^2-\mu_2)^2-2VR(2)(R_t^1-\mu_1)^2 \\ (R_t^3-\mu_3)^2-3VR(3)(R_t^1-\mu_1)^2 \\ \vdots \\ (R_t^L-\mu_L)^2-LVR(L)(R_t^1-\mu_1)^2 \end{pmatrix}.$$

Let $g_T(\boldsymbol{\theta}) = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{h}(\boldsymbol{z_t},\boldsymbol{\theta})$ and $\boldsymbol{W_T}$ is some positive definite symmetric matrix. The GMM estimator of $\boldsymbol{\theta}$ maximizes the quadratic form $J_T(\boldsymbol{\theta}) = \boldsymbol{g}_T(\boldsymbol{\theta})'\boldsymbol{W_T}\boldsymbol{g}_T(\boldsymbol{\theta})$. In our case, $J_T(\hat{\boldsymbol{\theta}}) = 0$ because the estimator is exactly identified. It can be proved that $\sqrt{T}(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta_0}) \xrightarrow{d} N(0,\boldsymbol{V}(\hat{\boldsymbol{\theta}}))$. $\boldsymbol{V}(\hat{\boldsymbol{\theta}})$ is the asymptotic covariance matrix and is given by $(\boldsymbol{D_0}\boldsymbol{S_0}^{-1}\boldsymbol{D_0})^{-1}$, where $\boldsymbol{D_0} = E[\frac{\partial\boldsymbol{h}(\boldsymbol{z_t},\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}]$, and $\boldsymbol{S_0} = \sum_{i=-\infty}^{\infty}E[\boldsymbol{h}(\boldsymbol{z_t},\boldsymbol{\theta_0})]E[\boldsymbol{h}(\boldsymbol{z_t},\boldsymbol{\theta_0})]'$. $\boldsymbol{V}(\hat{\boldsymbol{\theta}})$ is estimated by $\hat{\boldsymbol{V}}(\hat{\boldsymbol{\theta}}) = (\boldsymbol{D_T}\boldsymbol{S_T}^{-1}\boldsymbol{D_T})^{-1}$, where

$$\boldsymbol{D_T} = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial\boldsymbol{h}(\boldsymbol{z_t},\hat{\boldsymbol{\theta}})}{\partial\boldsymbol{\theta}},$$

and $\boldsymbol{S_T}$ is estimated using the Newey and West (1987) method i.e.

$$\boldsymbol{S_T} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{h}(\boldsymbol{z_t},\hat{\boldsymbol{\theta}})\boldsymbol{h}(\boldsymbol{z_t},\hat{\boldsymbol{\theta}})' + \sum_{i=1}^{n}\left[1-\frac{i}{n+1}\right]$$
$$\times\left[\frac{1}{T}\sum_{t=1+i}^{T}\boldsymbol{h}(\boldsymbol{z_t},\hat{\boldsymbol{\theta}})\boldsymbol{h}(\boldsymbol{z_{t-i}},\hat{\boldsymbol{\theta}})' + \frac{1}{T}\sum_{t=1}^{T-i}\boldsymbol{h}(\boldsymbol{z_t},\hat{\boldsymbol{\theta}})\boldsymbol{h}(\boldsymbol{z_{t+i}},\hat{\boldsymbol{\theta}})'\right],$$

where the number of lags n is equal to 15 [3]. The estimates of $\hat{\boldsymbol{\theta}}$ are reported in Table 1 for the S&P Index at yearly, quarterly, and monthly frequencies for $L = 10^4$. The data sources are described in the Data Appendix. When annual data are used, the variance ratio for a two-year investment horizon equals one. However, $VR(2)$ can be greater than one if different data span is used. For investment horizons greater than 2 years, variance ratios are lower than one which corresponds to findings of Cecchetti et al. (1990) and Poterba and Summers (1988). For quarterly returns, positive autocorrelation coefficients overweight the negative ones up to the investment horizon of seven quarters (1.75 years). Finally, variance ratios for monthly returns are greater than one for all periods. Estimates of $\boldsymbol{\theta}$ are consistent with the following stylized facts: equity returns display positive serial correlation at horizons shorter than one year and negative serial correlation for longer horizons.

Using the fact that $\hat{\boldsymbol{\theta}}$ is asymptotically normally distributed with covariance matrix $\boldsymbol{V}(\hat{\boldsymbol{\theta}})$, one can write

$$Q = T(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta_0})'\boldsymbol{V}(\hat{\boldsymbol{\theta}})^{-1}(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta_0}) \sim \chi^2_{2L-1}. \tag{2}$$

The above relationship also holds when $\boldsymbol{V}(\hat{\boldsymbol{\theta}})$ is replaced by its consistent estimate, $\hat{\boldsymbol{V}}(\hat{\boldsymbol{\theta}})$. The vector $\boldsymbol{\theta}$ can be thought of as a function of a vector of underlying parameters of an asset pricing model. In this case, one might falsely conclude that the number of degrees of freedom in the Q-statistic is the difference between the number of elements in $\boldsymbol{\theta}$ (which equals the number of restrictions) and the number of model parameters. However, since no estimation with respect to model parameters is conducted, the number of degrees of freedom is simply the number of elements in $\boldsymbol{\theta}$. The Q statistic will be employed to test the CCAPM with time non-separable preferences.

The CCAPM with Time Non-Separable Preferences

The framework adopted here is that of Lucas (1978). The model is solved for the case with time-dependent preferences. The lifetime utility function of the representative consumer takes the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t + \alpha c_{t-1})^{1-\gamma}}{(1-\gamma)}, \tag{3}$$

where c_t is the consumption of services at time t and β and α denote the discount factor and the time non-separability parameter, respectively. α is the coefficient of interest since it affects the autocorrelation structure of the model returns (see Zemčík 2000) as well as the size of the equity premium implied by the model (see Constantinides 1990). γ is approximately equal to the expected value of the RRA coefficient (see Ferson and Constantinides 1991) for $\alpha \neq 0$ and equals the RRA for $\alpha = 0$.

The present version of the CCAPM has been estimated and tested extensively. Ferson and Constantinides (1991) estimate the CCAPM by GMM at annual, quarterly, and monthly frequencies to find negative time non-separability parameter for annual and quarterly data and a positive one for monthly data. Estimation of alternative versions of the CCAPM by Eichenbaum and Hansen (1990) and Heaton (1995) also supports durability in monthly data. Cecchetti et. al (1994) test the CCAPM using tests based on volatility bounds. Their results favor habit persistence for annual and monthly data frequencies. A certain degree of habit persistence is necessary to generate a more volatile pricing kernel which in turn increases the equity premium implied by the model.

Let s_t, p_t, and d_t be the amount of assets (trees) held, the market price of the asset, and the dividend, respectively. The representative agent then maximizes (3) with respect to the following budget constraint:

$$c_t + p_t s_{t+1} \leq (p_t + d_t) s_t.$$

The first order necessary conditions for the optimization problem imply

$$p_t = E_t m_{t+1}(p_{t+1} + d_{t+1}), \tag{4}$$

where m_{t+1} is the Intertemporal Marginal Rate of Substitution ($IMRS$) and is given by

$$m_{t+1} = \frac{\beta[(1+\alpha x_{t+1}^{-1})^{-\gamma} + \beta\alpha E_{t+1}(x_{t+2}+\alpha)^{-\gamma}]}{(1+\alpha x_t^{-1})^{-\gamma} + \beta\alpha E_t(x_{t+1}+\alpha)^{-\gamma}} x_{t+1}^{-\gamma},$$

where $x_{t+1} = \frac{c_{t+1}}{c_t}$. Using the expression for the $IMRS$, the Euler equation (4) can be written as

$$v_t = E_t m_{t+1} h_{t+1}(1 + v_{t+1}), \tag{5}$$

where v_t denotes the price-dividend ratio and h_t the gross growth rate of the dividend, respectively.

Shocks and Solution Method

This section derives variance ratios implied by the CCAPM. At first, the state dependent continuous distribution of the consumption growth rate is discretized by the Gaussian N-point quadrature rule for $N = 6$[5]. Then, the $IMRS$ is discretized as well and so is the Euler equation which can be written in the terms of price-dividend ratios. Next, the Markov process forcing equilibrium returns is derived using the price-dividend ratios and the transition matrix of the consumption process. Finally, variance ratios are calculated from the moments of model returns[6].

Bonomo and Garcia (1994) argue that the two-state Markov switching model with one mean and two variances (2SMS1M2V) is the most parsimonious, statistically acceptable model for consumption growth rate. I follow Bonomo and Garcia (1994) and estimate parameters of this process using the method of maximum likelihood.[7]

The 2SMS1M2V model is defined as:

$$\ln(x_{t+1}) = \delta_0 + (\omega_0 + \omega_1 u_t)\epsilon_{t+1},$$

where $u_t = 1$ or 0, depending on the state of the economy. $\epsilon_t \sim$ N(0,1). The transpose of the transition matrix between the states 0 and 1 is:

$$\boldsymbol{G} = \begin{pmatrix} g_{00} & (1-g_{00}) \\ (1-g_{11}) & g_{11} \end{pmatrix},$$

where g_{00} is the probability of remaining at state 0, while g_{11} is the probability of remaining at state 1. To conduct the estimation, consumption data on non-durables and services in the U.S. are used - see the Data Appendix for description of the data and Table 2 for the summary statistics. The maximum likelihood estimates of $\delta_0, g_{11}, g_{00}, \omega_0$ and ω_1 are reported in Table 3.

Let $\mathbf{ln(\boldsymbol{x}^0)}$ be an $(N \times 1)$ vector with elements

$$\ln(x_i^0) = \delta_0 + \omega_0 a_i, \quad i = 1, 2, \ldots, N,$$

where a_i is the abscissa for an N-point quadrature rule for the standard normal density. Also, let $\mathbf{ln(\boldsymbol{x}^1)}$ denote an $(N \times 1)$ vector with elements

$$\ln(x_i^1) = \delta_0 + (\omega_0 + \omega_1) a_i, \quad i = 1, 2, \ldots, N.$$

Then, the consumption growth rate $\ln(x_{t+1})$ is approximated by a Markov chain, where the vector of possible values of the consumption process is defined as

$$\mathbf{ln(\boldsymbol{x})} = \begin{pmatrix} \mathbf{ln(\boldsymbol{x}^0)} \\ \mathbf{ln(\boldsymbol{x}^1)} \end{pmatrix},$$

with the transpose of the transition matrix

$$\boldsymbol{T} = \begin{pmatrix} g_{00}\mathbf{\Pi} & (1-g_{00})\mathbf{\Pi} \\ (1-g_{11})\mathbf{\Pi} & g_{11}\mathbf{\Pi} \end{pmatrix}, \tag{6}$$

where $\Pi_{ij} = w_j, i, j = 1, 2, \ldots, N$, and w_j's are the weights of an N-point quadrature rule for the standard normal density.

The $IMRS$ is written in the terms of x_{t+1} and discretized. Let $\boldsymbol{M}$ denotes the $(2N \times 2N)$ matrix of values of the $IMRS$. Thus, the Euler equation (5) can be expressed as:

$$\boldsymbol{v} = \boldsymbol{K}\boldsymbol{\iota} + \boldsymbol{K}\boldsymbol{v}, \tag{7}$$

where $\boldsymbol{v}$ is a $(2N \times 1)$ vector of price-dividend ratios and $\boldsymbol{\iota}$ is a $(2N \times 1)$ vector of ones. The $(2N \times 2N)$ matrix $\boldsymbol{K}$ is defined as

$$\{K_{ij}\} = M_{ij} x_j T_{ij}, \quad i, j = 1, 2, \ldots, 2N.$$

(7) is a system of $2N$ equations in $2N$ unknowns and can be easily solved for $\boldsymbol{v}$. The equity returns then are:

$$R_{ij} = \frac{p_j + d_j}{p_i} = \frac{v_j + 1}{v_i} x_j, \quad i, j = 1, ..., 2N. \tag{8}$$

Using $\boldsymbol{T}$ and (8), the equilibrium path for returns is characterized by another Markov chain with $(4N^2 \times 1)$ vector of possible values of returns, $\boldsymbol{R}$, the $(4N^2 \times 4N^2)$ transition matrix , $\boldsymbol{P_R}$, and the $(4N^2 \times 1)$ vector of unconditional probabilities, $\mathbf{\Pi_{\boldsymbol{R}}}$ (see Hamilton 1994, Section 22.4. for an example of a similar approach). The implied moments of returns are

$$E[R_t] = \mathbf{\Pi_{\boldsymbol{R}}}'\boldsymbol{R} = \mu, \tag{9}$$

$$\text{Var}[R_t] = \mathbf{\Pi_{\boldsymbol{R}}}'(\boldsymbol{R}.\boldsymbol{R}) - \mu^2,$$

and

$$E[R_{t+s}R_t] = (\boldsymbol{R}.\mathbf{\Pi_{\boldsymbol{R}}})'\boldsymbol{P_R}^s\boldsymbol{R}.$$

Thus, the s-th autocorrelation coefficient of the equilibrium returns can be calculated using

$$\rho_s = \frac{E[R_{t+s}R_t] - \mu^2}{\text{Var}[R_t]}. \tag{10}$$

The variance ratios implied by the CCAPM are calculated using (10) and (1). $\boldsymbol{\theta_0}$ is defined as $(\mu, 2\mu, ..., L\mu, VR(2), VR(3), ..., VR(L))$, where $L = 10$. The Q-statistic then can be computed for a given set of parameter values by substituting for $\boldsymbol{\theta_0}$ in (2).

Testing the CCAPM

In this section, contour maps of p-values associated with the Q-statistic are constructed. Annual, quarterly and monthly data are used. At each data frequency, the CCAPM is fully characterized by the endowment process parameters $\delta, g_{11}, g_{00}, \omega_0, \omega_1$ and by the utility function parameters α, β, γ. The consumption process parameters are set equal to their maximum-likelihood estimates given in Table 3. Preference parameters are chosen to be within a range of 'reasonable' values. Values for β being examined for annual data are 1.03, 1.00, 0.97, and 0.90, respectively. Kocherlakota (1990) provides a theoretical justification for $\beta > 1$. β's for the other two data frequencies are adjusted to account for a different time-period. α varies from -1 to 1 and γ from 0 to 25. Based on the outcome, a narrower parameter space is then examined in finer intervals. Finally, parameter combinations with the highest p-values are presented for each data frequency. The described calibration exercise can be thought of as an informal estimation procedure. A rigorous estimation of the identical version of the CCAPM is conducted in Ferson and Constantinides (1991) and is beyond the scope of this paper.

Results for annual data are summarized in Figure 1. Contour diagrams A, B, and C in Figure 1 differ only by the value of β. Admissible values for the model are the ones for which the null hypothesis of the equilibrium model being valid cannot be rejected. For instance when the p-value taken from one of contour diagrams is higher than 10% it is not possible to reject the model at 10% level of significance. Since there are no p-values higher than 1% for $\beta = 0.90$, no contour diagram is drawn. Interestingly, in all the other cases, both durability and habit persistence are plausible. In addition, time separability is not rejected either, though corresponding p-values are somewhat lower. The highest p-value across all evaluated preference parameters is 0.99999 for $\beta = 1.00$, $\alpha = -0.04$, and $\gamma = 4.30$.

The outcome of testing the CCAPM using quarterly data is depicted in Figure 2. β's for quarterly data are 1.0074 in the part A of the picture, 1.0000 in B, 0.9924 in C, and 0.9740 in D, respectively. Contours are similar to their counterparts in Figure 1 in admitting durability, habit persistence and time-separability for all β's considered, even for $\beta = 0.9740$, which is equivalent to $\beta = 0.90$ at a yearly frequency. The parameter combination with the highest p-value of 0.99995757 is $\beta = 1.00742$, $\alpha = 0$, and $\gamma = 6.4$.

Results for monthly data are illustrated in Figure 3. Contrary to Figures 1 and 2, Figure 3 favors durability, though time-separable preferences and habit persistence mostly cannot be rejected. When $\beta = 0.99126$, no p-value is greater than 1% and no picture is drawn. The highest p-value of 1 is obtained for $\beta = 1.00000$, $\alpha = 0.10$, and $\gamma = 2.80$.

Power of the Modified Variance-Ratio Test

The null hypothesis is defined by the maximum-likelihood estimates of the consumption process parameters (see Table 3) in combination with preference parameters with the highest p-value according to the Q-statistic (see Section "Testing the CCAPM"). The null hypothesis is used to parameterize the Data Generating Process, DGP_0. The DGP_0 is employed to produce a sequence of 20,000 repetitions of the Monte-Carlo experiment where a series of returns is generated under the null hypothesis of the CCAPM model being true. The number observations for each data frequency is taken from Table 2. Simple means of returns and variance ratios are estimated for each series of returns generated by the DGP_0. The estimates are used to calculate the Q-statistic. An empirical distribution of Q is derived. The empirical distribution provides the size corrected critical value of the test. Then an alternative hypothesis is formulated by varying utility function parameters. The DGP_1, where the preference parameters are supplied by the alternative hypothesis, is formed. The DGP_1 is used to generate 1,000 time series of returns. Mean simple returns and variance ratios are estimated so that the Q-statistic can be calculated. The Q-statistic is compared to its critical value. Both the size corrected and asymptotic critical values are considered. The size corrected critical value is obtained from the distribution of Q under the null hypothesis. The alternative hypothesis is either accepted or rejected using the Q-statistic. Finally, the ratio of the number of rejections to the number of repetitions determines the power of the modified variance-ratio test.

A more formal discussion follows closely Spanos (1993). Let

$$\boldsymbol{b} = (\beta, \alpha, \gamma, \delta, g_{11}, g_{00}, \omega_0, \omega_1)' \in B,$$

where B is the set of plausible values for parameters of the CCAPM. Suppose we have two competing hypotheses, $H_0 : \boldsymbol{b} \in B_0$ and $H_1 : \boldsymbol{b} \in B_1$, where $B_0 \cup B_1 = B$ and $B_0 \cap B_1 = \emptyset$. Let us also define the acceptance region $C_0 = \{\boldsymbol{b} : Q(\boldsymbol{b}) < \chi_{s,2L-1}\}$ and the rejection region $C_1 = \{\boldsymbol{b} : Q(\boldsymbol{b}) > \chi_{s,2L-1}\}$, where s is the level of significance of the test. Thus, if $\boldsymbol{b} \in C_0$ we accept H_0 at the s level of significance and if $\boldsymbol{b} \in C_1$ we reject H_0 at the s level of significance. Q is defined by equation (2). Then the *power of the test* is the probability of rejecting H_0 when false i.e. $Pr(\boldsymbol{b} \in C_1 | \boldsymbol{b} = \boldsymbol{b}_1)$ for $\boldsymbol{b}_1 \in B_1$.

To compute the empirical distribution of Q, an artificial dataset has to be constructed. We know from Section "Shocks and Solution Method" that returns satisfying restrictions given by the CCAPM are driven by a Markov chain with vector of possible values $\boldsymbol{R}$ and the transition matrix $\boldsymbol{P_R}$. The Markov chain is fully characterized by the parameters of the CCAPM, combined in a vector $\boldsymbol{b}$. Let $\boldsymbol{d} = (\delta, g_{11}, g_{00}, \omega_0, \omega_1)'$ denote a vector of consumption process parameters. Table 3 defines $\boldsymbol{d}$ for annual data ($\boldsymbol{d^a}$), quarterly data ($\boldsymbol{d^q}$), and monthly data ($\boldsymbol{d^m}$), respectively. Values of preference parameters are taken from the calibration exercise in Section "Testing the CCAPM". Thus, we can formulate the null hypotheses as follows:

Annual Data $H_0 : \boldsymbol{b_0^a} = (1.00, -0.04, 4.30, \boldsymbol{d^{a\prime}})'$,
Quarterly Data $H_0 : \boldsymbol{b_0^q} = (1.00742, 0, 6.4, \boldsymbol{d^{q\prime}})'$,
Monthly Data $H_0 : \boldsymbol{b_0^m} = (1.00, 0.1, 2.8, \boldsymbol{d^{m\prime}})'$.

To see the difference between the asymptotic and empirical distributions of Q I calculate the Q-statistic for each series produced by the DGP_0. The model implied means of simple returns and variance ratios are defined using $\boldsymbol{b_0}$ or in other words we can express $\boldsymbol{\theta}$ in (2) as a function of the underlying parameters of the CCAPM represented by $\boldsymbol{b}$. Under H_0, $\boldsymbol{\theta(b)} = \boldsymbol{\theta(b_0^a)}$ for annual data and similarly for quarterly and monthly data. Graph 4 compares the two distributions for each data frequency. The theoretical relative frequency for 10-unit intervals is computed as $F(Q_2) - F(Q_1)$, where $Q_2 - Q_1 = 10$ and $F(.)$ is the cumulative distribution function of χ_{19}. To account for the difference I conduct two kinds of tests to assess the power of the modified variance-ratio test: the asymptotic test and the size corrected test. The 10% critical value for χ_{19} is 27.2036. So, the rejection region for the asymptotic test is $C_1 = \{\boldsymbol{b} : Q(\boldsymbol{b}) > 27.2036\}$. Using size corrected critical values, rejection regions are

Annual Data: $C_1 = \{\boldsymbol{b} : Q(\boldsymbol{b}) > 89.108211\}$,
Quarterly Data: $C_1 = \{\boldsymbol{b} : Q(\boldsymbol{b}) > 50.610095\}$,
Monthly Data: $C_1 = \{\boldsymbol{b} : Q(\boldsymbol{b}) > 40.614630\}$.

Not surprisingly, as the number of observations increases, the empirical distribution of Q gets closer to the asymptotic distribution. However, the empirical distribution remains fat-tailed for all frequencies.

Finally, the power is calculated. The alternatives are intentionally constructed so that only one element in the parameter vector $\boldsymbol{b}$ differs from the null hypothesis. The alternative hypothesis is formulated by varying either the RRA coefficient γ, or the time non-separability parameter α. The hypothesis is either rejected or accepted based on the Q-statistic and the corresponding rejection region. The power of the test is estimated by

$$\hat{Pr}(\boldsymbol{b} \in C_1 | \boldsymbol{b} = \boldsymbol{b_1}) = \frac{\text{number of rejections}}{\text{number of repetitions}}.$$

The results of power calculations are depicted in Figure . For annual data, see figures A and B where the alternative hypotheses are respectively:

$$\text{Annual Data, } \gamma \text{ varies } H_1^{\gamma}: \boldsymbol{b_1^a} = (1.00, -0.04, \gamma, \boldsymbol{d^{a\prime}})', \gamma \in [0, 5],$$

$$\text{Annual Data, } \alpha \text{ varies } H_1^{\alpha}: \boldsymbol{b_1^a} = (1.00, \alpha, 2.5, \boldsymbol{d^{a\prime}})', \alpha \in [-0.5, 0.5].$$

The pattern in both figures clearly corresponds to what can be seen in Figure 1. As the null hypothesis is approached, the power decreases in both cases. However, aside from the close proximity to $\boldsymbol{b_0^a}$, the power is unity or close to unity. The power of the size corrected test is somewhat lower by construction (size corrected critical value is greater than the asymptotic critical value) but it reaches unity as well for $\gamma < 1.0$ and $\gamma > 3.8$ in Figure 5A and for $\alpha = -0.6$ in Figure 5B.

Alternatives for quarterly data are given as:

$$\text{Quarterly Data, } \gamma \text{ varies } H_1^{\gamma}: \boldsymbol{b_1^q} = (1.00742, 0, \gamma, \boldsymbol{d^{q\prime}})', \gamma \in [6, 7],$$

$$\text{Quarterly Data, } \alpha \text{ varies } H_1^{\alpha}: \boldsymbol{b_1^q} = (1.00742, \alpha, 6.4, \boldsymbol{d^{q\prime}})', \alpha \in [-0.5, 1.0].$$

Results are summarized in Figures 5C and 5D and correspond to Figure 2. There are two peaks with high p-values in Figure 2 and a through for $\alpha = 0.1$. The power is relatively low for the peaks and high for the through. Alternative hypotheses for monthly data are as follows:

$$\text{Monthly Data, } \gamma \text{ varies } H_1^{\gamma}: \boldsymbol{b_1^m} = (1.00, 0.1, \gamma, \boldsymbol{d^{m\prime}})', \gamma \in [0, 7],$$

$$\text{Monthly Data, } \alpha \text{ varies } H_1^{\alpha}: \boldsymbol{b_1^m} = (1.00, \alpha, 2.8, \boldsymbol{d^{m\prime}})', \alpha \in [-0.5, 0.5].$$

The power for monthly data is shown in Graphs 5E and 5F. Again, results are compatible with testing the CCAPM using the Q-statistic in Figure 3.

The power of the test is high when γ deviates from it's null hypothesis' value. However, the power for $\alpha > -0.1$ is rather low since the data cannot clearly distinguish between habit persistence and durability.

Summary

Various forms of the variance-ratio test are utilized to document predictability and to test asset pricing models. This paper introduces the modified variance-ratio test where a joint distribution of variance ratios is derived together with mean simple returns to construct a test statistic, which may be employed to test a variety of models generating asset returns. The intuition behind the metric originates from the autocorrelation pattern in the U.S. equity data i.e. a positive autocorrelation in shorter investment horizons and negative autocorrelation in longer investment horizons. The statistic evaluates whether the difference between the model implied variance ratios and the model implied mean simple returns and their historical counterparts is statistically significant.

The CCAPM with time non-separable preferences is tested using the modified variance-ratio test. The results reflect what we observe in the U.S. stock market data. Variance ratios lower than one translate into habit persistence for annual data though neither durability nor the time-separable model can be rejected. For quarterly data frequency, the variance ratios are at first higher and then lower than one and consequently, the highest p-value is found for the time separable model. However, there are parameter combinations representing respectively habit persistence and durability, which cannot be rejected either. Finally, variance ratios greater than one in monthly data imply test results favoring durability.

Finally, the power of the modified variance-ratio test is examined to evaluate the extent of information contained in test results. The null hypothesis is formed using parameter combinations with highest p-values according to the modified variance-ratio test. Both the asymptotic and size corrected critical values are calculated from empirical distribution of the test statistic under the null hypothesis of the CCAPM being the true model. An alternative hypothesis is either accepted or rejected using both the asymptotic and size corrected tests. The ratio of rejections to the number of repetitions determines the power. Alternative hypotheses are formed by varying one parameter of the model at a time. The parameters changing their values are the RRA coefficient and the time non-separability parameter, respectively. When the RRA coefficient varies, the power of the test is mostly one across all data frequencies and is lower only in the neighborhood of the parameter combination defined by the null hypothesis. While the test rules out successfully values of the RRA that are different from the null hypothesis, it does not distinguish among habit persistence, time-separability, and durability very well. The low power in this case is caused by data supporting a wide range of values of the time non-separability coefficient.

Notes

1. For recent development regarding volatility bounds, see Hansen, Heaton and Luttmer (1995), Hansen and Jagannathan (1997), and Balduzzi and Kallal (1997).

2. Poterba and Summers (1988) offer an alternative definition of the variance-ratio statistic for monthly data: $VR(k) = \frac{\text{Var}(R_t^k)}{k} / \frac{\text{Var}(R_t^{12})}{12}$ i.e. they compare variances of simple returns in relation to the variation over a one-year period. For quarterly data, 12 would be replaced by 4. This approach is not adopted here because of potential problems with variance ratios which arise when the time horizon is large relative to the total time span of the data (see Campbell et al. 1997, Chapter 2).

3. The Gauss code for GMM written by Hansen, Heaton and Okagi is available upon request.

4. $L = 10$ is chosen since Cecchetti et al. (1990), Bonomo and Garcia (1994), and Zemčík (2000) examine $VR(k)$ for $k = 2, 3, \ldots, 10$.

5. The resulting number of corresponding rates of returns is $(\# \text{ states} \times N)^2$ i.e. 144.

6. For the sake of brevity, some details of the solution method are omitted. The details are available upon request from the author.

7. Cecchetti et al. (1994) use a random walk model for annual data and an AR(1) model for monthly data. The random walk model is explicitly ruled out by Bonomo and Garcia (1994). For the sake of comparison, I also solve the model using the AR(1) process at all data frequencies and conducte the calibration exercise described in on testing the CCAPM below. The CCAPM then generates autocorrelated returns even for time separable preferences, contrary to calibration of the model by the 2SMS1M2V process. However, differences in results seem minor when the Q-statistic is used and uncertainty regarding point estimates of historical variance ratios is accounted for.

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Data Appendix

Annual Data

The yearly data considered here were used in Cecchetti et al. (1993). Cecchetti et al. (1990) provides a detailed account of the data sources. There are four series:

1. *Consumption*: The real per capita consumption of non-durables and services, 1889-1987.
2. *CPI*: Both the annual average and end of year observations from 1890 to 1987.
3. *Dividends*: The nominal dividends, 1890-1987, deflated by the annual average CPI.
4. *Standard and Poor's Composite Stock Price Index*: January observations, 1890-1988, adjusted to inflation by the end of period CPI.

Quarterly Data

The quarterly data I use contain observations form 1947 II to 1993 I and consist of two series:

1. *Consumption*: Real per capita consumption of non-durables and services - CITIBASE series $(GCNQ + GCSQ)/GPOP$).
2. *Stock Return*: Quarterly value weighted Standard and Poor's 500 returns taken from the CRSP tape, adjusted for inflation.

Monthly Data

The monthly data considered start from February 1959 and end in March 1993. They include the following series:

1. *Consumption*: The real per capita consumption of non-durables and services in 1987 dollars - CITIBASE series $(GMCSQ + GMCNQ)/POP$.
2. *Price Index*: Computed as $(GMCS + GMCN)/(GMCSQ + GMCNQ)$, where $GMCS$, $GMCN$, $GMCSQ$, $GMCNQ$ are respectively nominal consumption expenditures on services, nominal consumption expenditures on non-durables, real consumption expenditures in 1987 dollars on services, and real consumption expenditures in 1987 dollars on non-durables.
3. *Standard and Poor's Composite Common Stock Price Index*: CITIBASE series FSPCOM adjusted for inflation by the above price index.
4. *Dividends*: Constructed using the dividend yield on Standard and Poor's Composite Common Stock (CITIBASE series FSDXP), Standard and Poor's Composite Common Stock Price Index, and the price index, both defined above.

Table 1: Summary Statistics for Real Historical Returns

	Yearly		1890-	1987	Quarterly		1947:II-	1993:I	Monthly		1959:2-	1993:3
Observations	98				184				410			
Mean	0.07978				0.02195				0.00495			
St.dev.	0.19500				0.07670				0.03488			
First Autocorr.	−0.00998				0.13309				0.28204			
k	$\hat{\mu}_k$	s.e.	$\widehat{VR(k)}$	s.e.	$\hat{\mu}_k$	s.e.	$\widehat{VR(k)}$	s.e.	$\hat{\mu}_k$	s.e.	$\widehat{VR(k)}$	s.e.
1	1.078	0.021			1.021	0.006			1.005	0.002		
2	2.156	0.030	1.000	0.153	2.043	0.009	1.147	0.154	2.010	0.003	1.283	0.092
3	3.234	0.034	0.849	0.147	3.065	0.011	1.137	0.187	3.015	0.004	1.375	0.132
4	4.310	0.039	0.859	0.140	4.087	0.012	1.119	0.193	4.019	0.004	1.426	0.146
5	5.390	0.043	0.846	0.152	5.110	0.014	1.105	0.196	5.024	0.005	1.473	0.152
6	6.471	0.046	0.798	0.148	6.132	0.015	1.095	0.202	6.029	0.005	1.525	0.159
7	7.552	0.049	0.763	0.157	7.155	0.016	1.059	0.204	7.034	0.006	1.545	0.164
8	8.635	0.052	0.757	0.161	8.177	0.017	0.997	0.195	8.038	0.006	1.534	0.167
9	9.719	0.054	0.739	0.154	9.200	0.017	0.938	0.184	9.043	0.006	1.520	0.169
10	10.799	0.056	0.708	0.145	10.223	0.018	0.904	0.176	10.048	0.007	1.502	0.167

Notes
The mean simple gross returns, variance ratios and their standard errors are based on the joint GMM estimation.

Table 2: Summary Statistics for Per Capita Consumption Growth Rate

	Annual Data 1890-1987	Quarterly Data 1947:II-1993:I	Monthly Data 1959:02-1993:03
Observations	98	184	410
Mean	0.0172	0.00444	0.00160
St. Dev.	0.0342	0.00563	0.00395
Skewness	-0.4045	-0.44717	0.003436
Kurtosis	3.9773	3.6211	3.5229
First Autocorrelation	-0.1331	0.1986	-0.2424

Table 3: Maximum Likelihood Estimates of the 2SMS1M2V Process

	Annual Data	Quarterly Data	Monthly Data
δ	0.0187	0.0046	0.0017
	(10.416)	(11.813)	(8.258)
g_{11}	0.9885	0.9832	0.4510
	(3.500)	(2.396)	(-0.231)
g_{00}	0.9854	0.9942	0.9247
	(3.086)	(4.316)	(2.878)
ω_0	0.0113	0.0050	0.0042
	(8.436)	(17.091)	(19.226)
ω_1	0.0315	0.0037	-0.0028
	(7.523)	(2.557)	(-4.482)
LF	2.20788	3.78981	4.12347

Notes

(1) Asymptotic t-ratios in parentheses. For $g_{ii}, i = 0, 1$, the reported t-ratios are those of the transformation $ln(g_{ii}/(1 - g_{ii})), i = 0, 1$, respectively. The transformation was employed to restrict probability estimates to the interval $(0, 1)$. (2) LF refers to the mean log-likelihood function as calculated by the GAUSS Constrained Maximum Like-lihood Estimation module.

Figure 1: P-values; Annual Data

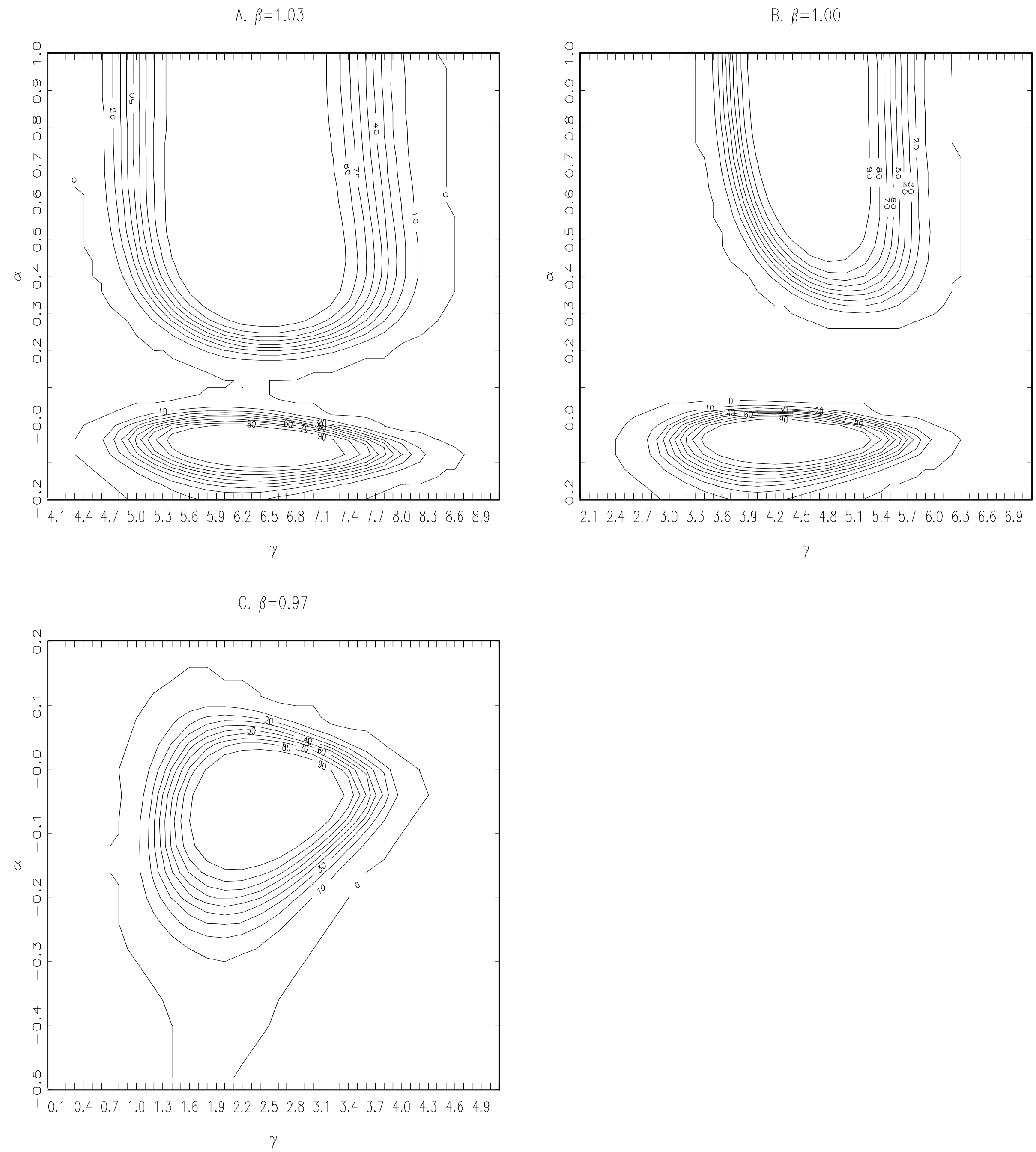


Figure 2: P-values; Quarterly Data

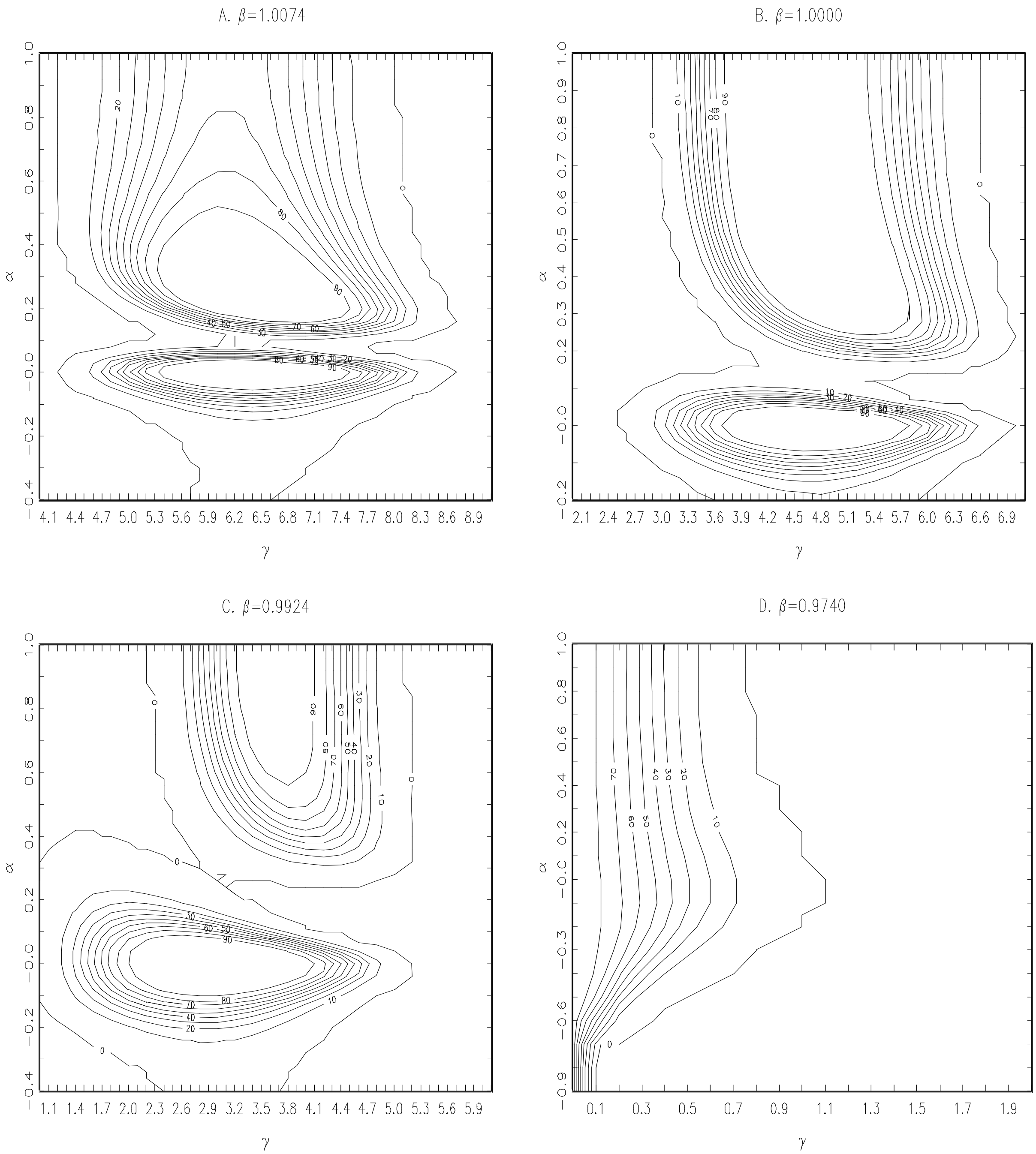


Figure 3: P-values; Monthly Data

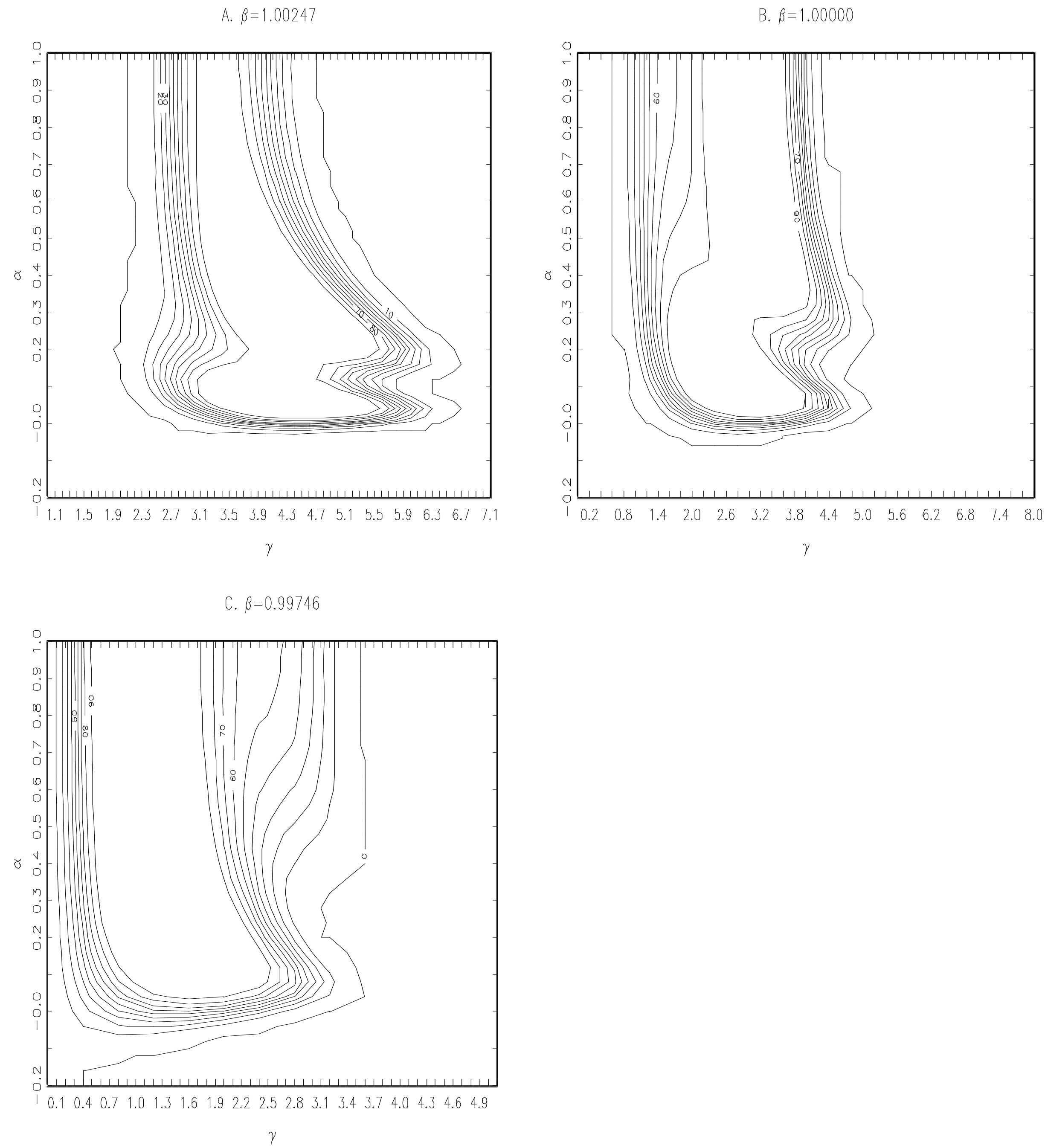
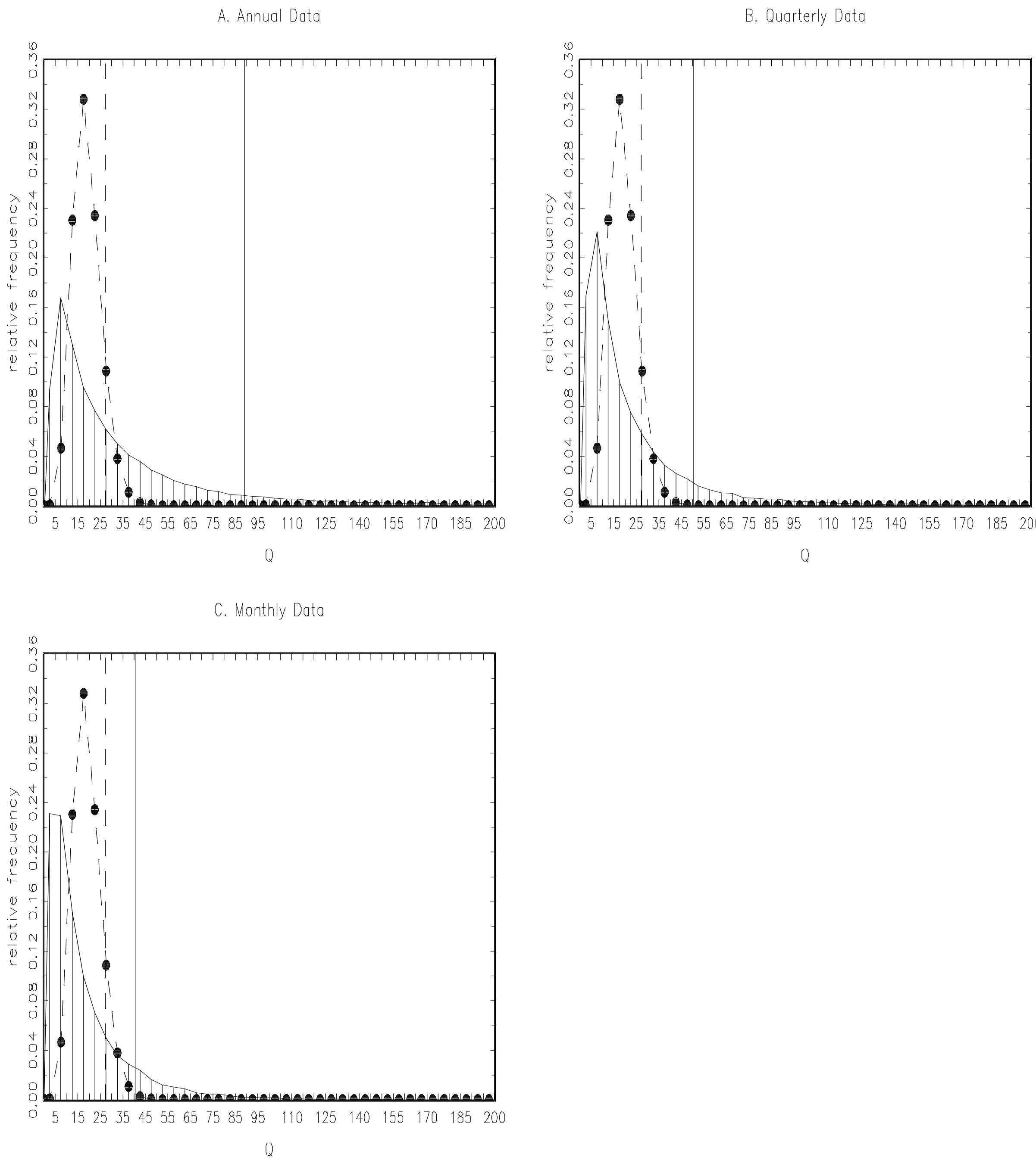
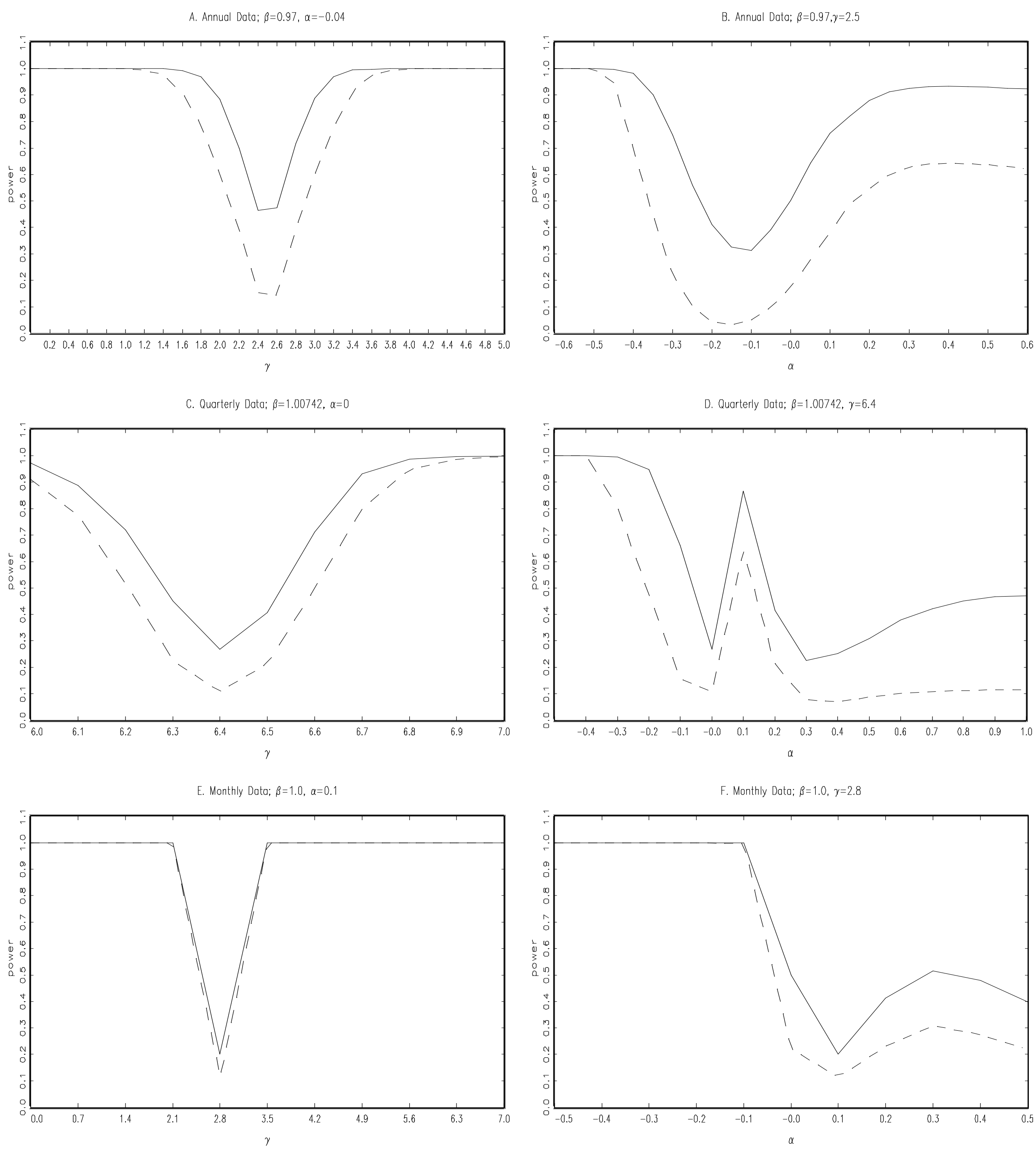


Figure 4: Empirical Distribution of Q Compared to χ_{19}



Notes
Dashed lines with solid circles illustrate χ_{19}. Vertical dashed lines denote the critical value of χ_{19} at 10% level of significance. Solid lines represent the empirical distribution of Q using the DGP_0. Vertical solid lines are size corrected critical values at 10% level of significance.

Figure 5: Power of the Modified Variance-Ratio Test



Notes
Solid lines represent the power calculated using asymptotic critical values; Dashed lines represent the power calculated using size corrected critical values.