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Mean reversion in asset returns and time non-separable preferences

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Abstract

Time non-separable preferences are used in combination with various specifications of the endowment process to calibrate the Capital Asset Pricing Model (CAPM). Time non-separability is caused either by habit persistence or durability. It is demonstrated that the model can indeed produce the amount of mean reversion detected in historical returns. Specifically, habit persistence is required to match negative autocorrelation of annual asset returns and durability is needed to replicate positive autocorrelation detected in monthly asset returns. In addition, the CAPM with habit persistence can predict negative expected returns when calibrated to monthly data. © 2001 Elsevier Science Inc. All rights reserved.

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1. Introduction

Various studies of the US stock market report evidence that equity returns display positive serial correlation at horizons shorter than 1 year and negative serial correlation at longer horizons (see Campbell, Lo, & MacKinlay, 1997, Chapter 2 for a survey). Though autocorrelation of asset returns does not imply a violation of market efficiency, it does raise the question of whether the behavior of security markets can be explained by a rational

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expectations asset pricing model. This paper argues that it can, provided that consumer preferences are time non-separable.

The hypothesis of serially independent returns (the random walk hypothesis) is often tested using the variance ratio test which, according to Poterba and Summers (1988), has a higher power than alternatives such as the likelihood ratio test or the regression of current returns on lagged returns. Since the volatility of asset returns changes over time, it would be of no interest to reject the random walk model due to heteroskedasticity. Lo and MacKinlay (1988) thus derive the asymptotic distribution of variance ratios under the null hypothesis of random walk allowing for changing variances. Their specification test is applied here to both annual and monthly stock market returns and the random walk hypothesis is strongly rejected for all considered time horizons. As expected, returns with holding periods less than 1 year are positively autocorrelated and returns with longer holding periods are negatively autocorrelated.

Several studies make successful attempts to rationalize deviations of asset returns from random walk. Typically, they employ the Lucas (1978) Capital Asset Pricing Model (CAPM) with time-separable, constant relative risk aversion (RRA) utility function. Kandel and Stambaugh (1990) use the CAPM to replicate autocorrelations of equity returns as well as other unconditional moments. Their model is calibrated to the quarterly consumption growth rate, which is assumed to follow a four-state Markov switching process. Parameters of the Markov process are selected jointly with preference parameters to reflect various characteristics of the consumption and returns data. A similar approach is adopted in Cecchetti, Lam, and Mark (1990) who model the endowment process by a two-mean, one-variance Markov chain whose parameters are estimated using the consumption data only. Cecchetti et al. (1990) generate the distribution of variance ratios implied by the CAPM and then test the null hypothesis of the model being true using point estimates of variance ratios from historical returns. Comparison of variance ratios of model returns with variance ratios of historical returns demonstrates that historical returns could have, in fact, been generated by this type of a model. However, Bonomo and Garcia (1994) use variance ratios to show that the degree of mean reversion in Kandel and Stambaugh and in Cecchetti et al. (1990) is sensitive to the choice of the Markov switching model for the endowment process. They conclude that the CAPM cannot account for the magnitude of mean reversion observed in the data once the proper Markov specification for the endowment process is chosen, the proper specification being the two-state, one-mean, and two-variance Markov switching model for yearly data. Based on likelihood ratio tests, this specification is considered superior to both the four-state process in Kandel and Stambaugh and to the two-state process with two means and one variance in Cecchetti et al. Bonomo and Garcia also show that the CAPM with time additive preferences is unable to generate negative excess returns.

In this paper, I employ the Markov switching model of Bonomo and Garcia (1994) to model endowment at annual frequency. For monthly data, a more general two-state, two-mean, and two-variance specification is used. In addition — following Cecchetti, Lam and Mark (1994) — a first-order autoregressive process is considered to investigate the robustness of results at both yearly and monthly frequencies. Since the Lucas CAPM does not provide any guideline to distinguish among consumption, dividends and output, historical series on real consumption, dividends, and GNP growth rates are all, respectively, used to estimate parameters of the endowment process.[1]

In an attempt to account for autocorrelation patterns found in the US stock market, time non-separable preferences are introduced. Time non-separability can be brought in the CAPM by making current utility dependent on past consumption in two ways: either utility depends on aggregate consumption or on an individual's own consumption. We speak about *external habit* in the case of the former and about *internal habit* in the case of the latter. Campbell and Cochrane (1999) use a slow-moving external habit to replicate (among other things) the long horizon forecastability of stock returns. In their study, dynamics in asset returns is produced by interaction between the surplus consumption ratio, which evolves as a heteroskedastic AR(1), and endowment, which is assumed to follow a random walk. The current study, on the other hand, focuses on the relatively more popular internal habit formulation where there is a basis for comparison to results based on estimation (see Eichenbaum & Hansen, 1990; Ferson & Constantinides, 1991; Heaton, 1995) and on volatility bounds (see Balduzzi & Kallal, 1997; Cecchetti et al., 1994). This formulation also allows one to investigate durability in addition to habit persistence.

The exact specification of the preference structure is taken from Ferson and Constantinides (1991). The consumer values how much more she can consume today in comparison with how much she consumed yesterday. So, what matters is not level of current consumption, but the difference between current consumption and lagged consumption. If yesterday's consumption increases the agent's utility — one can think of a vacation or of a haircut — preferences display durability. If the lagged consumption lowers utility, there is habit persistence. For instance, it is hard to go back to junk food after one has become accustomed to eating in good restaurants. There is an ongoing dispute of which effect dominates. The evidence from testing the overidentifying restrictions is somewhat ambiguous, but seems to suggest that habit prevails in the long run and durability in the short run. Ferson and Constantinides use Generalized Method of Moments (GMM) to test the CAPM and conclude that the complementarity effect is strong for quarterly and annual data even if time averaging is accounted for. Heaton (1995) exploits a more complicated form of the utility function by adding more lags of consumption. He finds that the first few coefficients on consumption are positive, and then the sign switches. Eichenbaum and Hansen (1990) use monthly data and GMM to show that local substitutability dominates. Testing the CAPM based on volatility bounds tends to favor habit persistence to durability (see Balduzzi & Kallal, 1997; Cecchetti et al., 1994). To investigate whether either of the two effects can generate mean reversion, several versions of the model are examined: strong habit persistence, modest habit persistence, modest habit persistence, time separability, modest durability, and strong durability.

The CAPM with a time non-separability parameter is calibrated using estimated parameters of the corresponding endowment process. Then, the equilibrium returns are solved for. The solution method is based on discretization of the first-order conditions using the

[1] GNP data are not available at monthly frequency.

Gaussian quadrature rule (see Tauchen & Hussey, 1991) and enables one to calculate model variance ratios without the small sample bias characteristics to Monte Carlo simulations. The results demonstrate that the amount of mean reversion in historical returns can be matched by the CAPM with time non-separable preferences for all considered endowment models, time series, and data frequencies. Specifically, habit persistence generates negative and durability positive autocorrelation of model returns. Therefore, habit persistence is necessary to replicate negative serial correlation in yearly historical returns and durability is needed to reproduce positive serial correlation observed at monthly frequency. As established by Bonomo and Garcia (1994), time-separable preferences do not imply mean reversion in model returns for a two-variance Markov switching model. Finally, the CAPM calibrated to monthly data can predict negative expected returns when consumption is complementary over time.

The paper is organized as follows. Section 2 uses the asymptotic distribution of variance ratios derived in Lo and MacKinlay (1988) to show that asset returns do not follow random walk. In Section 3, parameters of the endowment processes are estimated. The estimates are used to calibrate the CAPM. Section 4 describes the CAPM with time non-separable preferences and indicates how equilibrium price−dividend ratios can be used to calculate model variance ratios and expected excess returns. Section 5 presents results and Section 6 concludes. Appendix A gives a detailed account of data sources and Appendix B demonstrates how the Gaussian quadrature method is employed to solve the CAPM with time non-separable utility function.

2. Mean reversion in historical returns

The random walk hypothesis of asset returns has been tested extensively in the financial literature. The consensus is that asset returns tend to be positively serially correlated for horizons shorter than 1 year and negatively serially correlated for longer horizons. To test the random walk hypothesis for equity returns, I adopt the framework of Lo and MacKinlay (1988), who develop a specification test based on the asymptotic distribution of variance ratios that is robust to the presence of heteroskedasticity.

The variance ratio test exploits the fact that if the stock return follows a random walk, the return variance should be proportional to the return horizon. The variance ratio statistic is defined as:

$$\mathrm{VR}(q) = \frac{\mathrm{Var}(R_t^q)}{q\mathrm{Var}(R_t^l)} = 1 + \frac{2}{q}\sum_{j=1}^{q-1}(q-j)\rho_j, \quad q = 1, 2, \ldots, \tag{1}$$

where R_t^q is the simple q period return and ρ_j is the jth serial correlation coefficient of returns. Alternatively, the variance ratio statistic for monthly data can be defined as $\mathrm{VR}(q)=[\mathrm{Var}(R_t^q)/q]/[\mathrm{Var}(R_t^{12})/12]$, i.e., variances of simple returns are compared to the variation over a 1-year period (e.g., see Poterba & Summers, 1988). Campbell et al. (1997, Chapter 2) argue that this approach might be problematic if the time horizon is large relative to the time period covered by the available data. Therefore, the formulation in Eq. (1) is employed.

Table 1
Variance ratios for historical returns; yearly data 1870–1987

q	VR(q)	$z(q)$
2	1.0275	2.9952
3	0.8891	− 7.9440
4	0.8923	− 6.0742
5	0.8760	− 5.9204
6	0.8205	− 7.5561
7	0.7918	− 7.9245
8	0.8013	− 6.9658
9	0.7928	− 6.7778
10	0.7705	− 7.0959

The random walk hypothesis allowing for heteroskedasticity is rejected in all cases at 1% level.

The specification test in Lo and MacKinlay (1988) (see Section 1.2 of their paper) is designed to test the random walk hypothesis allowing for dependent, but uncorrelated, increments. The asymptotic distribution of the variance ratio statistic based on Eq. (1) is derived under a compound null hypothesis that imposes rather general restrictions on the type and degree of heteroskedasticity present. Under this null hypothesis, the variance ratio estimator still approaches unity asymptotically and importantly, estimators of autocorrelation coefficients in Eq. (1) are asymptotically uncorrelated. Consequently, estimates of their variances can be summed up using squared weights in Eq. (1) to estimate the variance of the variance ratio statistic. The variance then can be used for statistical inference.

Let $\hat{VR}(q)$ and $\hat{\vartheta}(q)$ denote the variance ratio estimator and the heteroskedasticity-consistent estimator of its variance, respectively.[2] The statistic $z(q)=\sqrt{Tq}(\widehat{VR(q)})-1/\sqrt{\widehat{\vartheta(q)}}$ is asymptotically standard normal. The estimates of variance ratios and the $z(q)$ statistics are computed using both annual and monthly data on real returns of the S&P Composite Index (see Appendix A for details).

Results for the yearly frequency are reported in Table 1. The variance ratios are greater than one for the second period and they are lower than one from the third period on. Since the variance ratios can be expressed as a function of the autocorrelations, this means that real returns display the pattern of at first positive and then negative serial correlation. A variance ratio lower than one for the time periods beyond 2 years indicates very strong negative autocorrelation at long horizons. The autocorrelation has to be large in absolute terms to make up for the first period when the returns are positively serially correlated. So, long-term returns are, to some extent, predictable. The variance ratios are slightly higher than those reported in Bonomo and Garcia (1994) since the sample bias is taken into account. Nevertheless, they are significantly different from one in all cases and the random walk hypothesis is strongly rejected.

Variance ratios calculated using monthly returns are displayed in Table 2. Since they are all significantly greater than one, the random walk hypothesis is again rejected in all cases. Variance ratios greater than one indicate positive serial correlation in monthly returns.

[2] See Lo and MacKinlay (1988), the second expression in Eq. (13) for the variance ratio estimator and Eq. (20) for its variance.

Table 2
Variance ratios for historical returns; monthly data 1947:02–1994:03

q	VR(q)	$z(q)$
2	1.2652	111.4259
3	1.3629	106.7755
4	1.4248	103.2105
5	1.4902	104.2021
6	1.5669	108.5213
7	1.6150	107.9693
8	1.6339	103.3748
9	1.6491	99.4246
10	1.6636	96.0809

The random walk hypothesis allowing for heteroskedasticity is rejected in all cases at 1% level.

Tables 1 and 2 confirm stylized facts regarding equity returns. The identified autocorrelation pattern stands as a challenge for the CAPM.

3. The endowment process

In equilibrium of a typical representative agent economy, the consumption stream equals the dividend stream. In addition, the output (perishable 'fruits') is equivalent to the dividend payment. Therefore, the following time series is considered for the empirical analysis: the real per capita consumption of non-durables and services,[3] the dividend growth rate, and the real per capita GNP growth rate. Appendix A describes both annual and monthly series. Tables 3 and 4 provide corresponding summary statistics. Since the data for output are not collected monthly, only series for consumption and dividends are used in the case of monthly frequency.

Bonomo and Garcia (1994) consider the following L-state Markov switching model for the endowment process:

$$x_t = \alpha_0 + \alpha_1 S_{l,t-1} + \cdots + \alpha_{L-1} S_{L-1,t-1} + (\omega_0 + \omega_1 S_{L-1,t-1} + \cdots + \omega_{L-1} S_{L-1,t-1})\varepsilon_t, \tag{2}$$

where x_t is the natural logarithm of the endowment process and $S_{i,t}=1$ whenever the state of economy is i and 0 otherwise. ε_t is an i.i.d. $N(0,1)$ error term.

Specification given by Eq. (2) encompasses the two-state Markov switching model with two means and one variance (2SMS2M1V) used in Cecchetti et al. (1990) as well as the

[3] Most studies based on monthly data employ consumption of non-durables and services to analyze the performance of the CAPM. Non-durable and services consumption can be used under the assumption that preferences over durables, and non-durables and services are separable. Among the cited studies using lower frequency data, Cecchetti et al. (1990) and Bonomo and Garcia (1994) use total consumption, while Kandel and Stambaugh (1990) use consumption of non-durables and services. Hence, both types of consumption data are used in the case of yearly frequency.

Table 3
Summary statistics for growth rates in sample; yearly data

	Total consumption	Consumption of non-durables and services	Dividends	GNP
Time period	1890–1987	1890–1987	1872–1987	1890–1987
Observation	98	98	116	118
Mean	0.0182	0.0172	0.0112	0.0178
S.D.	0.0374	0.0342	0.1262	0.0514
Skewness	− 0.4097	− 0.4045	− 0.8228	− 0.7574
Kurtosis	3.8750	3.9773	6.3321	7.6627
Maximum	0.0990	0.0994	0.4168	0.1613
Minimum	− 0.0987	− 0.0874	− 0.4314	− 0.2216
First autocorrelation	− 0.0679	− 0.1343	0.2089	0.3908

four-state Markov switching model with two means and two variances (4SMS2M2V) employed in Kandel and Stambaugh (1990). Bonomo and Garcia (1994) use the likelihood ratio test to reject the 2SMS2M1V model when the two-state Markov switching model with two means and two variances (2SMS2M2V) is used as an alternative. However, the 2SMS1M2V model cannot be rejected against the same alternative. In addition, the 2SMS1M2V model cannot be rejected neither against the alternative of the three-state, three-mean, and three-variance Markov switching model nor against the alternative of the 4SMS2M2V model. Therefore, for reasons of parsimony, Bonomo and Garcia adopt the 2SMS1M2V model as the model according to which the endowment growth rate evolves.

For the 2SMS1M2V model, $L=2$ and $\alpha_1=0$. α_0 is both the conditional and unconditional mean of x_t. If $S_t=0$, the conditional variance of x_t is $\omega_0{}^2$ and $(\omega_0+\omega_1)^2$ otherwise. The transpose of the transition matrix for the Markov process S is defined as follows:

$$P = \begin{pmatrix} p_{00} & (1-p_{00}) \\ (1-p_{11}) & p_{11} \end{pmatrix}.$$

Table 4
Summary statistics for growth rates in sample; monthly data

	Consumption	Dividends
Time period	1959:02–1993:03	1947:02–1993:03
Observation	410	554
Mean	0.00159	0.000768
S.D.	0.00394	0.005666
Skewness	0.0195	1.73730
Kurtosis	3.5174	16.72803
Maximum	0.01598	0.03945
Minimum	− 0.010795	− 0.0341
First autocorrelation	− 0.2442	0.1992

Table 5
Maximum likelihood estimates of the 2SMS1M2V process; yearly data

	Total consumption	Consumption of non-durables and services	Dividends	GNP
α_0	0.0197 (8.087)	0.0187 (10.416)	0.0144 (2.304)	0.0179 (5.701)
p_{11}	0.9897 (3.742)	0.9885 (3.500)	0.8193 (1.746)	0.9281 (2.707)
p_{00}	0.9874 (3.338)	0.9854 (3.086)	0.8165 (2.228)	0.9834 (3.966)
ω_0	0.0165 (8.714)	0.0113 (8.436)	0.0381 (7.569)	0.0303 (10.913)
ω_1	0.0299 (6.328)	0.0315 (7.523)	0.01350 (6.922)	0.0698 (4.161)

Asymptotic t ratios in parentheses. For p_{ii}, $i=0,1$, the reported t ratios are those of the transformation $\ln(p_{ii}/(1-p_{ii}))$, $i=0,1$, respectively. The transformation was employed to restrict probability estimates to the interval (0,1).

As the notation suggests, p_{00} is the probability of the remaining state 0 while p_{11} is the probability of remaining at the state 1. I replicate the maximum likelihood estimation[4] undertaken in Bonomo and Garcia (1994) (see their Table 1, p. 23) and report the results in Table 5.[5]

For the monthly data, a more general 2SMS2M2V process is used to characterize the endowment processes for consumption and dividends. $\alpha_1 \neq 0$ allows for autocorrelation in the endowment process. Estimates of the parameters of the 2SMS2-M2V process are summarized in Table 6. α_1 is significantly different from zero only for dividends.

Sometimes, an AR(1) model is used to characterize the endowment process, especially for the monthly data frequency (see Cecchetti et al., 1994 for instance). Even though the AR(1) model does not capture heteroskedasticity implied by findings of Bonomo and Garcia (1994), I use it to evaluate the performance of the CAPM as well. At annual frequency, estimates of the autocorrelation coefficient by GMM are spurious and differ greatly depending on what instruments are used. Thus, OLS estimates are used instead. For monthly data, the GMM estimation is robust and GMM estimates are used to calibrate the CAPM. Even so, the AR(1) model is rejected for consumption using the Hansen and Singleton (1982) J statistic. The AR(1) process does not seem to capture the time series properties of endowment series very well and consequently, its implications will be only briefly mentioned in the text where they are different from results based on the Markov switching models.[6] An alternative to AR(1) could be a higher-order autoregressive process. However, this is prohibitive due to restrictions imposed by the used solution method. Moreover, Cecchetti et al. (1994) use monthly consumption data to show that coefficients on

[4] The likelihood function for autoregressive processes in which the parameters of the autoregression can change as the result of a regime-shift variable is derived, e.g., in Hamilton (1994, Chapter 22).

[5] Estimates are not identical to those of Bonomo and Garcia (1994) because their dataset is updated by two observations.

[6] A detailed description of results for the AR(1) model is available upon request.

Table 6
Maximum likelihood estimates of the 2SMS2M2V process; monthly data

	Consumption	Dividends
α_0	0.0015 (5.940)	0 (0.180)
α_1	0.0003 (0.331)	0.007 (3.237)
p_{11}	0.5377 (0.139)	0.6037 (0.898)
p_{00}	0.8483 (1.216)	0.9516 (7.712)
ω_0	0.0034 (8.588)	0.0033 (19.030)
ω_1	0.0020 (2.085)	0.0095 (6.858)

Asymptotic t ratios in parentheses. For p_{ii}, $i=0,1$, the reported t ratios are those of the transformation $\ln(p_{ii}/(1-p_{ii}))$, $i=0,1$, respectively. The transformation was employed to restrict probability estimates to the interval (0,1).

the second through 12th lags of consumption growth are not significantly different from zero.

4. The asset pricing model with time non-separability

In this section, a version of the Lucas (1978) tree model is presented where the utility at time t depends on the utility at time $t-1$. The consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, C_{t-1}),$$

subject to the budget constraint

$$C_t + P_t^{\mathrm{E}} A_{t+1}^{\mathrm{E}} + P_t^{\mathrm{F}} A_{t+1}^{\mathrm{F}} \leq (P_t^{\mathrm{E}} + D_t) A_t^{\mathrm{E}} + A_t^{\mathrm{F}},$$

where A_t^{E}, P_t^{E}, and D_t are the amount of risky assets (equity or 'trees') held, the market prices of the risky asset, and the dividend, respectively. A_t^{F} and P_t^{F} are the investment in the riskless asset and its price, respectively. C_t is consumption. The value of the utility function depends on both consumption at time t and $t-1$ and is assumed to have standard properties. β is the discount factor. Let Λ_t and M_t denote the Lagrange multiplier of the maximization problem connected with the budget constraint and the Intertemporal Marginal Rate of Substitution (IMRS), respectively. Then it follows from the first-order conditions that:

$$M_{t+1} = \frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta[U_1(C_{t+1}, C_t) + \beta E_{t+1} U_2(C_{t+2}, C_{t+1})]}{U_1(C_t, C_{t-1}) + \beta E_t U_2(C_{t+1}, C_t)}.$$

To make sure that IMRS is stationary and that there exists a representative consumer with the same preference specification over aggregate consumption, the following class of utility functions is adopted:

$$U(C_t, C_{t-1}) = \frac{(C_t + \delta C_{t-1})^{1-\gamma}}{(1-\gamma)}, \tag{3}$$

One can think of utility being derived from a good called *services* where services are linear in both current and past consumption. δC_{t-1} is the internal habit. The sign of δ determines whether the consumption is substitutable or complementary over time. If the consumption is substitutable, then the utility from the flow of services dominates the effect of habit persistence. On the other hand, if δ is negative, the habit developed by the consumer is stronger than durability. The coefficient was estimated with different results. Eichenbaum and Hansen (1990) report a positive sign for monthly data while Ferson and Constantinides (1991) find evidence in monthly, quarterly, and annual data that habit persistence prevails. Tests based on volatility bounds also support negative δ (see Balduzzi & Kallal, 1997 for a monthly frequency and Cecchetti et al., 1994 for monthly and annual frequencies). To investigate implications of both durability and habit persistence, performance of the model is evaluated for positive as well as negative values of δ. γ is approximately equal to the expected value of the RRA coefficient and if $\delta = 0$, then γ is exactly equal to the RRA coefficient.

Using the specification in Eq. (3), the IMRS can be expressed as:

$$M_{t+1} = \frac{\beta\left[1 + \delta X_{t+1}^{-1}\right)^{-\gamma} + \beta\delta E_{t+1}(X_{t+2} + \delta)^{-\gamma}\right]}{(1 + \delta X_t^{-1})^{-\gamma} + \beta\delta E_t(X_{t+1} + \delta)^{-\gamma}} X_{t+1}^{-\gamma}, \tag{4}$$

where $X_{t+1} = C_{t+1}/C_t$. The Euler equation for the risky asset:

$$P_t^{\mathrm{E}} = E^t M_{t+1}(P_{t+1}^{\mathrm{E}} + D_{t+1})$$

can be written as:

$$V_t = E_t M_{t+1} H_{t+1}(1 + V_{t+1}), \tag{5}$$

where V_t is the price–dividend ratio and H_t is the gross growth rate of the dividend. The gross return on the risky asset is defined as:

$$R_{t+1}^{\mathrm{E}} = \frac{P_{t+1}^{\mathrm{E}} + D_{t+1}}{P_t^{\mathrm{E}}} = \frac{V_{t+1} + 1}{V_t} H_{t+1}. \tag{6}$$

If one solves for autocorrelations of the model implied equity returns, variance ratios can be calculated using Eq. (1).

The Euler equation for the risk-free asset is:

$$P_t^{\mathrm{F}} = E_t M_{t+1}.$$

The return on the riskless asset can be written as:

$$R_{t+1}^{\mathrm{F}} = \frac{1}{P_t^{\mathrm{F}}} = \frac{1}{E_t M_{t+1}}. \tag{7}$$

Bonomo and Garcia (1994) argue that the CAPM with time-separable preferences cannot generate negative excess returns. To address the issue, conditional expected excess returns can be expressed as:

$$E_t(R_{t+1}^{\mathrm{E}} - R_{t+1}^{\mathrm{F}}). \tag{8}$$

The presence of time non-separability makes the model more difficult to solve. To solve for the value of the model, a version of the method described in Tauchen and Hussey (1991) is used. They develop a discrete space approximation to solutions of non-linear asset pricing models which is based on the quadrature method (also known as Nystrom's method). The solution method is described thoroughly in Appendix B. Briefly, the conditional normal distribution from the continuous part of the 2SMS2M2V process x_t is approximated using the Gaussian N-point quadrature rule. The difference Eq. (5) is discretized accordingly and solved for price–dividend ratios. Price–dividend ratios are used to calculate equity returns (see Eq. (6)) and their variance ratios. Finally, conditional expected excess returns are computed (see Eq. (8)). The solution algorithm is easily modified for the AR(1) process.

5. Empirical results

The CAPM can be calibrated using consumption (both total and of non-durables and services), dividends, and GNP as the endowment process. For the calibration to be complete, preference parameters have to be set as well. The considered parameter set has the following structural interpretations: strong habit persistence, modest habit persistence, time-separable preferences, modest durability, and strong durability. It is demonstrated that time non-separable preferences can indeed generate mean reversion of the degree observed in the data for all endowment processes. The negative autocorrelation detected in yearly frequency is matched when preferences exhibit modest habit persistence. The positive serial correlation in monthly returns is replicated by the CAPM with durability in utility function. The CAPM calibrated to monthly data can also produce negative expected returns but only for preferences displaying strong habit persistence.

The endowment parameters are α_0, α_1, p_{11}, p_{00}, ω_0, and ω_1. Their maximum likelihood estimates are given in Tables 5 and 6. By definition, $\alpha_1 = 0$ for the 2SMS1M2V process. Chosen values of the utility function parameters are in accordance with both Cecchetti et al. (1990) and Bonomo and Garcia (1994), i.e., $\beta = 0.97$ and $\gamma = 1.70$.[7] In addition to the discount factor and the RRA coefficient, a time non-separability parameter δ is introduced to evaluate the impact of time non-separability in preferences on the (potential) autocorrelation of model returns. It can take values from -1 to 1. Negative δ generates negative values at some states of the discretized IMRS process. The appearance of negative values of the IMRS depends on the process used as the endowment as well as on the particular combination of parameters. δ for the large degree of habit persistence is the lowest value that implies non-negative values of the IMRS given $\beta = 0.97$, $\gamma = 1.70$, and corresponding estimates of the parameters of the endowment process. δ is set to -0.07 for a small degree of habit persistence. For $\delta = 0$, preferences are time-separable and results from this paper should be directly comparable to those of Bonomo and Garcia. The small degree of durability is represented by $\delta = 0.07$. Finally, $\delta = 0.60$ for the large degree of durability. Model variance ratios are calculated using Eqs. (1) and (13). Means and standard deviations are calculated using Eqs. (11) and (12), respectively. Equity premiums are computed by taking the unconditional expectation of Eq. (14)[8].

[7] The value for the RRA coefficient corresponds to results of various empirical studies that report estimated values of γ between 1 and 2 for stocks (see, for instance, a maximum likelihood estimation of the RRA coefficient in Neely, Roy, & Whiteman, 1996).

[8] Eqs. (11)–(14) can be found in Appendix B.

Bonomo and Garcia (1994) argue that the CAPM cannot produce expected excess returns that are negative. Since they only consider a model with time-separable preference, introduction of the time non-separability parameter δ can potentially render a model with expected returns being negative at some states for a favorable combination of parameters. The expected excess returns are calculated according to Eq. (14).

5.1. Annual data

Table 7 reports variance ratios, means, standard deviations, and equity premiums of the model returns for annual consumption data. The IMRS becomes negative at the 16th state for $\delta = -0.66$ in the case of total consumption and for $\delta = -0.67$ in the case of

Table 7
Variance ratios for historical and equilibrium returns endowment calibrated to total consumption and to consumption of non-durables and services of the 2SMS1M2V process; yearly data

Total consumption

	Actual	$\delta = -0.65$	$\delta = -0.07$	$\delta = 0$	$\delta = 0.07$	$\delta = 0.60$
VR(2)	1.0275	0.9100	0.8831	1.0001	1.1120	1.4576
VR(3)	0.8891	0.8835	0.8442	1.0001	1.1493	1.6101
VR(4)	0.8923	0.8729	0.8248	1.0002	1.1680	1.6864
VR(5)	0.8760	0.8685	0.8132	1.0003	1.1792	1.7322
VR(6)	0.8205	0.8672	0.8055	1.0003	1.1867	1.7627
VR(7)	0.7918	0.8677	0.8000	1.0004	1.1921	1.7845
VR(8)	0.8013	0.8692	0.7959	1.0005	1.1961	1.8009
VR(9)	0.7928	0.8715	0.7928	1.0005	1.1993	1.8136
VR(10)	0.7705	0.8741	0.7903	1.0006	1.2018	1.8238
Mean	0.0818	0.1912	0.0666	0.0664	0.0663	0.0661
S.D.	0.1871	1.2891	0.0439	0.0386	0.0350	0.0284
Equity premium	0.0529	0.1459	0.0029	0.0024	0.0020	0.0011

Consumption of non-durables and services

	Actual	$\delta = -0.66$	$\delta = -0.07$	$\delta = 0$	$\delta = 0.07$	$\delta = 0.60$
VR(2)	1.0275	0.9651	0.8830	1.0001	1.1121	1.4577
VR(3)	0.8891	0.9550	0.8440	1.0001	1.1495	1.6103
VR(4)	0.8923	0.9511	0.8246	1.0002	1.1682	1.6866
VR(5)	0.8760	0.9496	0.8130	1.0003	1.1794	1.7324
VR(6)	0.8205	0.9494	0.8053	1.0003	1.1869	1.7629
VR(7)	0.7918	0.9498	0.7998	1.0004	1.1923	1.7847
VR(8)	0.8013	0.9506	0.7957	1.0005	1.1964	1.8011
VR(9)	0.7928	0.9517	0.7926	1.0005	1.1995	1.8138
VR(10)	0.7705	0.9530	0.7901	1.0006	1.2020	1.8240
Mean	0.0818	0.1904	0.0647	0.0645	0.0664	0.0643
S.D.	0.1871	2.0772	0.0399	0.0351	0.0318	0.0257
Equity premium	0.0529	0.1444	0.0024	0.0020	0.0017	0.0009

$\beta = 0.97$ and $\gamma = 1.70$; δ values represent, respectively, strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

consumption of non-durables and services. Thus, values -0.65 and -0.66 are used, respectively. The degree of mean reversion as measured by variance ratios is low compared to that of the actual returns, especially for the consumption of non-durables and services. This is perhaps surprising, given the magnitude of habit persistence. Nonetheless, variance ratios for $\delta=-0.07$, which represents modest habit persistence, can in fact generate the mean reversion observed in the data for both consumption processes. For $\delta=0$, the model has a structural interpretation of being time-separable. The variance ratios are equal to unity in all the cases, which confirms results of Bonomo and Garcia (1994) who report variance ratios in the range from 0.9987 to 0.9852 for the one due to small sample bias since they are calculated for a sample size 1160, whereas the

Table 8
Variance ratios for historical and equilibrium returns endowment calibrated to dividends and to GNP of the 2SMS1M2V process; yearly data

Dividends						
	Actual	$\delta=-0.46$	$\delta=-0.07$	$\delta=0$	$\delta=0.07$	$\delta=0.60$
VR(2)	1.0275	0.8611	0.8866	1.0013	1.1100	1.4484
VR(3)	0.8891	0.8219	0.8496	1.0022	1.1471	1.5980
VR(4)	0.8923	0.8057	0.8314	1.0030	1.1658	1.6729
VR(5)	0.8760	0.7977	0.8208	1.0035	1.1771	1.7179
VR(6)	0.8205	0.7933	0.8137	1.0040	1.1847	1.7479
VR(7)	0.7918	0.7906	0.8088	1.0043	1.1902	1.7694
VR(8)	0.8013	0.7889	0.8051	1.0046	1.1943	1.7855
VR(9)	0.7928	0.7878	0.8023	1.0049	1.1975	1.7980
VR(10)	0.7705	0.7869	0.8000	1.0051	1.2000	1.8080
Mean	0.0818	0.3255	0.0632	0.0608	0.0593	0.0570
S.D.	0.1871	1.5981	0.1552	0.1359	0.1231	0.0987
Equity premium	0.0529	0.3886	0.0346	0.0282	0.0238	0.0133

GNP						
	Actual	$\delta=-0.54$	$\delta=-0.07$	$\delta=0$	$\delta=0.07$	$\delta=0.60$
VR(2)	1.0275	0.7406	0.8845	1.0006	1.1115	1.4541
VR(3)	0.8891	0.6755	0.8466	1.0013	1.1489	1.6055
VR(4)	0.8923	0.6576	0.8280	1.0018	1.1679	1.6813
VR(5)	0.8760	0.6576	0.8172	1.0024	1.1793	1.7268
VR(6)	0.8205	0.6657	0.8102	1.0029	1.1871	1.7571
VR(7)	0.7918	0.6778	0.8054	1.0034	1.1927	1.7789
VR(8)	0.8013	0.6920	0.8019	1.0038	1.1970	1.7952
VR(9)	0.7928	0.7071	0.7993	1.0042	1.2004	1.8079
VR(10)	0.7705	0.7225	0.7973	1.0046	1.2031	1.8180
Mean	0.0818	0.1335	0.0639	0.0635	0.0633	0.0629
S.D.	0.1871	0.5558	0.0624	0.0548	0.0498	0.0402
Equity premium	0.0529	0.0946	0.0058	0.0047	0.0040	0.0022

$\beta=0.97$ and $\gamma=1.70$; values of δ represent respectively strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

variance ratios reported here can be thought of as asymptotic values. For δ higher than one, the model displays durability and the variance ratios are greater than one.

For strong habit persistence, the model mean return is higher than the actual mean return and so is the deviation. The equity premium for the model returns is also relatively high, mainly due to negative riskless returns at some states. A gradual increase of the time non-separability parameter implies a lower variation in the IMRS and results in lower mean returns, standard deviations of returns, and equity premiums, respectively.

For yearly dividends and GNP, the results are given in Table 8. The negative values of the IMRS first appear at the 16th state for $\delta = -0.47$ and $\delta = -0.55$, respectively. Therefore, $\delta = -0.46$ and $\delta = -0.54$ are used in calculation of the model returns for the CAPM displaying strong habit persistence. Mean reversion for the model with strong habit persistence is more pronounced compared to the case where consumption is used as the endowment process. Modest habit persistence again implies variance ratios with a pattern resembling that of the data, i.e., variance ratios are closer to one for the first few periods and then they drop. The variance ratios again tend to increase with the increasing non-separability parameter δ and are greater than one for $\delta > 0$. Note that means and standard deviations of returns are slightly lower for dividends compared to means and standard deviations of returns resulting from using either consumption or GNP as the endowment process. Otherwise, the model mean returns, standard deviations, and equity premiums are again decreasing as δ increases.

Equilibrium variance ratios of returns for the same parameter combinations as in the case of the 2SMS1M2V model are calculated for the AR(1) model. Variance ratios lower than one

Table 9
Equilibrium expected excess returns of the 2SMS1M2V process; yearly data

State	Total consumption, $\delta = -0.65$	Consumption of non-durables and services, $\delta = -0.66$	Dividends, $\delta = -0.46$	GNP, $\delta = -0.54$
1	0.0429	0.0226	0.2465	0.0540
2	0.0366	0.0201	0.2125	0.0457
3	0.0318	0.0181	0.1878	0.0396
4	0.0277	0.0164	0.1679	0.0347
5	0.0242	0.0148	0.1508	0.0305
6	0.0209	0.0134	0.1356	0.0268
7	0.0178	0.0119	0.1214	0.0233
8	0.0145	0.0103	0.1071	0.0198
9	0.9887	0.9584	34.5342	2.8414
10	0.6129	0.6061	3.1920	1.1976
11	0.4124	0.4124	1.2929	0.6751
12	0.2819	0.2840	0.6835	0.4166
13	0.1878	0.1901	0.3906	0.2611
14	0.1151	0.1167	0.2193	0.1560
15	0.0554	0.0559	0.1060	0.0785
16	0.0023	0.0013	0.0223	0.0156

$\beta = 0.97$ and $\gamma = 1.70$; values of δ represent strong habit persistence.

appear already for the power utility model for both total consumption and consumption of non-durables and services. For GNP, strong habit persistence is necessary to match historical variance ratios. For dividends, equilibrium variance ratios are close to one or greater than one for all parameter combinations considered.

Table 9 reports the expected excess returns for all four endowment processes and the values of utility function parameters $\beta = 0.97$ and $\alpha = 1.70$. δ takes the lowest values admissible, i.e., $\delta = -0.65$ for total consumption, $\delta = -0.66$ for consumption of non-durables and services, $\delta = -0.46$ for dividends, and $\delta = -0.54$ for GNP. There are 16 possible states of the economy: the first eight correspond to the lower and the other eight to the higher conditional standard deviation of the endowment process. As seen in

Table 10
Variance ratios for historical and equilibrium returns endowment calibrated to consumption and to dividends of the 2SMS2M2V process; monthly data

Consumption						
	Actual	$\delta = -0.84$	$\delta = -0.07$	$\delta = 0$	$\delta = 0.07$	$\delta = 0.60$
VR(2)	1.2652	0.6113	0.8808	1.0000	1.1141	1.4600
VR(3)	1.3629	0.4820	0.8411	1.0000	1.1522	1.6134
VR(4)	1.4248	0.4174	0.8212	1.0000	1.1712	1.6900
VR(5)	1.4902	0.3787	0.8093	1.0000	1.1826	1.7360
VR(6)	1.5669	0.3529	0.8013	1.0000	1.1902	1.7667
VR(7)	1.6150	0.3344	0.7957	1.0000	1.1956	1.7886
VR(8)	1.6339	0.3206	0.7914	1.0000	1.1997	1.8050
VR(9)	1.6491	0.3099	0.7881	1.0000	1.2029	1.8178
VR(10)	1.6636	0.3013	0.7854	1.0000	1.2054	1.8280
Mean	0.006759	0.1073	0.0339	0.0339	0.0339	0.0339
S.D.	0.03431	0.4575	0.0047	0.0041	0.0037	0.0030
Equity premium	0.002612	0.0751	0.0000	0.0000	0.0000	0.0000
Dividends						
	Actual	$\delta = -0.77$	$\delta = -0.07$	$\delta = 0$	$\delta = 0.07$	$\delta = 0.60$
VR(2)	1.2652	0.6916	0.8807	1.0000	1.1143	1.4601
VR(3)	1.3629	0.5928	0.8409	1.0000	1.1524	1.6135
VR(4)	1.4248	0.5450	0.8210	1.0000	1.1714	1.6901
VR(5)	1.4902	0.5170	0.8091	1.0000	1.1828	1.7361
VR(6)	1.5669	0.4986	0.8011	1.0000	1.1905	1.7668
VR(7)	1.6150	0.4856	0.7954	1.0000	1.1959	1.7887
VR(8)	1.6339	0.4759	0.7912	1.0000	1.2000	1.8052
VR(9)	1.6491	0.4684	0.7879	1.0000	1.2032	1.8179
VR(10)	1.6636	0.4624	0.7852	1.0000	1.2057	1.8282
Mean	0.006759	0.0483	0.0313	0.0313	0.0313	0.0313
S.D.	0.03431	0.2407	0.0059	0.0059	0.0053	0.0043
Equity premium	0.002612	0.0182	0.001	0.0000	0.0000	0.0000

$\beta = 0.97$ and $\gamma = 1.70$; values of δ represent, respectively, strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

Table 11
Equilibrium expected excess returns of the 2SMS1M2V process; monthly data

State	Consumption, $\delta = -0.84$	Dividends, $\delta = -0.77$
1	0.8598	0.3498
2	0.5826	0.2443
3	0.3577	0.1563
4	0.1583	0.0764
5	− 0.0269	0.0006
6	− 0.2954	− 0.0740
7	− 0.3846	− 0.1505
8	0.5787	− 0.2353
9	1.4583	1.6300
10	0.9560	1.0240
11	0.5659	0.5834
12	0.2327	0.2267
13	− 0.0666	− 0.0790
14	− 0.3458	− 0.3523
15	− 0.6176	− 0.6078
16	− 0.9026	0.8647

$\beta = 0.97$ and $\gamma = 1.70$; values of δ represent strong habit persistence.

Table 9, none of the expected excess returns is negative though the expected excess return in the 16th state is close to zero in all the cases. The negative excess returns appear when $\delta = -0.66$ for consumption, $\delta = -0.67$ for consumption of non-durables and services, $\delta = -0.47$ for dividends, and $\delta = -0.55$ for GNP; i.e., only when the IMRS is negative. The model calibrated using the AR(1) process is also unable to generate negative excess returns.

5.2. Monthly data

Table 10 compares historical and model variance for the calibration based on monthly data. The lowest acceptable time non-separability coefficient δ is -0.84 for consumption of non-durables and services and -0.77 for the dividends, respectively. The pattern of equilibrium variance ratios is similar to the one found in annual data, i.e., they are lower than one for habit persistence, equal to unity for time-separable preferences, and greater than one for durability. Contrary to findings in annual data, monthly returns are positively serially correlated, with variance ratios significantly greater than one. Consequently, one needs $\delta > 0$ to match model variance ratios with historical ones. Again, habit persistence is necessary to generate sufficiently large equity premium.

The AR(1) model for the consumption process implies variance ratios lower than one for both strong and modest degrees of habit persistence, time separability, and modest durability. $\delta = 0.60$ results in variance ratios greater than one. For dividends, variance ratios are lower than one only for $\delta = -0.77$ and greater than one, otherwise.

Table 11 provides expected returns for strong habit persistence in both consumption dividends. Interestingly, there are negative expected rates of returns at some states. The negative expected rates of return only appear for $\delta < 0$. When the endowment processes are modeled by AR(1), the CAPM does not generate negative expected rates of return for any parameter combination.

6. Summary

In this paper, I examine an equilibrium asset pricing model with time non-separable preferences from the prospective of its ability to match the magnitude of mean reversion detected in the data on asset returns.

The mean reversion in asset returns is documented using the variance ratio test. The null hypothesis is that of the random walk and is rejected for all holding periods considered. The variance ratios of long horizon returns (with the exception of the 2-year variance ratio) imply negative autocorrelation and returns with holding periods between 1 and 10 months are positively autocorrelated.

Two types of models for the endowment process are considered: the Markov switching model allowing for heteroskedasticity and AR(1) model. At the annual frequency, parameters of the models are estimated using data on total consumption, consumption of non-durables and services, dividends, and GNP, respectively. Consumption of non-durables and services is utilized at the monthly frequency. Parameter estimates of the endowment process are employed together with utility function parameters to calibrate the CAPM. The model variance ratios and expected excess returns are then solved for.

Evidence regarding time separability is inconclusive since implications of the CAPM are sensitive to the choice of the endowment process. On the other hand, the results clearly indicate that there is a connection between time non-separability in preferences and mean reversion. A sufficient degree of habit persistence can produce negatively autocorrelated asset returns. Similarly, strong-enough durability implies positively serially correlated returns. This result is robust across all endowment models, times series, and frequencies considered. To match the pattern of at first positive and then negative serial correlation in historical returns, one needs a combination of local substitution and long-run habit persistence. Heaton (1995) finds evidence that such a combination is also consistent with Hansen and Jagannathan (1991) bounds. So, the endowment process could be approximated by a higher-order autoregressive model with the first few autoregressive coefficients positive and the others negative. This approach posits two problems, however. First, it is difficult to solve the CAPM given the current framework and second, the autoregressive model might not be acceptable from the statistical point of view. For example, Cecchetti et al. (1994) rule out higher-order autoregressive processes in favor of the AR(1) model using monthly consumption data.

Finally, the CAPM with consumption complementary over time is shown to generate negative conditional expected returns when calibrated to monthly data.

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Appendix A. Data

A.1. Annual data

The annual data considered here are those used by Cecchetti, Lam, and Mark (1993) and by Bonomo and Garcia (1994). A detailed description of the data sources is given in Cecchetti et al. (1990). The data consist of the following series:

1. *Consumption*: The real per capita total consumption and consumption of non-durables and services, 1889–1987.
2. *GNP*: The real per capita GNP, 1869–1987.
3. *CPI*: Both the annual average and end of year observations from 1870 to 1987.
4. *Dividends (D)*: The nominal dividends, 1871–1987, deflated by the annual average CPI.
5. *Standard and Poor's Composite Stock Price Index (P)*: January observations, 1871–1988, adjusted to inflation by the end of period CPI.
6. *Risk-free yield (R^F)*: The nominally riskless yields on Treasury securities, 1871–1987. Adjusted to inflation by the end of period CPI.

The summary statistics for growth rates of consumption, dividends, and GNP are reported in Table 3. Real annual returns on equity are constructed using the series P and D as $R_{t+1}{}^{\mathrm{E}}=[(P_{t+1}+D_t)/P_t]$. The mean equity premium is computed as $E[R_t^{\mathrm{E}}-R_t^{\mathrm{F}}]$.

A.2. Monthly data

The monthly data include the following series:

1. *Consumption*: The real per capita consumption of non-durables and services in 1987 dollars — CITIBASE series (GMCSO + GMCNO)/POP, 1959:02–1993:03.
2. *Price index*: Calculated as (GMCS + GMCN)/(GMCSO + GMCNO), where GMCS, GMCN, GMCSO, GMCNO are, respectively, nominal consumption expenditures on services, nominal consumption expenditures on non-durables, real consumption expenditures in 1987 dollars on services, and real consumption expenditures in 1987 dollars on non-durables, 1947:02–1993:03.
3. *Standard and Poor's Composite Common Stock Price Index*: CITIBASE series FSPCOM adjusted for inflation by the above price index, 1947:02–1993:03.
4. *Risk-free rate*: Monthly collected interest rate on 3-month Treasury Bills (CITIBASE series FYGM3) adjusted for inflation by the above price index, 1947:02–1993:03.
5. *Dividends*: Calculated using the dividend yield on Standard and Poor's Composite Common Stock (CITIBASE series FSDXP), Standard and Poor's Composite Common Stock Price Index, and the price index, both defined above, 1947:02–1993:03.

Table 4 provides summary statistics for monthly consumption and dividends. Real returns and mean equity premium are calculated in a manner similar to annual data.

Appendix B. Solution method for non-linear asset pricing models

B.1. Price–dividend ratios

The first part of Appendix B derives price–dividend ratios implied by the joint hypothesis of the CAPM and the forcing process driving endowment.

Let us construct a Markov process for x_t with the number of states given by $2N$ and let $\boldsymbol{x}$ be a $(2N \times 1)$ vector of values corresponding to the $2N$ states, i.e.,

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x^0} \\ \boldsymbol{x_1} \end{pmatrix}.$$

$\boldsymbol{x}^0$ is an $(N \times 1)$ vector with elements:

$$\boldsymbol{x}_i^0 = \alpha_0 + \omega_0 \alpha_i, \quad = 1, 2, \ldots, N,$$

where α_i is the abscissa for an N-point quadrature rule for the standard normal density.[9] Similarly, $\boldsymbol{x}^1$ is an $(N \times 1)$ vector with elements:

$$\boldsymbol{x}_i^1 = \alpha_0 + \alpha_1 + (\omega_0 + \omega_1)\alpha_1, \quad i = 1, 2, \ldots, N.$$

The transpose of the transition matrix for $\boldsymbol{x}$ is:

$$\mathbf{T} = \begin{pmatrix} p_{00}\Pi_{00} & (1 - p_{00})\Pi_{01} \\ (1 - p_{11})\Pi_{10} & p_{11}\Pi_{11} \end{pmatrix}. \tag{9}$$

x_t is normally distributed with the conditional mean v_t and the conditional variance σ_t^2. $v_t = \alpha_0$ for $S_{t-1} = 0$ and $v_t = \alpha_0 + \alpha_1$, otherwise. $\sigma_t^2 = \omega_0^2$ for $S_{t-1} = 0$ and $\sigma_t^2 = (\omega_0 + \omega_1)^2$, otherwise. Let us define $z = (x_t - v_t)/\sigma_t$. Since z is a random variable with the standard normal density, we can write the conditional probability density function $f(x_t | x_{t-1})$ as $\phi(z)/\sigma_t$, where $\phi(\cdot)$ denotes the standard normal density function. Also, the cumulative density function $F(x_t = y | x_{t-1}) = \int_{-\infty}^{y} \frac{f(x_t|x_{t-1})}{\sigma_t} \mathrm{d}x_t = \int_{-\infty}^{\frac{y-v_t}{\sigma_t}} \phi(z)\mathrm{d}z = \Phi\left(\frac{y-v_t}{\sigma_t}\right)$, where $\Phi(\cdot)$ denotes the standard normal cumulative density function. So, the conditional mean of x_t does not depend on x_{t-1} and $\boldsymbol{\Pi}_{00} = \boldsymbol{\Pi}_{01} = \boldsymbol{\Pi}_{10} = \boldsymbol{\Pi}_{11} = \boldsymbol{\Pi}$, where:

$$\boldsymbol{\Pi}_{\mathrm{ij}^-} = \omega_j, \quad i,j = 1, 2, \ldots, N.$$

[9] As N increases, the approximate solution converges to the exact solution uniformly. In most applications, accuracy does not increase much beyond $N = 5$. I use $N = 8$, which is a compromise between desired prevision and computational tractability.

ω_j's are the weights of an N-point quadrature rule for the standard normal density. Note that the IMRS (see Eq. (4)) can be written as:

$$M_{t+1} = \frac{\beta + \beta^2 \delta E_{t+1} B_{t+2}}{1 + \beta\delta E_t B_{t+1}} B_{t+1},$$

where

$$B_{t+1} = \left(\frac{\delta + X_{t+1}}{\delta + X_t} X_{t+1} \right)^{-\gamma}.$$

Let us define elements of a $(2N \times 2N)$ matrix $\mathbf{B}$ as:

$$B_{ij} \left(\frac{\delta + \mathrm{e}^{x_j}}{\delta + \mathrm{e}^{x_i}} e^{x_i} \right)^{-\gamma}, \quad i,j = 1,2,\ldots,2N.$$

$\mathbf{B}$ can be used to discretize the IMRS by defining a $(2N \times 2N)$ matrix $\mathbf{M}$ with elements:

$$\mathrm{M}_{ij} = \frac{\beta + \delta\beta^2 E[B_{ij}|j]}{1 + \delta\beta E[B_{ij}|i]} B_{ij}, \quad i,j = 1,2,\ldots,2N.$$

Using Eq. (9), $E[\mathrm{B}_{ij}|j] = \sum_{j=1}^{2N} \mathrm{B}_{ij} T_{ij}$. Finally, the Euler equation (Eq. (5)) can be discretized as well:

$$\boldsymbol{v} = \mathbf{K}\boldsymbol{\iota} + \mathbf{K}\boldsymbol{v},$$

where $\boldsymbol{v}$ is a $(2N \times 1)$ vector of price–dividend ratios and $\boldsymbol{\iota}$ is a $(2N \times 1)$ vector of ones. Elements of the $(2N \times 2N)$ matrix $\mathbf{K}$ are defined as:

$$\mathrm{K}_{ij} = M_{ij} \mathrm{e}^{x_j} T_{ij}, \quad i,j = 1,2,\ldots,2N.$$

Solving for $\boldsymbol{v}$, one gets:

$$\boldsymbol{v} = (\mathbf{I} - \boldsymbol{K})^{-1} \boldsymbol{K}\boldsymbol{\iota},$$

where $\mathbf{I}$ is the $(2N \times 2N)$ identity matrix.

B.2. Model returns

The tomorrow's return to the equity conditioned on today's state is:[10]

$$R_{ij}^{\mathrm{E}} = \frac{P_j^{\mathrm{E}} + D_j}{P_i^{\mathrm{E}}} = \frac{v_j + 1}{v_t} \mathrm{e}^{x_j}, \quad i,j = 1,\ldots 2N. \tag{10}$$

The returns implied by the model calibrated to the process of the growth rate of endowment will be used for the derivation of the model variance ratios.

In Section B.1, the endowment growth rate is approximated by a Markov chain with $2N$ states where the transition probabilities are given by $\mathbf{T}$. The equilibrium real return at time t depends on the endowment growth rates at times $\boldsymbol{t}$ and $t-1$ and is given by Eq. (10). Thus, a Markov chain for the returns can be constructed where the number of states is $4N^2$. Using the

[10] Note that when $N = 8$, there are 256 values for the rate of return.

transition matrix of the equilibrium returns, one is able to compute autocorrelations of those returns, and consequently, the variance ratios implied by the model. The transpose of the transition matrix for the model returns is:

$$
Q = \begin{pmatrix}
T_{1,1}, & T_{1,2}, & \dots & T_{1,2N}, & 0, & 0, & \dots & 0, & 0, & 0, & \dots & 0 \\
0, & 0, & \dots & 0, & T_{2,1}, & T_{2,2}, & \dots & T_{2,2N}, & 0, & 0, & \dots & 0 \\
 & & \dots & & & \dots & & & \dots & & & \\
0, & 0, & \dots & 0, & 0, & 0, & \dots & 0, & T_{2N,1}, & T_{2N,2}, & \dots & T_{2N,2N} \\
T_{1,1}, & T_{1,2}, & \dots & T_{1,2N}, & 0, & 0, & \dots & 0, & 0, & 0, & \dots & 0 \\
0, & 0, & \dots & 0, & T_{2,1}, & T_{2,2}, & \dots & T_{2,2N}, & 0, & 0, & \dots & 0 \\
 & & \dots & & & \dots & & & \dots & & & \\
0, & 0, & \dots & 0, & 0, & 0, & \dots & 0, & T_{2N,1}, & T_{2N,2}, & \dots & 0 \\
 & & \dots & & & \dots & & & \dots & & & \\
T_{1,1}, & T_{1,2}, & \dots & T_{1,2N}, & 0, & 0, & \dots & 0, & 0, & 0, & \dots & 0 \\
0, & 0, & \dots & 0, & T_{2,1}, & T_{2,2}, & \dots & T_{2,2N}, & 0, & 0, & \dots & 0 \\
 & & \dots & & & \dots & & & \dots & & & \\
0, & 0, & \dots & 0, & 0, & 0, & \dots & 0, & T_{2N,1}, & T_{2N,2}, & \dots & T_{2N,2N}
\end{pmatrix}.
$$

Let ψ denote the $(4N^2 \times 1)$ vector of unconditional probabilities of the returns. The following procedure delivers the unconditional expected value of the product of today's and lagged returns: (i) compute the unconditional unexpected value of returns by:

$$E[R_t] = \varphi' R = \kappa, \tag{11}$$

where $\boldsymbol{R}$ is the $(4N^2 \times 1)$ vector of possible values of the returns and κ is the expected value; (ii) compute the variance of returns by:

$$\mathrm{Var}[R_t] = \varphi'(\boldsymbol{R} \cdot \boldsymbol{R}) - \kappa^2 = \eta^2, \tag{12}$$

(iii) get the unconditional expected value of the product of today's and lagged return:

$$E[R_{t+s} R_t] = (\boldsymbol{R} \cdot \varphi)' \boldsymbol{Q}^s \boldsymbol{R}.$$

Equilibrium values of the variance ratios are then computed using Eq. (1) and:

$$\rho_s = \frac{E[R_{t+s}R_t] - \kappa^2}{\eta^2}. \tag{13}$$

The expected excess returns can be computed using the transition matrix **T** (Eq. (10)) and the risk-free returns. The risk-free return is simply one over the price of the risk-free asset (see Eq. (7)) and can be expressed as:

$$R_i^{\mathrm{F}} = \frac{1}{\sum_{j=1}^{2N} T_{ij} M_{ij}}, \quad i = 1, 2, \ldots, 2N.$$

The expected excess returns then are:

$$E[R_i^{\mathrm{E}} - R_i^{\mathrm{F}} | i] = \sum_{j=1}^{2N} T_{ij}(R_{ij}^{\mathrm{E}} - R_i^{\mathrm{F}}). \tag{14}$$

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