# Mean Reversion in Asset Returns and Time Non-Separable Preferences


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## Mean Reversion

Equity returns display negative serial correlation at horizons longer than one year.

The variance ratio test exploits the fact that if the stock return follows a random walk, the return variance should be proportional to the return horizon. The variance ratio statistic is defined as

$$VR(q) = \frac{Var(R_t^q)}{qVar(R_t)} = 1 + \frac{2}{q}\sum_{j=1}^{q-1}(q-j)\rho_j, \quad q = 1, 2, ..., \qquad (1)$$

where $R_t^q$ is the simple q-period return, $R_t$ is the simple one period return, and $\rho_j$ is the j-th serial correlation coefficient of returns.

Poterba and Summers (1988) show that the variance ratio test has a higher power than alternatives such as the likelihood-ratio test and the regression of current returns on lagged returns.

Lo and MacKinlay (1988) develop a specification test of the random walk hypothesis that is robust to the presence of heteroskedasticity.

## Mean Reversion and the Capital Asset Pricing Model (CAPM)

Cecchetti, Lam, and Mark (1990) show using variance ratios that the CAPM with time separable preferences can generate mean-reverted returns.

Kandel and Stambaugh (1990) demonstrate using autocorrelation coefficients that the CAPM with time separable preferences can generate mean-reverted returns.

Bonomo and Garcia (1994) show that the results of Cecchetti, Lam, and Mark (1990) and Kandel and Stambaugh (1990) are due to misspecified endowment process and that the CAPM with time separable preferences CANNOT produce mean reversion. In addition, they demonstrate that the CAPM is unable to generate negative expected exess returns.

## Time Non-separable Preferences

Constantinides (1990) uses the time non-separable utility function to resolve the equity premium puzzle identified by Mehra and Prescott (1985).

Ferson and Constantinides (1991) use GMM to test the CAPM and conclude that habit persistence is strong for the quarterly and annual data.

Heaton (1995) exploits a more complicated form of the utility function by adding more lags of consumption. He estimates the first couple of coefficients on consumption to be positive (durability) and then the sign switches (habit persistence).

Hansen and Eichenbaum (1990) use monthly data and GMM to show that durability dominates.

## Research Question

Can time non-separability improve the performance of the CAPM provided we use the proper specification of the endowment process?

## Summary

The CAPM with time non-separable preferences can generate mean reversion in asset returns - habit persistence for annual data, durability for monthly data.

The CAPM with time non-separable preferences still can produce negative expected excess returns, when calibrated to monthly data.

## Variance Ratio test of the Random Walk Hypothesis

Let us define the process for asset returns as

$$R_t = \mu + \varsigma_t,$$

where $\mu$ is an arbitrary drift parameter and $\varsigma_t$ is the random disturbance term.

To allow for rather general forms of heteroskedasticity, Lo and MacKinlay (1988) consider the following hypothesis $H_0$:

1. For all $t$, $E[\varsigma_t] = 0$, and $E[\varsigma_t \varsigma_{t-\tau}] = 0$ for any $\tau \neq 0$.

2.&3. Restrictions on the maximum degree of dependence and heteroskedasticity allowable.

4. For all $t$, $E[\varsigma_t \varsigma_{t-i} \varsigma_t \varsigma_{t-j}] = 0$ for any nonzero $i$ and $j$ where $i \neq j$. This condition implies that the sample autocorrelations of $\varsigma_t$ are asymptotically uncorrelated.

Under $H_0$, the statistic $z(q) = \sqrt{Tq}(V\widehat{R}(q) - 1)/\sqrt{\widehat{\vartheta(q)}}$ is asymptotically standard normal. $V\widehat{R}(q)$ is a variance-ratio estimator with favorable finite sample properties and $\widehat{\vartheta(q)}$ is a heteroskedasticity-consistent estimator of its variance.

The random walk hypothesis is still strongly rejected and the rejection poses a challenge for the CCAPM.

## Model

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t + \delta C_{t-1})^{1-\gamma}}{(1-\gamma)},$$

subject to the budget constraint

$$C_t + P_t^E A_{t+1}^E + P_t^F A_{t+1}^F \leq (P_t^E + D_t) A_t^E + A_t^F,$$

$A_t^E$, $P_t^E$, and $D_t$ are the amount of risky assets (equity or 'trees') held, the market price of the risky asset, and the dividend, respectively.
$A_t^F$ and $P_t^F$ are the investment in the risk-less asset and its price, respectively.
$C_t$ is consumption.

$\beta$ is the discount factor.

$\delta$ is the time non-separability parameter
$\delta > 0$ durability (substitutability)
$\delta = 0$ time separability
$\delta < 0$ habit persistence (complementarity)

$\gamma$ is approximately equal to the expected value of the Relative Risk Aversion ($RRA$) coefficient. When $\delta = 0$ then $\gamma$ is exactly equal to the $RRA$ coefficient.

The $IMRS$ can be expressed as

$$M_{t+1} = \frac{\beta[(1+\delta X_{t+1}^{-1})^{-\gamma} + \beta\delta E_{t+1}(X_{t+2}+\delta)^{-\gamma}]}{(1+\delta X_t^{-1})^{-\gamma} + \beta\delta E_t(X_{t+1}+\delta)^{-\gamma}} X_{t+1}^{-\gamma},$$

where $X_{t+1} = \frac{C_{t+1}}{C_t}$.

The Euler equation for the risky asset

$$P_t^E = E_t M_{t+1}(P_{t+1}^E + D_{t+1})$$

can be written as

$$V_t = E_t M_{t+1} H_{t+1}(1 + V_{t+1}),$$

where $V_t$ is the price-dividend ratio and $H_t$ is the gross growth rate of the dividend.

The Euler equation for the risk-free asset is

$$P_t^F = E_t M_{t+1}.$$

## Endowment Process

Consider the following $L$-state Markov switching (MS) model for the endowment process:

$$
\begin{aligned}
x_t &= \alpha_0 + \alpha_1 S_{1,t-1} + \cdots + \alpha_{L-1} S_{L-1,t-1} \\
&+ (\omega_0 + \omega_1 S_{1,t-1} + \cdots + \omega_{L-1} S_{L-1,t-1})\epsilon_t,
\end{aligned}
$$

where $x_t$ is the natural logarithm of the endowment process and $S_{i,t} = 1$ if whenever the state of the economy is $i$ and 0 otherwise. $\epsilon_t$ is an i.i.d. $N(0,1)$ error term.

Cecchetti at al. (1990): $L = 2$ and $\omega_1 = 0$ - the two-state MS model with two means and one variance.

Kandel and Stambaugh (1990): $L = 4$ but their specification also imposes that for any state with a specific mean and variance, there exists another state which has the same mean or the same variance.

Bonomo and Garcia (1994): $L = 2$ and $\alpha_1 = 0$ - the two state MS model with one mean and two variances (2SMS1M2V). The transpose of the transition matrix for the Markov process $S$ is defined as follows:

$$
\boldsymbol{P} = \begin{pmatrix} p_{00} & (1 - p_{00}) \\ (1 - p_{11}) & p_{11} \end{pmatrix},
$$

where $p_{00}$ is the probability of remaining at the state 0 while $p_{11}$ is the probability of remaining at the state 1.

## Solution Method

Let us construct a Markov process for $x_t$ with the number of states given by $2N$ and let $\boldsymbol{x}$ be a $(2N \times 1)$ vector of values corresponding to the $2N$ states i.e.

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x^0} \\ \boldsymbol{x^1} \end{pmatrix}.$$

$\boldsymbol{x^0}$ is an $(N \times 1)$ vector with elements

$$x_i^0 = \alpha_0 + \omega_0 a_i, \quad i = 1, 2, \ldots, N,$$

where $a_i$ is the abscissa for an $N$-point quadrature rule for the standard normal density. Similarly, $\boldsymbol{x^1}$ is an $(N \times 1)$ vector with elements

$$x_i^1 = \alpha_0 + (\omega_0 + \omega_1) a_i, \quad i = 1, 2, \ldots, N.$$

The transpose of the transition matrix for $\boldsymbol{x}$ is

$$\boldsymbol{T} = \begin{pmatrix} p_{00}\mathbf{\Pi_{00}} & (1 - p_{00})\mathbf{\Pi_{01}} \\ (1 - p_{11})\mathbf{\Pi_{10}} & p_{11}\mathbf{\Pi_{11}} \end{pmatrix}.$$

Since the conditional mean of $x_t$ does not depend on $x_{t-1}$, $\mathbf{\Pi_{00}} = \mathbf{\Pi_{01}} = \mathbf{\Pi_{10}} = \mathbf{\Pi_{11}} = \mathbf{\Pi}$, where

$$\Pi_{ij} = w_j, \quad i, j = 1, 2, \ldots, N.$$

$w_j$'s are the weights of an $N$-point quadrature rule for the standard normal density.

The Euler equation can be now discretized as:

$$\boldsymbol{v} = \boldsymbol{K}\boldsymbol{\iota} + \boldsymbol{K}\boldsymbol{v},$$

where $\boldsymbol{v}$ is a $(2N \times 1)$ vector of price-dividend ratios and $\boldsymbol{\iota}$ is a $(2N \times 1)$ vector of ones. Elements of the $(2N \times 2N)$ matrix $\boldsymbol{K}$ are defined as

$$K_{ij} = M_{ij} x_j T_{ij}, \quad i, j = 1, 2, \ldots, 2N,$$

where $M_{ij}$ is an element of $(2N \times 2N)$ matrix $M$, the discretized version of the IMRS. Solving for $\boldsymbol{v}$, one gets

$$\boldsymbol{v} = (\boldsymbol{I} - \boldsymbol{K})^{-1}\boldsymbol{K}\boldsymbol{\iota},$$

where $\boldsymbol{I}$ is the $(2N \times 2N)$ identity matrix.

## Model returns

The tomorrow's return to the equity conditioned on today's state is

$$R_{ij}^E = \frac{P_j^E + D_j}{P_i^E} = \frac{v_j + 1}{v_i} a_j, \quad i, j = 1, ..., 2N. \tag{2}$$

The transpose of the $(4N^2 \times 4N^2)$ transition matrix for the model returns is denoted $\boldsymbol{Q}$. Let $\boldsymbol{\psi}$ denote the $(4N^2 \times 1)$ vector of unconditional probabilities of the returns.

1. Compute the unconditional expected value of returns by

$$E[R_t] = \boldsymbol{\psi}' R = \kappa,$$

where $R$ is the $(4N^2 \times 1)$ vector of possible values of the returns and $\kappa$ is the expected value;

2. Compute the variance of returns $(\eta^2)$ by

$$Var[R_t] = \boldsymbol{\psi}'(R.R) - \kappa^2 = \eta^2;$$

3. Get the unconditional expected value of the product of the today's and lagged return:

$$E[R_{t+s} R_t] = (R.\boldsymbol{\psi})' \boldsymbol{Q}^s R.$$

Equilibrium values of the variance ratios are then computed using (1) and

$$\rho_s = \frac{E[R_{t+s} R_t] - \kappa^2}{\eta^2}.$$

## Expected Excess Returns

The risk-free return is simply one over the price of the risk-free asset from and can be expressed as

$$R_i^F = \frac{1}{\sum_{j=1}^{2N} T_{ij} M_{ij}}, i = 1, 2, \ldots, 2N.$$

The expected excess returns then are

$$E[R_i^E - R_i^F | i] = \sum_{j=1}^{2N} T_{ij}(R_{ij}^E - R_i^F).$$

## Annual Data

Consumption: The real per capita total consumption and consumption of non-durables and services, 1889-1987.

GNP: The real per capita GNP, 1869-1987.

CPI: Both the annual average and end of year observations from 1870 to 1987.

Dividends (D): The nominal dividends, 1871-1987, deflated by the annual average CPI.

Standard and Poor's Composite Stock Price Index (P): January observations, 1871-1988, adjusted to inflation by the end of period CPI.

Risk-free yield ($R^F$): The nominally risk-less yields on Treasury securities, 1871-1987. Adjusted to inflation by the end of period CPI.

Real annual returns on equity: Constructed using the series $P$ and $D$ as $R^E_{t+1} = \frac{P_{t+1}+D_t}{P_t}$.

The mean equity premium: Computed as $E[R^E_t - R^F_t]$.

## Monthly Data

Consumption: The real per capita consumption of non-durables and services in 1987 dollars - CITIBASE series

$$(GMCSQ + GMCNQ)/POP,$$

1959:02 1993:03.

Price Index: $(GMCS + GMCN)/(GMCSQ + GMCNQ)$, where $GMCS$, $GMCN$, $GMCSQ$, $GMCNQ$ are respectively nominal consumption expenditures on services, nominal consumption expenditures on non-durables, real consumption expenditures in 1987 dollars on services, and real consumption expenditures in 1987 dollars on non-durables, 1947:02 1993:03.

Standard and Poor's Composite Common Stock Price Index: CITIBASE series FSPCOM adjusted for inflation by the above price index, 1947:02 1993:03.

Risk-Free Rate: Monthly collected interest rate on the three-months Treasury Bills (CITIBASE series FYGM3) adjusted for inflation by the above price index, 1947:02 1993:03.

Dividends: Calculated using the dividend yield on Standard and Poor's Composite Common Stock (CITIBASE series FSDXP), Standard and Poor's Composite Common Stock Price Index, and the price index, both defined above, 1947:02 1993:03.

Table 1: Variance Ratios for Historical Returns; Yearly Data 1870-1987

| q | VR(q) | z(q) |
|---|---|---|
| 2 | 1.0275 | 2.9952 |
| 3 | 0.8891 | -7.9440 |
| 4 | 0.8923 | -6.0742 |
| 5 | 0.8760 | -5.9204 |
| 6 | 0.8205 | -7.5561 |
| 7 | 0.7918 | -7.9245 |
| 8 | 0.8013 | -6.9658 |
| 9 | 0.7928 | -6.7778 |
| 10 | 0.7705 | -7.0959 |

*Note*
The random walk hypothesis allowing for heteroskedasticity is rejected in all cases at 1% level.

Table 2: Variance Ratios for Historical Returns; Monthly Data 1947:02 1994:03

| q | VR(q) | z(q) |
|---|---|---|
| 2 | 1.2652 | 111.4259 |
| 3 | 1.3629 | 106.7755 |
| 4 | 1.4248 | 103.2105 |
| 5 | 1.4902 | 104.2021 |
| 6 | 1.5669 | 108.5213 |
| 7 | 1.6150 | 107.9693 |
| 8 | 1.6339 | 103.3748 |
| 9 | 1.6491 | 99.4246 |
| 10 | 1.6636 | 96.0809 |

*Note*
The random walk hypothesis allowing for heteroskedasticity is rejected in all cases at 1% level.

Table 3: Summary Statistics for Growth Rates in Sample; Yearly data

| | Total Consumption | Consumption of Non-durables and Services | Dividends | GNP |
|---|---|---|---|---|
| Time Period | 1890-1987 | 1890-1987 | 1872-1987 | 1870-1987 |
| Obs. | 98 | 98 | 116 | 118 |
| Mean | 0.0182 | 0.0172 | 0.0112 | 0.0178 |
| St.Dev. | 0.0374 | 0.0342 | 0.1262 | 0.0514 |
| Skewness | -0.4097 | -0.4045 | -0.8228 | -0.7574 |
| Kurtosis | 3.8750 | 3.9773 | 6.3321 | 7.6627 |
| Maximum | 0.0990 | 0.0994 | 0.4168 | 0.1613 |
| Minimum | -0.0987 | -0.0874 | -0.4314 | -0.2216 |
| First Autocor. | -0.0679 | -0.1343 | 0.2089 | 0.3908 |

Table 4: Summary Statistics for Growth Rates in Sample; Monthly Data

| | Consumption | Dividends |
|---|---|---|
| Time Period | 1959:02 1993:03 | 1947:02 1993:03 |
| Obs. | 410 | 554 |
| Mean | 0.00159 | 0.000768 |
| St.Dev. | 0.00394 | 0.005666 |
| Skewness | 0.0195 | 1.73730 |
| Kurtosis | 3.5174 | 16.72803 |
| Maximum | 0.01598 | 0.03945 |
| Minimum | -0.010795 | -0.0341 |
| First Autocor. | -0.2442 | 0.1992 |

Table 5: Maximum Likelihood Estimates of the 2SMS1M2V Process, Yearly Data

| | Total Consumption | Consumption of Non-durables and Services | Dividends | GNP |
|---|---|---|---|---|
| $\alpha_0$ | 0.0197 | 0.0187 | 0.0144 | 0.0179 |
| | (8.087) | (10.416) | (2.304) | (5.701) |
| $p_{11}$ | 0.9897 | 0.9885 | 0.8193 | 0.9281 |
| | (3.742) | (3.500) | (1.746) | (2.707) |
| $p_{00}$ | 0.9874 | 0.9854 | 0.8165 | 0.9834 |
| | (3.338) | (3.086) | (2.228) | (3.966) |
| $\omega_0$ | 0.0165 | 0.0113 | 0.0381 | 0.0303 |
| | (8.714) | (8.436) | (7.569) | (10.913) |
| $\omega_1$ | 0.0299 | 0.0315 | 0.1350 | 0.0698 |
| | (6.328) | (7.523) | (6.922) | (4.161) |

*Note*
Asymptotic t-ratios in parentheses. For $p_{ii}, i = 0, 1$, the reported t-ratios are those of the transformation $ln(p_{ii}/(1 - p_{ii})), i = 0, 1$, respectively. The transformation was employed to restrict probability estimates to the interval $(0, 1)$.

Table 6: Maximum Likelihood Estimates of the 2SMS2M2V Process; Monthly Data

| | Consumption | Dividends |
|---|---|---|
| $\alpha_0$ | 0.0015 | 0 |
| | (5.940) | (0.180) |
| $\alpha_1$ | 0.0003 | 0.007 |
| | (0.331) | (3.237) |
| $p_{11}$ | 0.5377 | 0.6037 |
| | (0.139) | (0.898) |
| $p_{00}$ | 0.8483 | 0.9516 |
| | (1.216) | (7.712) |
| $\omega_0$ | 0.0034 | 0.0033 |
| | (8.588) | (19.030) |
| $\omega_1$ | 0.0020 | 0.0095 |
| | (2.085) | (6.858) |

*Note*

Asymptotic t-ratios in parentheses. For $p_{ii}, i = 0, 1$, the reported t-ratios are those of the transformation $ln(p_{ii}/(1 - p_{ii})), i = 0, 1$, respectively. The transformation was employed to restrict probability estimates to the interval $(0, 1)$.

Table 7: Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Total Consumption and to Consumption of Non-durables and Services, the 2SMS1M2V Process, Yearly Data

Total Consumption

| | Actual | $\delta = -0.65$ | $\delta = -0.07$ | $\delta = 0$ | $\delta = 0.07$ | $\delta = 0.60$ |
|---|---|---|---|---|---|---|
| VR(2) | 1.0275 | 0.9100 | 0.8831 | 1.0001 | 1.1120 | 1.4576 |
| VR(3) | 0.8891 | 0.8835 | 0.8442 | 1.0001 | 1.1493 | 1.6101 |
| VR(4) | 0.8923 | 0.8729 | 0.8248 | 1.0002 | 1.1680 | 1.6864 |
| VR(5) | 0.8760 | 0.8685 | 0.8132 | 1.0003 | 1.1792 | 1.7322 |
| VR(6) | 0.8205 | 0.8672 | 0.8055 | 1.0003 | 1.1867 | 1.7627 |
| VR(7) | 0.7918 | 0.8677 | 0.8000 | 1.0004 | 1.1921 | 1.7845 |
| VR(8) | 0.8013 | 0.8692 | 0.7959 | 1.0005 | 1.1961 | 1.8009 |
| VR(9) | 0.7928 | 0.8715 | 0.7928 | 1.0005 | 1.1993 | 1.8136 |
| VR(10) | 0.7705 | 0.8741 | 0.7903 | 1.0006 | 1.2018 | 1.8238 |
| mean | 0.0818 | 0.1912 | 0.0666 | 0.0664 | 0.0663 | 0.0661 |
| st.dev. | 0.1871 | 1.2891 | 0.0439 | 0.0386 | 0.0350 | 0.0284 |
| eq. premium | 0.0529 | 0.1459 | 0.0029 | 0.0024 | 0.0020 | 0.0011 |

Consumption of Non-durables and Services

| | Actual | $\delta = -0.66$ | $\delta = -0.07$ | $\delta = 0$ | $\delta = 0.07$ | $\delta = 0.60$ |
|---|---|---|---|---|---|---|
| VR(2) | 1.0275 | 0.9651 | 0.8830 | 1.0001 | 1.1121 | 1.4577 |
| VR(3) | 0.8891 | 0.9550 | 0.8440 | 1.0001 | 1.1495 | 1.6103 |
| VR(4) | 0.8923 | 0.9511 | 0.8246 | 1.0002 | 1.1682 | 1.6866 |
| VR(5) | 0.8760 | 0.9496 | 0.8130 | 1.0003 | 1.1794 | 1.7324 |
| VR(6) | 0.8205 | 0.9494 | 0.8053 | 1.0003 | 1.1869 | 1.7629 |
| VR(7) | 0.7918 | 0.9498 | 0.7998 | 1.0004 | 1.1923 | 1.7847 |
| VR(8) | 0.8013 | 0.9506 | 0.7957 | 1.0005 | 1.1964 | 1.8011 |
| VR(9) | 0.7928 | 0.9517 | 0.7926 | 1.0005 | 1.1995 | 1.8138 |
| VR(10) | 0.7705 | 0.9530 | 0.7901 | 1.0006 | 1.2020 | 1.8240 |
| mean | 0.0818 | 0.1904 | 0.0647 | 0.0645 | 0.0644 | 0.0643 |
| st.dev. | 0.1871 | 2.0772 | 0.0399 | 0.0351 | 0.0318 | 0.0257 |
| eq. premium | 0.0529 | 0.1444 | 0.0024 | 0.0020 | 0.0017 | 0.0009 |

*Note*

$\beta = 0.97$ and $\gamma = 1.70$; values of $\delta$ represent respectively strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

Table 8: Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Dividends and to GNP, the 2SMS1M2V Process, Yearly Data

Dividends

| | Actual | $\delta=-0.46$ | $\delta=-0.07$ | $\delta=0$ | $\delta=0.07$ | $\delta=0.60$ |
|---|---|---|---|---|---|---|
| VR(2) | 1.0275 | 0.8611 | 0.8866 | 1.0013 | 1.1100 | 1.4484 |
| VR(3) | 0.8891 | 0.8219 | 0.8496 | 1.0022 | 1.1471 | 1.5980 |
| VR(4) | 0.8923 | 0.8057 | 0.8314 | 1.0030 | 1.1658 | 1.6729 |
| VR(5) | 0.8760 | 0.7977 | 0.8208 | 1.0035 | 1.1771 | 1.7179 |
| VR(6) | 0.8205 | 0.7933 | 0.8137 | 1.0040 | 1.1847 | 1.7479 |
| VR(7) | 0.7918 | 0.7906 | 0.8088 | 1.0043 | 1.1902 | 1.7694 |
| VR(8) | 0.8013 | 0.7889 | 0.8051 | 1.0046 | 1.1943 | 1.7855 |
| VR(9) | 0.7928 | 0.7878 | 0.8023 | 1.0049 | 1.1975 | 1.7980 |
| VR(10) | 0.7705 | 0.7869 | 0.8000 | 1.0051 | 1.2000 | 1.8080 |
| mean | 0.0818 | 0.3255 | 0.0632 | 0.0608 | 0.0593 | 0.0570 |
| st.dev. | 0.1871 | 1.5981 | 0.1552 | 0.1359 | 0.1231 | 0.0987 |
| eq. premium | 0.0529 | 0.3886 | 0.0346 | 0.0282 | 0.0238 | 0.0133 |

GNP

| | Actual | $\delta=-0.54$ | $\delta=-0.07$ | $\delta = 0$ | $\delta = 0.07$ | $\delta = 0.60$ |
|---|---|---|---|---|---|---|
| VR(2) | 1.0275 | 0.7406 | 0.8845 | 1.0006 | 1.1115 | 1.4541 |
| VR(3) | 0.8891 | 0.6755 | 0.8466 | 1.0013 | 1.1489 | 1.6055 |
| VR(4) | 0.8923 | 0.6576 | 0.8280 | 1.0018 | 1.1679 | 1.6813 |
| VR(5) | 0.8760 | 0.6576 | 0.8172 | 1.0024 | 1.1793 | 1.7268 |
| VR(6) | 0.8205 | 0.6657 | 0.8102 | 1.0029 | 1.1871 | 1.7571 |
| VR(7) | 0.7918 | 0.6778 | 0.8054 | 1.0034 | 1.1927 | 1.7789 |
| VR(8) | 0.8013 | 0.6920 | 0.8019 | 1.0038 | 1.1970 | 1.7952 |
| VR(9) | 0.7928 | 0.7071 | 0.7993 | 1.0042 | 1.2004 | 1.8079 |
| VR(10) | 0.7705 | 0.7225 | 0.7973 | 1.0046 | 1.2031 | 1.8180 |
| mean | 0.0818 | 0.1335 | 0.0639 | 0.0635 | 0.0633 | 0.0629 |
| st.dev. | 0.1871 | 0.5558 | 0.0624 | 0.0548 | 0.0498 | 0.0402 |
| eq. premium | 0.0529 | 0.0946 | 0.0058 | 0.0047 | 0.0040 | 0.0022 |

*Note*

$\beta = 0.97$ and $\gamma = 1.70$; values of $\delta$ represent respectively strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

Table 9: Equilibrium Expected Excess Returns, the 2SMS1M2V Process, Yearly Data

| State | Total Consumption<br>$\delta = -0.65$ | Consumption of Non-durables and Services<br>$\delta = -0.66$ | Dividends<br>$\delta = -0.46$ | GNP<br>$\delta = -0.54$ |
|---|---|---|---|---|
| 1 | 0.0429 | 0.0226 | 0.2465 | 0.0540 |
| 2 | 0.0366 | 0.0201 | 0.2125 | 0.0457 |
| 3 | 0.0318 | 0.0181 | 0.1878 | 0.0396 |
| 4 | 0.0277 | 0.0164 | 0.1679 | 0.0347 |
| 5 | 0.0242 | 0.0148 | 0.1508 | 0.0305 |
| 6 | 0.0209 | 0.0134 | 0.1356 | 0.0268 |
| 7 | 0.0178 | 0.0119 | 0.1214 | 0.0233 |
| 8 | 0.0145 | 0.0103 | 0.1071 | 0.0198 |
| 9 | 0.9887 | 0.9584 | 34.5342 | 2.8414 |
| 10 | 0.6129 | 0.6061 | 3.1920 | 1.1976 |
| 11 | 0.4124 | 0.4124 | 1.2929 | 0.6751 |
| 12 | 0.2819 | 0.2840 | 0.6835 | 0.4166 |
| 13 | 0.1878 | 0.1901 | 0.3906 | 0.2611 |
| 14 | 0.1151 | 0.1167 | 0.2193 | 0.1560 |
| 15 | 0.0554 | 0.0559 | 0.1060 | 0.0785 |
| 16 | 0.0023 | 0.0013 | 0.0223 | 0.0156 |

*Note*

$\beta = 0.97$ and $\gamma = 1.70$; values of $\delta$ represent strong habit persistence.

Table 10: Variance Ratios for Historical and Equilibrium Returns - Endowment Calibrated to Consumption and to Dividends, the 2SMS2M2V Process, Monthly Data

Consumption

| | Actual | $\delta = -0.84$ | $\delta = -0.07$ | $\delta = 0$ | $\delta = 0.07$ | $\delta = 0.60$ |
|---|---|---|---|---|---|---|
| VR(2) | 1.2652 | 0.6113 | 0.8808 | 1.0000 | 1.1141 | 1.4600 |
| VR(3) | 1.3629 | 0.4820 | 0.8411 | 1.0000 | 1.1522 | 1.6134 |
| VR(4) | 1.4248 | 0.4174 | 0.8212 | 1.0000 | 1.1712 | 1.6900 |
| VR(5) | 1.4902 | 0.3787 | 0.8093 | 1.0000 | 1.1826 | 1.7360 |
| VR(6) | 1.5669 | 0.3529 | 0.8013 | 1.0000 | 1.1902 | 1.7667 |
| VR(7) | 1.6150 | 0.3344 | 0.7957 | 1.0000 | 1.1956 | 1.7886 |
| VR(8) | 1.6339 | 0.3206 | 0.7914 | 1.0000 | 1.1997 | 1.8050 |
| VR(9) | 1.6491 | 0.3099 | 0.7881 | 1.0000 | 1.2029 | 1.8178 |
| VR(10) | 1.6636 | 0.3013 | 0.7854 | 1.0000 | 1.2054 | 1.8280 |
| mean | 0.006759 | 0.1073 | 0.0339 | 0.0339 | 0.0339 | 0.0339 |
| st.dev. | 0.03431 | 0.4575 | 0.0047 | 0.0041 | 0.0037 | 0.0030 |
| eq. premium | 0.002612 | 0.0751 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Dividends

| | Actual | $\delta = -0.77$ | $\delta = -0.07$ | $\delta = 0$ | $\delta = 0.07$ | $\delta = 0.60$ |
|---|---|---|---|---|---|---|
| VR(2) | 1.2652 | 0.6916 | 0.8807 | 1.0000 | 1.1143 | 1.4601 |
| VR(3) | 1.3629 | 0.5928 | 0.8409 | 1.0000 | 1.1524 | 1.6135 |
| VR(4) | 1.4248 | 0.5450 | 0.8210 | 1.0000 | 1.1714 | 1.6901 |
| VR(5) | 1.4902 | 0.5170 | 0.8091 | 1.0000 | 1.1828 | 1.7361 |
| VR(6) | 1.5669 | 0.4986 | 0.8011 | 1.0000 | 1.1905 | 1.7668 |
| VR(7) | 1.6150 | 0.4856 | 0.7954 | 1.0000 | 1.1959 | 1.7887 |
| VR(8) | 1.6339 | 0.4759 | 0.7912 | 1.0000 | 1.2000 | 1.8052 |
| VR(9) | 1.6491 | 0.4684 | 0.7879 | 1.0000 | 1.2032 | 1.8179 |
| VR(10) | 1.6636 | 0.4624 | 0.7852 | 1.0000 | 1.2057 | 1.8282 |
| mean | 0.006759 | 0.0483 | 0.0313 | 0.0313 | 0.0313 | 0.0313 |
| st.dev. | 0.03431 | 0.2407 | 0.0067 | 0.0059 | 0.0053 | 0.0043 |
| eq. premium | 0.002612 | 0.0182 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |

*Note*

$\beta = 0.97$ and $\gamma = 1.70$; values of $\delta$ represent respectively strong habit persistence, modest habit persistence, time separability, modest durability, and strong durability. Means, standard deviations, and equity premiums are reported in addition to variance ratios for both historical and equilibrium returns.

Table 11: Equilibrium Expected Excess Returns, the 2SMS1M2V Process, Monthly Data

| State | Consumption $\delta = -0.84$ | Dividends $\delta = -0.77$ |
|---|---|---|
| 1 | 0.8598 | 0.3498 |
| 2 | 0.5826 | 0.2443 |
| 3 | 0.3577 | 0.1563 |
| 4 | 0.1583 | 0.0764 |
| 5 | -0.0269 | 0.0006 |
| 6 | -0.2054 | -0.0740 |
| 7 | -0.3846 | -0.1505 |
| 8 | -0.5787 | -0.2353 |
| 9 | 1.4583 | 1.6300 |
| 10 | 0.9560 | 1.0240 |
| 11 | 0.5659 | 0.5834 |
| 12 | 0.2327 | 0.2267 |
| 13 | -0.0666 | -0.0790 |
| 14 | -0.3458 | -0.3523 |
| 15 | -0.6176 | -0.6078 |
| 16 | -0.9026 | -0.8647 |

*Note*

$\beta = 0.97$ and $\gamma = 1.70$; values of $\delta$ represent strong habit persistence.