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Default Risk in Equity Returns

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ABSTRACT

This is the first study that uses Merton's (1974) option pricing model to compute default measures for individual firms and assess the effect of default risk on equity returns. The size effect is a default effect, and this is also largely true for the book-to-market (BM) effect. Both exist only in segments of the market with high default risk. Default risk is systematic risk. The Fama–French (FF) factors SMB and HML contain some default-related information, but this is not the main reason that the FF model can explain the cross section of equity returns.

A FIRM DEFAULTS WHEN IT FAILS to service its debt obligations. Therefore, default risk induces lenders to require from borrowers a spread over the risk-free rate of interest. This spread is an increasing function of the probability of default of the individual firm.

Although considerable research effort has been put toward modeling default risk for the purpose of valuing corporate debt and derivative products written on it, little attention has been paid to the effects of default risk on equity returns.[1] The effect that default risk may have on equity returns is not obvious, since equity holders are the residual claimants on a firm's cash flows and there is no promised nominal return in equities.

Previous studies that examine the effect of default risk on equities focus on the ability of the default spread to explain or predict returns. The default spread is usually defined as the yield or return differential between long-term BAA corporate bonds and long-term AAA or U.S. Treasury bonds.[2] However,

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[1] For papers that model default risk see for instance, Madan and Unal (1994), Duffie and Singleton (1995, 1997), Jarrow and Turnbull (1995), Longstaff and Schwartz (1995), Zhou (1997), Lando (1998), and Duffee (1999), among others.

[2] For instance, many studies have shown that the yield spread between BAA and AAA corporate bond spread can predict expected returns in stocks and bonds. Such studies include those of Fama and Schwert (1977), Keim and Stambaugh (1986), Campbell (1987), and Fama and French (1989),

as Elton et al. (2001) show, much of the information in the default spread is unrelated to default risk. In fact, as much as 85 percent of the spread can be explained as reward for bearing systematic risk, unrelated to default. Furthermore, differential taxes seem to have a more important influence on spreads than expected loss from default. These results lead us to conclude that, independent of whether the default spread can explain, predict, or otherwise relate to equity returns, such a relation cannot be attributed to the effects that default risk may have on equities. In other words, we still know very little about how default risk affects equity returns.

The purpose of this paper is to address precisely this question. Instead of relying on information about default obtained from the bonds market, we estimate default likelihood indicators (DLI) for individual firms using equity data. These DLI are nonlinear functions of the default probabilities of the individual firms. They are calculated using the contingent claims methodology of Black and Scholes (1973) (BS) and Merton (1974). Consistent with the Elton et al. (2001) study, we find that our measure of default risk contains very different information from the commonly used aggregate default spreads. This occurs despite the fact that our DLI can indeed predict actual defaults.

We find that default risk is intimately related to the size and book-to-market (BM) characteristics of a firm. Our results point to the conclusion that both the size and BM effects can be viewed as default effects. This is particularly the case for the size effect.

The size effect exists *only* within the quintile with the highest default risk. In that segment of the market, the return difference between small and big firms is of the order of 45 percent per annum (p.a.). The small stocks in the high-default-risk quintile are typically among the smallest of the small firms and have the highest BM ratios. Furthermore, even within the high-default-risk quintile, small firms have much higher default risk than big firms, and default risk decreases monotonically as size increases.

A similar result is obtained for the BM effect. The BM effect exists only in the two quintiles with the highest default risk. Within the highest default risk quintile, the return difference between value (high BM) and growth (low BM) stocks is around 30 percent p.a., and goes down to 12.7 percent for the stocks in the second highest default risk quintile. There is no BM effect in the remaining stocks of the market. Again, the value stocks in these categories have the highest BMs of all stocks in the market, and the smallest size. Value stocks have much higher default risk than growth stocks, and there is a monotonic relation between BM and default risk.

We also find that high-default-risk firms earn higher returns than low default risk firms, only to the extent that they are small in size and high BM. If these firm characteristics are not met, they do not earn higher returns than low default risk firms, even if their risk of default is actually very high.

among others. In addition, Chen, Roll, and Ross (1986), Fama and French (1993), Jagannathan and Wang (1996), and Hahn and Lee (2001) consider variations of the default spread in asset-pricing tests.

We finally examine whether default risk is systematic. We find that it is indeed systematic and therefore priced in the cross section of equity returns.

Fama and French (1996) argue that the SMB and HML factors of the Fama and French (1993) (FF) model proxy for financial distress. Our asset-pricing results show that, although SMB and HML contain default-related information, this is not the reason that the FF model can explain the cross section. SMB and HML appear to contain important priced information, unrelated to default risk.

Several studies in the corporate finance literature examine whether default risk is systematic, but their results are often conflicting. Denis and Denis (1995), for example, show that default risk is related to macroeconomic factors and that it varies with the business cycle. This result is consistent with ours since our measure of default risk also varies with the business cycle. Opler and Titman (1994) and Asquith, Gertner, and Sharfstein (1994), on the other hand, find that bankruptcy is related to idiosyncratic factors and therefore does not represent systematic risk. The asset-pricing results of the current study show that default risk is systematic.

Contrary to the current study, previous research has used either accounting models or bond market information to estimate a firm's default risk and in some cases has produced different results from ours.

Examples of papers that use accounting models include those of Dichev (1998) and Griffin and Lemmon (2002). Dichev examines the relation between bankruptcy risk and systematic risk. Using Altman's (1968) *Z*-score model and Ohlson's (1980) conditional logit model, he computes measures of financial distress and finds that bankruptcy risk is not rewarded by higher returns. He concludes that the size and BM effects are unlikely to proxy for a distress factor related to bankruptcy. A similar conclusion is reached in the case of the BM effect by Griffin and Lemmon (2002), who use Olson's model and conclude that the BM effect must be due to mispricing.

There are several concerns about the use of accounting models in estimating the default risk of equities. Accounting models use information derived from financial statements. Such information is inherently backward looking, since financial statements aim to report a firm's past performance, rather than its future prospects. In contrast, Merton's (1974) model uses the market value of a firm's equity in calculating its default risk. It also estimates its market value of debt, rather than using the book value of debt, as the accounting models do. Market prices reflect investors' expectations about a firm's future performance. As a result, they contain forward-looking information, which is better suited for calculating the likelihood that a firm may default in the future.

In addition, and most importantly, accounting models do not take into account the volatility of a firm's assets in estimating its risk of default. Accounting models imply that firms with similar financial ratios will have similar likelihoods of default. This is not the case in Merton's model, where firms may have similar levels of equity and debt, but very different likelihoods to default, if the volatilities of their assets differ. Clearly, the volatility of a firm's assets provides crucial information about the firm's probability to default. Campbell et al. (2001) demonstrate that firm level volatility has trended upward since the mid-1970s.

Furthermore, using data from 1995 to 1999, Campbell and Taksler (2003) show that firm level volatility and credit ratings can explain equally well the cross-sectional variation in corporate bond yields. Clearly, a firm's volatility is a key input in the Black–Scholes option-pricing formula.

As mentioned, an alternative source of information for calculating default risk measures is the bonds market. One may use bond ratings or individual spreads between a firm's debt issues and an aggregate yield measure to deduce the firm's risk of default. When a study uses bond downgrades and upgrades as a measure of default risk, it usually relies implicitly on the following assumptions: that all assets within a rating category share the same default risk and that this default risk is equal to the historical average default risk. Furthermore, it assumes that it is impossible for a firm to experience a change in its default probability, also without experiencing a rating change.[3]

Typically, however, a firm experiences a substantial change in its default risk prior to its rating change. This change in its probability of default is observed only with a lag, and measured crudely through the rating change. Bond ratings may also represent a relatively noisy estimate of a firm's likelihood to default because equity and bond markets may not be perfectly integrated, and because the corporate bond market is much less liquid than the equity market.[4] Merton's model does not require any assumptions about the integration of bond and equity markets or their efficiencies, since all the information needed to calculate the default risk measures is obtained from equities.

Examples of studies that use bond ratings to examine the effect of upgrades and downgrades on equity returns include those of Holthausen and Leftwich (1986), Hand, Holthausen, and Leftwich (1992), and Dichev and Piotroski (2001), among others. The general finding is that bond downgrades are followed by negative equity returns. The effect of an increase in default risk on the subsequent equity returns is not examined in the current study.

The remainder of the paper is organized as follows. Section I discusses the methodology used to compute DLI for individual firms. Section II describes the data and provides summary statistics. Section III examines the ability of the DLI to predict actual defaults. In Section IV we report results on the performance of portfolios constructed on the basis of default-risk information. In Section V, we provide asset-pricing tests that examine whether default risk is priced. We conclude in Section VI with a summary of our results.

I. Measuring Default Risk

A. Theoretical Model

In Merton's (1974) model, the equity of a firm is viewed as a call option on the firm's assets. The reason is that equity holders are residual claimants on

[3] See also, Kealhofer, Kwok, and Weng (1998).

[4] For instance, Kwan (1996) shows that lagged stock returns can predict current bond yield changes. However, Hotchkiss and Ronen (2001) find that although the correlation between bond and stock returns is positive and significant, there is no causal relation between the two markets.

the firm's assets after all other obligations have been met. The strike price of the call option is the book value of the firm's liabilities. When the value of the firm's assets is less than the strike price, the value of equity is zero.

Our approach in calculating default risk measures using Merton's model is very similar to the one used by KMV and outlined in Crosbie (1999).[5] We assume that the capital structure of the firm includes both equity and debt. The market value of a firm's underlying assets follows a geometric Brownian motion (GBM) of the form:

$$dV_A = \mu V_A \, dt + \sigma_A V_A \, dW, \tag{1}$$

where V_A is the firm's assets value, with an instantaneous drift μ, and an instantaneous volatility σ_A. A standard Wiener process is W.

We denote by X_t the book value of the debt at time t, that has maturity equal to T. As noted earlier, X_t plays the role of the strike price of the call, since the market value of equity can be thought of as a call option on V_A with time to expiration equal to T. The market value of equity, V_E, will then be given by the Black and Scholes (1973) formula for call options:

$$V_E = V_A N(d_1) - X e^{-rT} N(d_2), \tag{2}$$

where

$$d_1 = \frac{\ln(V_A/X) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}, \quad d_2 = d_1 - \sigma_A\sqrt{T}, \tag{3}$$

r is the risk-free rate, and N is the cumulative density function of the standard normal distribution.

To calculate σ_A we adopt an iterative procedure. We use daily data from the past 12 months to obtain an estimate of the volatility of equity σ_E, which is then used as an *initial value* for the estimation of σ_A. Using the Black–Scholes formula, and for each trading day of the past 12 months, we compute V_A using V_E as the market value of equity of that day. In this manner, we obtain daily values for V_A. We then compute the standard deviation of those V_A's, which is used as the value of σ_A, for the next iteration. This procedure is repeated until the values of σ_A from two consecutive iterations converge. Our tolerance level for convergence is 10E-4. For most firms, it takes only a few iterations for σ_A to converge. Once the converged value of σ_A is obtained, we use it to back out V_A through equation (2).

[5] There are two main differences between our approach and the one used by KMV. They use a more complicated method to assess the asset volatility than we do, which incorporates Bayesian adjustments for the country, industry, and size of the firm. They also allow for convertibles and preferred stocks in the capital structure of the firm, whereas we allow only equity, as well as short- and long-term debt.

The above process is repeated at the end of every month, resulting in the estimation of monthly values of σ_A. The estimation window is always kept equal to 12 months. The risk-free rate used for each monthly iterative process is the 1-year T-bill rate observed at the end of the month.

Once daily values of V_A are estimated, we can compute the drift μ, by calculating the mean of the change in $\ln V_A$.

The default probability is the probability that the firm's assets will be less than the book value of the firm's liabilities. In other words,

$$P_{def,t} = \text{Prob}\left(V_{A,t+T} \leq X_t | V_{A,t}\right) = \text{Prob}\left(\ln\left(V_{A,t+T}\right) \leq \ln\left(X_t\right) | V_{A,t}\right). \quad (4)$$

Since the value of the assets follows the GBM of equation (1), the value of the assets at any time t is given by:

$$\ln\left(V_{A,t+T}\right) = \ln\left(V_{A,t}\right) + \left(\mu - \frac{\sigma_A^2}{2}\right)T + \sigma_A\sqrt{T}\varepsilon_{t+T}, \quad (5)$$

$$\varepsilon_{t+T} = \frac{W(t+T) - W(t)}{\sqrt{T}}, \quad \text{and} \quad \varepsilon_{t+T} \sim N(0,1). \quad (6)$$

Therefore we can rewrite the default probability as follows:

$$P_{def,t} = \text{Prob}\left(\ln\left(V_{A,t}\right) - \ln\left(X_t\right) + \left(\mu - \frac{\sigma_A^2}{2}\right)T + \sigma_A\sqrt{T}\varepsilon_{t+T} \leq 0\right)$$

$$P_{def,t} = \text{Prob}\left(-\frac{\ln\left(\frac{V_{A,t}}{X_t}\right) + \left(\mu - \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}} \geq \varepsilon_{t+T}\right). \quad (7)$$

We can then define the distance to default (DD) as follows:

$$DD_t = \frac{\ln(V_{A,t}/X_t) + \left(\mu - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}. \quad (8)$$

Default occurs when the ratio of the value of assets to debt is less than 1, or its log is negative. The DD tells us by how many standard deviations the log of this ratio needs to deviate from its mean in order for default to occur. Notice that although the value of the call option in (2) does not depend on μ, DD does. This is because DD depends on the future value of assets which is given in equation (3).

We use the theoretical distribution implied by Merton's model, which is the normal distribution. In that case, the theoretical probability of default will be given by:

$$P_{def} = N(-DD) = N\left(-\frac{\ln(V_{A,t}/X_t) + \left(\mu - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right). \tag{9}$$

Strictly speaking, P_{def} is not a default probability because it does not correspond to the true probability of default in large samples. In contrast, the default probabilities calculated by KMV are indeed default probabilities because they are calculated using the empirical distribution of defaults. For instance, in the KMV database, the number of companies times the years of data is over 100,000, and includes more than 2,000 incidents of default. We have a much more limited database. For that reason, we do not call our measure default probability, but rather DLI.[6]

It is important to note that the difference between our measure of default risk and that produced by KMV is not material for the purpose of our study. The DLI of a firm is a positive nonlinear function of its default probability. Since we use our measure of default risk to examine the relation between default risk and equity returns rather than price debt or credit risk derivatives, this nonlinear transformation cannot affect the substance of our results.

II. Data and Summary Statistics

We use the COMPUSTAT annual files to get the firm's "Debt in One Year" and "Long-Term Debt" series for all companies. COMPUSTAT starts reporting annual financial data in 1963. However, prior to 1971, only a few hundred firms have debt data available. Therefore, we start our analysis in 1971.

As book value of debt we use the "Debt in One Year" plus half the "Long-Term Debt." It is important to include long-term debt in our calculations for two reasons. First, firms need to service their long-term debt, and these interest payments are part of their short-term liabilities. Second, the size of the long-term debt affects the ability of a firm to roll over its short-term debt, and therefore reduce its risk of default. How much of the long-term debt should enter our calculations is arbitrary, since we do not observe the coupon payments of the individual firms. KMV uses 50 percent and argues that this choice is sensible, and captures adequately the financing constraints of firms.[7] We do the same.

[6] Our procedure also differs from the one used in KMV with respect to the way we calculate the distance to default. Whereas we use the formula that follows from the Black-Scholes model, KMV uses the one below: DD = (Market value of Assets − Default Point)/(Market value of Assets × Asset Volatility).

[7] To obtain an idea of how sensitive our results would be to our choice about the proportion of long-term debt included in our calculations of DLI, we performed the following test. We examined the variation of the ratio of long-term debt to total debt across size and BM quintiles. If there is no substantial variation, our results should not be influenced by the choice we make. We find that there is virtually no variation across BM portfolios. There is a small variation across size portfolios, with the small firms having a somewhat smaller ratio than the big firms. However, the small firms have also a larger standard deviation than the big firms. Overall, the difference in the ratios is not deemed large enough to alter the qualitative results of the paper.

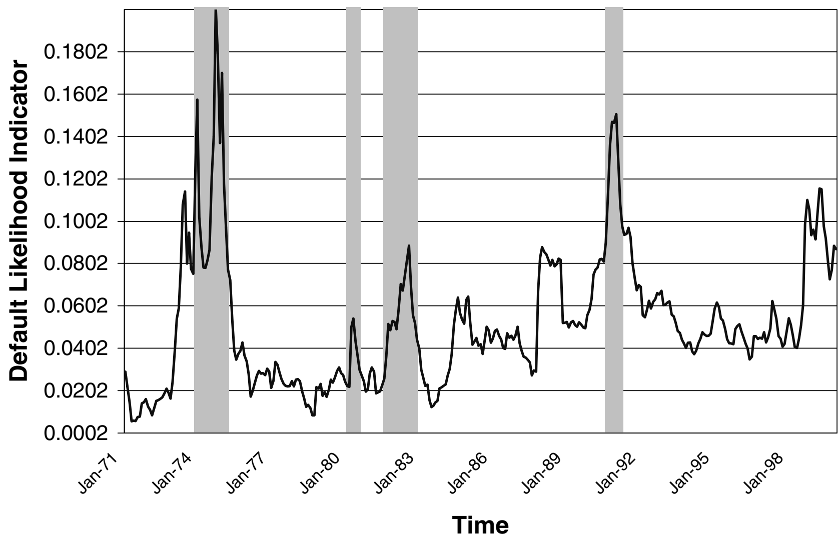

Figure 1. Aggregate default likelihood indicator. The aggregate DLI is defined as the simple average of the DLI of all firms. The shaded areas denote recession periods, as defined by NBER.

We use annual data for the book value of debt. To avoid problems related to reporting delays, we do not use the book value of debt of the new fiscal year, until 4 months have elapsed from the end of the previous fiscal year.[8] This is done in order to ensure that all information used to calculate our default measures was available to the investors at the time of the calculation.

We get the daily market values for firms from the CRSP daily files. The book value of equity information is extracted from COMPUSTAT. Each month, the BM ratio of a firm is the 6-month prior book value of equity divided by the current month's market value of equity. Firms with negative BM ratios are excluded from our sample.

As risk-free rate for the computation of DLI, we use monthly observations of the 1-year Treasury Bill rate obtained from the Federal Reserve Board Statistics.

Table I reports the number of firms per year for which DLI could be calculated, as well as the number of firms that filed for bankruptcy (Chapter 11) or were liquidated.

The aggregate default likelihood measure $P(D)$ is defined as a simple average of the DLI of all firms. A graph of the $P(D)$ is provided in Figure 1 for the whole sample period (January 1971 to December 1999). The shaded areas represent recession periods as defined by the NBER. The graph shows that default probabilities vary greatly with the business cycle and increase substantially during recessions.

[8] The SEC requires firms to report 10K within three months after the end of the fiscal year, but a small percentage of firms report it with a longer delay.

Table I

Firm Data

The second column of the table reports the number of firms each year for which DLI could be calculated. The third column reports the number of firms that filed for bankruptcy (Chapter 11), while the fourth reports the number of liquidations.

Year	No. of Stocks in Sample	No. of Bankruptcy	No. of Liquidations
1971	1,355	13	1
1972	1,532	8	4
1973	2,347	15	4
1974	2,490	13	4
1975	2,612	13	6
1976	2,885	18	8
1977	2,952	12	9
1978	2,957	17	10
1979	2,956	14	26
1980	2,928	18	23
1981	2,958	8	22
1982	3,054	13	23
1983	3,083	24	13
1984	3,311	12	17
1985	3,386	19	16
1986	3,343	60	29
1987	3,425	24	16
1988	3,577	45	16
1989	3,515	42	8
1990	3,408	11	6
1991	3,379	52	20
1992	3,461	42	31
1993	3,570	70	37
1994	3,830	48	24
1995	4,004	48	14
1996	4,177	41	11
1997	4,462	33	17
1998	4,495	54	11
1999	4,250	67	13

We define the aggregate survival rate, SV as $1 - P(D)$. The change in aggregate survival rate Δ(SV) at time t is given by $SV_t - SV_{t-1}$. Summary statistics for SV and Δ(SV) are presented in Panel A of Table II.

The default return spread is from Ibbotson Associates, and it is defined as the return difference between BAA Moody's rated bonds and AAA Moody's rated bonds. Similarly, the default yield spread is defined as the yield difference between Moody's BAA bonds and Moody's AAA bonds. The series is obtained from the Federal Reserve Bank of St. Louis. The change in spread Δ(spread) is obtained from Hahn and Lee (2001). The spread in Hahn and Lee is defined as the difference in the yields between Moody's BAA bonds and 10-year government bonds. Δ(spread) is the change in that spread.

Panel B of Table II provides the correlation coefficients between the above-defined default spreads and Δ(SV). The correlations are very low and reveal

Table II

Summary Statistics

In this table, SV denotes the survival rate and it is equal to 1 minus the aggregate DLI. The variable Δ(SV) is the change in the survival rate. Mean, Std, Skew, Kurt, and Auto refer to the mean, standard deviation, skewness, kurtosis and autocorrelation at lag one, respectively. The variable RDEF is the return difference between Moody's BAA corporate bonds and AAA corporate bonds. The variable YDEF is the yield difference between Moody's BAA bonds and Moody AAA corporate bonds. The variable Δ(spread) is the default measure used in Hahn and Lee (2001) which is defined as: $\Delta(spread) = (y_t^{BAA} - y_t^{TB}) - (y_{t+1}^{BAA} - y_{t+1}^{TB})$, where y_t^{BAA} is the yield of the Moody's BAA corporate bonds, and y_t^{TB} is yield on 10-year government bonds. The variable EMKT denotes the value-weighted excess return on the stock market portfolio over the risk-free rate; SMB and HML are the Fama and French (1993) factors. Size denotes the firm's market capitalization and BM its book-to-market ratio. DLI is the firm's DLI. T-values are calculated from Newey and West (1987) standard errors, which are corrected for heteroskedasticity and serial correlation up to three lags. The R^2's are adjusted for degrees of freedom. In Panel F, SMB and HML are the Fama–French factors. When the expression (within sample) appears next to SMB and HML, it means that these factors are calculated using the data in the current study and following exactly the same methodology as in Fama and French. "Auto" refers to the first-order autocorrelation.

Panel A: Summary Statistics on Aggregate Survival Indicator (SV)

	Mean	Std	Skew	Kurt	Auto
SV	0.9579	0.0292	−1.8956	7.9054	0.9384
Δ(SV)	−0.0004	1.0472	−0.1785	13.2094	0.1657

Panel B: Correlation between Δ(SV) and Other Default Measures

	Δ(SV)	RDEF	YDEF	Δ(Spread)
Δ(SV)	1			
RDEF	0.0758	1		
YDEF	0.1424	0.0702	1	
Δ(Spread)	0.0998	0.1416	−0.113	1

Panel C: Correlation between Δ(SV) and Other Factors

	Δ(SV)	EMKT	SMB	HML
Δ(SV)	1			
EMKT	0.5375	1		
SMB	0.5214	0.2839	1	
HML	−0.1709	−0.4382	−0.1422	1

Panel D: Time-Series Regression of Fama–French Factors on Δ(SV)

Factor		Constant	Δ(SV)	R-squared
EMKT	coef	0.0064	2.321	0.2869
	t-value	−3.4197	−6.0689	
SMB	coef	0.0009	1.4331	0.2697
	t-value	−0.6299	−5.0854	
HML	coef	0.0031	−0.4509	0.0264
	t-value	−1.815	−2.0671	

Table II—*Continued*

Panel E: Firm Characteristic and Default Risk			
	SIZE	BM	DLI
Average cross-sectional correlation between firm characteristics			
Size	1		
BM	−0.3165	1	
DLI	−0.3084	0.4332	1
Average time-series correlation between firm characteristics			
Size	1		
BM	−0.7155	1	
DLI	−0.4119	0.432	1

Panel F: SMB and HML within Sample				
	Mean	t-value	Std	Auto
SMB	0.0864	(0.5600)	2.8783	0.1374
SMB within Sample	0.0730	(0.4763)	2.8634	0.1451
HML	0.3076	(2.0770)	2.7627	0.1850
HML within Sample	0.3345	(2.4816)	2.5181	0.2000

that the information captured by our measure is markedly different from that captured by the commonly used default spreads. This is consistent with the findings in Elton et al. (2001).

The Fama–French factors HML and SMB, and the market factor EMKT are obtained from Kenneth French's web page.[9] From the same web page we also obtain data for the 1-month T-bill rate used in our asset-pricing tests. Panel C of Table II reports the correlation coefficients between Δ(SV) and the Fama–French factors. The correlations of Δ(SV) with EMKT and SMB are positive and of the order of 0.5, whereas that with HML is negative and equal to −0.18. This suggests that EMKT and SMB contain potentially significant default-related information. The regressions of Panel D in Table II show that Δ(SV) can explain a substantial portion of the time-variation in EMKT and SMB. This does not mean, however, that the priced information in EMKT and SMB is related to default risk. The default-related content of the priced information in SMB and HML will be examined in Section V.

Finally, given that the need to compute DLI for each stock constrains us to use only a subset of the U.S. equity market as presented in Table I, it is important to verify that our results are representative of the U.S. market as a whole. To this end, we construct the Fama–French factors HML and SMB within our sample, and compare them with those constructed by Fama and French using a much larger cross section of U.S. equities. The results are reported in Panel E

[9] We thank Ken French for making the data available. Details about the data, as well as the actual data series, can be obtained from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

of Table II. The distributional characteristics of the HML and SMB factors constructed within our sample are similar to those of the HML and SMB factors provided by Fama and French. Furthermore, their correlations are quite large and of the order of 0.95 for SMB and 0.86 for HML. The above comparisons reveal that the subsample we use in our study is largely representative of the U.S. equity samples used in other studies of equity returns.

III. Measuring Model Accuracy

In this section, we evaluate the ability of our default measure to capture default risk. To do that, we employ Moody's Accuracy Ratio. In addition, we compare the DLI of actually defaulted firms with those of a control group that did not default.

A. Accuracy Ratio

The accuracy ratio (AR) proposed by Moody's reveals the ability of a model to predict actual defaults over a 5-year horizon.[10]

Let us suppose a model that ranks the firms according to some measure of default risk. Suppose there are N firms in total in our sample and M of those actually default in the next 5 years. Let $\theta = \frac{M}{N}$ be the percentage of firms that default. For every integer λ between 0 and 100, we look at how many firms actually defaulted within the λ percent of firms with the highest default risk. Of course, this number of defaults cannot be more than M. We divide the number of firms that actually defaulted within the first λ percent of firms by M and denote the result by $f(\lambda)$. Then $f(\lambda)$ takes values between 0 and 1, and is an increasing function of λ. Moreover, $f(0) = 0$ and $f(100) = 1$.

Suppose we had the "perfect measure" of future default likelihood, and we were ranking stocks according to that. We would then have been able to capture all defaults for each integer λ, and $f(\lambda)$would be given by

$$f(\lambda) = \frac{\lambda}{\theta} \text{ for } \lambda < \theta \quad \text{and} \quad f(\lambda) = 1 \text{ for } \lambda \geq \theta. \tag{10}$$

Suppose we also calculate the average $f(\lambda)$ for all months covered by the sample. The graph of this function of average $f(\lambda)$ is shown as the kinked line in Figure 2, graph B.

At the other extreme, suppose we had zero information about future default likelihoods, and we were ranking the stocks randomly. If we did that a large number of times, $f(\lambda)$ would be equal to λ. Graphically, the average $f(\lambda)$ would correspond to the 45° line in the graphs of Figure 2.

We measure the amount of information in a model by how far the graph of the average $f(\lambda)$ function lies above the 45° line. Specifically, we measure it by

[10] See, "Rating Methodology: Moody's Public Firm Risk Model: A Hybrid Approach to Modeling Short Term Default Risk," Moody's Investors Service, March 2000. The AC ratio is somewhat related to the Kolmogorov-Smirnov test.

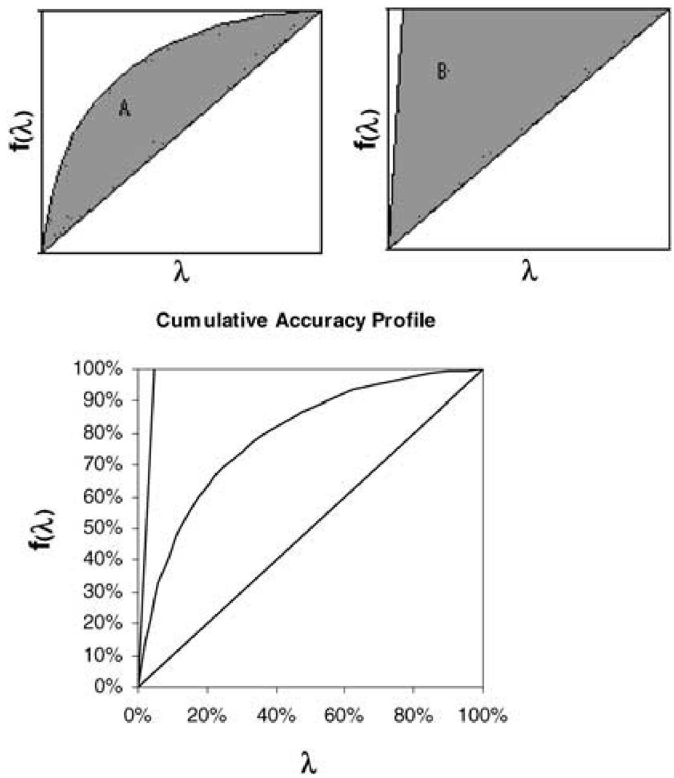

Figure 2. Accuracy ratio. Accuracy Ratio = 0.59231 (defined as the ratio of Area A over Area B).

the area between the 45° line and the graph of average $f(\lambda)$. The accuracy ratio of a model is then defined as the ratio between the area associated with that model's average $f(\lambda)$ function and the one associated with the "perfect" model's average $f(\lambda)$ function. Under this definition, the "perfect" model has accuracy ratio of 1, and the zero-information model has an accuracy ratio of 0.

The measure implied by Merton's model is the distance-to-default (DD). Therefore, if we rank stocks according to DD, the accuracy ratio we obtain is equal to 0.592. This means that our measure contains substantial information about future defaults.

By construction, our measure of default risk is related to size. It is therefore tempting to conclude that it contains virtually the same information as the market value of equity. This is not the case, however. If we rank stocks on the basis of their market value of equity and compute the corresponding accuracy ratio, this will be equal to only 0.089. Therefore, DD contains much more information than that conveyed by the size of the firms. This is an important point, since part of our analysis in Section IV provides an interpretation of the size effect, based on the information contained in DLI.

Finally, an important parameter in the DD measure is the volatility of assets. Therefore, one may conjecture that what we capture with our default measure is simply the volatility of assets. This is again not the case. If we rank stocks on the basis of their volatility of assets, the accuracy ratio we obtain is 0.290, which is much lower than that based on DD (0.592). In other words, our measure of default risk captures important default information beyond what is conveyed by the market value of equity or the volatility of the firm's assets alone.

B. Comparison between Defaulted Firms and Non-defaulted Firms

As a further test of the ability of our measure to capture default risk, we compare the DLI of firms that actually defaulted with those of a control group of firms that did not default. Similar comparisons have been performed in the past in Altman (1968) and Aharony, Jones, and Swary (1980). To make the comparison meaningful, we choose firms in the control group that have similar size and industry characteristics as those in the experimental group. In particular, for every firm that defaults, we select a firm with a market capitalization similar to that of the firm in the experimental group before it defaulted. In addition, the firm in the control group shares the same two-digit industry code as the one in the experimental group.

We compute the average DLI for each group. Figure 3 presents the results. We find that the average DLI of the experimental group goes up sharply in the 5 years prior to default. In contrast, the average DLI of the control group stays at the same level throughout the 5-year period. Note that in the graph, $t = 0$ corresponds to about 2 to 3 years prior to default, since the database does not

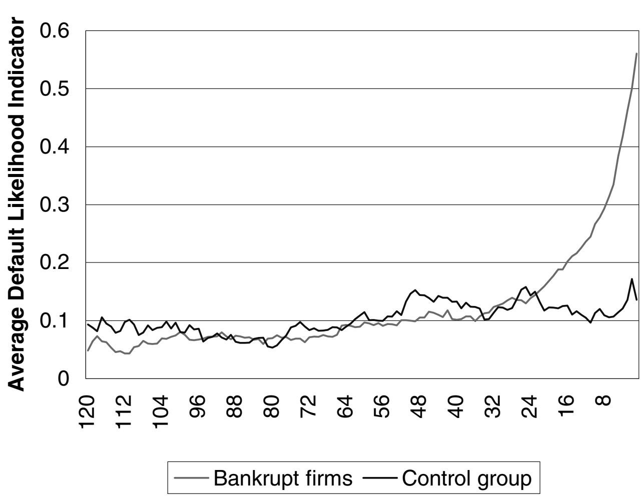

Figure 3. Average default likelihood indicators of bankrupt firms and firms in a control group. The control group contains firms with the same size and industry characteristics as those in the experimental group that did not default. Firms are delisted 2 to 3 years prior to bankruptcy. Numbers in x-axis denote months prior to delisting, and not prior to the actual default.

provide data up to the date of default. Therefore, an average DLI of 0.57 for the experimental group can be considered high. The results of this test provide further assurance that our DLIs do indeed capture default risk.

IV. Default Risk and Variation in Equity Returns

We start our analysis of the relation between default risk and equity returns by examining whether portfolios with different default risk characteristics provide significantly different returns. A significant difference in the returns would indicate that default risk may be important for the pricing of equities.

Table III reports simple sorts of stocks based on their DLI. At the end of each month from December 1970 to November 1999, we use the most recent monthly default probability for each firm to sort all stocks into portfolios. We first sort stocks into five portfolios. We examine their returns when the portfolios are equally weighted or value-weighted and report the average DLI for each one of them. Evidently, the lower the average DLI, the lower the risk of default.

Table III

Portfolios Sorted on the Basis of DLI

From December 1970 to November 1999, at each month end, we use the most recent monthly DLI of each firm to sort all portfolios into quintiles and deciles. We then compute the equally and value-weighted returns over the next month. "Return" denotes the average portfolio return and "ADLI" the average portfolio DLI. Portfolio 1 is the portfolio with the highest default risk and portfolio 10 is the portfolio with the lowest default risk. When stocks are sorted in quintiles, Portfolio 5 contains the stocks with the lowest default risk. "High–Low" is the difference in the returns between the high and low default risk portfolios. T-values are calculated from Newey–West standard errors. The value of the truncation parameter q was selected in each case to be equal to the number of autocorrelations in returns that are significant at the 5 percent level.

	High 1	2	3	4	5	6	7	8	9	Low 10	High–Low	t-value
Equally weighted												
Return	1.72	1.29	1.41	1.38	1.19						0.53	(1.96)
ADLI	19.38	1.61	0.24	0.04	0.01						19.37	
Value-weighted												
Return	1.26	1.27	1.28	1.36	1.12						0.14	(0.46)
ADLI	14.92	1.38	0.21	0.03	0.03						14.89	
Average size	2.56	3.52	4.24	4.89	5.59							
Average BM	1.64	0.99	0.82	0.74	0.64							
Equally weighted												
Return	2.12	1.32	1.25	1.32	1.44	1.39	1.37	1.39	1.24	1.14	0.98	(2.71)
ADLI	31.74	7.25	2.35	0.86	0.34	0.14	0.06	0.03	0.01	0.01	31.73	
Value-weighted												
Return	1.20	1.21	1.19	1.30	1.19	1.37	1.29	1.41	1.31	1.04	0.16	(0.44)
ADLI	29.18	6.44	2.12	0.86	0.33	0.11	0.06	0.02	0.01	0.04	29.15	
Average size	2.24	2.87	3.32	3.71	4.08	4.40	4.73	5.06	5.40	5.78		
Average BM	2.01	1.27	1.05	0.92	0.84	0.79	0.75	0.72	0.68	0.61		

Note that in calculating the returns of portfolios in Section IV, we use the following procedure. Every time a stock gets delisted due to default, we set the return of the portion of the portfolio invested in that stock equal to −100 percent. In other words, we assume that the recovery rate for equity holders is zero. In this way, we fully take into account the cost of default in our calculations of average portfolio returns. In fact, the returns we report may be considered as the lower bounds of returns (before transaction costs) earned by equity-holders. The reason is that often, the recovery rate is not zero.

The t-values of all tests in Section IV are computed from Newey and West (1987) standard errors. In particular, they are corrected for White (1980) heteroskedasticity and serial correlation up to the number of lags that are statistically significant at the 5 percent level.

The return difference between the equally weighted high-default-risk portfolio and low-default-risk portfolio is 53 basis points (bps) per month or 6.36 percent per annum (p.a.). The difference is statistically significant at the 5 percent level. This is not the case for the value-weighted portfolios whose difference in returns is only 14 bps per month.

When we sort stocks into 10 portfolios, the results we obtain are similar. The difference in returns between the high-default-risk portfolio and the low-default-risk portfolio is statistically significant for the equally weighted portfolios but not for the value-weighted portfolios. The return differential for the equally weighted portfolios is 98 bps per month or 11.76 percent p.a.

Notice though that the aggregate default measure for the equally weighted portfolios assumes bigger values than it does for the value-weighted portfolios. It appears that small-capitalization stocks have on average higher default risk, and as a result, they earn higher returns than big-capitalization stocks do. In addition, both in the case of default quintiles and deciles, the average market capitalization of a portfolio (size) and its BM ratio vary monotonically with the average default risk of the portfolio. In particular, the average size increases as the default risk of the portfolio decreases, whereas the opposite is true for BM. These results suggest that the size and BM effects may be linked to the default risk of stocks. Recall that both effects are considered stock market anomalies according to the literature of the Capital Asset-Pricing Model (CAPM). The reason for their existence remains unknown. The remainder of the paper investigates further the possible link between default risk and those effects. Our analysis will focus on equally weighted portfolios, since this is the weighting scheme typically employed in studies that consider the size and BM effects.[11] However, all the results of the paper remain *qualitatively the same* when portfolios are value-weighted.

A. Size, BM, and Default Risk

To examine the extent to which the size and BM effects can be interpreted as default effects, we perform two-way sorts and examine each of the two effects within different default risk portfolios.

[11] For recent references, see for instance Chan, Hamao, and Lakonishok (1991) and Fama and French (1992).

Table IV

Size Effect Controlled by Default Risk

From January 1971 to December 1999, at the beginning of each month, stocks are sorted into five portfolios on the basis of their DLI in the previous month. Within each portfolio, stocks are then sorted into five size portfolios, based on their past month's market capitalization. The equally weighted average returns of the portfolios are reported in percentage terms. "Small–Big" is the return difference between the smallest and biggest size portfolios within each default quintile. BM stands for book-to-market ratio. The rows labeled "Whole Sample" report results using all stocks in our sample. T-values are calculated from Newey–West standard errors. The value of the truncation parameter q was selected in each case to be equal to the number of autocorrelations in returns that are significant at the 5 percent level.

	Small 1	2	3	4	Big 5	Small–Big	t-stat
			Panel A: Average Return				
High DLI 1	4.6256	1.7233	1.1105	0.7801	0.8048	3.8208	(9.5953)
2	1.5333	1.2293	1.0915	1.2269	1.2865	0.2468	(1.0464)
3	1.4725	1.4583	1.2988	1.3268	1.3978	0.0747	(0.3481)
4	1.2973	1.3970	1.4683	1.3446	1.2946	0.0027	(0.0129)
Low DLI 5	1.2755	1.2216	1.1997	1.0520	1.1286	0.1469	(0.5730)
Whole sample	2.1207	1.1591	1.2032	1.2837	1.2238	0.8969	(3.2146)
			Panel B: Average Size				
High DLI 1	0.6883	1.6858	2.3936	3.1619	4.7013		
2	1.4885	2.5637	3.3076	4.1511	5.7973		
3	2.0103	3.2055	4.0250	4.9553	6.6873		
4	2.4612	3.7715	4.6935	5.7503	7.4202		
Low DLI 5	2.9161	4.4122	5.4394	6.5299	8.2456		
Whole sample	1.5312	2.9019	3.9120	5.0684	7.0886		
			Panel C: Average DLI				
High DLI 1	27.4500	20.6530	17.8550	16.0280	14.2960		
2	2.0050	1.7930	1.6770	1.5870	1.4260		
3	0.3170	0.2670	0.2510	0.2600	0.2200		
4	0.0590	0.0510	0.0420	0.0380	0.0380		
Low DLI 5	0.0140	0.0110	0.0090	0.0060	0.0070		
Whole sample	11.6100	4.9351	2.5953	1.3932	0.6141		
			Panel D: Average BM				
High DLI 1	2.2378	1.6810	1.5307	1.5022	1.3275		
2	1.2604	1.0476	0.9803	0.9191	0.8581		
3	1.0365	0.8571	0.7971	0.7426	0.7462		
4	0.9507	0.7476	0.6963	0.6698	0.6952		
Low DLI 5	0.9150	0.6977	0.5991	0.5498	0.5059		
Whole sample	1.5111	1.0802	0.8994	0.7490	0.6646		

A.1. The Size Effect

Table IV presents results from sequential sorts. Stocks are first sorted into five quintiles according to their default risk. Subsequently, the stocks within each default quintile are sorted into five size portfolios. This procedure produces

25 portfolios in total. In what follows, we examine whether the size effect exists in all default risk quintiles, as well as in the whole sample.

The results of Panel A show that the size effect is present only within the quintile that contains the stocks with the highest default risk (DLI 1). The effect is very strong with an average return difference between small and big firms of 3.82 percent per month or a staggering 45.84 percent p.a. Notice that the difference in returns drops to close to zero for the remaining default-sorted portfolios. There is a statistically significant size effect in the whole sample, but the return difference between small and big firms is more than four times smaller than in DLI 1.

The results of Panel A suggest that the size effect exists only within the segment of the market that contains the stocks with the highest default risk. To what extent, however, are we truly capturing the size effect? Is there really substantial variation in the market capitalizations of stocks within the DLI 1 portfolio? Panel B addresses this question. We see that there is indeed large variation in the market caps of stocks within the highest default risk portfolio. But in terms of the average market caps for the size quintiles formed using the whole sample, the biggest firms in DLI 1 are rather medium to large firms. On the other hand, the DLI 1-Small portfolio contains the smallest of the small firms compared to the small size quintile formed on the basis of the whole sample. These results imply that the size effect is concentrated in the smallest of the small firms, which also happen to be among those with the highest default risk.

How much riskier are the stocks in DLI 1 compared to the other default risk quintiles? Panel C of Table IV shows that they are a lot riskier. The small firms in DLI 1 are almost 14 times riskier in terms of likelihood of default than the small firms in DLI 2. They are also on average more than twice as risky in terms of default than the stocks in the small size quintile constructed using the whole sample. Therefore, the large average returns that small high-default stocks earn compared to the rest of the market can be considered to be compensation for the large default risk they have.

To see that, notice also that in the high DLI quintile, DLI decreases monotonically as size increases. In other words, the large difference in returns between small and big stocks in the DLI 1 quintile can be explained by the large difference in the default risk of those portfolios. In the remaining default quintiles where there is no evidence of a size effect, the difference in default risk between small and big stocks is also very small.

Panel D reports the average BM of the default- and size-sorted portfolios. These results are useful in order to understand the extent to which size, default risk, and BM are interrelated. Panel D shows that the average BMs in the size-sorted portfolios of DLI 1 are the highest in the sample. The BM decreases monotonically with DLI, which suggests that the BM effect may also be related to default risk.

The conclusion that emerges from Table IV is that the size effect is in fact a default effect. There is a size effect only in the segment of the market with the highest default risk. Within that segment, the difference in returns between small and big stocks can be explained by the difference in their default risk. In the remaining stocks in the market, where there is no significant size effect, the difference in default risk between small and big stocks is minimal. BM seems also to be related to default risk and size, and we will examine these relations in the following section.

A.2. The BM Effect

Table V presents results from portfolio sortings in the same spirit as those of Table IV. Stocks are first sorted into five default risk quintiles, and then each of the five default quintiles is sorted into five BM portfolios. In what follows, we will examine the BM effect within each default quintile, as well as for the market as a whole.

Panel A shows that the BM effect is prominent only in the two quintiles with the highest default risk, with the return differential between value (high BM) and growth (low BM) stocks being almost two and a half times bigger in DLI 1 than in DLI 2. There is a BM effect in the whole sample, but the return differential is about half as big as that found in DLI 1.

Notice that within DLI 1, the average DLI is much higher for value stocks than it is for growth stocks. In DLI 2, where the BM effect is weaker, the difference in default risk between value and growth stocks is also small. These results imply that, similar to the size effect, the BM effect seems to be due to default risk. The only difference is that the BM effect is significant within the two-fifths of the stocks with the highest default risk, whereas the size effect is present only in the one-fifth of stocks with the highest default risk. In other words, the interrelation between size and default risk seems to be a bit tighter. This is confirmed in Section IV.C using regression analysis.

There is a lot of dispersion in the average BM ratios within the DLI portfolios. This is particularly true for DLI 1 and 2, which means that indeed the return differential we examine captures a BM effect. In fact, the average BM ratio varies more across portfolios in DLI 1 than it does across BM portfolios formed using the whole sample. In DLI 1 and 2 where default risk is higher than in the other quintiles and the market as a whole, the average BM ratios of the BM-sorted portfolios are also higher. This result underlines again the interrelation between BM and default risk discussed above. Furthermore, the average DLIs in Panel C exhibit a monotonic relation with BM only in the DLI 1 and 2 quintiles, that is, the two quintiles with the highest default risk, where the BM effect is significant. For the rest of the sample, the relation between default risk and BM ratios does not appear to be linear. A similar result emerges from Table IV, Panel C. Default risk varies monotonically with size only within the two highest default risk quintiles. It seems that there are linear relations between default risk and size, and default risk and BM, only to the extent that default risk is sizeable. When the risk of default of a company is very small, the linearity in the relation between default and size and default and BM disappears, probably because defaults are very unlikely to occur in those cases.

Table V

BM Effect Controlled by Default Risk

From January 1971 to December 1999, at the beginning of each month, stocks are sorted into five portfolios on the basis of their DLI in the previous month. Within each portfolio, stocks are then sorted into five BM portfolios, based on their past month's BM ratio. The equally weighted average returns of the portfolios are reported in percentage terms. "High–Low" is the return difference between the highest BM and lowest BM portfolios within each default quintile. The rows labeled "Whole Sample" report results using all stocks in our sample. T-values are calculated from Newey–West standard errors. The value of the truncation parameter q was selected in each case to be equal to the number of autocorrelations in returns that are significant at the 5 percent level.

	High BM 1	2	3	4	Low BM 5	High–Low	t-stat
			Panel A: Average Returns				
High DLI 1	3.3636	2.0412	1.5164	1.2047	0.8170	2.5466	(9.8984)
2	1.7981	1.5438	1.2955	0.9946	0.7282	1.0699	(3.4716)
3	1.7420	1.4287	1.3053	1.2381	1.2338	0.5083	(1.5026)
4	1.6284	1.4604	1.1840	1.1864	1.3414	0.2870	(0.9575)
Low DLI 5	1.4415	1.2669	1.0932	1.0688	1.0074	0.4341	(1.5134)
Whole sample	2.1572	1.4893	1.2267	1.0963	1.0128	1.1445	(4.5879)
			Panel B: Average BM				
High DLI 1	3.7233	1.8967	1.3310	0.9007	0.4191		
2	2.0395	1.2307	0.8848	0.6070	0.2949		
3	1.6616	1.0184	0.7399	0.5065	0.2462		
4	1.4547	0.9154	0.6782	0.4705	0.2339		
Low DLI 5	1.2970	0.8052	0.5733	0.3858	0.2009		
Whole sample	2.2137	1.1258	0.7861	0.5243	0.2472		
			Panel C: Average DLI				
High DLI 1	30.9210	19.4650	16.2910	14.7660	14.6620		
2	2.0460	1.7450	1.6340	1.5400	1.5180		
3	0.3150	0.2580	0.2500	0.2590	0.2320		
4	0.0510	0.0470	0.0420	0.0460	0.0410		
Low DLI 5	0.0130	0.0070	0.0110	0.0080	0.0080		
Whole sample	12.0360	3.6598	2.2206	1.6334	1.5062		
			Panel D: Average Size				
High DLI 1	2.0112	2.4445	2.6701	2.7970	2.7742		
2	2.9754	3.4027	3.5753	3.6821	3.6893		
3	3.6649	4.1947	4.3284	4.4044	4.3099		
4	4.2412	4.8918	5.0060	5.0645	4.9220		
Low DLI 5	4.4908	5.3668	5.6338	6.0028	6.0942		
Whole sample	2.8680	3.9437	4.3643	4.6518	4.7197		

Panel D shows again that DLI 1 contains mainly small firms. However, size does not vary monotonically with BM, except within the two highest default risk quintiles. The same conclusion can be reached from Panel D of Table IV. The average BM ratios vary monotonically with size only within the two highest default risk quintiles. In both cases the variation is small.

It seems that size and BM proxy to some extent for each other only within the segment of the market with the highest default risk. This implies that they are not identical phenomena. Furthermore, the return premium of small firms over big firms is more than 1 percent larger than that of high BM stocks over low BM stocks. In addition, the size effect is present in a subset of the segment of the market in which the BM effect exists. Both are linked, however, to a common risk measure, which is default risk.

B. The Default Effect

Tables IV and V show that size and BM are intimately related to default risk. But does this mean that there is also a default risk in the data? And if there is, is it confined only within certain size and BM quintiles? In other words, is default risk rewarded differently depending on the size and BM characteristics of the stock? These are the questions we address in this section. We define the default effect as a positive average return differential between high and low default risk firms.

B.1. The Default Effect in Size-sorted Portfolios

Table VI examines whether there is a default effect in size-sorted portfolios by reversing the sorting procedure of Table IV. In particular, we first sort stocks into five size quintiles, and then sort each size quintile into five default portfolios. As we will see below, this exercise also allows us to obtain a better understanding of small firms as an asset class.

Panel A shows that there is a statistically significant default effect only within the small size quintile. The average monthly return is 2.2 percent or 26.4 percent p.a. In most of the remaining size quintiles, the difference in returns between high and low default risk portfolios is in fact negative. This means that high-default-risk firms earn a higher return than low default risk firms, only if they are also small in size.

To verify this point, see Panel B of Table VI. All high-DLI portfolios have substantial default risk, independent of the market capitalization of the stocks. Similarly, all low-DLI portfolios have virtually no default risk. However, only small high-default-risk stocks earn higher returns than low default risk stocks.

This result may indicate that firms differ in their ability to re-emerge from Chapter 11, depending on their size. If small firms, for instance, are less likely to emerge from the restructuring process as public firms, investors may require a bigger risk premium to hold them, compared to what they require for bigger size high-default-risk firms. This will induce the average returns of small high-DLI firms to be higher than those of bigger high-DLI firms.[12] Empirical evidence

[12] This interpretation assumes that default risk is systematic, and therefore, not diversifiable. In Section V we test whether default risk is priced in the cross section of equity returns. Our results show that default risk is indeed priced, and therefore, it constitutes a systematic source of risk.

Table VI

Default Effect Controlled by Size

From January 1971 to December 1999, at the beginning of each month, stocks are sorted into five portfolios on the basis of their market capitalization (size) in the previous month. Within each portfolio, stocks are then sorted into five portfolios, based on past month's DLI. Equally weighted average portfolio returns are reported in percentage terms. "HDLI-LDLI" is the return difference between the highest and lowest default risk portfolios within each size quintile. T-values are calculated from Newey–West standard errors. The value of the truncation parameter q was selected in each case to be equal to the number of autocorrelations in returns that are significant at the 5 percent level.

	High DLI 1	2	3	4	Low DLI 5	High–Low	t-stat
			Panel A: Average Returns				
Small 1	3.7315	2.1580	1.8666	1.4127	1.5020	2.2295	(5.9430)
2	0.7852	1.0599	1.3095	1.3212	1.3200	−0.5348	(−1.8543)
3	0.8748	1.2387	1.3406	1.3623	1.1947	−0.3198	(−1.7375)
4	1.1115	1.2662	1.4690	1.3171	1.2542	−0.1427	(−0.8505)
Big 5	1.3714	1.2954	1.2391	1.1717	1.0428	0.3286	(1.7074)
			Panel B: Average DLI				
Small 1	41.5360	12.7980	3.8906	0.8832	0.0955		
2	20.4190	3.4020	0.7731	0.1516	0.0239		
3	11.6090	1.1100	0.2276	0.0528	0.0091		
4	6.3550	0.4880	0.1014	0.0284	0.0096		
Big 5	2.9220	0.1200	0.0315	0.0075	0.0063		
			Panel C: Average Size				
Small 1	1.2008	1.4570	1.5742	1.6668	1.7450		
2	2.8306	2.8830	2.9113	2.9332	2.9510		
3	3.8612	3.8901	3.9172	3.9374	3.9537		
4	4.9955	5.0381	5.0718	5.1008	5.1357		
Big 5	6.7779	6.9570	7.0820	7.2114	7.4129		
			Panel D: Average BM				
Small 1	2.4472	1.5668	1.3194	1.1538	1.1031		
2	1.6172	1.1213	0.9548	0.8645	0.8460		
3	1.3290	0.9027	0.8036	0.7345	0.7286		
4	1.0048	0.7531	0.7028	0.6765	0.6087		
Big 5	0.8774	0.7187	0.6731	0.6013	0.4538		

from the corporate bankruptcy literature shows that indeed large firms are more likely to survive Chapter 11 than small firms.[13]

Panel B of Table VI also provides insights into the profile of small firms as an asset class. Notice that within the small size quintile, DLI varies between 41.53 percent and 0.09 percent. This implies that small firms can differ a lot with respect to their (default) risk characteristics. They can also differ significantly with respect to their returns, as Panel A reveals. These results suggest that small firms do not constitute a homogenous asset class, as is commonly believed.

[13] See for instance, Moulton and Thomas (1993) and Hotchkiss (1995).

Finally, Panel B shows that default risk decreases monotonically as size increases, confirming the close relation between size and default risk observed in Table IV. Panels C and D show that the small–high DLI portfolio contains the smallest of the small stocks and those with the highest BM ratio.

Two important conclusions emerge from this table. First, default risk is rewarded only in small value stocks. Firms that have high default risk, but are not categorized as small and high BM, will not earn higher returns than firms with low default risk and similar size and BM characteristics. This result further underlines the close link among size, default risk, and BM. Second, small firms are not made equal. They differ substantially in terms of both their return and (default) risk characteristics. This result reveals that small firms do not constitute a homogeneous asset class.

B.2. The Default Effect in BM-sorted Portfolios

To further examine the link between default risk and BM, Table VII examines the presence of a default effect in BM-sorted portfolios. Assets are first sorted in five BM quintiles, and subsequently, each BM-sorted quintile is subdivided into five default-sorted portfolios.

Panel A reveals that the default effect is again present only within the high BM quintile. This result is consistent with that of Table VI. Since the smallest high-DLI firms are also typically the highest BM firms, the same interpretation applies here. Specifically, default risk is rewarded only for small, value stocks, and not for any other stocks in the market, independently of their risk of default. This is confirmed in Panels C and D.

Once again, Panel B shows that value stocks can differ a lot with respect to their default risk characteristics. Given that they also differ significantly in terms of their returns, Panels A and B suggest that, similar to small firms, value stocks do not constitute a homogeneous asset class either.

The results of Table VII are consistent and analogous to those of Table VI. High-default-risk stocks earn a higher return than low default risk stocks, only to the extent that they are small and high BM. If the size and BM criteria are not fulfilled, they will not earn higher returns than low default risk stocks, even if their default risk is very high. Furthermore, our analysis implies that small firms and value stocks do not constitute homogeneous asset classes.[14]

C. Examining the Interaction of Size and Default, and BM and Default Using Regression Analysis

In this section, we summarize and quantify the degree of interaction between size and default and BM and default using regression analysis. Two different methodologies are employed. The first one is a portfolio-based regression

[14] The results presented in Section IV based on sequential sorts hold also when independent sorts are performed. To conserve space, we do not report those results here. The main insight offered by the independent sorts is that most small stocks are also high-DLI stocks, whereas most big stocks are low-DLI stocks. Similarly, most value stocks are high default risk stocks, whereas most growth stocks have low risk of default.

Table VII

Default Effect Controlled by BM

From January 1971 to December 1999, at the beginning of each month, stocks are sorted into five portfolios on the basis of their BM ratio in the previous month. Within each portfolio, stocks are then sorted into five portfolios, based on past month's DLI. Equally weighted average portfolio returns are reported in percentage terms. "HDLI-LDLI" is the return difference between the highest and lowest default risk portfolios within each size quintile. T-values are calculated from Newey–West standard errors. The value of the truncation parameter q was selected in each case to be equal to the number of autocorrelations in returns that are significant at the 5 percent level.

	High DLI 1	2	3	4	Low DLI 5	High–Low	t-stat
Panel A: Average Returns							
High BM 1	3.2285	2.1825	1.9488	1.8361	1.6243	1.6042	(3.9785)
2	1.3880	1.4370	1.5597	1.5544	1.5098	−0.1218	(−0.4580)
3	1.1506	1.2602	1.3190	1.1712	1.2307	−0.0802	(−0.3317)
4	0.9077	1.1734	1.1679	1.1064	1.1246	−0.2169	(−0.8294)
Low BM 5	0.7044	0.9765	1.2983	1.1074	0.9711	−0.2667	(−0.8369)
Panel B: Average DLI							
High BM 1	42.0930	13.1080	4.4229	1.3284	0.2008		
2	15.6030	2.1510	0.5362	0.1186	0.0149		
3	10.1880	0.7840	0.1623	0.0389	0.0104		
4	7.7070	0.4180	0.0895	0.0244	0.0066		
Low BM 5	7.3560	0.2140	0.0534	0.0152	0.0063		
Panel C: Average BM							
High BM 1	3.1427	2.2544	2.0319	1.8890	1.7795		
2	1.1569	1.1390	1.1245	1.1105	1.0985		
3	0.8018	0.7886	0.7854	0.7798	0.7748		
4	0.5375	0.5280	0.5237	0.5219	0.5107		
Low BM 5	0.2464	0.2473	0.2477	0.2493	0.2453		
Panel D: Average Size							
High BM 1	1.9664	2.4581	2.8438	3.3376	3.7075		
2	2.7494	3.4317	4.0187	4.5993	4.9060		
3	3.0165	3.8540	4.5044	5.0304	5.4005		
4	3.2014	4.1530	4.7369	5.3104	5.8360		
Low BM 5	3.1744	4.1586	4.6966	5.2828	6.2550		

approach developed in Nijman, Swinkels, and Verbeek (2002). The second one uses the Fama and MacBeth (1973) methodology on individual stock returns.

C.1. The Portfolio-based Regression Approach

The regression methodology in Nijman, Swinkels, and Verbeek (2002) is an extension of the methodology in Heston and Rouwenhorst (1994), which allows for the presence of interaction terms between the variables of interest. In the current application, we analyze average returns of portfolios grouped on the basis of DLI, size, and BM, and examine the relative magnitudes of the individual effects, as well as their interactions.

Similar to Daniel and Titman (1997), Nijman, Swinkels, and Verbeek (2002) assume that the conditional expected return of a stock can be decomposed into several effects. In other words,

$$E_t\left(R_{i,t+1}\right) = \sum_{a=1}^{N_a} \sum_{b=1}^{N_b} \alpha_{a,b} X_{i,t}\left(a,b\right) \tag{11}$$

where $E_t(\cdot)$ denotes the expectation conditional on the information available at time t, $R_{i,t+1}$ is the return of the stock at time $t+1$, $X_{i,t}(a,b)$ is a dummy variable that indicates the membership of the stock in a particular portfolio, and $\alpha_{a,b}$ the expected return of a stock with characteristics a and b. In our application, a and b are either size and default risk or BM and default risk. Therefore, equation (7) simply states the conditional expected return of a stock, given its size/BM and default risk characteristics that grant it membership to a particular portfolio.[15]

The conditional expected return on a portfolio p of N stocks with weights $w_{i,t}^{p}$, can then be written as:

$$E_t\left(R_{t+1}^{p}\right) = \sum_{a=1}^{N_a} \sum_{b=1}^{N_b} \alpha_{a,b} X_{i,t}^{p}\left(a,b\right), \tag{12}$$

where $X_{i,t}^{p}(a,b) = w_{i,t}^{p} X_{i,t}(a,b)$. Since the portfolios we use for our tests are all equally weighted and sorted on the basis of the characteristics a and b, we can simplify the above equation as follows:

$$E_t\left(R_{t+1}^{p}\right) = \sum_{a=1}^{N_a} \sum_{b=1}^{N_b} \alpha_{a,b} X_t\left(a,b\right). \tag{13}$$

The regression equation implied can be written as:

$$R_{t+1}^{p} = \sum_{a=1}^{N_a} \sum_{b=1}^{N_b} \alpha_{a,b} X_t\left(a,b\right) + \varepsilon_{t+1}, \tag{14}$$

where $\varepsilon_{t+1}^{P} \equiv R_{t+1}^{p} - E_t(R_{t+1})$, which is by construction orthogonal to the regressors. The only assumption made is that the cross-autocorrelation structure is zero, that is, $E(\varepsilon_{t+h}^{p}\varepsilon_{t}^{p}) = 0$. However, equation (10) can be written in a more parsimonious way by imposing an additive structure similar to that in Roll (1992)

[15] Note that, in principle, we could examine all three effects simultaneously, that is the size, BM, and default effects. This, however, would increase the parameters to be estimated considerably, at the expense of efficiency. For that reason, we concentrate on two effects at a time.

and Heston and Rouwenhorst (1994). In that case, the conditional expected return of portfolio p will be given by:

$$E_t\left(R_{t+1}^p\right) = \alpha_{1,1} + \sum_{a=2}^{N_a} \phi_a X_t\,(a,\cdot) + \sum_{b=2}^{N_b} \phi_b X_t\,(\cdot,b) + \sum_{a=2}^{N_a}\sum_{b=2}^{N_b} \alpha_{a,b} X_t\,(a,b), \quad (15)$$

where $X_t(.,b)$, for instance, denotes that only the argument b is considered. In that case, all stocks in group b are considered, irrespectively of their a characteristic. The constant $\alpha_{1,1}$ denotes the return on the reference portfolio. The reference portfolio is arbitrarily chosen and is used to avoid the dummy trap. When we examine the interaction of size and default effects, the reference portfolio we use is the portfolio that contains big cap and low-DLI stocks. In the tests of the interaction of BM and default effects, the reference portfolio is the one that contains stocks with low BM and low DLI.

The estimated coefficients ϕ can be interpreted as the difference in return between portfolio p and the reference portfolio attributed to a particular effect. Similarly, the coefficients α denote the additional expected return for portfolio p due to the interaction of two effects. The total expected return on portfolio p is given by the sum of the returns of the reference portfolio, the individual effects, and the interaction effects.

Each set of estimations uses 15 left-hand-side portfolios. In the case of the size-default effects test, they are comprised of three size portfolios, three default-sorted (DLI) portfolios, and nine portfolios created from the intersection of two independent sorts on three size and three DLI portfolios. In the case of the BM-default effects test, the portfolios include three BM portfolios, three default-sorted portfolios, and nine portfolios from the intersection of two independent sorts on three BM portfolios and three DLI portfolios. In both sets of tests, there are eight parameters to be estimated.

The results are reported in Table VIII. The first panel refers to the tests of the size-default effects, whereas the second panel contains the results for the BM-default effects.

Panel A shows that the economically and statistically most important coefficients for the individual effects are for small size and high DLI. In addition, the strongest interaction effect refers to the interaction of small size and high DLI. In other words, a portfolio will earn higher return, the smaller its market cap, and the higher its default risk. It will also earn an additional return from the interaction of high default risk and small size. This additional return is zero if the small firms have medium default risk. These results are consistent with our earlier finding that the size effect exists only among high-default-risk stocks. Note also that the coefficient on the interaction term between high DLI and medium size is negative and statistically insignificant. This is again in line with our previous result that the default effect exists only within small firms.

The return on the reference portfolio (big firms, low DLI) is 1.1363 percent per month (p.m.). This means that a portfolio of small firms with high DLI will earn 2.37(1.136 + 0.4287 + 0.50 + 0.31) percent per month, compared to 1.79

Table VIII

A Decomposition of Returns in Size, BM, and DLI Portfolios Using Regression Analysis

Panel A provides results using 15 size- and DLI-sorted portfolios. Out of these 15 portfolios, 3 are sorted on the basis of size, 3 on the basis of DLI, and 9 portfolios are created from the intersection of two independent sorts on three size and three DLI portfolios. The reference portfolio contains big firms with low DLI. Its average return is 1.1363 percent per month. Panel B provides results based on 15 BM- and DLI-sorted portfolios. The portfolios are constructed in an analogous fashion to that of the portfolios of Panel A. The reference portfolio contains now low BM and low DLI firms, and has an average return of 1.0529 percent per month. The results presented are from Fama–MacBeth regressions. T-values are computed from standard errors corrected for White (1980) heteroskedasticity and serial correlation up to three lags using the Newey–West estimator. The Wald test examines the hypothesis that the coefficients of each individual effect are jointly zero.

Panel A: Size Effect and Default Effect

	Size (M)	Size (S)	DLI (M)	DLI (H)	Size (M) DLI (M)	Size (M) DLI (H)	Size (S) DLI (M)	Size (S) DLI (H)
Coefficient	0.1069	0.4287	0.2158	0.5003	−0.0939	−0.1201	0.0087	0.3078
t-value	0.7474	1.8661	2.0895	2.2291	−1.0289	−1.2345	0.1321	2.3895

	Size	DLI
Wald test	5.9102	5.0468
p-value	0.0151	0.0247

Panel B: BM Effect and Default Effect

	BM (M)	BM (H)	DLI (M)	DLI (H)	BM (M) DLI (M)	BM (H) DLI (M)	BM (M) DLI (H)	BM (H) DLI (H)
Coefficient	0.1533	0.7257	0.2512	0.3626	−0.0073	−0.1915	0.0096	0.2427
t-value	1.3377	4.0523	2.4627	1.5327	−0.0866	−1.5060	0.0678	1.9991

	BM	DLI
Wald test	48.9252	6.6891
p-value	0.0000	0.0097

percent p.m. that a portfolio of small firms of medium DLI will earn. Similarly, a portfolio of medium firms of high DLI will earn 1.62 percent per month, whereas a portfolio of medium size firms of medium DLI will earn only 1.37 percent per month.

Notice that the returns above are smaller than those in Tables IV and VI. The reason is that stocks here are classified into tertiles of size and DLI portfolios rather than quintiles as in Table IV to VII. The pattern of returns and the conclusions remain the same: the highest returns are earned by stocks with the highest DLI and smallest size.

Similar conclusions emerge for the BM-DLI portfolios in Panel B. The stocks that earn the highest returns are stocks that are both high BM and high DLI. The return of the reference portfolio here (low BM, low DLI) is 1.05 percent p.m. Therefore, the high BM, high DLI portfolio will earn a total return of

Table IX

Fama–MacBeth Regressions on the Relative Importance of Size, BM, and DLI Characteristics for Subsequent Equity Returns

The Fama–MacBeth regression tests are performed on individual equity returns. The variables size and BM are rendered orthogonal to DLI. The regressions relate individual stock returns to their past month's size, BM, and DLI characteristics. Size2, BM2, DLI2 denote the characteristics squared, whereas SizeDLI and BMDLI denote the products of the respective variables. Those products aim to capture the interaction effects of each pair of variables.

	Constant	DLI	DLI2	Size	Size2	BM	BM2	SizeDLI	BMDLI
Coef	1.3087	−4.8980	17.8748	−0.0030	0.0000	0.5710	−0.0293	−0.6800	0.1071
t-value	4.4352	−2.7120	4.3832	−0.5061	−0.2406	5.5091	−1.5762	−3.8740	1.9802
Coef	1.3027	−6.2470	19.7108	−0.0072	0.0000			−0.7869	
t-value	4.3906	−3.3818	4.6873	−1.1187	0.2159			−4.2910	
Coef	1.2905	0.7063	2.1471			0.5899	−0.0477		0.1345
t-value	4.3421	0.5158	3.6537			5.7721	−2.4581		2.1236

2.38 percent p.m. as opposed to the 1.84 percent earned by the high BM, medium DLI portfolio. Medium default risk firms earn an extra return for default risk, but it is smaller than that earned by high-default-risk firms. In addition, the only positive and statistically significant interaction coefficient is the one referring to high BM and high DLI stocks. By the same token, a portfolio of medium BM and high DLI stocks will earn 1.58 percent per month compared to 1.45 percent per month earned by a portfolio of medium BM and medium DLI firms. In both cases, the interaction term is economically and statistically equal to zero.

C.2. The Fama–MacBeth Regression Approach on Individual Stock Returns

Table IX presents results from Fama–MacBeth regressions of individual stocks on their past month's size, BM, and DLI characteristics. The regressions consider both a linear relation between stock returns and characteristics, as well as a nonlinear relation by including the characteristics squared (size2, BM2, DLI2). In addition, there are interaction terms proxied by the product of size with DLI (sizeDLI) and BM with DLI (BMDLI). We render size and BM orthogonal to DLI before performing the tests, in order to avoid possible problems in the interpretation of the results.

The results show that what explains next month's equity returns is the current default risk of securities, their BM, and the interaction of default risk and size. Size per se does not appear to play any role. This is confirmed in tests where only the DLI and size variables are considered. Indeed, only DLI, DLI2, and sizeDLI are important for explaining the next period's equity returns. In contrast, BM seems to contain incremental information about next period's returns, over and above that contained in DLI. The regressions that consider only DLI and BM variables show that the BM variables and DLI2, in addition to the interaction term, are important for explaining next period's equity returns. The regression results in Table IX also highlight the importance of the squared terms, and therefore, the nonlinearity in the relations between equity returns, DLI, and BM.

The bottom line from these tests is the following. The observed relation in the literature between size and equity returns is completely due to default risk. Size proxies for default risk and this is why small caps earn higher returns than big caps. They do so because small caps have higher default risk than big caps. BM also proxies partially for default risk. Default risk is not however all the information included in BM.

D. Conclusions About the Size, BM, and Default Effects

The results in Section IV point to the following conclusions. The size effect is a default effect as it exists only within the quintile of firms with the highest default risk. The BM effect is also related to default risk, but it exists among firms with both high and medium default risk. Default risk is rewarded only to the extent that high-default-risk firms are also small and high BM and in no other case. In other words, default risk and size share a nonlinear relation, and the same is true for default risk and BM. The exact functional form of these relations is not completely mapped out here. Rather, we highlight some of the principal characteristics of these relations. It is clear that the highest returns are earned by stocks that are either both small in size and high DLI, or both high DLI and high BM. It is also clear though that default risk is a variable worth considering above and beyond size and BM, and the asset-pricing tests of the following section confirm that.

V. The Pricing of Default Risk

The results of the previous section imply that the size and BM effects are compensations for the high default risk that small and high BM stocks exhibit. But does this mean that default risk is systematic? The answer to this question is not obvious, since defaults are rare events and seem to affect only a small number of firms. However, the default of a firm may have ripple effects on other firms, which may give rise to a systematic component in default risk.

The purpose of this section is to investigate through asset-pricing tests, whether default risk is systematic, and therefore whether it is priced in the cross section of equity returns.

A. The Tested Hypotheses

Two hypotheses are examined as part of our asset-pricing tests. First, we test whether default risk is priced. To do so, we need to consider a plausible empirical asset-pricing specification in which default risk appears as a factor.

It is clear that an asset-pricing model that includes only default as a risk factor would certainly be mis-specified, since even if default risk is priced, it is unlikely to be the only risk factor that affects equity returns. For that reason, we consider an asset-pricing model that includes as factors the excess return on the market portfolio (EMKT) and the aggregate survival measure Δ(SV). The empirical asset-pricing specification is given below.

$$R_t = a + bEMKT_t + d\,\Delta(SV)_t + \varepsilon_t \tag{16}$$

where R_t represents the return at time t of a stock in excess of the risk-free rate.

Such a model can be understood in the context of an Intertemporal Capital Asset-Pricing Model (ICAPM) as in Merton (1974). One can postulate a version of ICAPM where default risk affects the investment opportunity set, and therefore, investors want to hedge against this source of risk.

The second hypothesis examined is whether the FF factors SMB and HML proxy for default risk. Recall that the FF model is empirical in nature, and includes apart from the market factor, a factor related to size (SMB) and a factor related to BM (HML). Fama and French (1996) argue that SMB and HML proxy for financial distress. We test this hypothesis here, by including Δ(SV) in the FF model. In other words, we test the following empirical specification:

$$R_t = a + bEMKT_t + sSMB_t + hHML_t + d\,\Delta(SV)_t + \varepsilon_t. \tag{17}$$

If indeed all the priced information in SMB and HML is related to financial distress, we would expect to find that in the presence of Δ(SV), SMB and HML lose all their ability to explain equity returns.

To get a sense of the performance of the two empirical specifications examined, we also present results from tests of the CAPM and FF model. These two models act as benchmarks for comparison purposes.

B. The Test Assets

As previously mentioned, two hypotheses are examined in our asset-pricing tests. First, whether default risk is priced, and second, whether SMB and HML proxy for default risk. This implies that there are three variables against which the test assets have to exhibit maximum dispersion: Δ(SV), size, and BM. By test assets we mean the portfolios whose returns the asset-pricing models will be called upon to explain.

To obtain maximum dispersion against all three variables, we perform a three-way independent sort. All equities in our sample are sorted in three portfolios according to Δ(SV). They are also sorted in three portfolios according to size. Finally, they are sorted in three portfolios according to BM. Twenty-seven equally weighted portfolios are formed from the intersection of the three independent sorts. Summary statistics of the 27 portfolios are provided in Table X.

C. Empirical Methodology of the Asset-Pricing Tests

To test the asset-pricing models of Section V.A, we use the Generalized Methods of Moments (GMM) methodology of Hansen (1982), and employ the asymptotically optimal weighting matrix. For each model considered, we also

Table X

Summary Statistics on the 27 Size, BM, and DLI Sorted Portfolios

The 27 portfolios are constructed from the intersection of three independent sorts of all stocks into three size, three BM, and three default risk portfolios. Default risk is measured by the DLI. The second, third, and fourth columns describe the characteristics of each portfolio in terms of its size, BM, and DLI. Size refers to the market value of equity. Equally weighted average returns are reported in percentage terms.

	SIZE	BM	DLI	Average Return	Size	BM	DLI
1	Small	High	High	2.4229	1.8015	2.2192	18.9380
2	Small	High	Medium	1.6977	2.1021	1.6630	0.4960
3	Small	High	Low	1.6124	2.0410	1.6420	0.0240
4	Small	Medium	High	1.3834	2.0606	0.7734	10.2640
5	Small	Medium	Medium	1.4333	2.3183	0.8019	0.4410
6	Small	Medium	Low	0.9525	2.2164	0.8777	0.0290
7	Small	Low	High	0.8020	2.0956	0.3068	9.4810
8	Small	Low	Medium	1.1139	2.4143	0.3099	0.2850
9	Small	Low	Low	1.0843	2.3665	0.3453	0.0430
10	Medium	High	High	1.1913	3.7834	1.8675	12.0920
11	Medium	High	Medium	1.6750	3.9597	1.4046	0.3380
12	Medium	High	Low	1.5653	4.0343	1.3088	0.0170
13	Medium	Medium	High	0.7646	3.8382	0.8152	5.9680
14	Medium	Medium	Medium	1.3332	4.0316	0.7673	0.2930
15	Medium	Medium	Low	1.3354	4.1092	0.7711	0.0200
16	Medium	Low	High	0.6980	3.8611	0.3363	4.8230
17	Medium	Low	Medium	1.0774	4.0249	0.3248	0.2180
18	Medium	Low	Low	1.1680	4.1068	0.3315	0.0220
19	Big	High	High	1.6955	5.8582	1.7436	10.0360
20	Big	High	Medium	1.6261	6.3154	1.3537	0.2530
21	Big	High	Low	1.5171	6.7343	1.1435	0.0180
22	Big	Medium	High	0.9546	5.8168	0.8355	5.9050
23	Big	Medium	Medium	1.3203	6.1914	0.7767	0.2560
24	Big	Medium	Low	1.2019	6.5926	0.7227	0.0140
25	Big	Low	High	0.8634	5.8207	0.3560	3.5250
26	Big	Low	Medium	1.3465	6.1598	0.3510	0.2250
27	Big	Low	Low	1.1793	6.7712	0.3373	0.0150

compute Hansen's J-statistic on its overidentifying restrictions. In addition, we report a Wald test (Wald(b)) on the joint significance of the coefficients of the pricing kernel implied by each model.

To compare the alternative models, we use the Hansen and Jagannathan (1997) (HJ) distance measure. To calculate the p-value of the HJ-distance, we simulate the weighted sum of $n - k\,\chi^2(1)$ random variables 100,000 times, where n is the number of test assets, and k is the number of factors in the model examined.[16]

D. Asset-Pricing Results

The results from the asset-pricing tests are reported in Table XI. The rows labeled "coefficient" refer to the coefficient(s) of the factor(s) in the pricing

[16] See Jagannathan and Wang (1996).

Table XI

Optimal GMM Estimation of Competing Asset-pricing Models

The GMM estimations use Hansen's (1982) optimal weighting matrix. The tests are performed on the excess returns of the 27 equally weighted portfolios of Table IX. EMKT refers to the excess return on the stock market portfolio over the risk-free rate. Δ(SV) is the change in the survival rate, which is defined as 1 minus the aggregate DLI. HML is a zero-investment portfolio, which is long on high BM stocks and short on low BM stocks. Similarly, SMB is a zero-investment portfolio, which is long on small market capitalization (size) stocks and short on big size stocks. The J-test is Hansen's test on the overidentifying restrictions of the model. The Wald(b) test is a joint significance test of the b coefficients in the pricing kernel. The J-test and Wald(b) tests are computed in GMM estimations that use the optimal weighting matrix. We denote by HJ the Hansen–Jagannathan (1997) distance measure. It refers to the least-square distance between the given pricing kernel and the closest point in the set of pricing kernels that price the assets correctly. The p-value of the measure is obtained from 100,000 simulations. The estimation period is from January 1971 to December 1999. In Panel A we test the hypothesis that default risk is priced in the context of a model that includes the EMKT along with a measure of default risk (Δ(SV)). Panels B and C present results from tests of the CAPM and Fama–French models, which are used as benchmarks for comparison purposes. Finally, in Panel D we test the hypothesis that the Fama–French factors SMB and HML include default-related information, by including in the Fama–French model our aggregate measure of default risk.

	Constant	EMKT	SMB	HML	Δ(SV)	Test	J-test	Wald(b)	HJ
				Panel A: The EMKT + Δ(SV) Model					
Coefficient	1.0200	1.5398			−44.3823	Statistic	63.6054		0.8678
t-value	(39.2795)	(0.8804)			(−3.8607)	p-value	(0.0000)	(0.0001)	(0.0000)
Premium		0.0079			0.0043				
t-value		(2.8024)			(4.2752)				

Panel B: The CAPM									
Coefficient	1.0030	−2.2689				Statistic	69.4761		0.8991
t-value	(85.2437)	(−2.0283)				p-value	(0.0000)	(0.0425)	(0.0000)
Premium		0.0047							
t-value		(2.0283)							
Panel C: The Fama–French Model									
Coefficient	1.0325	−5.1332	0.0561	−8.2368		Statistic	67.3000		0.8766
t-value	(44.4182)	(−4.0217)	(0.0270)	(−3.4076)		p-value	(0.0000)	(0.0001)	(0.0000)
Premium		0.0061	0.0009	0.0036					
t-value		(2.5597)	(0.5317)	(2.0602)					
Panel D: The Fama–French Model Augmented by Δ(SV)									
Coefficient	0.9322	4.6068	24.7941	−12.0076	−135.2905	Statistic	46.8368		0.8032
t-value	(15.7444)	(1.6395)	(4.0315)	(−2.8654)	(−4.7691)	p-value	(0.0024)	(0.0000)	(0.0000)
Premium		0.0098	−0.0025	0.0082	0.0097				
t-value		(2.1551)	(−0.6916)	(2.6620)	(4.4788)				

kernel, whereas the rows labeled "premium" refer to the risk premium(s) implied for the factor(s).

The first panel shows the results of the model that includes the market and $\Delta(SV)$ as factors. We see that $\Delta(SV)$ commands a positive and statistically significant risk premium. This implies that default risk is systematic and it is priced in the cross section of equity returns. As expected, the *J*-test, and the *HJ*-distance measure have both very small *p*-values, which means that the model cannot price assets correctly. Even though both the EMKT and Δ(SV) are priced, it appears that there are other factors that may be important for explaining the cross-sectional variation in equity returns, and which are not considered here. Despite this implication, the model considered has a smaller *HJ* distance than both the CAPM (Panel B) and the FF model (Panel C). This means that, any mis-specification present in this model translates into at least as small an annualized pricing error as those resulting from the two standard asset-pricing models in the literature, the CAPM and FF model.[17]

Panel D reports the results from testing the hypothesis that SMB and HML proxy for default risk. In particular, we test the model of equation (8). The results show that Δ(SV) continues to receive a positive and statistically significant risk premium, even when it is considered part of the augmented model. HML is also priced again, as in Panel C, and SMB is not priced either in Panel C or Panel D.

Notice, however, that the coefficients of SMB and HML are very different in Panel D than they are in Panel C, and this is particularly the case for SMB. The fact that the coefficients of SMB and HML change in the presence of Δ(SV) suggests that SMB and HML share some common information with Δ(SV). The dramatic change in the coefficient of SMB between Panels C and D is an indication that SMB shares more common information with Δ(SV) than HML does. In general, we expect the coefficients to change when the factors in the pricing kernel are not orthogonal. Table II shows that Δ(SV) is positively and highly correlated with EMKT and SMB, but has a small and negative correlation with HML.

Recall that statistically significant coefficients in the pricing kernel imply that the corresponding factors help price the test assets, whereas a statistically significant premium means that the corresponding factor is priced.[18] The results in Panel D show that although all factors help price the test assets, SMB is not a priced factor.

Notice also that the coefficient on SMB is not statistically significant in Panel C, whereas it is in Panel D. This may be the case if the FF model is more misspecified than the model in Panel D. It seems that SMB needs the presence of Δ(SV) in the pricing kernel in order for its coefficient to become significant. The fact that the coefficient of SMB becomes significant in this case further shows that although there is some common information between SMB and

[17] For an interpretation of the *HJ*-distance as the maximum annualized pricing error, see Campbell and Cochrane (2000).

[18] See Cochrane (2001), Section 13.5.

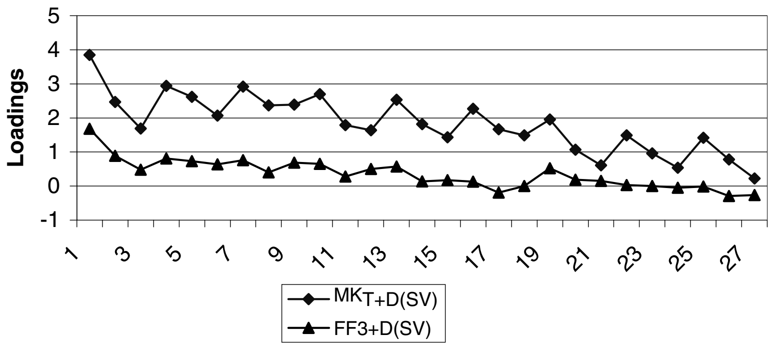

Figure 4. Loadings of the 27 portfolios on Δ(SV). This graph shows the loadings of the 27 portfolios of Table IX on the survival indicator Δ(SV). The portfolios are ordered in the same way as in Table IX. *EMKT* + Δ(*SV*) labels the loadings on Δ(SV) from the model that includes the market factor and Δ(SV) in the pricing kernel. Similarly, FF3 + Δ(*SV*) labels the loadings on Δ(SV) from the augmented Fama–French (1993) model by the Δ(SV) factor. The sample period is from January 1971 to December 1999.

Δ(SV), there is also residual information in both factors, which is important for pricing the test assets.

This interpretation is also supported by the values of the *HJ*-distance measures for the models of Panels C and D. The *HJ* distance for the FF model is larger in value than that of the model in Panel D. This suggests that the FF model may be more mis-specified than the model in Panel D. An implication of this result is that although there is some common information between Δ(SV) and the FF factors, there is also a lot of additional important information in SMB and HML which helps explain the test assets, but is unrelated to default risk.[19]

Figure 4 plots the loadings of the 27 portfolios on Δ(SV) from the models of Panels A and D. The portfolios are ordered in the same way as in Table X. It is interesting to note that the loadings on Δ(SV) for the model of Panel A are equal or larger than 1, for 20 of the 27 portfolios. This means that default risk is important for a large segment of the cross section that includes not just small firms but also medium-sized and big firms. In other words, the pricing of default risk is not driven by only a handful of portfolios.

Once SMB and HML are included in the pricing kernel, the loadings of Δ(SV) are reduced substantially for all portfolios. For the two portfolios that include small, high BM stocks with high or medium level of default risk, the loadings are also significantly reduced, but they remain around 1. The fact that the

[19] Vassalou (2003) shows, for instance, that a model which includes the market factor along with news about future GDP growth absorbs most of the priced information in SMB and HML. In the presence of news about future GDP growth in the pricing kernel, SMB and HML lose virtually all their ability to explain the cross section. Furthermore, Li, Vassalou, and Xing (2000) show that the investment component of GDP growth can price equity returns very well, and can completely explain the priced information in the Fama–French factors.

loadings of Δ(SV) are so drastically reduced for all 27 portfolios suggests again that SMB and HML include important default-related information.

The conclusion that emerges from the asset-pricing tests is that default risk is priced, and it is priced even when Δ(SV) is included in the FF model. SMB and HML contain some default-related information. However, this information does not appear to be the reason that the FF model is able to explain the cross section of equity returns.

VI. Conclusions

This paper uses for the first time the Merton (1974) model to compute monthly DLI for individual firms, and examine the effect that default risk may have on equity returns.

Our analysis provides a risk-based interpretation for the size and BM effects. It shows that both effects are intimately related to default risk. Small firms earn higher returns than big firms, only if they also have high default risk. Similarly, value stocks earn higher returns than growth stocks, if their risk of default is high. In addition, high-default-risk firms earn higher returns than low default risk firms, only if they are small in size and/or high BM. In all other cases, there is no significant difference in the returns of high and low default risk stocks.

We also examine through asset-pricing tests whether default risk is systematic, and we find that it is indeed. Fama and French (1996) argue that their factors SMB and HML proxy for default risk. Our results show that, although SMB and HML contain some default-related information, this is not the reason that the Fama–French model is able to explain the cross section of equity returns. SMB and HML appear to contain other significant price information, unrelated to default risk. Risk-based explanations for this information are provided in Vassalou (2003) and Li, Vassalou, and Xing (2000). Our results show that default is a variable worth considering in asset-pricing tests, above and beyond size and BM.

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