Financial Markets II, Spring 2009
Prof. Petr Zemčík

HOMEWORK ASSIGNMENT THREE

1. [0.5 point] **Essentials of Diffusion Processes**

Use notes *The Essentials of Diffusion Processes and Itô's Lemma.* Demonstrate that the expression (20) holds.

2. [0.5 point] **Itôs Lemma**

Suppose that x is the yield to maturity with continuous compounding on a zero-coupon bond that pays off \$1 at time T. Assume that x follows the process

$$dx = a(x_0 - x)\, dt + sx\, dz,$$

where a, x_0, and s are positive constants and dz is a Wiener process. What is the process followed by the bond price $B = e^{-x(T-t)}$?

3. [2 points] **Black-Scholes Formula**

With the notation used in the notes OPTION PRICING IN CONTINUOUS TIME AND THE BLACK SCHOLES EQUATION:
a. What is $N'(x)$?
b. Show that $S\ N'(d_1) = X\ e^{-r(T-t)} N'(d_2)$.
c. Calculate $\partial d_1/\partial S$ and $\partial d_2/\partial S$.
d. Show that

$$\frac{\partial c}{\partial t} = -rX\ e^{-r(T-t)} N(d_2) - SN'(d_1) \frac{\sigma}{2\sqrt{T-t}}.$$

e. Show that $\partial c/\partial S = N(d_1)$.
f. Show that c satisfies the Black-Scholes differential equation.

4. [1 point] **Equity Premium Puzzle**

This question follows Mehra and Prescott (1985). Let $\beta = 0.97$.

a. Calculate the weights $\omega_i, i = 1, 2$, corresponding to the system of linear equations (9) for $\alpha = 4$ and for $\alpha = 27$.

b. Using $\beta = 0.97$ and the weights calculate in a, calculate the expected return on equity, the expected return on the risk-free asset, and the expected equity premium for $\alpha = 4$ and $\alpha = 27$, respectively. Use equations (10)-(14). A word of caution: ordinarily, Figure 4 of the paper should serve you as a way of checking your results. However, my calculations show that the risk-free rate is somewhat higher than indicated in the figure.

c. The files in hw3_matlab.zip contain Matlab programs and procedures to solve the consumption based asset pricing model from Mehra and Prescott (1985) using a quadrature method. You need to use program hw3.m (with all the other files in the same directory). Before you run it, you need to alter the procedure returns.m to calculate a vector of conditional risk free rates in addition to the matrix of conditional risky returns. Also, you need to create and include a procedure, which calculates the unconditional risky and risk free returns. After these alternations, run the program hw3.m for different number of quadrature points, np=2,4,6,8,10 for $\beta = 0.97$ and $\alpha = 4$ and $\alpha = 27$, respectively. Does the equity premium and the risk-free rates differ from your results in part b? (Organize your results in a suitable table and include computer printouts for both the new Matlab program hw3.m plus at least one printed output)

d. Using your results from part b, explain what is meant by terms *equity premium puzzle* and *risk-free rate puzzle*.

5. [1 point] **Hansen and Jagannathan Volatility Bounds**

This question follows Burnside (1994). You will also need the data in a file hw3_data.xls. The data come from Mehra and Prescott (1985) and cover period 1890-1978. The data consist of the following series: year, c_t/c_{t-1}, the return on equity re and the risk-free return rf.

a. calculate the mean and the standard deviation of the intertemporal marginal rate of substitution $m_t = \beta(c_t/c_{t-1})^{-\gamma}$ for $\beta = 0.97$ and $\gamma = 4$ and $\gamma = 27$, respectively. Check your results using Figure 1 of Burnside (1994).

b. Derive the inequality (1.4) for one risk-free and one risky asset.

c. Using the data on equity returns and risk-free rate returns and formula (1.4) in Burnside (1994), calculate the Hansen and Jagannathan bound for $Em = 0.914$ and $Em = 1.041$, respectively. Again, check your results using Figure 1 of Burnside (1994).

c. Using your results from parts a and c, when is the Hansen and Jagannathan bound violated? Also, compare the results with your results from question 4. Note that you used the same coefficients of risk aversion in both questions.