Financial Markets II, Spring 2009
Prof. Petr Zemčík

HOMEWORK ASSIGNMENT TWO

1. [1 point] *Redundant Securities*

Re-program in Matlab the algorithm described in a solution to the problem 2.8 and programmed in Gauss in the problem 2.9 in the Černý textbook - see
http://martingales.info/mtfweb/index.html

Test your program on the pay-offs matrix

$$A = \begin{pmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}.$$

As a solution, provide a printout of your program and a printout of your results.

2. [1 point] *Arbitrage and State Prices*

Consider the following price, pay-off pairs:

(a) $S' = (1\ \ 1)$ and

$$A' = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix},$$

(b) $S' = (3\ \ 3)$ and

$$A' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix},$$

(b) $S' = (3\ \ 3)$ and

$$A' = \begin{pmatrix} 2 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}.$$

i. Which of these pairs is arbitrage-free?
ii. Compute, for each example, the complete set of state prices ψ satisfying $S = A'\psi$.
iii. Use your answers to (i) and (ii) to verify that absence of arbitrage implies positive state prices.

3. [1 point] *Option Pricing 1/3*

Let S_0 denote the current stock price, K the strike price, T the time to expiration, r the (continuously compounded) risk-free interest rate, C the value of American call option, and P the value of American put option, respectively. Show that

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

(*Hint:* For the first part of the relationship consider (a) a portfolio consisting of a European call plus an amount of cash equal to K and (b) a portfolio consisting of an American put option plus one share.)

4. [1 point] *Option Pricing 2/3*

Explain why an American call option is always worth at least as much as as its intrinsic value. Is the same true of a European call option? Explain your answer.

5. [1 point] *Option Pricing 3/3*

A European call option and put option on a stock both have a strike price of $20 and an expiration date in three months. Both sell for $3. The risk-free rate is 10% per annum, the current stock price is $19, and a $1 dividend is expected in one month. Identify the arbitrage opportunity open to a trader.