Financial Markets II, Spring 2009
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HOMEWORK ASSIGNMENT ONE

1. [1 point] *Risk aversion*

Let $\tilde{\epsilon}$ be a random variable with a zero mean. Assume that it can take only two values, x with probability p and y with probability $(1-p)$ and further assume that $x \in [0,1]$. Suppose that a risk averse individual is facing a bet where his current wealth W can become $(1+\tilde{\epsilon})W$. Derive the portion of his current wealth, which he would be willing to pay to avoid participating in this bet. Comment on your result in the context of risk aversion.

2. [1 point] *Risk Aversion*

a. Consider a power utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}$. Show that θ is a coefficient of relative risk aversion.

b. Show that for $\theta = 25$, individuals would rather accept a 18% reduction in consumption with certainty than risk a 50-50 chance of a 20% reduction.

c. Show that as θ approaches to one, the utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}$ approaches the log utility function $u(c) = ln(c)$.(Hint: use the L'Hospital rule.)

3. [0.5 point] *Convex indifference curves in the* $(\bar{R}_P, \sigma_P)$ *space*

Let $\bar{R}_P$ and σ_P denote the mean return and standard deviation of a portfolio P. Show that if a utility function is strictly concave and asset returns normal, the indifference curves in the $(\bar{R}_P, \sigma_P)$ space are convex.

4. [0.5 point] *Efficient frontier with a risk-less asset*

Demonstrate that the point A in Figure 0-3: Efficient Frontier of the notes on Mean Variance Analysis by Penati and Pennacchi is unique.

5. [1 point] *Computation of the mean-variance efficient set and the CAPM.*
There are 10 states of the world, which are equally likely. The following table gives firm-specific cash flows in each state:

Path	1	2	3	4	5	6	7	8	9	10
Firm 1	5	5	5	24	25	30	32	68	75	75
Firm 2	4	5	10	21	66	65	65	20	20	20
Firm 3	55	55	49	22	22	20	10	10	10	10

Prices of firm 1 to 3's shares are 28, 26, and 25 respectively.
a) Characterize the efficient frontier. What are the minimum-variance portfolio weights

associated with an expected return of 11%?
b) Assume that there is a risk-free asset with a rate of return of 8%. Characterize the efficient frontier in this case.
c) Calculate the weights of the market portfolio and its expected return and standard deviation.
d) Calculate the firm specific betas.
e) Draw the market security line (i.e. $\bar{R}_i$ on the vertical axis is related to β_i on the horizontal axis via the expression (8) of the CAPM notes). Compute the market price of risk.

6. [1 point] *Multi-factor asset pricing models*

The objective of this question is to update and to some extent expand results from Fama, E.F. and K.R. French, 1993, "Common risk factors in the returns on stocks and bonds," Journal of Financial Economics 33, 3-56 (FF henceforth). The updated dataset is from the web site of K.R. French:
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

a) I have replicated time-series (TS) regressions from FF using the updated dataset of monthly returns from 1946:01 to 2005:8. The regressions have the following form:

$$R_{ij} - RF_t = a_{ij} + b_{ij}(RM_t - RF_t) + s_{ij}SMB_t + h_{ij}HML_t + \epsilon_t \;\; i = 1, ..., 5, \; j = 1, ..., 5. \quad (1)$$

where R_{ijt} is a stock return on a stock of size quintile i, book-to-market quintile j and time t. RM_t is the market return and RF_t is the risk-free return. SMB is the small-minus-big FF-factor and HML is the high-minus-low book-to-market FF factor. Results of the regressions are in the sheet *TS Regressions* in the file Hw01_FF.xls (available on my web site). Complete the table (the yellow spaces) - this amounts to running two regressions using the data series in the sheet *TS Data*. Compare your results with Table 6 in FF (just for stocks) and Table 9a (iv). Briefly comment on whether major conclusions of FF hold up.

b) Consider a (second pass) cross-sectional (CS) regression:

$$\bar{R}_{ij} = \lambda_{0,ij} + \lambda_{1,ij}\hat{b}_{ij} + \lambda_{2,ij}\hat{s}_{ij} + \lambda_{3,ij}\hat{h}_{ij} + v_{ij} \;\; i = 1, ..., 5, \; j = 1, ..., 5. \quad (2)$$

The data are estimates from (a) plus mean returns, all of which are organized for your convenience in the sheet *CS Data*. The yellow spaces need to be filled in using your results from (a). Run the regression (2) and comment on the results. Namely, focus on the intercept and on which factors are priced.

Instructions: Your answers should consist of three parts: printouts of the filled Excel tables, computer printouts from your regressions, and your comments on the results.