Presidential Address: Issuers, Underwriter Syndicates, and Aftermarket Transparency

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ABSTRACT
I model strategic interaction among issuers, underwriters, retail investors, and institutional investors when the secondary market has limited price transparency. Search costs for retail investors lead to price dispersion in the secondary market, while the price for institutional investors is infinitely elastic. Because retail distribution capacity is assumed to be limited for each underwriter-dealer, Bertrand competition breaks down in the primary market and new issues are underpriced in equilibrium. Syndicates emerge in which underwriters bid symmetrically, with quantities allocated internally to efficiently utilize retail distribution capacity.

Underwriters and dealers appear to compete vigorously for new security issues, and yet also appear to earn high profits in this business. What value are they generating that warrants high compensation? Are there obstacles that keep this value from flowing through to security issuers, despite apparently fierce competition in the market for these financial services?

The model in this paper combines three elements: a lack of price transparency in the secondary market, limited retail distribution capacity, and Bertrand competition in the primary market. Dealer-underwriters bid for new securities as Bertrand competitors. They then sell these securities in a secondary market with limited price transparency. Retail investors face costs of gathering price information, while institutional investors, who are frequent repeat customers, do not. This leads to price dispersion in the secondary market, where some retail customers get attractive terms and others do not. The rents that accrue to broker-dealers, who can sell to retail customers, are limited by their distribution capacity. This, in turn, limits their incentives to bid aggressively in the primary market. In equilibrium, they share the new issues through arrangements that resemble underwriter syndicates. New issues are underpriced relative to average subsequent transactions prices.

The results illustrate how noncompetitive outcomes in the secondary market can lead to breakdowns in competition in the primary market, and why issuers may have little to gain from improved price transparency. While more transparency limits the rents dealers can extract from retail investors in the secondary market, the lost rents are not absorbed by issuers. Mechanisms

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that divide rents between issuers and underwriters, whether through explicit
price concessions by underwriters or through golf games and closing din-
ers, will lead issuers to oppose institutional reforms aimed at improved price
transparency.

The benefits of transparent pricing in the securities markets and the costs of
mandating transparency have become central concerns for researchers, regula-
tors, and investors in the United States and abroad. Many important securities
trade over the counter. In some of these markets, such as those for currency
or credit swaps, trading is dominated by professional, sophisticated interme-
diaries, and the benefits of mandated price transparency would seem to be
minimal. In other cases, such as the markets for municipal or corporate bonds,
there are significant retail holdings of the securities, or at least there might be
if terms of trade for these investors were not so punitive.

Regulatory bodies have recently been encouraging or mandating centralized
reporting of trade prices to facilitate greater posttrade transparency. Advocates
of these policies ask how easier access to information can be a bad thing. They
point to the attributes of information as a public good, to reduced adverse se-
lection, and to the regulatory goal of “investor protection.” For example, it can
be argued that in decentralized, opaque markets intermediaries will find more
occasions to exploit local monopoly power and engage in opportunistic self-
dealing. Investors concerned about the resulting costs will trade less or avoid
the market altogether. If some investors avoid the market for these reasons,
positive congestion externalities in liquidity provision increase costs for those
who must trade. Ultimately, the costs of the reduced liquidity may be borne by
issuers in their cost of funds. The public will pay less for their securities.

On the other side of the debate are those who are generally suspicious of
regulatory authorities determining how, when, and where people can trade.
They argue that protecting retail investors from the consequences of their own
poor decisions will only make inattentive investors even more inattentive, and
thus hurt efficiency more broadly. The costs associated with reporting and cen-
trally publishing prices will be wasted if the securities in question are naturally
illiquid. More transparency can, indeed, hurt liquidity. Institutional and sophis-
ticated traders, who value anonymity, will respond to increased transparency
by migrating to other trading venues, leaving retail investors no one to trade
with. For example, the introduction of the TRACE reporting system for corpo-
rate bonds has reportedly driven some large traders to the credit derivatives
markets.

Recent empirical research in the corporate and municipal bond market sug-
gests that the costs of trading for retail investors are often extremely high.
Harris and Piwowar (2006) use time-series methods to estimate trading costs
for municipal bonds, and show that these are decreasing in trade size. Green,
Hollifield, and Schöff (2007a) match buys and sales in seasoned bonds to
estimate the profits to dealers from the liquidity provision. Their structural
model yields estimates of relative bargaining power for customers and dealers
that depend on trade size. Biais and Green (2006) compare trading costs in
the current environment to costs during historical periods when bonds were
actively traded on exchanges. Bessimbinder, Maxwell, and Venkataraman (2007), Edwards, Harris, and Piwowar (2007), and Goldstein, Hotchkiss, and Sirri (2007) study the costs of trade in corporate bonds, and show that increased posttrade transparency has lowered trading costs.

Green, Hollifield, and Schürhoff (2007b), who examine price dispersion in the market for new municipal bond issues, provide the empirical study most pertinent to the model developed here. The new issues market is of particular interest because it is one time in the life of the bonds when there is a good deal of trading volume for exogenous reasons: There must be trade to move the bonds from the inventories of the dealer-underwriters to the portfolios of investors. For issues with high levels of retail participation, Green, Hollifield, and Schürhoff (2007b) show that trades occur at a wide range of prices virtually simultaneously. Some of the dispersion is attributable to trade size. Large investors, presumably institutions, get more attractive terms of trade than smaller retail investors. Some retail investors, however, trade at very attractive terms. There is underpricing of new issues relative to the average price buyers pay for the bonds. Green, Hollifield, and Schürhoff (2007b) argue that the high level of price dispersion observed for retail size trades is likely due to the differences in the access investors have to price information. While it is relatively easy to find this information, many investors are unsophisticated about its availability, not knowing where to look for it. Increased pretrade and posttrade transparency, by making the information immediately available, would presumably narrow the range of prices at which retail investors buy. This paper captures these features of prices in the secondary market by adapting to this setting the models developed to study price dispersion in consumer markets. Early examples are Shilony (1977), Varian (1980), and Burdett and Judd (1983). An application of this type of model in a financial context can be found in Carlin (2007), who considers intermediaries choosing both a price and the costs to be imposed on investors attempting to ascertain what the price is. Mixed strategies lead to dispersion both in the price and in price transparency, as appears evident in mutual funds or brokerage accounts with “hidden fees.”

Price dispersion in the secondary market, however, is only part of the puzzle. As in any underpricing problem, one wonders why issuers appear willing to sell their securities for less than the market value. A lack of price transparency, per se, is also not particularly satisfactory as an answer. Retail investors are occasional participants in these markets, and hence lack incentives and opportunities to gain sophistication. Almost by construction, in a market that lacks price transparency it will be difficult or costly for such investors to evaluate the terms of trade and determine whether they are getting a bad deal. Issuers, on the other hand, have professional staff or advisors managing their dealings with the market and with the intermediaries, who seem to compete aggressively for their business. It is more difficult to believe that they would be unconcerned or unaware that many investors appear to pay substantial premia over the reoffering price established in their negotiations with the underwriters.

It is similarly curious that the issuers do not take a more active role in the policy debates about price transparency, as they appear to have a great deal
at stake. If intermediaries are capturing sustainable rents through trading that is shrouded due to lack of price transparency, why do issuers not seek alternative means of raising funds? Why do they not list their securities in more transparent trading venues, such as exchanges? The model developed here is consistent with these outcomes. The lack of price transparency does not deceive issuers. They know they are getting a bad deal. Since unsophisticated investors are not available in infinitely elastic supply, however, issuers may be unable to capture the benefits of these high-valuation investors through Bertrand competition by the intermediaries.

Two frictions play central roles in the model, namely, limited capacity of retail distribution networks and search costs for retail investors. Both seem to me to be of obvious importance in explaining the institutional features of financial markets.

The ability to reach retail customers is the scarce resource and source of economic rents in the model. The distribution networks through which broker-dealers reach retail customers can be viewed as installed capacity that is difficult to expand quickly at a low cost. In last year’s presidential address, “Household Finance,” Campbell (2006) emphasized the difficulty competitors have in reaching retail investors with information as a barrier to reform and innovation in these markets. We also observe brokerage firms investing large amounts of human, advertising, and technological resources in providing high levels of personal and advisory services to retail customers.

The difficulties in reaching retail investors are mirrored in the costs these investors face when searching for information. Often the cost of acquiring information about prices for securities and financial services is not high “if you know where to look,” but time and effort are needed to educate investors about where to look. The market for new issues of municipal bonds, studied by Green, Hollifield, and Schürhoff (2007b), illustrates this. Information about the reoffering price of new bond issues is not difficult to acquire, but many retail customers do not even know a bond is recently issued when they purchase it through their broker.

Along with the empirical papers mentioned above, the ideas developed here are related to various theoretical arguments in the economics literature, both long-standing and recent. Economists have long recognized that capacity constraints or decreasing returns interfere with Bertrand competition.1 My model shares with the classic formulations the general notion that decreasing profitability of the marginal unit discourages aggressive price competition. Pricing more aggressively, for example, does you no good if you are already operating at capacity. The channels through which the tradeoffs work, however, are quite different in my setting due to the two-stage nature of the game. For example, the zero-profit competitive outcome is one of the equilibria here because if an underwriter bids less aggressively at the zero-profit point he loses the entire issue to his rival. In the standard setting, the competitive outcome is not an equilibrium because, by deviating to a price above marginal cost, a firm retains some of the market and earns positive profits.

1 See, for example, Mas-Colell, Whinston, and Green (1995), pp. 394–395, for a discussion.
The behavior studied in this paper has some similarities with the “shrouded equilibria” considered by Garbaix and Laibson (2006), which Campbell (2006) argues can be important in explaining barriers to information flow in retail finance. Garbaix and Laibson (2006) study nontransparent pricing for add-ons, such as phone charges or mini-bar items in hotels, and show that in equilibrium competitors lack incentives to educate unsophisticated customers about opaque pricing. Such informative advertising may make consumers better shoppers, but the firms investing in these activities will not be able to attract the customers they educate, since sophisticated shoppers are subsidized by naive shoppers. The base good is priced at a loss, and sophisticated shoppers can substitute away from the add-ons. In my model educated customers are similarly less profitable customers. In other respects, however, the models are very different in their concerns and methods. This paper concerns why the rents intermediaries gain through opaque pricing are not passed through to issuers, despite apparently vigorous Bertrand-like competition for new business. Issuers do not gain from increased transparency since it only serves to concentrate prices around the valuation of the marginal buyers, rather than that of the inframarginal buyers. In Garbaix and Laibson (2006) competition between firms eliminates rents. Whether in the shrouded or transparent equilibria, industry profits are the same.

The remainder of the paper is organized as follows. In the next section I model the interaction between a single issuer and multiple underwriters bidding for the new securities. To focus on the central ideas, I initially simplify the environment the underwriter-dealers face in the secondary market. Each dealer has exclusive access to a set of retail customers with high valuations, and shared access to sophisticated institutional investors with perfectly elastic demand at a lower price. In equilibrium dealers quote prices to retail investors using a mixed strategy, leading to price dispersion of the sort documented in Green, Hollifield, and Schürhoff (2007b). Section III ties the primary and secondary markets together. It shows that the demand functions the search model implies for the secondary market support noncompetitive outcomes as equilibria in the primary market. I also discuss the benefits for issuers of reducing the cost of search for investors. The final section briefly concludes.

I. A Model of Underwriting Syndicates

This section develops a simple model that illustrates the basic forces at work in the competition between potential underwriters. Two intermediaries, denoted i and j, function as both underwriters and dealers. The dealers each have limited capacity to reach a set of retail investors with relatively high valuations. In Section II these retail buyers are modeled as investors with a relatively high cost of search. For the moment I assume these search costs are infinite. In
addition, underwriters can sell their inventory to a large set of informed institutional investors who have infinitely elastic demand at a lower price. The size of the typical issue exceeds the retail capacity of the underwriters, so that some portion of any issue must be sold to institutional investors. Underwriters first compete in a Bertrand manner for the business of an issuer. They then sell their inventory in a secondary market, where they are free to price discriminate between retail and institutional investors because pricing is not transparent.

In this section, I abstract from uncertainty and I do not model the source of the differences in valuation between retail and institutional investors. The model is free of any frictions (or rents) except for the dealers’ monopoly power with a subset of their customers. The critical assumption is that their ability to reach these high-valuation investors is limited by some previously installed capacity, which we can view as the infrastructure associated with building and maintaining a retail distribution network.

There are two dealers. Each has monopoly access to a continuum of retail customers, with unit measure. Each such retail investor will pay $\bar{p}$ for a unit of the security. Dealers also have access to an institutional market where they operate competitively. Institutional investors have perfectly elastic demand at a price $v$, which we will call the “institutional price.”

Suppose a particular dealer has an inventory of $Q$ securities to sell in the aftermarket, which were purchased from the issuer at price $b$. I refer to the price dealers pay the issuer as the “offering price.” The profit dealer $i$ will earn selling the securities, denoted $\pi(b, Q_i)$, will be

$$
\pi(b, Q_i) = \begin{cases} 
(\bar{p} - b)Q_i & Q_i < 1 \\
(\bar{p} - b) + (v - b)(Q_i - 1) & Q_i \geq 1 
\end{cases}
$$

(1)

Given this profit function in the aftermarket, each dealer competes in a Bertrand fashion to buy the securities from an issuer. Assume the quantity of securities the issuer wishes to float is $K$, where $K > 2$. Thus, even if the underwriters evenly split the issue, some of it must be sold to the institutional customers. I refer to any allocation of the securities that absorbs the entire issue into dealer inventories, $Q_i + Q_j = K$, as feasible.

The issuer gives the entire issue to the dealer who offers the lowest price. Dealers play a Bertrand game with each other, but as is usual in auction settings they account for the issuer’s response in their bidding strategies. Accordingly, I define an equilibrium as a set of bids and quantities $\{b^*_i, b^*_j, Q^*_i, Q^*_j\}$ such that the allocation is feasible and no dealer would change his bid, taking as given the bid of the other dealer and the response of the issuer.

Note that dealers only compete in the price space. If the dealers bid identical prices for the issue, I will be agnostic, for the time being, regarding how the securities are allocated across dealers and the coordination mechanism through which this is achieved. In this sense, the model can be seen as one of endogenous syndicate formation. One could view the quantity allocation, for example, as proposed by a lead underwriter. I show below that it is in the collective interests
of the underwriter-dealers to control the allocation, rather than letting the
issuer exercise this control or playing individually in quantity space. A given
dealer’s alternative is to defect from the syndicate, and bid independently for
the issue.

I do not limit attention to equilibria that are symmetric. Thus, \( b^*_1 = b^*_2 = b^* \)
and \( Q^*_1 + Q^*_2 = K \) are an equilibrium, unless one of the dealers can earn higher
profit by bidding a higher price and capturing the entire issue, or bidding a
lower price and earning zero. Formally, the following conditions must be met:

\[
Q^*_1 + Q^*_2 = K \\
\pi(b^*, Q^*_h) \geq \pi(b^* + \varepsilon, K) \text{ for } \varepsilon > 0, h = \{i, j\} \\
\pi(b^*, Q^*_h) \geq 0 \quad h = \{i, j\}. \tag{2}
\]

The next result follows immediately from the Bertrand setting, and ensures
we can limit attention to equilibria of the sort described in conditions (2) above.

**Proposition 1:** In any equilibrium with nonnegative profits for either dealer,
both dealers bid the same price.

**Proof:** If the dealers bid different prices, the one with the lower bid receives
no securities from the issuer and earns zero profit. Assume this is dealer \( i \).

Then dealer \( j \) must be earning a positive profit on the entire allocation, \( Q_j = K \), at his bid \( b_j \), so that

\[
\pi(b_j, K) = (\bar{p} - b_j) + (v - b_j)(K - 1) > 0. \tag{3}
\]

By bidding \( b_j + \varepsilon \), with \( \varepsilon > 0 \), dealer \( i \) captures the entire issue and earns profit,

\[
\pi(b_j + \varepsilon, K) = \left[ \bar{p} - (b_j + \varepsilon) \right] + \left[ v - (b_j + \varepsilon) \right](K - 1) \\
= \pi(b_j, K) - K\varepsilon, \tag{4}
\]

which must be positive for sufficiently small \( \varepsilon \). Thus, it cannot be an equilibrium
for the dealers to bid different prices with either dealer earning positive profits.

A single dealer bidding for the entire issue in a zero-profit equilibrium needs
to be ruled out. Suppose that \( b_i < b_j \) and dealer \( j \) earns zero profit in equilibrium. Then,

\[
\pi(b_j, K) = (\bar{p} - b_j) + (v - b_j)(K - 1) = 0. \tag{5}
\]

By slightly reducing his bid by \( \varepsilon < b_j - b_i \), dealer \( j \) can earn strictly positive
profits while still outbidding dealer \( i \) and not losing the issue. Q.E.D.

The above proposition can be viewed as a model of syndicate formation. In
any equilibrium, the dealers offer the issuer the same price and divide the
securities between themselves in some way that does not affect the issuer. There
is no collusion with regard to the offering price. The dealers are competing in
a Bertrand fashion. As we will see, however, many of these equilibria are quite
profitable for the dealers.
In fact, there is a continuum of Nash equilibria in this setting, which are of various levels of profitability to the dealers and thus differ in cost to the issuer. At one extreme is the most profitable equilibrium, where both dealers offer the institutional price to the issuer and the allocation of securities between the two dealers ensures that they both fully utilize their retail distribution capacity. At the other extreme is a unique zero-profit equilibrium, where the dealers split the issue symmetrically. There are also Nash equilibria at every price between these two, and these include equilibria that are asymmetric in both quantity and profitability.

PROPOSITION 2: There is a unique zero-profit equilibrium. In this equilibrium each dealer receives an allocation of

\[ Q_i = Q_j = \frac{K}{2} \]  

and the bonds are purchased from the issuer at a price of \( v + \hat{\Delta} \), where

\[ \hat{\Delta} = \frac{2(\bar{p} - v)}{K}. \]  

Proof: This allocation is clearly feasible, since \( Q_i + Q_j = K \). By the assumption that \( K > 2 \), these allocations exceed the retail capacity of unity. At this allocation dealer profits will be zero if each dealer bids \( v + \hat{\Delta} \), where \( \hat{\Delta} \) solves

\[ [\bar{p} - (v + \hat{\Delta})] + \left( \frac{K}{2} - 1 \right) [v - (v + \hat{\Delta})] = 0. \]  

This can be simplified to yield the expression in the proposition for \( \hat{\Delta} \).

At this price and quantity, neither dealer can increase profits by bidding less than \( v + \hat{\Delta} \), as he would receive no securities, and earn zero profit. If he bids \( \varepsilon \) more, he would receive the entire allocation, and earn profits of

\[ \pi(v + \hat{\Delta} + \varepsilon, K) = [\bar{p} - (v + \hat{\Delta} + \varepsilon)] - (K - 1)(\hat{\Delta} + \varepsilon) \\ = [\bar{p} - (v + \hat{\Delta})] - \left( \frac{K}{2} - 1 \right) \hat{\Delta} - \frac{K}{2} \hat{\Delta} - K \varepsilon \]  

Since \( \pi(v + \Delta, \frac{K}{2}) = 0 \), and the remaining terms are negative, the last line is negative and is decreasing in \( \varepsilon \). Thus, increasing the bid yields negative profits, which establishes that the conjectured prices and quantities are a Nash equilibrium.

It remains to show that the equilibrium is unique. Proposition 1 establishes that in any zero-profit equilibrium, both bidders bid the same. At a fixed price, the profit function is monotonically decreasing in quantity, given that each dealer receives more than his retail capacity. If the two dealers receive unequal
allocations, then they must earn different profits, and so cannot both be earning zero profits. If one receives less than his retail capacity and earns zero profit, then $\bar{p} - (v + \hat{\Delta}) = 0$, which would imply the other dealer earns negative profits. At equal quantities, the profit function is monotonic in price, and so only the price $v + \hat{\Delta}$ can deliver zero profits. Q.E.D.

In the most profitable equilibrium, the dealers pay the issuer the lowest price that breaks even in the institutional market. This equilibrium need not be symmetric in quantities, but will be symmetric in price.

**Proposition 3:** The institutional price $v$ is an equilibrium price, and at this price both dealers fully utilize their retail capacity in equilibrium.

*Proof:* Given that each dealer fully utilizes retail capacity, profits in this equilibrium are $\bar{p} - v$ and do not depend on the marginal quantity. Thus, the dealers are indifferent to any feasible allocation with $Q_i > 1$, $Q_j > 1$. At this price and such an allocation, no dealer will raise the price to obtain the full issue, since he would lose money in the institutional market and earn less in the retail market. Since both dealers are earning positive prices, neither will wish to bid less and earn zero.

It remains to show that there is no equilibrium at this price where one dealer, say dealer $i$, receives $Q_i < 1$. In such a case, dealer $i$’s profit would be $Q_i(\bar{p} - v)$. By increasing his bid by $\varepsilon$, he captures the entire issue and increases his profit by

$$
\pi(v + \varepsilon, K) - \pi(v, Q_i) = (\bar{p} - (v + \varepsilon)) + (K - 1)(v - (v + \varepsilon)) - Q_i(\bar{p} - v) = (\bar{p} - v)(1 - Q_i) - K\varepsilon,
$$

which will be positive for sufficiently small $\varepsilon$. Q.E.D.

The equilibria where the dealers pay the institutional price are the only equilibria where they do not lose money on the marginal bonds they hold. I will argue that within the institutional setting under consideration, this makes such equilibria particularly natural. There is, however, a continuum of equilibria, both symmetric and asymmetric, between the institutional and the zero-profit price, as the next result demonstrates.

**Proposition 4:** Any price $v + \Delta$, where $0 \leq \Delta \leq \hat{\Delta}$, is an equilibrium with associated allocations satisfying

$$Q_i + Q_j = K
$$

and, for $h = \{i, j\}$,

$$\max \left\{ K - \frac{\bar{p} - v}{\Delta}, 1 \right\} \leq Q_h \leq \min \left\{ \frac{\bar{p} - v}{\Delta}, K - 1 \right\}.
$$

*Proof:* Consider the inequalities in (12) when $\Delta \to 0$. The left-hand side will equal 1 and the right-hand side will equal $K - 1$. Then Proposition 3 establishes the result for this case.
The other extreme, when \( \Delta = \hat{\Delta} \), corresponds to Proposition 2. To see this, substitute \( \Delta = \frac{2(\bar{p} - v)}{K} \) into (12). Simplifying yields

\[
\max \left\{ K - \frac{K}{2}, 1 \right\} \leq Q_h \leq \min \left\{ \frac{K}{2}, K - 1 \right\}.
\]

The left-hand side and the right-hand side are both equal to \( \frac{K}{2} \), since by assumption \( K > 2 \). Proposition 2 establishes equal allocations with \( \Delta = \frac{2(\bar{p} - v)}{K} \) as an equilibrium.

Consider, then, any price \( v + \Delta \), where \( 0 < \Delta < \hat{\Delta} \), and any \( Q_i \) satisfying (12). Allocate to dealer \( j \) a quantity \( Q_j = K - Q_i \). This allocation is feasible, and by (12) \( Q_i \geq 1 \) and \( Q_j \geq 1 \), so that both dealers fully exploit their retail capacity. Dealer \( i \) earns profit

\[
\pi(v + \Delta, Q_i) = \bar{p} - v - Q_i \Delta \\
\geq \bar{p} - v - \left( \frac{\bar{p} - v}{\Delta} \right) \Delta \\
= 0,
\]

where the inequality follows from the right-hand side of (12). Thus, dealer \( i \) earns nonnegative profit at the conjectured price and quantity and has no reason to bid a lower price that would yield a payoff of zero. If he bids a higher price, he gains the entire issue. Given that he is fully exploiting retail capacity, however, his profit is decreasing in both price and quantity, so it will not be optimal for him to bid more.

Dealer \( j \)'s profit is

\[
\pi(v + \Delta, Q_j) = \bar{p} - v - (K - Q_i) \Delta \\
\geq \bar{p} - v - K \Delta + \left( K - \frac{\bar{p} - v}{\Delta} \right) \Delta \\
= 0,
\]

where the inequality follows from the left-hand side of (12). Dealer \( j \) therefore earns positive profit at the conjectured equilibrium and will not bid less than \( v + \Delta \). If he bids more, he pays a higher price and gains a larger quantity, but profits are decreasing in quantity given that the dealer fully uses retail capacity. Q.E.D.

Figure 1 illustrates the set of possible equilibria, assuming that the size of the issue, \( K \), is 5; the institutional price, \( v \), is par; and the retail price, \( \bar{p} \), is 103. At more profitable equilibria (low \( \Delta \)), allocations between dealers are indeterminate as long as each dealer receives an allocation in excess of his retail capacity (\( Q_i \geq 1 \) and \( Q_j \leq K - 1 \)). Once the price exceeds \( v + (\bar{p} - v)/(K - 1) \), the set of equilibria at which both dealers can earn positive profits begins to shrink, until at \( v + 2(\bar{p} - v)/K \) only a symmetric allocation is profitable for both dealers.
Figure 1. Equilibrium quantity allocations and prices. The plot shows possible equilibrium allocation and price outcomes for $K = 5$, $v = 100$, and $\bar{p} = 103$. The zero-profit equilibrium is unique and symmetric, with the price equal to $v + \Delta = 101.2$. In the maximum profit equilibrium, dealer allocations are indeterminate, between 1 and $K - 1 = 4$, and the price paid to the issuer is the institutional price of $v = 100$.

All of the equilibria described by Proposition 4 are consistent with the Bertrand competition between dealers when they can only play in price space. The equilibrium described by Proposition 3, in which the reoffering price is the institutional price, is clearly the one that maximizes both their individual and joint profits. It also corresponds to the empirical outcomes in Green, Hollifield, and Schürhoff (2007b). They show that the reoffering price, which is the offering price plus the negotiated underwriter spread, is a good predictor of the price at which large institutions trade, while smaller retail traders face considerable price dispersion and substantial markups over the reoffering price. Interdealer trades are also centered on the reoffering price.

Indeed, there are good theoretical reasons to focus on the most profitable equilibria, as they are the only outcomes that are robust to even minimal freedom for the dealers to play in quantity space. In any other equilibria the dealers are losing money on the marginal security in their allocations. A natural way to refine the set of equilibria, given the institutional setting, is to allow each dealer
to decline to accept bonds. Underwriter-dealer syndicates negotiate quantity allocations internally, with the lead underwriter coordinating the negotiations. It collectively benefits the underwriter-dealers to control the quantity allocation, rather than letting the issuer propose allocations. It would be easier for the issuer to bargain for price concessions in situations in which the allocation is symmetric.

If we enlarge the dealers’ strategy space in this way, only \( v \), the institutional price, survives as an equilibrium price. I say an equilibrium is \( Q \)-robust if dealers do not increase their profits by declining to accept a part of their allocation.

**Proposition 5:** In any \( Q \)-robust equilibria both dealers bid \( v \), and receive an allocation that fully utilizes their retail distribution capacity.

**Proof:** Suppose \( \Delta > 0 \). In any equilibrium meeting the conditions in Proposition 4, which imply \( Q_i \geq 1 \) and \( Q_j \geq 1 \), each dealer earns

\[
\bar{p} - (v + \Delta) + (Q_h - 1)[v - (v + \Delta)].
\]

(16)

The term multiplying \((Q_h - 1)\) is clearly negative, and by declining this quantity of securities the dealer will raise his profits. Q.E.D.

**II. Price Dispersion and Rents in the Aftermarket**

The model developed in the previous section relies on each firm facing a demand curve in the secondary market for securities that is initially downward sloping. The marginal unit, however, must be sold at a perfectly elastic price. While I assumed there were two types of investors, and dealers could perfectly discriminate between them, all that is essential to the argument is that there is some range over which dealers have local monopoly power, while they must price the marginal unit competitively.

For securities traded over the counter, a natural source of such market power is a lack of transparency in the trading venue, coupled with search costs for retail-level investors. A number of recent empirical studies show that smaller traders in over-the-counter bond markets can face extremely high costs of trade, and relate these costs to the degree of price transparency. Harris and Piwowar (2006) estimate trading costs for municipal bonds. The costs decrease dramatically with trade size. Green, Hollifield, and Schürhoff (2007a) match purchases and sales in seasoned municipal bonds to estimate the profits to dealers from liquidity provision. Their structural model yields estimates of relative bargaining power for customers and dealers that depend on trade size. Biais and Green (2006) compare trading costs in the current environment to costs during historical periods when bonds were actively traded on exchanges. Exchanges clearly represent a more transparent trading venue. Bessimbiner, Maxwell, and Venkataraman (2007), Edwards, Harris, and Piwowar (2007), and Goldstein, Hotchkiss, and Sirri (2007) study the costs of trade in corporate bonds, where increases in posttrade transparency appear to have lowered trading costs.
In this section, I model the secondary market as one with nonsequential search by retail investors. I will then show in Section III that the resulting demand functions can support an equilibrium in the primary market such as the one developed above. The arguments leading to equilibrium search and pricing strategies closely parallel models of consumer search and price dispersion, such as Varian (1980), Burdett and Judd (1983), and Nilsson (1999).

In all these models, firms compete in price space to sell goods that are produced at a constant cost. Consumers incur a cost initially that allows them to observe multiple prices, and they buy at the lowest observed price. They incur the cost before observing any prices, hence the search is “nonsequential.” The mixed-strategy equilibria that lead to price dispersion consist of a distribution from which firms choose their posted price, and probabilities that determine whether consumers observe multiple prices. Firms have some local monopoly power. Even if they quote a very high price, some consumers, who optimally choose not to search, will pay it. Firms are disciplined, however, by the equilibrium response of consumers and other firms. The higher the price they quote, the more likely they are to lose sophisticated shoppers to other firms. If their strategy places more weight on high prices, in equilibrium more consumers will invest in search costs.

Some issues regarding interpretation warrant discussion before proceeding. First, I use a model of nonsequential rather than sequential search. This seems appropriate in the institutional setting under study, where the costs retail-level investors face are less a matter of finding information about specific prices than they are a matter of set-up costs associated with learning about the market generally, so that one knows how and where to get such information inexpensively. For example, information about the reoffering yields of the new municipal bond issues studied by Green, Hollifield, and Schürhoff (2007b) is easily available in the financial press and through brokerage firms, if the investor knows where to look or knows to ask for it.

Second, I interpret the model in this section as one of competition, not just between dealers in a particular security, but also between dealers in the security that has been issued and close substitutes to it. Again, this seems appropriate to the institutional setting at hand. For example, the municipal bond market at a given point in time consists of thousands of bonds, which, because they are all insured by the same small number of bond insurance firms, are perfect substitutes in terms of credit risk. A dealer who prices bonds too aggressively risks losing sophisticated retail investors, not only to other dealers in that issue, but also to other dealers handling the many similar issues coming to market. Similarly, as more dealers price aggressively, investors have stronger incentives to incur the costs that enable them to search for good prices.

Interpreting the model in this way allows me to treat the shape of the demand function a dealer faces as independent of his allocation in a particular issue. What I wish to rule out is the possibility that by bidding successfully for an entire issue in the primary market, a dealer can become a perfect monopolist in dealing with retail investors. This would lead to existence problems in the game between dealers in the primary market. At any proposed equilibrium in
which the dealers share the issue, and fully utilize their retail capacity, one
dealer could capture the entire issue by raising his bid slightly, and then price
the issue as a monopolist in the secondary market. If the dealer is pricing the
issue as a monopolist, however, and earning positive profits, competing dealers
would underbid him in the primary market. Thus, there may be no equilibrium.
The notion that just by gaining full control of an issue a dealer is completely free
of price competition for sophisticated retail investors seems quite unrealistic in
the over-the-counter markets, where many close substitutes are available for a
given security.

Third, the traditional models of price dispersion consider multiple firms, each
posting a single price, but in a symmetric equilibrium these prices differ because
dealers follow mixed strategies. I will recast the model as one where dealers
encounter retail investors individually and quote each investor a price. The
price may be randomly determined by a mixed strategy. Dealers cannot a priori
discriminate between sophisticated and naïve retail investors, but different
retail investors may still receive different prices from the same dealer. Note that
this alternative specification does not alter the basic logic behind the arguments
and proofs. It seems more natural in the over-the-counter (OTC) market setting.
In a typical retail setting, the prices quoted to customers in a particular store
or advertisement are transparent to all customers who visit the store or read
the advertisement. In OTC markets, the dealings of a particular customer with
a dealer are not transparent to other customers.

I will focus in this section on symmetric equilibria, which considerably sim-
plifies the analysis. The model has two stages. First, I solve for equilibrium
dealer pricing and customer search strategies in the secondary market, assum-
ing $N$ dealers have symmetric retail distribution capacity and inventories of
securities that are perfect substitutes. I then show these quantities can be sup-
ported as equilibrium outcomes when two of the dealers bid for the issue in the
primary market as in the previous section, taking as given the demand function
they will face in the secondary market.

Suppose, then, that each dealer has preferential access to a set of retail cus-
tomers with measure $\mu$. Any inventory not sold to retail customers can be sold at
the institutional price of $v$. Dealers quoting the same price to a given customer
will attract him with equal probability. Because we consider only symmetric
equilibria, in equilibrium each dealer will earn an equal share of the aggregate
retail market.

The dealers come to the secondary market with $Q_i = Q_j = Q > \mu$. Once deal-
ers have purchased the securities from the issuer, the price they have paid is
sunk and the opportunity cost of selling them to retail investors at price $p$ is
the price they could have received in the institutional market, $v$.

Retail customers each face a cost, $c$, of acquiring pricing information in the
market. If they do not incur this cost, they observe only the price quote of the
dealer who has preferential access to them. If they pay the cost, they observe
the price quotes of all dealers and choose the lowest price. This cost can be
interpreted as a measure of market transparency. For example, decisions by
policy makers to require public posting of quotations in a central location, or
to provide low cost access to transactions prices, serve to decrease the cost of acquiring information about prices.

Retail investors can follow mixed strategies in deciding to acquire information. Let $q$ denote the probability a given investor does not pay the information cost. As long as a dealer prices the securities below $\bar{p}$, which we now interpret as a reservation price above which no retail investor will buy, he can attract a proportion $q$ of the mass of retail investors in his network. Whether he attracts the rest of his customers, who have measure $1 - q$, depends on whether the price he offers exceeds that offered by the other dealers.

Dealers can also follow mixed strategies over prices in the interval $[v, \bar{p}]$. Let $F(p)$ denote the distribution function of prices quoted by a dealer. Of course, $q$ is endogenous. If the dealer’s pricing strategy is too aggressive—$F(p)$ puts too much weight on high values of $p$—investors will respond by choosing a lower $q$ and becoming informed more frequently.

Consider, then, the possible outcomes when a dealer is approached by a retail investor, and the dealer quotes price $p$. A dealer will encounter all the unsophisticated investors from his own distribution network, who have mass $q \mu$, and also all the sophisticated investors from his own and other dealers’ networks, who have mass $(N + 1)(1 - q)\mu$. If the investor he encounters is unsophisticated, he makes a sale. If the investor is sophisticated, he makes a sale only if his price is the lowest quoted, which has probability $[1 - F(p)]^N$.

The expected profits, per customer encountered from quoting price $p$ to a customer are thus,

$$\Pi(p) = (p - v)\frac{q}{1 + N(1 - q)} + (p - v)(N + 1)(1 - q)\frac{1 - F(p)^N}{1 + N(1 - q)}.$$  \hspace{1cm} (17)

The next result relies on arguments that are standard in the literature on price dispersion for consumer goods. See, for example, Varian (1980) and Burdett and Judd (1983). The proof is deferred to the Appendix. The intuition behind the result is simple: If any price is chosen with positive point mass, then there is some positive probability that both dealers will offer the same price. Bertrand competition then leads them to offer slightly less.

**Lemma 1:** If $q \neq 1$, then

(a) $F(p)$ is continuous $[v, \bar{p}]$ or $F(p + \epsilon) = 1$ for all $\epsilon > 0$,

(b) the support of $F(p)$ is connected.

It is now straightforward to solve for $F(p)$. For this to be a symmetric mixed-strategy equilibrium, each price in the support of $F(p)$ must be equally profitable. Let $\Pi \equiv \Pi(p)$ denote this expected profit per customer encountered. At the upper bound in the price support, $\bar{p}, F(\bar{p}) = 1$ and the dealer is sure to lose all informed sales. This implies

$$\Pi \equiv \Pi(\bar{p}) = \frac{(\bar{p} - v)q}{1 + N(1 - q)}.$$  \hspace{1cm} (18)
Equating (17) and (18) yields
\[(\bar{p} - v)q = (p - v)q + (p - v)(N + 1)(1 - q)[1 - F(p)]^N, \tag{19}\]
which, upon simplification, gives
\[F(p) = 1 - \left[\frac{q}{(N + 1)(1 - q)} \left( \frac{\bar{p} - p}{p - v} \right) \right]^{\frac{1}{N}}. \tag{20}\]

Let \(p_m\) denote the lowest price in the support of \(F(p)\). Since at this price the dealer attracts all informed retail investors, it is defined implicitly by the condition
\[(\bar{p} - v)q = (p_m - v)q + (p_m - v)(N + 1)(1 - q), \tag{21}\]
or alternatively by the condition \(F(p_m) = 0\). Either condition gives \(p_m\) as a weighted average of the upper and lower bounds on the prices,
\[p_m = \frac{q}{q + (N + 1)(1 - q)} \bar{p} + \frac{(N + 1)(1 - q)}{q + (N + 1)(1 - q)} v. \tag{22}\]

Figure 2 illustrates how \(F(p)\) depends on investors’ search behavior, \(q\), and the competition between dealers, \(N\). It plots the density associated with \(F(p)\) for different values of \(q\), assuming the institutional price, \(v\), is 100, and the upper bound on retail valuations, \(\bar{p}\), is 103. When most retail investors search (lower \(q\)), the range of possible prices is larger, but relatively more mass is put on the extreme values in the range. The distribution becomes more distinctly bimodal. As fewer investors search, there is less of a peak in the distribution at the bottom end, but more even dispersion of prices within the range of equilibrium outcomes. Greater competition, in the form of a larger number of dealers, increases the bimodal shape of the density, much as more search by investors does, and it mitigates the impact more search has on the distribution of prices.

In an equilibrium, dealers’ choices of \(F(p)\), given \(q\), and retail investors’ choice of \(q\), given \(F(p)\), must be mutually consistent. There is always a monopoly price equilibrium. If no investors search, then dealers can all charge \(\bar{p}\). If all dealers charge \(\bar{p}\), it is in no investor’s interest to incur the costs of search.

The competitive outcome is not an equilibrium. If all dealers charge \(v\), it is in no investor’s interest to search, but in that case a dealer can raise the price to \(\bar{p}\) without losing any sales. Indeed, this argument rules out any pure-strategy equilibrium at a price less than the monopoly price.

There will also be dispersed price equilibria, as long as the cost of search, \(c\), is not too high. The expected gain from search for a retail investor is
\[G(q) \equiv E[p_i] - E[\min\{p_1, \ldots, p_{N+1}\}]
= \int_{p_m(q)}^{\bar{p}} p \, dF(p) - \int_{p_m(q)}^{\bar{p}} p(N + 1)[1 - F(p)]^N \, dF(p), \tag{23}\]
Figure 2. Price densities for differing investor search probabilities. The plot shows the probabilities with which dealers quote different prices in dispersed price equilibria, assuming \( v = 100 \) and \( \bar{p} = 103 \).
where the second term uses the standard expression for the distribution for the minimum.\(^2\) A dispersed price equilibrium will exist when there is a \( q, 0 < q < 1 \), such that \( G(q) = c \).

Evidently, in a symmetric mixed-strategy equilibrium, the expected value of a given price cannot be less than the expected value of the minimum price, and \( G(q) \) is weakly positive. At the two extreme values, the gains to search are zero. If dealer strategies reflect the expectation that all investors search, they will set prices competitively. For this case, equation (22) tells us \( p_m = v \) and (20) that \( F(p) = 1 \) for any \( p \). But with no price dispersion, it is not in the private interest of any agent to search. If no investors search, and \( q \to 1 \), then from (22) \( p_m = \bar{p} \), dealers all price monopolistically, and again there can be no gain to searching. So long as the gains to search are not identically zero for all values of \( q \), there will exist sufficiently small search costs for a dispersed price equilibrium to exist. This will be the case unless, for all \( q \), the dealers put unit mass on either \( p_m \) or \( \bar{p} \). To see this, integrate (23) by parts, noting that \( F(p_m) = 0 \) and \( F(\bar{p}) = 1 \), to obtain

\[
G(q) = \int_{p_m(q)}^{\bar{p}} p dF(p) - \int_{p_m(q)}^{\bar{p}} p(N + 1)[1 - F(p)]^N dF(p)
\]

\[
= \bar{p} - p_m - \int_{p_m(q)}^{\bar{p}} F(p) dp - \int_{p_m(q)}^{\bar{p}} [1 - F(p)]^{N+1} dp
\]

(24)

Unless \( p_m(q) = \bar{p} \), which requires \( q = 1, 0 < F(p) < 1 \) and the integrand in the last expression is strictly positive. As long as \( c \) is sufficiently low, there will be a \( q \) such that \( G(q) = c \), and we will have a dispersed price equilibrium.

### III. Primary Market Competition and Retail Price Dispersion

It is a relatively straightforward matter to combine the arguments of Sections I and II. Consider how dealers will bid in the primary market for the bonds in the issue, anticipating a dispersed price equilibrium in the secondary market. At this point, the profit on each bond purchased will be the difference between the price the investor pays and the price the dealer offers the issuer. (The price paid to the issuer is not yet “sunk.”)

Assume that two dealers, \( i \) and \( j \), are bidding for an issue of size \( K > 2\mu \). After purchasing the securities from the issuer, they will sell them in a secondary market where they compete with \( N + 1 \) dealers, all offering securities that are close substitutes for the issue in question. Our task is to show that \( Q_i = Q_j = K/2 \) and \( b_i = b_j = v \) are equilibrium outcomes in the primary market.

If dealer \( i \) bids for \( Q_i > \mu \), he will encounter \( q\mu \) unsophisticated retail customers from his own distribution network and will sell a security to each. He

\(^2\) See, for example, Degroot (1975), p. 132.
will also encounter sophisticated retail customers with measure \((N + 1)(1 - q)\mu\). Since all \(N + 1\) dealers are playing symmetric mixed strategies, they have equal chance of quoting the minimal price to any one sophisticated retail investor. Accordingly, they will each attract an equal share, \((1 - q)\mu\), of these customers. The total mass of customers the dealers each sell bonds to will be \(\mu\). The next lemma, which is proved in the Appendix, uses these facts to write the dealer's expected profit in a simple form.

**Lemma 2:** If \(Q_i > \mu\), then

\[
\pi(b_i, Q_i) = (\bar{p} - v)q\mu + (v - b_i)Q_i.
\]  

(25)

The main result of this section now follows almost directly from the expression above for dealer profits and the arguments in Sections I and II.

**Proposition 6:** For sufficiently low cost of search and \(K > 2\mu\), there is a symmetric equilibrium in which each dealer assumes half the issue at price \(b_i = b_j = v\) and earns profits \((\bar{p} - v)q\mu\).

**Proof:** Suppose that each dealer offers the issuer \(b_i = b_j = v\), and that each is allocated half the issue. By (25), profits for each dealer will be

\[
\pi \left( v, \frac{K}{2} \right) = (\bar{p} - v)q\mu,
\]  

(26)

which is positive. If the dealer bids \(v + \varepsilon, \varepsilon > 0\), and is allocated \(Q_i = K\), his profits are\(^3\)

\[
\pi (K, v) = (\bar{p} - v)q\mu - \varepsilon K.
\]  

(27)

If he bids less, he earns zero profit. Thus, bidding \(v\) is a best response. Q.E.D.

### IV. Conclusion

The model developed in this paper explains a number of anomalous facts and institutional features observed in new bond issues. Price dispersion is sustained in the secondary market as an endogenous response by dealers to search costs for retail investors. Limited retail distribution capacity leads to breakdowns of Bertrand competition in the primary market, so that new issues are underpriced in equilibrium. Dealer-underwriters maximize their surplus through syndicates: They bid the same price and coordinate internally to allocate inventory in a manner that efficiently utilizes their retail distribution capacity.

\(^3\) Note that this assumes, as discussed earlier, that the securities are not unique in the secondary market, so that the dealer cannot revert to monopoly pricing simply by controlling this particular issue. If he prices his inventory at \(\bar{p}\) he will lose informed retail investors to close substitutes offered by other dealers.
Issuers have no incentive to advocate for reforms to increase price transparency, or to participate in institutional innovations that would increase it. Increased transparency simply commoditizes the bonds they issue, reducing the surplus available jointly to issuers and underwriters.

While the model corresponds most closely to issues with retail participation in OTC markets, it may offer broader lessons. It illustrates how noncompetitive outcomes in one sphere of an intermediary’s activities can lead to, and help sustain, noncompetitive outcomes in other related activities. This approach may help us to better understand the boundaries of the firm in financial services, why certain services are bundled and others are not, and why rents appear to be so persistent and sustainable in this industry.

**Appendix**

**Proof of Lemma 1:** Suppose \( F(\cdot) \) has a discontinuity at \( \hat{p} \), where \( v < \hat{p} \leq \bar{p} \). If \( q \neq 1 \), then there is a strictly positive probability that sophisticated investors will observe price \( \hat{p} \) for all other dealers. If this happens, each dealer attracts the informed demand with equal probability, along with their uninformed demand of \( q \). Denote the probability that all other dealers quote the same price as \( \psi = [F(\hat{p}^+) - F(\hat{p}^-)]^N \). The expected profit to a dealer quoting \( \hat{p} \) is

\[
\Pi(\hat{p}) = (\hat{p} - v) \frac{q}{1 + N(1 - q)} + \frac{(N + 1)(1 - q)}{1 + N(1 - q)} [1 - F(\hat{p}^+)]^N (\hat{p} - v)
\]

\[
+ \frac{1}{N} \frac{(N + 1)(1 - q)}{1 + N(1 - q)} (\hat{p} - v).
\]

The expected profit to a dealer quoting \( \hat{p} - \varepsilon \) is

\[
\Pi(\hat{p} - \varepsilon) = (\hat{p} - \varepsilon - v) \frac{q}{1 + N(1 - q)} + \frac{(N + 1)(1 - q)}{1 + N(1 - q)} [1 - F(\hat{p}^+)]^N (\hat{p} - \varepsilon - v)
\]

\[
+ \frac{1}{N} \frac{(N + 1)(1 - q)}{1 + N(1 - q)} (\hat{p} - \varepsilon - v).
\]

The gain, therefore, from deviating from the proposed equilibrium is

\[
\Pi(\hat{p} - \varepsilon) - \Pi(\hat{p})
\]

\[
= -\varepsilon \left[ \frac{q}{1 + N(1 - q)} + \frac{(N + 1)(1 - q)}{1 + N(1 - q)} [1 - F(\hat{p}^+)]^N + \frac{1}{N} \frac{(N + 1)(1 - q)}{1 + N(1 - q)} \right]
\]

\[
+ \psi \left( 1 - \frac{1}{N} \right) \frac{(N + 1)(1 - q)}{1 + N(1 - q)} (\hat{p} - v),
\]

which must be positive for sufficiently small \( \varepsilon \). This proves the first claim, (a).

To prove (b), suppose that for \( v < \hat{p} < \bar{p} < \hat{p}, F(\hat{p}) = F(\bar{p}) \). The expected profit to a dealer quoting \( \hat{p} \) is

\[
\Pi(\hat{p}) = (\hat{p} - v) \frac{q}{1 + N(1 - q)} + \frac{(N + 1)(1 - q)}{1 + N(1 - q)} [1 - F(\hat{p})]^N (\hat{p} - v).
\]
By offering $\hat{p} + \epsilon$, for $\epsilon < (\bar{p} - \hat{p})$, the dealer increases expected profits on a sale without in any way altering the chances he loses sales to other dealers. This implies that $\hat{p}$ cannot be in the equilibrium support of $F(p)$, contradicting our assumption. Q.E.D.

Proof of Lemma 2: When dealers view the institutional price, $v$, as their cost per bond, expected profit per retail customer encountered is given by (18) and (19) as

$$\Pi = \frac{(\bar{p} - v)q}{1 + N(1 - q)}.$$  \hspace{1cm} (A5)

The total measure of retail customers encountered is $[1 + N(1 - q)]\mu$, so that total expected retail profits at cost $v$ are $(\bar{p} - v)q\mu$. Because investors are non-atomic and the law of large numbers holds, the expected and actual number of securities sold in the retail market in the second-stage game are the same. These have measure $\mu$, so the profits per security are just $(\bar{p} - v)q$, and the revenue per security is $(\bar{p} - v)q + v$.

In the primary market, therefore, the expected profit per security purchased by intermediary $h$ and sold in the retail market is

$$(\bar{p} - v)q + v - b_h$$  \hspace{1cm} (A6)

and the total profit for $Q_h > \mu$ will be

$$\pi(b_h, Q_h) = [(\bar{p} - v)q + v - b_h]\mu + (v - b_h)(Q_h - \mu).$$  \hspace{1cm} (A7)

This immediately simplifies to the desired expression. Q.E.D.

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