Comparing Guessing Games with Homogeneous and Heterogeneous Players: Experimental Results and a CHM Explanation*

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We investigate the decisions of individuals in simple and complex environments. As vehicle for our investigation we use a recent version of the Guessing Game first explored by Güth et al. (2002). We find that individuals in complex environments think more carefully before making the decisions. We rationalize our findings with the Cognitive Hierarchy Model (CHM) proposed by Camerer et al. (2002). We relate our findings to the emerging literature on the decision making of collective actors.

Keywords: Guessing Game, experiment, steps of reasoning, heterogeneity
Classification code: C72, C91, C92

* The authors would like to thank Colin Camerer, Dirk Engelmann, Werner Güth, Martin Kocher, and Matthias Sutter for helpful comments, Peter Košinár for his contributions in the early stages of this project, and math camps chief instructors Alexander Erdélyi and Peter Novotný for allowing us to run the experiments at the camps. We also appreciate the help of František Brázdik with this project. We are grateful to Bank Austria for providing the funds. All errors are ours.
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> Human rational behavior is shaped by a scissors whose two blades are the structure of task environments and the computational capabilities of the actor. (Simon 1990, p. 7)

1. Introduction

In the Guessing Game (Beauty-Contest Game) participants are asked to choose a number from a closed interval. The winner is the person who picks the number closest to a given proportion of the average of all chosen numbers, e.g., Nagel (1995). The simplicity and flexibility of this game, and the fact that it captures nicely the strategic interaction of actors in financial markets, has made it a frequent topic of experimental studies of depth of reasoning (Camerer 2003).

Güth et al. (2002) have investigated experimentally an interesting modification of the Guessing Game that allows individuals to make decisions in "homogeneous" (simple) and "heterogeneous" (complex) environments. For simplification, we call these individuals homogeneous and heterogeneous players, respectively.

There are persuasive theoretical arguments, and there is ample experimental evidence, that the heterogeneity of actors and the complexity of the environment are important. For example, economic theory predicts that collusion among firms is more difficult to sustain when firms are heterogeneous (Tirole 1992, p. 243). Experimentally, the importance of the heterogeneity of actors and/or the complexity of the environment has been demonstrated, for example, by Wilcox (1993), Wilcox (2004), Camerer et al. (2002), Palfrey and Prisbrey (1997), and Harrison and Johnson (2004).

We are interested in the interaction of the two blades of Simon's scissors. Specifically, we like to better understand whether a (relatively) complex environment triggers deeper thinking. Our working hypothesis (for which there is no theoretical rationale yet) suggests it does. The Guessing Game is an ideal vehicle for an experimental test for that conjecture.

Güth et al. (2002) also conjectured that players in heterogeneous groups guess closer to equilibrium than those in homogeneous groups. In their experiment they found, however, no evidence in support of this conjecture. In fact, they found that individuals in homogeneous environments converged faster than those in heterogeneous environments, contradicting their conjecture.

Replication is the hallmark of experimental research. It was therefore our first purpose to replicate an experimental result that both Güth et al. (2002) and the present authors found counterintuitive. Our results conform to their and our intuition that the heterogeneity of players should induce the players to consider each others' strategies more thoroughly. The second purpose of this paper is to rationalize our findings with the Cognitive Hierarchy Model (CHC) by Camerer et al. (2002) and thus to take a step towards a better understanding of decision making of heterogeneous actors.

The structure of the paper is as follows. In the next section we describe our Guessing Game treatments and the working hypothesis. In Section 3 we describe the Cognitive Hierarchy Model. In Section 4 we discuss the implementation and design of our experiment. In Section 5 we describe the experimental results that we use in Section 6 to rationalize our findings. Section 7 relates our work to an important emerging literature that Camerer (2003) has identified as one of his Top Ten Research Questions. Section 8 concludes.

2. The game and the working hypothesis

Let n (where $n > 2$) be the number of players participating in the game. Each player $i \in \{1, \ldots, n\}$ chooses a real number $s_i \in S_i = [0, 100]$. A choice s_i is a pure strategy and the interval $[0, 100]$ is the set of all possible strategies (guesses) for each player. For any strategy vector $s = (s_1, \ldots, s_n)$,

$$\overline{s} = \frac{1}{n} \sum_{i=1}^{n} s_i$$

denotes the average of numbers chosen by all players. Let the payoff function $u_i(s)$ be given by

$$u_i(s) = C - c|s_i - q_i \overline{s}|,$$

where $q_i \in (0, 1)$ is the proportion of $\overline{s}$ which determines player i's best guess.[1] If player i guesses exactly the number $q_i \overline{s}$, he receives the payoff C. If he does not, the amount c (where $c > 0$) is deducted from C for each unit by which the numbers s_i and $q_i \overline{s}$ differ.[2]

Following the advice of Davis and Holt (1993) to facilitate comparison with previous experiments, we use the boundary equilibrium parametrization from Güth et al. (2002). Specifically, we set $n = 4$, $C = 50$, and $c = 1$ and we study two treatments that we call HOM and HET. In the HOM treatment, all four players are given $q_i = 1/2$ (for $i = 1, 2, 3, 4$). We will call the players in that treatment homogeneous. In the HET treatment, two of the players are given $q_i = 1/3$ (for $i = 1, 2$) and two of the players are given $q_i = 2/3$ (for $i = 3, 4$). We will call the players in that treatment heterogeneous. As in the standard version of the Guessing Game, it is easy to prove that iterated elimination of strictly dominated strategies yields a unique equilibrium $s_i^* = 0$ in both treatments.

Güth et al. (2002) conjectured that heterogeneous players think harder about other players' behavior and hence their guesses are closer to the equilibrium. We find this an intuitive and persuasive conjecture that was, however not confirmed in their earlier experiment.

[1] This specification is different from the standard version of the Guessing Game, where only the player who is closest to $q_i \overline{s}$ wins a fixed payoff; see Nagel (1995). Our form of the Guessing Game is necessary to implement heterogenous environments.

[2] Here $|x|$ denotes the absolute value.

3. Cognitive hierarchy model (CHM)

Camerer et al. (2002) propose a Cognitive Hierarchy Model, where each player assumes his strategy is the most sophisticated. The CHM builds on a probability distribution $f(k)$ of players where k denotes the number of steps of thinking the player takes. Step 0 players randomize their guesses and do not assume anything about their opponents, step k players assume that they are the smartest and also assume that other player types are distributed according to some probability distribution on $\{0, 1, \ldots, k-1\}$. They furthermore assume that there are no other players capable of higher steps of thinking than $k-1$.

Camerer et al. (2002) require the distribution $f(k)$ to fulfill the following properties. It should be a discrete probability distribution and $f(k)$ should be decreasing when k is high enough (higher than some k_0). Both these properties are obviously satisfied if $f(k)/f(k-1)$ is proportional to $1/k$. Camerer et al. (2002) show that the only discrete distribution satisfying this assumption is the Poisson distribution with parameter τ, i.e., $f(k) = e^{-\tau}\tau^k/k!$. It is well-known that both mean and variance of this distribution are equal to τ. Thus, $f(k)$ is a parsimonious model of the distribution of depth of reasoning in a subject pool. The value of τ can be specified by additional assumptions[3] or estimated from data. Camerer et al. (2002) argue, that the Poisson distribution is able to fit data almost as well as more parametrized models but it is easier to compute and work with.

With increasing τ the prediction of the CHM for the Guessing Game converges to the Nash equilibrium because a higher τ means more players with higher order of thinking capabilities. Rational players do not play the Nash equilibrium, but they pick guesses according to their depth of reasoning. Below we estimate τ from first-round data.

From the CHM it follows that a higher average τ of players' steps of thinking leads to guesses that are closer to the Nash equilibrium. The CHM also suggests

[3] Using intuitive principles, the authors derive the golden ratio $\frac{1}{2}(1+\sqrt{5}) \approx 1.62$ as a plausible value of τ.

that for two games with players from the same population but different values of τ for these games, the one with higher value reveals more complex reasoning and deeper thinking. It follows that, if our working hypothesis is valid, the estimated parameter for the HET treatment should be higher than the one for the HOM treatment.

4. Design and implementation

The experiments were run in June 2002 at two camps for young mathematicians, organized by the students of the Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava, Slovakia. The participants of these camps were chosen according to their performance in two independent national correspondence competitions in mathematics.[4] Nearly all of them were students of secondary schools, aged 14–18; it was their first experience with experimental economics. None of the participants had taken a course on game theory before.

In both camps, we first ran an experiment with the 'Choose Game'[5] and paid all participants their payoffs. In one camp we then ran the HOM treatment with seven groups of four homogeneous players; in the other camp we ran the HET treatment with seven groups of four heterogeneous players.

At every camp, the participants were read the instructions (see Appendix A). The instructions specified that in each of five rounds the subjects would be matched randomly so as to yield 7 four-player groups. The groups were rematched in each

[4] These competitions were of comparable quality and reputation. Indeed almost identical numbers (84 and 86) of participants took part in them. 28 participants took part in both competitions, among them 7 participated in the first camp and 10 in the second camp. The intersection of the participants in both camps was empty. While, ideally, we would have liked to run both treatments in each camp, it was not possible.

[5] The Choose Game is a Guessing Game Redux. In its simplest form, players form groups of two or three and choose only between numbers 0,1 or 0,1,2. Results of this experiment are reported in Ortmann and Ostatnický (2004).

round. In the HET treatment, the subjects were told that in each round there would be two subjects of each type in each group.

After each round we collected the record sheets, calculated averages (using calculators) for each group, and also publicly gave information about all the averages. After the experiment, for each round two groups were randomly selected and earnings were paid out to every selected participant. This payment mode had been announced as a part of the instructions.

The maximum amount that participants could earn in each round was 50 SKK.[6] For every unit of difference from the target number they lost 1 SKK. However, they were informed in the instructions that the minimal outcome was 0 SKK. So the actual payoff function was

$$u_i(s) = \max\{0,\ 50 - |s_i - q_i\bar{s}|\}.$$

5. Results of the experiment

Table 1 in Appendix B shows average payoffs[7] and Table 2 in Appendix B shows means, standard deviations, medians, as well as the minimum and the maximum of guesses, separately for each round and each treatment at the math camps. Average payoffs increased continually with the number of rounds. The increase in average payoff was smallest for homogeneous players. The standard deviation of guesses, as well as the maximum of guesses, dropped substantially from round to round.

Figure 1 in Appendix B shows the distribution of first-round guesses.[8] Figure 2 in Appendix B shows the average guesses in the course of the experiment for

[6] The amount of 50 SKK was, according to the official exchange rate at the end of June 2002, about 1.13 EURO. In Slovakia, for 50 SKK, it is possible to buy 2–3 beers or 3 loaves of bread. Given the age of the participants, the payoffs were not insignificant.

[7] These are potential payoffs. As specified in Section 4 in each round we selected randomly two groups and paid them their earnings. We paid off 1802 SKK in the HET treatment and 1769 SKK in the HOM treatment.

[8] Note, that the total number of participants for the HOM treatment was as large as the total number of participants for each HET treatment.

both treatments. We split the asymmetric treatment data into two clusters, with $q_i = 1/3$ and $q_i = 2/3$. Clearly, all guesses converge to the equilibrium.

As to our hypothesis, namely that the heterogeneity of players induces guessing closer to the equilibrium, heterogeneous agents guessed on average closer to the equilibrium in each round[9] (Figure 2 in Appendix B). The only exception was the second round, where the guesses of players with $q_i = 2/3$ were a bit higher on average than those of homogeneous players. As expected, session averages for players with $q_i = 1/3$ were lower than averages for players with $q_i = 2/3$.

Our results confirm the hypothesis and contrast with the findings of Güth et al. (2002) who found that session averages with homogenous groups were significantly lower than average guesses in heterogeneous groups.

Comparing our results to those of Güth et al. (2002), we see also faster convergence to the equilibrium for our data. In our treatments, the averages were well below 5 in the fifth round while in Güth et al. (2002), the averages in the fifth round were approximately 2.5, 7, and 12 for players with q_i equal to 1/2, 1/3, and

[9] We tested our hypothesis using the Mann-Whitney test (one-tailed test with the null hypothesis that the distributions of guesses for both homogeneous and heterogeneous players are the same against the alternative that they differ and heterogeneous players had lower guesses) for each round. Starting from the second round, the distributions were significantly different and heterogeneous players' guesses were closer to the equilibrium (with $p < 0.1$ in round 2; $p < 0.05$ in round 3, and $p < 0.01$ in rounds 4 and 5). The detailed statistics are available at `http://home.cerge-ei.cz/kovac/papers/xp`.

2/3, respectively[10]. While the speed of convergence is an interesting issue, it was not our focus in this study. Hence we do not pursue the issue here further.

6. Application of the CHM

We apply the Cognitive Hierarchy Model to the first-round guesses to provide theoretical support for our hypothesis. Following Camerer et al. (2002), we assume that the steps of thinking are distributed according to the Poisson distribution with parameter τ (which is equal to its mean). We estimate the value of τ for each treatment separately.[11] We conclude that the estimated value of τ in the HET treatment is higher than the one in the HOM treatment. Because the subjects in both camps were comparable, higher value of τ means that those in the HET treatment think more carefully about their strategy confirming our hypothesis.

Specifically, we consider a grid of values of τ. For each τ we compute numerically

[10] This difference in speed of convergence may be due to one or several of the following four factors. First, the public information may have sped up the process of learning. Players could see what happened in the other groups and thus they could eliminate dominated strategies. Second, our participants were the best mathematical talents in the Slovak Republic for their age category. We can assume that their reasoning process, and process of learning, is quicker than is typical for subjects. (Using the Choose Game data from the experiment that preceded the one we report here, Ortmann and Ostatnický (2004) find that increasing mathematical ability results in more equilibrium play.) Third, the faster convergence may have been influenced by the Choose Game experiment. In this experiment, participants could see that the optimal strategy is to bid 0. Our participants may have been able to carry over the idea of iterated elimination of dominated strategies to the Guessing Game. Finally, the experiment was not computerized and the calculation of averages for participants took about 3–5 minutes. The subjects could use this time for deeper thinking about the game.

[11] In the HET we assume that the steps follow the same distribution, i.e., we apply only one value of τ for both types of players.

the strategies (guesses) of players resulting from various steps of thinking.[12] In this way we obtain for each τ a distribution of guesses which allows us to predict the mean and the standard deviation of guesses depending on τ. We then compare the prediction and our data to estimate τ, as explained below.

Let us denote $b_q(k)$ the strategy of a k-step player i with $q_i = q$ (where $k = 1, 2, \dots$).[13] The predicted strategies show several interesting patterns. The strategies of 1-step players are obviously independent on the distribution. In the HOM treatment we receive $b_{1/2}(1) = 21$; in the HET treatment $b_{1/3}(1) = 14$ and $b_{2/3}(1) = 30$. For each τ, the strategies are non-increasing in k and become stationary for a high enough k. Moreover, for a fixed k the strategies are also non-increasing in τ. For the values of τ in our grid we conjecture that they do not converge to zero as $k \to \infty$. This is intuitively clear, because it is obviously not optimal to play zero (Nash equilibrium), when there is a high probability of some of the opponents being a 1-step player. For example, for $\tau = 1.64$ we receive $b_{1/2}(2) = 9$, $b_{1/2}(3) = b_{1/2}(4) = 7$, $b_{1/2}(5) = b_{1/2}(6) = \cdots = 6$. Using these strategies we can compute statistics of the distribution of guesses.

To estimate τ in the HOM treatment we use the method of moments proposed by Camerer et al. (2002) and estimate τ up to two decimals. For our sample mean 20.28 (standard deviation 22.56), we obtain $\tau = 1.64$ (yielding mean 20.29 and standard deviation 20.26). Note that this value of τ is close to the golden ratio (approximately 1.62) derived theoretically by Camerer et al. (2002).

The estimation of τ in the HET treatment is more difficult. Because of the assumption that the distribution of players of both types is the same we receive two

[12] For the computations we use the actual payoff function as discussed in Section 4. We consider values of τ from a grid with difference 0.01 on the interval $[0, 4]$ and we compute the strategies of players up to 10-th step and for certain values of τ (of our interest) up to 100-th step. For comparison, the highest value of τ estimated by Camerer et al. (2002) is 3.7. The source code and the detailed results of the computations are available at `http://home.cerge-ei.cz/kovac/papers/xp`.

[13] Although we allowed the players to guess non-integers (up to two decimals), we restrict the theoretical analysis of the CHM model to integer strategies. Only 7 players (2 in HET, 5 in HOM) out of 56 submitted non-integers in the first round. Hence the restriction seems warranted.

means (one for the players with $q_i = 1/3$, the other for the players with $q_i = 2/3$) for each value of τ. As we are not able to fit the means of both types, we do not provide any ultimate estimation method, but rather offer four methods shown in Table 3 in Appendix B. Those are: $\text{MM}_{1/3}$, $\text{MM}_{2/3}$, MM, and LS which denote the minimization of $|m_{1/3} - d_{1/3}|$, $|m_{2/3} - d_{2/3}|$, $|(m_{1/3} + m_{2/3}) - (d_{1/3} + d_{2/3})|$, and $(m_{1/3} - d_{1/3})^2 + (m_{2/3} - d_{2/3})^2$, respectively (where $d_{1/3}, d_{2/3}$ denote the sample means; $m_{1/3}, m_{2/3}$ the theoretical means predicted by the CHM). For comparison we provide also the statistics for $\tau = 1.64$ estimated in the HOM treatment (last column of Table 3).

Our sample means exhibit a smaller spread $(20.05 - 17.39)$ than the predicted means (e.g., $22.32 - 15.11$ for method MM). Table 3 in Appendix B shows that methods $\text{MM}_{1/3}$ and $\text{MM}_{2/3}$ provide extreme values of τ which causes asymmetrically large error (e.g., in method $\text{MM}_{1/3}$ the error for the $q_i = 1/3$ group is 0.06, but for the $q_i = 2/3$ group the error is 4.69). Therefore, methods MM and LS are more suitable for estimation. They both yield a higher value of τ than we estimated for the HOM treatment. Comparing the data with the predicted means for $\tau = 1.64$ estimated in the HOM treatment the means are similar for the players with $q_i = 1/3$, but the mean in the data is lower for players with $q_i = 2/3$.

The above results show that the players in the HET treatment really do think more carefully about the game which supports our hypothesis and rationalizes our results.

7. A quick review of related literature

Recently an important piece of literature on the decision making of collective actors has emerged (Camerer 2003, p. 475). One key finding is that collective actors in guessing games converge faster to the Nash equilibrium than individual actors in a statistically significant manner (Kocher and Sutter 2005). As we do, these authors do not find a statistically significant difference in first-round choices although the mean and median of chosen numbers is consistently lower for collective actors (groups of these individuals) than it is for individuals.

Sutter (2004) finds furthermore that teams with four members outperform teams with two members and single persons significantly, whereas the latter two types of decision makers do not differ.

We thus see that groups of certain size are, like heterogeneous groups, more efficient in their convergence behavior. Interestingly, subjects in our groups were not allowed to communicate, therefore the increased efficiency that we found seems due to harder thinking, triggered by a more complex environment. In other words, the complexity of an environment to some extent seems able to substitute for group deliberation, at least in Guessing Games.

8. Conclusion

We have experimentally explored a recent version of the Guessing Game introduced by Güth et al. (2002). Our subjects were mathematically talented youths and the experiments confirmed our hypothesis that heterogeneous players guess closer to the equilibrium. This conclusion is supported by the data and further rationalized by the use of the Cognitive Hierarchy Model proposed by Camerer et al. (2002). We also find that heterogeneous players' guesses converge faster to the equilibrium. While it is unclear why our results do not replicate those by Güth et al. (2002), it would be interesting, indeed, to model more formally their persuasive intuition that seems to be confirmed by our data.

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Appendix

A. Instructions

Instructions were written in Slovak. Based on the instructions by Güth et al. (2002), we first created an English version which was later translated into Slovak. The instructions below are for heterogeneous players with $q = \frac{1}{3}$. The instructions for heterogeneous players with $q = \frac{2}{3}$ differ only in the last sentence of paragraph 4 and in the formal expression for the payoff. In the instructions for homogeneous players (i.e., $q = \frac{1}{2}$) the whole of paragraph 4 was replaced by: "The target number for you (and everyone else in your group) is one-half of the average of all 4 chosen numbers in your group." Additionally, the formal expression for the earnings contained $\frac{1}{2}$ instead of $\frac{1}{3}$.

Sample instructions

Welcome to our experiment and thank you for participating. From now on please stop talking to your neighbor(s). If you have a question, please raise your hand.

You will be randomly divided into groups of 4 persons. Each person in your group chooses a number between zero (0) and one hundred (100). Zero and 100 are also possible. It is not necessary to choose an integer. However, numbers with more than two decimals are excluded.

Your potential earnings depend on how close your chosen number is to a target number. The closer your chosen number is to the target number, the higher are your earnings.

Your group consists of two participants of type A and two participants of type B. Target numbers of type A and type B participants are different. If you are a type A, your target number is one-third of the average of all 4 numbers chosen in the group. If you are type B, your target number is two-thirds of the average of all 4 numbers chosen in the group. **You are type A**, so the target number in your case is **one-third of the average of all 4 chosen numbers** in your group.

The potential earnings in each round depend on the difference between your chosen number and the target number. If your chosen number in that round is identical with the target number, your earnings will be 50 crowns. If the two numbers differ, their distance will be deducted from the 50 crowns. Formally, your potential earnings per round are calculated as follows:

$$\text{earnings (per round)} = 50 - \left| x - \frac{1}{3}\,\text{average} \right| .$$

If your earnings are negative, we will treat them as zero.

The experiment will last 5 rounds. Groups are rematched in each round. (You can see from the Table at the bottom of these Instructions the number of the group to which you belong in a particular round.) In each round you will write the chosen number on one of the attached Record Sheets and we will collect it.

After each round, we will write on the "blackboard" the average of each group. We advise you to enter in the Table at the end of these Instructions your chosen number; we also urge you to keep track of the average in your group. We recommend that you calculate your earnings after each round (using the above formula).

After the experiment proper, we will collect these Instructions (and the Table) and then will draw randomly one quarter of all participants to pay them off. All earnings will be paid in cash and privately at the end of the experiment.

Round	**1**	**2**	**3**	**4**	**5**
Group					
Chosen number					
Average					
Earnings					

B. Tables and Figures

Table 1

Average payoffs in the experiment (in SKK)

Group	Round				
	1	2	3	4	5
$q_i = 2/3$	34.82	44.14	45.38	47.98	48.11
$q_i = 1/3$	38.41	42.04	45.57	48.21	49.10
$q_i = 1/2$	37.63	42.54	44.10	47.17	47.54

Table 2

Descriptive statistics for the experiment

Group	Statistics	Round				
		1	2	3	4	5
$q_i = 2/3$	mean	20.05	13	5.51	2.70	1.89
$(n = 14)$	std. dev.	22.46	14.08	7.32	4.54	4.14
	median	12.50	8.77	2.54	1.14	0
	min	0	0	0	0	0
	max	66	47	26	15	15
$q_i = 1/3$	mean	17.39	7.35	4.76	1.85	0.72
$(n = 14)$	std. dev.	13.67	6.22	3.65	1.98	1.61
	median	14.50	5.50	4	1.06	0.10
	min	0	1	0	0	0
	max	54.50	21	12	7	6
$q_i = 1/2$	mean	20.28	12.55	9.24	4.36	2.99
$(n = 28)$	std. dev.	22.56	9.70	8.04	4.15	4.77
	median	13.50	10	7.51	4	2
	min	0	0	0	0	0
	max	100	38	35	19.73	24

Table 3

Estimation of τ for the HET treatment

Method	DATA	$MM_{1/3}$	$MM_{2/3}$	MM	LS	HOM
τ		1.62	2.09	1.86	1.89	1.64
$q_i = 1/3$	17.39	17.45	13.27	15.11	14.86	17.11
	(13.67)	(21.00)	(17.59)	(19.23)	(19.01)	(20.92)
$q_i = 2/3$	20.05	24.74	20.07	22.32	22.03	24.53
	(22.46)	(19.92)	(17.53)	(18.63)	(18.47)	(19.81)

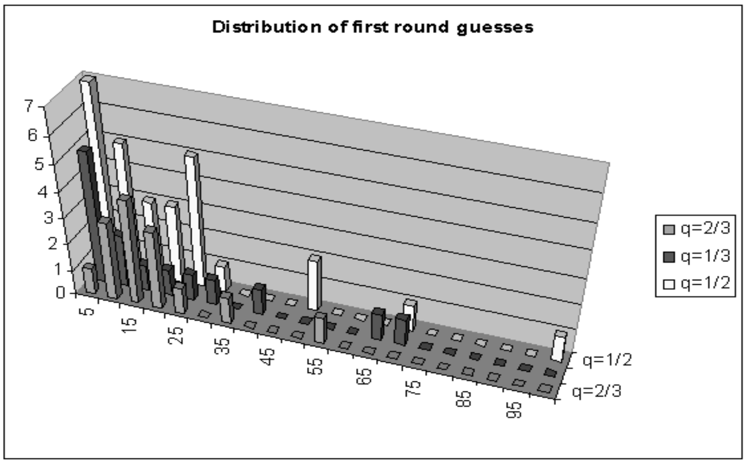

Fig. 1. *Distribution of first-round guesses*

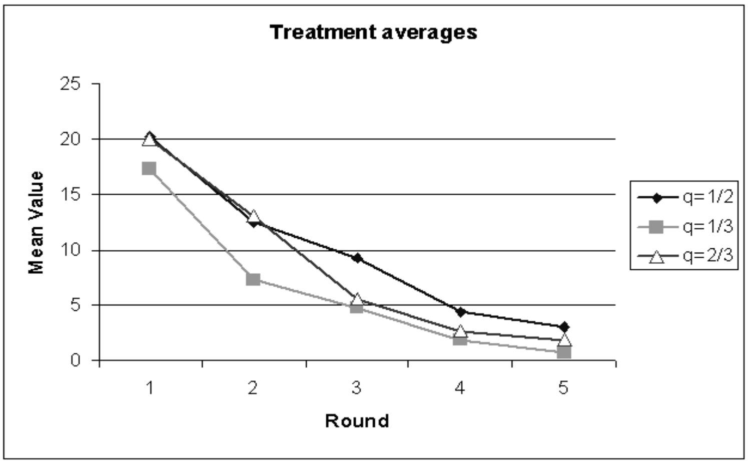

Fig. 2. *Treatment averages*