E-Learning: A Way to Solve the Human Capital Bootstrapping Problem in Transitional Economies in Central Europe?*

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Abstract

We propose a general equilibrium model of endogenous growth in which human capital investment is the engine of growth. Within that model we analyze the potential role of e-learning in the elimination of the human capital boostrapping problem so typical for transition economies. We find that e-learning can indeed speed up convergence to the frontier of knowledge. The intuitive idea behind this result is the ability of a transition economy to access external knowledge sources that do not require local teachers to first learn the requisite skills before they can teach them. Our results are derived in an economy with two education sectors – traditional public classroom education and private e-learning – which are modelled in a unified manner, with relative teacher quality and class size being the key variables. The endogenous class size creates a negative externality in the public education sector and is responsible for multiple equilibria: 'bad' or 'passive parents' equilibrium, 'good' or 'active parents' equilibrium, and corner 'zero e-learning trap'. We show that there are typically two transitions: 'catch-up transition' along good equilibria to balanced growth path equilibrium with sustained growth, and 'stagnation transition' along bad equilibria to autarchic zero e-learning equilibrium. If the economy is currently at a 'good' ('bad') equilibrium, then a pro-e-learning policy can have positive (negative) effects on the performance of the economy. Similarly, a pro-public education policy can have positive (negative) effects on the economy only if the economy is currently at a 'bad' ('good') equilibrium.

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1 Introduction

Jeong and Kejak [9] theoretically and empirically study the problem of "human capital mismatch" in Central European transition economies, i.e., the well-documented fact that the composition of the human capital stock inherited from socialism is not well matched with the requirements of a modern market economy. Their project is guided by two key questions: (1) what has been the effect of this human capital mismatch on the growth experience of four Central European economies that are scheduled to gain accession into the EU; and (2) how fast will the composition of the human capital stock and the levels of per capita income of these "accessing economies" converge to those of current EU countries.

Judson [11] analyzed the effect of human capital allocation on growth in a static and partial equilibrium framework. Kejak [12], in a related work, proposes a model of stages of growth of economic development which is based on the standard assumptions of such classic models as Lucas [10], Azariadis and Drazen [2], and Zilibotti [24]. Specifically, he assumes that there is a frontier of "theoretical knowledge" that is given exogenously and represents large advances in knowledge such as those produced by an industrial revolution. In contrast to Zilibotti's model, Kejak's model makes human capital the engine of growth. Furthermore, the growth engine does not have to be started through a structural one-off shock. Most importantly, and in contrast to most papers on endogenous growth, his model provides a mathematical analysis of both steady state(s) and transitional dynamics.

None of these models answers the question of how exactly the problem of human capital mismatch can be overcome. Specifically, none addresses the important bootstrapping problem that transition economies face by their very nature: In order to overcome the skills mismatch, new skills have to be taught by someone. However, teachers who are qualified to do the job typically do not exist locally. Local teachers first have to learn the requisite skills before they can teach them. In the following, we shall call teachers who have such skills 'teachers with superior knowledge'. Intuitively, bootstrapping makes it very unlikely that transition economies can catch up with the frontier of knowledge. Indeed, there are – we believe, justified – concerns that the theoretical frontier of knowledge and the actual frontier of knowledge currently drift apart ([5]; [13].

It is here where e-learning, which is typically web-based and hence not location-bound, comes in. The on-line delivery of content allows students to see, hear, and speak to professors and fellow students without ever having to leave their local learning center, or even home or office. No longer do students have to come to the university; the university comes to them. Clearly, this development is a significant paradigm shift [15].

In the present context, the most interesting aspect of e-learning is that it overcomes constraints on location. Hence, bootstrapping is no longer the only available option. It is widely agreed that e-learning has filled a void in the USA in that it has made accessible teachers with superior knowledge to people who otherwise would not have access to such knowledge ([1];[19]; see also [22];[23]

). So far the impact of e-learning has been most pronounced in postsecondary education, which has seen the emergence of a new breed of for-profit providers that uses a distinctly different business model (e.g., multiple locations similar to chainstores or franchisers rather than the single location model of traditional providers), often caters to a distinctly different clientele (adults rather than adolescents), and has pioneered both a distinctly different mode of authentication (certificates rather than degrees) as well as new delivery modes (e-learning both in addition to, and instead of, brick-and-mortar facilities) that make available the frontier of knowledge (e.g., [1]; [16],[17],[18]). [1] While e-learning has made inroads in other areas such as high schools, it is for the present purpose useful to think about e-learning as being an alternative delivery mode of post-secondary and/or continuing education. In the present paper we explore to what extent e-learning can play a similar, or possibly even greater, role in transition economies.

Our paper is structured as follows. In the following section 2 we present our overlapping generations model and analyze its equilibria. In section 3 we analyze the balanced growth path equilibria and study the effects of changes in tax rate, government spending on (telecommunications) infrastructure, and the growth rate of the frontier of knowledge. In section 4 we study the dynamics of two typical transitions: catch-up and stagnation. Section 5 is devoted to a study of the effect of changes in key government policies on the dynamic equilibria. Section 6 concludes the paper.

2 The Model

2.1 Overlapping Generations Economy

Consider an overlapping generations economy in which individuals live for two periods, when they are young and when they are old. When they are old, each individual gives birth to another individual, thus becoming a parent. To simplify our model, we normalize the size of the population to one, $N_t \equiv N \equiv 1$.

The old are equipped with human capital h_t that they acquired while young. They work, receive income Ah_t (where A defines a productivity parameter), consume and decide whether to invest in the education of their offspring.[2] In what follows investment in education is understood to be investment in private e-learning; attending traditional public schools is free.[3]

[1] Adelman [1] is an eminently readable sketch of the emerging "parallel universe of post-secondary credentials ... an education and training enterprise that is transnational and competency-based, confers certifications not degrees, and exists beyond governments' notice or control."

[2] Given that our emphasis is on post-secondary education, the assumption of parental decision making might sound a bit curious. We believe this simplification of our model to be inconsequential in the context of the explicit purpose of our study: understanding whether e-learning can overcome the societal human capital bootstrapping problem.

[3] This assumption describes reasonably well the empirical reality in the USA and other developed countries as well as transition economies in Central Europe.

Following the literature on human capital (e.g.,[7]; [6];[21]), we represent the CES preferences of individuals in their old age by $\frac{1}{1-\gamma}(c_t^{1-\gamma} + \beta h_{t+1}^{1-\gamma})$, where c_t denotes their consumption and h_{t+1} is the stock of human capital of their offspring with $\gamma > 0$ and $\beta > 0$. The young at t are endowed with one unit of time a fraction of which, θ_t, is spent on traditional public education and another fraction of which, $1 - \theta_t$, is spent on private e-learning.

Education Sectors The schooling technology for human capital we use is similar to the one in [21].[4] Our technology allows us to model the traditional public education sector and the private e-learning sector in a unified way[5]. Specifically, e-learning makes teachers with superior knowledge accessible that would otherwise not be within reach:

$$h_{t+1}^i = B^i s_t^i h_t \left[(C_t^i)^\mu (Q_t^i)^{1-\mu}\right]^\eta \tag{1}$$

where s_t^i is the fraction of time an offspring spends in education sector i, with $i = \{p, e\}$ specifying the public education sector and the e-learning sector, respectively. Clearly, $s_t^p = \theta_t$ and $s_t^e = (1 - \theta_t)$. We denote by $h_{t+1}^i$the stock of human capital acquired in sector i by a young person. Average class size in sector i is C_t^i and relative teacher quality in sector i is Q_t^i; B^i is the productivity parameter of sector i and $0 < \mu < 1$, $0 < \eta \leq 1$.

Average class size[6] in sector i is defined as the ratio of the number of students N_t^i to the number of teachers T_t^i:

$$C_t^i = \frac{N_t^i}{T_t^i}. \tag{2}$$

Average relative school quality or relative teacher quality in sector i is proxied by the ratio of teachers' human capital H_t^i to the average level of human capital:[7]

$$Q_t^i = \frac{H_t^i}{h_t}. \tag{3}$$

Public education sector Since we assume that there is no heterogeneity in human capital among the agents, teachers without superior knowledge have the same level of human capital as parents, $H_t^p = h_t$, and school quality is $Q_t^p = 1$.

[4]... which is different from the usual technology in the public education literature.

[5]Specifically, it allows for a trade-off between class size and teacher quality.

[6]It is useful to think of class size as a proxy for other school ingredients such as school buildings, equipment, textbooks, etc. All of these contribute to the educational experience, with teachers traditionally being the most important, and costly, component.

[7]This assumption is a simplification but justifiable as the majority of expenses of colleges and universities are for teachers whose pay in turn is a function of their qualifications. We believe this simplification of our model to be inconsequential in the context of the explicit purpose of our study: understanding whether e-learning can overcome the societal human capital bootstrapping problem.

Thus according to (2) the human capital, h^p_{t+1}, of an offspring who spends θ_t of his time in public schools is

$$h^p_{t+1} = B^p \theta_t h_t \left[\left(\frac{N^p_t}{T^p_t} \right)^{-\mu} \right]^{\eta} \tag{4}$$

where B^p captures the productivity of the public education sector and where h_t is the stock of the parental human capital.

E-Learning Sector We assume that students have their own teacher; hence the class size $C^e_t = 1$. [8] Following from (2), the human capital, h^e_{t+1}, of an offspring who spends $1 - \theta_t$ of his time on e-learning is

$$h^e_{t+1} = B^e (1 - \theta_t) h_t \left(Q^e_t \right)^{\eta(1-\mu)} \tag{5}$$

where the productivity of the e-learning sector[9] $B^e = \Psi g^{\phi}$ depends on government spending on the infrastructure, G_t, and $g \equiv \frac{G_t}{Ah_t}$ with $1 > \phi > 0$ and $\Psi > 0$. We assume teacher quality, H^e_t, to be a function of the parents' expenditures on e-learning, e_t, and, importantly, of the frontier of knowledge, $\bar{h}_t$, which is not available (domestically) from the public education sector but is accessible through e-learning, $H^e_t = \varphi(e_t, \bar{h}_t)$ with $\varphi_1, \varphi_2 > 0$, $\varphi_{11}, \varphi_{22} < 0$. In addition, we assume φ to be linearly homogenous in $e_t, \bar{h}_t$; thus, the quality of e-learning is $Q^e_t = \frac{\varphi(e_t, \bar{h}_t)}{h_t}$.

2.2 Equilibrium

An old individual's decision problem at time t is

$$\max_{c_t, e_t, \theta_t, h_{t+1}} \frac{1}{1-\gamma} \left[c_t^{1-\gamma} + \beta h_{t+1}^{1-\gamma} \right] \tag{6}$$

[8] This may seem like an odd assumption, especially in light of the long-running debate about classroom size effects (e.g.,[8]). Lazear, however, has recently provided an intriguing model of educational production that has as its simple but effective point of departure the insight that classroom learning has congestion effects. His model predicts a high correlation between the behavior of students and the student-teacher ratio: the better students behave, the higher the ratio.We note that Lazear's model applies to traditional classroom instruction where the disruptive behavior of a student imposes negative externalities on other students. Such disruptions are much less likely in postsecondary education mediated through e-learning because it has more private, and less public good characteristics. Specifically, the various congestion effects are much less if they exist at all. See also [14], footnote 5.

[9] E-learning requires some infrastructure such as high-speed and reliable internet connections, computers with certain minimum specs, etc. The productivity parameter B^e is meant to capture this infrastructure. Typically such infrastructure is subject to congestion effects. Following [3], we model the spending on infrastructure as relative to output. Röller and Waverman [20] provide persuasive empirical evidence for the impact of spending on (telecommunications) infrastructure.

s.t.

$$\begin{aligned}
c_t + e_t &= (1-\tau)Ah_t && (7)\\
h_{t+1} &= h^p_{t+1} + h^e_{t+1} && (8)\\
h^p_{t+1} &= B^p \theta_t h_t \left(C^p_t\right)^{-\mu\eta} && (9)\\
h^e_{t+1} &= \Psi g_t^{\phi}(1-\theta_t) h_t \left(Q^e(e_t, \bar{h}_t)\right)^{\eta(1-\mu)} && (10)
\end{aligned}$$

Equation (7) expresses the budget constraint of parents who earn after-tax income $(1-\tau)Ah_t$, invest in the private education of their children[10], e_t, and consume the rest, c_t. The total human capital of an offspring, h_{t+1}, is the sum of the human capital acquired in the public education sector, h^p_{t+1}, and through e-learning, h^e_{t+1}, as summarized in equation (8). The remaining two constraints (9) and (10) specify the production of an offspring's human capital in the public education and e-learning sectors, respectively. To simplify matters, we assume that the government runs a balanced budget, uses the tax revenues to finance public schools, E_t, and invests in the e-learning infrastructure.

$$\tau A h_t = E_t + G_t. \tag{11}$$

The first-order conditions of this constrained maximization problem are

$$\begin{aligned}
c_t^{-\gamma} &= \lambda_{1,t} && (12)\\
\lambda_{4,t}\eta(1-\mu)B^e(1-\theta_t)h_t \left(Q^e_t\right)^{\eta(1-\mu)-1}\frac{\partial Q^e_t}{\partial e_t} &= \lambda_{1,t} && (13)\\
-\lambda_{3,t}B^p h_t (C^p_t)^{-\mu\eta} + \lambda_{4,t}B^e h_t \left(Q^e_t\right)^{\eta(1-\mu)} &= 0 && (14)\\
\lambda_{2,t} = \lambda_{3,t} = \lambda_{4,t} &= \beta h_{t+1}^{-\gamma} && (15)
\end{aligned}$$

where $\lambda_{j,t}$ is the shadow price of the jth constraint and $j = 1, ..., 4$.
Using these constraints we get the following results:

$$\begin{aligned}
\left(\frac{h_{t+1}}{c_t}\right)^{\gamma} &= \beta\eta(1-\mu)B^e(1-\theta_t)h_t \left(Q^e_t\right)^{\eta(1-\mu)-1}\frac{\partial Q^e_t}{\partial e_t} && (16)\\
B^p (C^p_t)^{-\mu\eta} &= B^e \left(Q^e_t\right)^{\eta(1-\mu)} && (17)\\
h_{t+1} &= B^p h_t \left(C^p_t\right)^{-\eta\mu} && (18)
\end{aligned}$$

Equation (16) shows that in equilibrium the marginal product of the parents' investment in the private education of their offspring ought to equal the

[10]In an extended model we analyzed a version of the model with government subsidies to private education, σe_t, where $0 < \sigma < 1$ is the subsidy rate. However such a modification makes the analysis of the government policies very cumbersome with similar main qualitative results. We can still contemplate a simple analysis of government subsidies proportional to household income in the current model. Such subsidies will simply offset the taxes, and the tax rate τ should then be viewed as the tax rate net of the subsidy rate.

marginal rate of substitution between the parents' consumption and their offspring's human capital. Equation (17) postulates that the marginal products of time spent on public education and e-learning ought to be equal. Since the human capital acquired in the two sectors are perfect substitutes, and since the productions of these human capitals are linear in the time spent on education, the growth rate of total human capital is equal to the marginal product of time spent in the education sectors, as stated in equation (18).

In equilibrium, the normalized number of students in the public education sector is equal to the share of time spent in the public sector, $N_t^p = \theta_t$. Assuming that all government spending on public education E_t is used to pay teachers, the number of teachers is $T_t^p = \tau - g$.[11] The class size in the public sector is then

$$C_t^p = \frac{\theta_t}{T_t^p}. \tag{19}$$

We furthermore assume that teacher quality in the e-learning sector is captured by the following specification of φ,

$$H_t^e = \varphi(e_t, \bar{h}_t) = \bar{h}_t^{\alpha} e_t^{1-\alpha}. \tag{20}$$

Substituting equations (19) and (20) into (17), we obtain the following expression for the share of time spent in the public education sector:

$$\theta_t = (\tau - g)\left(\frac{B^p}{\Psi g^{\phi}}\right)^{\frac{1}{\eta\mu}} \left(\frac{\bar{h}_t^{\alpha} e_t^{1-\alpha}}{h_t}\right)^{-\frac{1-\mu}{\mu}}. \tag{21}$$

This equation specifies θ_t and $\frac{e_t}{h_t}$, which satisfy the equilibrium condition (17) that the marginal product of time spent in public education is equal to that spent in the e-learning sector.

Substituting (3), (20), (17), and (18) together with (7) into (**??**) we obtain

$$\theta_t = 1 - \Gamma^{-1}\left[(1-\tau)A\left(\frac{e_t}{h_t}\right)^{-1} - 1\right]^{-1} \tag{22}$$

where we assumed $\gamma = 1$ and $\Gamma \equiv \beta\eta(1-\mu)(1-\alpha)$.

This equation provides us with θ_t and $\frac{e_t}{h_t}$, which satisfy the optimality condition (16) that the marginal product of the investment in private education is equal to the marginal rate of substitution between consumption and the offspring's human capital.

The equilibrium of the economy is a set of sequences $\{c_t\}_{t=0}^{\infty}$, $\{\theta_t\}_{t=0,}^{\infty}\{e_t\}_{t=0}^{\infty}$, and $\{h_{t+1}\}_{t=0}^{\infty}$ such that (i) $c_{t,}$ θ_t, and e_t are the optimal choices of a parent with human capital h_t at time t, who takes as given the government policies (τ, g), and the external frontier of knowledge $\bar{h}_t$ growing exogenously at the rate $\Omega > 1$, i.e. $\bar{h}_t = \bar{h}_0\Omega^t$, (ii) the human capital of offspring, h_{t+1}, is composed of a mix of human capital acquired either in the public education sector or the e-learning sector according to equations (9) and (10).

[11] The number of teachers is $T_t^p = \frac{E_t}{Ah_t}$.

2.3 Multiple Equilibria

The equilibrium values of θ_t and $\frac{e_t}{h_t}$ have to satisfy equations (21) and (22). In Proposition 3 we show that there can be three possible cases:

1. two interior equilibria: low growth and high growth, as well as zero e-learning equilibrium,
2. one interior equilibrium and one zero e-learning equilibrium,
3. one zero e-learning equilibrium.

Proposition 1 (Equality of MPs of Time) *For a given level of the human capital gap, $\frac{\bar{h}_t}{h_t}$, the time spent in the public education sector θ_t is according to (21) a function of $\frac{e_t}{h_t}$, i.e. $\theta_t = f_I(\frac{e_t}{h_t})$. The function is downward sloping, $f_I' < 0$ and convex, $f_I'' > 0$. Additionally, a bigger gap, $\frac{\bar{h}_t}{h_t}$,* ceteris paribus *makes e-learning more attractive and thus implies less time spent in the public education sector, $\frac{\partial f_I}{\partial\left(\frac{\bar{h}_t}{h_t}\right)} < 0$. Condition (21) holds for the interior equilibria $\frac{e_t}{h_t} \in [\widehat{\frac{e_t}{h_t}}, \infty)$ where $\widehat{\frac{e_t}{h_t}} = \left((\tau - g)\left(\frac{B^p}{\Psi g^\phi}\right)^{\frac{1}{\eta\mu}}\right)^{\frac{\mu}{(1-\alpha)(1-\mu)}} \left(\frac{\bar{h}_t}{h_t}\right)^{-\frac{\alpha}{1-\alpha}}$ and* $\lim_{\frac{e_t}{h_t}\to\infty} f_I(\frac{e_t}{h_t}) = 0$. *At the corner equilibria, $\frac{e_t}{h_t} \in [0, \widehat{\frac{e_t}{h_t}})$, the marginal product in the public education sector is larger than that of the e-learning sector and no time is spent in the e-learning sector, hence $\theta_t = 1$.*

Proof. By taking a derivative of θ_t from (21) with respect to e_t/h_t we obtain $f_I' = -\frac{(1-\alpha)(1-\mu)}{\mu}(\tau - g)\left(\frac{B^p}{\Psi g^\phi}\right)^{\frac{1}{\eta\mu}}\left(\frac{\bar{h}_t}{h_t}\right)^{\frac{-\alpha(1-\mu)}{\mu}}\left(\frac{e_t}{h_t}\right)^{-\frac{(1-\alpha)(1-\mu)}{\mu}-1} < 0$ and $f_I'' = \frac{(1-\alpha)(1-\mu)}{\mu}\left(\frac{(1-\alpha)(1-\mu)}{\mu} + 1\right)(\tau - g)\left(\frac{B^p}{\Psi g^\phi}\right)^{\frac{1}{\eta\mu}}\left(\frac{\bar{h}_t}{h_t}\right)^{-\alpha\frac{1-\mu}{\mu}}\left(\frac{e_t}{h_t}\right)^{-(1-\alpha)\frac{1-\mu}{\mu}-1} > 0$. Furthermore, $\frac{\partial f_I}{\partial\left(\frac{\bar{h}_t}{h_t}\right)} = -\frac{\alpha(1-\mu)}{\mu}\frac{f_I}{\left(\frac{\bar{h}_t}{h_t}\right)} < 0$ ■

Proposition 2 (MRS equals MP of expenditures on e-learning) *Time spent in the public education sector, θ_t, is according to condition (22) a function of $\frac{e_t}{h_t}$, i.e. $\theta_t = f_{II}(\frac{e_t}{h_t})$. The function f_{II} is downward sloping, $f_{II}' < 0$, and concave, $f_{II}'' < 0$. If all time is spent in the public education sector, $\theta_t = 1$, then spending on e-learning is zero and $f(0) = 1$.*

Proof. By taking a derivative of θ_t from (22) with respect to e_t/h_t we obtain $f_{II}' = -\Gamma^{-1}\frac{(1-\tau)A\left(\frac{e_t}{h_t}\right)^{-2}}{\left((1-\tau)A\left(\frac{e_t}{h_t}\right)^{-1}-1\right)^2} < 0$ and $f_{II}'' = -\Gamma^{-1}\frac{(1-\tau)A\left(\frac{e_t}{h_t}\right)^{-2}}{\left((1-\tau)A\left(\frac{e_t}{h_t}\right)^{-1}-1\right)^3}2\times \left(\left(\frac{e_t}{h_t}\right)^{-1}\left((1-\tau)A\left(\frac{e_t}{h_t}\right)^{-1} - 1\right) - 1\right) < 0$. Furthermore, $\frac{\partial f_I}{\partial\left(\frac{\bar{h}_t}{h_t}\right)} = 0$. ■

Any interior equilibrium has to satisfy the standard general equilibrium condition that the marginal rate of transformation (MRT) of time between the public and private education sector (or the ratio of the marginal products of time) has to be equal to the marginal rate of substitution (MRS) between the consumption of the old and the investment in the human capital of the young (or the ratios of the marginal utilities of consumption and human capital, respectively). As the preceding propositions show, both conditions can be expressed as relations between time spent in the public education sector, θ_t, and e-learning expenditures, e_t/h_t. In graphical terms, the interior equilibria are demonstrated in Figure 1 as the cross-sections of the MRT curve, f_I, and the MRS curve, f_{II}. As Proposition 3 states, there is a multiplicity of simultaneous equilibria in our economy (i.e. strategic complementarities in the sense of Cooper and John,[4], exist[12]): Suppose that there is a drop in public school attendance θ_t. This will increase the productivity of the public education sector because it decreases class size, resulting in higher human capital of children, h_{t+1}. However, higher human capital allows parents, who care about it in their preferences, to reduce consumption and increase spending on private education, e_t/h_t. This increases productivity in the e-learning sector and thus attracts people to spend more time there. Thus the decline in θ_t can be self-sustained.

The three possible scenarios of two, one, and no interior equilibrium are specified below and shown, together with the corner equilibrium, N, in Figure 1.

Proposition 3 (Multiple General Equilibria) *For every economy there exists a level of the gap,* $\widehat{\left(\frac{\bar{h}_t}{h_t}\right)}$*, such that:*

1. *There exist two interior general equilibria and one corner general equilibrium:* Low Growth equilibrium $\left(\theta_t^1, \left(\frac{e_t}{h_t}\right)^1\right)$, High Growth equilibrium $\left(\theta_t^2, \left(\frac{e_t}{h_t}\right)^2\right)$, *and* Zero E-learning equilibrium $\left(\theta_t^3, \left(\frac{e_t}{h_t}\right)^3\right) = (1,0)$ *where* $1 > \theta_t^1 > \theta_t^2$ *and* $0 < \left(\frac{e_t}{h_t}\right)^1 < \left(\frac{e_t}{h_t}\right)^2$ *when* $\left(\frac{\bar{h}_t}{h_t}\right) > \widehat{\left(\frac{\bar{h}_t}{h_t}\right)}$*; further* $\omega_t^3 < \omega_t^1 < \omega_t^2$ *where* ω_t^i *is the growth rate at equilibrium* $i \in \{1,2,3\}$.

2. *There exist one interior general equilibrium* $\left(\theta_t^1, \left(\frac{e_t}{h_t}\right)^1\right)$, *and one corner equilibrium with* Zero E-learning $\left(\theta_t^2, \left(\frac{e_t}{h_t}\right)^2\right) = (1,0)$ *where* $1 > \theta_t^1$ *and* $0 < \left(\frac{e_t}{h_t}\right)^1$ *when* $\left(\frac{\bar{h}_t}{h_t}\right) = \widehat{\left(\frac{\bar{h}_t}{h_t}\right)}$*; further* $\omega_t^2 < \omega_t^1$ *where* ω_t^i *is the growth rate at equilibrium* $i \in \{1,2\}$.

3. *There exists only one corner general equilibrium with* Zero E-learning $\left(\theta_t^1, \left(\frac{e_t}{h_t}\right)^1\right) = (1,0)$ *when* $\left(\frac{\bar{h}_t}{h_t}\right) < \widehat{\left(\frac{\bar{h}_t}{h_t}\right)}$.

[12]In our model these complementarities are caused by the negative externalities of class size in the public education sector. After the removal of endogenous class size the interior indeterminacy disappears. However, the corner solution will still be possible.

Proof. Since according to Proposition 1 the MRT locus depends negatively on the knowledge gap $\left(\frac{\bar{h}_t}{h_t}\right)$, $\frac{\partial f_I}{\partial\left(\frac{\bar{h}_t}{h_t}\right)} < 0$, and f_I is convex and f_{II} is concave, there is a unique critical level of $\widehat{\left(\frac{\bar{h}_t}{h_t}\right)}$ such that there is only one interior equilibrium as in 2 in the above Proposition. Then for any $\left(\frac{\bar{h}_t}{h_t}\right) > \widehat{\left(\frac{\bar{h}_t}{h_t}\right)}$ there will be two interior equilibria and for $\left(\frac{\bar{h}_t}{h_t}\right) < \widehat{\left(\frac{\bar{h}_t}{h_t}\right)}$ there will be no interior equilibria. ∎

One can think of the *Low Growth (High Growth)* equilibrium as *'Bad' ('Good')* or *'Passive Parents' ('Active Parents')* equilibrium with high (low) current consumption and low (high) human capital of their offspring.

3 Balanced Growth Path

Along the balanced growth path (BGP) the model variables $\left(\frac{c}{h}\right)_*$, $\left(\frac{e}{h}\right)_*$, θ_*, and $\omega_* = \left(\frac{h_{t+1}}{h_t}\right)_*$ stay constant. There are two possible BGP equilibria[13]: *an interior BGP equilibrium with e-learning* and *a corner equilibrium without.*

3.1 Interior BGP

On the interior balanced growth path, human capital in the public education sector, e-learning sector, their sum, and the external frontier of knowledge all have to grow at the same rate $\Omega = \omega_*$. Thus from equation (18), the interior BGP share of the public education sector $0 < \theta_* < 1$ has to satisfy

$$\theta_* = \left(\frac{B^p}{\Omega}\right)^{\frac{1}{\mu\eta}} (\tau - g). \tag{23}$$

Using equation (17) the BGP relative quality of e-learning is

$$Q_* = \left(\frac{\Omega}{B^e}\right)^{\frac{1}{\eta(1-\mu)}}. \tag{24}$$

For an economy to insure the existence of a unique interior BGP, several conditions have to be simultaneously satisfied: the condition of the equality of marginal products (21) and the condition of the equality between the marginal rate of substitution and the marginal rate of transformation (22) such that there is only one interior equilibrium (see Proposition 3). This implies that the slopes of MRS and MRT have to be equal. Moreover, the condition that the BGP rate

[13] Technically, there is also the possibility of perpetual oscillating between interior and corner equilibria. However, these unrealistic oscillatory equilibria are the outcome of our simplified version of the overlapping generations model. They could be removed by extending the set of decision variables of agents by including variables from more periods. In the current model they can be removed by the proper calibration.

equals the external growth (23) must be fulfilled as well. The above conditions impose a constraint on the model's parameters as specified in the following proposition.

Proposition 4 (Benchmark Interior BGP) *If the model parameters satisfy the condition*

$$\beta(1-\alpha)(1-\mu) = \frac{\theta_*(1+\Lambda_*) - 1 + \sqrt{(\theta_*(1+\Lambda_*)-1)^2 + 4\Lambda_*(1-\theta_*)}}{2\Lambda_*(1-\theta_*)} \quad (25)$$

where $\Lambda_* \equiv \frac{1-\mu}{\mu}(1-\alpha)\theta_*$ *and* θ_* *is given by (23), then there exists a unique (non-oscillatory) interior BGP equilibrium with the growth rate* $\omega_* = \Omega$*. Moreover, the e-learning expenditures, and the level of knowledge gap are given by*

$$\left(\frac{e}{h}\right)_* = (1-\tau)A\left(1 + \frac{1}{\Gamma(1-\theta_*)}\right)^{-1}, \quad (26)$$

$$\left(\frac{\bar{h}}{h}\right)_* = \left(\frac{\Omega}{\Psi g^{\phi}}\right)^{\frac{1}{\alpha\eta(1-\mu)}} \left(\frac{e}{h}\right)_*^{-\frac{1-\alpha}{\alpha}}, \quad (27)$$

respectively.

Proof. For such an equilibrium to exist it has to satisfy simultaneously two equilibrium conditions (21)-(22), $(\frac{e}{h})_* = (f_I)^{-1}(\theta_*) = (f_{II})^{-1}(\theta_*)$, and the condition for the BGP rate equal to Ω given by (23). The condition for the uniqueness additionally requires that the slopes of the MRS curve (22) and the MRT curve (21) are the same: $f_I'((\frac{e}{h})_*) = f_{II}'((\frac{e}{h})_*)$. This leads to a quadratic equation for Γ: $\Gamma^2\Lambda_*(1-\theta_*) + \Gamma\left((1+\Lambda_*)\theta - 1\right) - 1 = 0$ for which the positive root given in (25). Γ also has to satisfy its definition $\Gamma = \beta(1-\alpha)(1-\mu)$. ■

3.2 Autarchic BGP Equilibrium

The return on the time invested in e-learning is smaller than the return in the public education sector if the first-order condition (14) is not binding and thus $\theta_t = 1$. This means that no one is acquiring human capital in the e-learning sector. Such an economy is insulated from the frontier of knowledge and is hence restrained in its growth rate $\omega_* = B^p(\tau - g)^{\eta\mu} < \Omega$. Such an economy – by cutting itself off from the possibility of convergence to Ω – actually diverges from frontier economies.

3.3 Comparative Statics

In this section we analyze the comparative statics of the interior BGP equilibrium. The following discussion is based on our analytic results for the special case of $\gamma = 1$. We first analyze the effects of changes in tax rate, τ, and infrastructure expenditures, g. Then we analyze the effects of changes in the growth rate of the frontier of knowledge, Ω.

Effects of the tax rate, τ The effects of the tax rate on the model variables are summarized in Proposition 5. First, we analyze the effect of the tax rate on time spent in the public education sector, θ_*. We find that a higher tax rate (which translates into higher government expenditures on the public education sector and means more teachers and hence smaller class size) makes the public education sector more productive and thus entices more people to study there (see 28).

Second, we analyze the effect of the tax rate on expenditures on e-learning, $\left(\frac{e}{h}\right)_*$. We find that a higher tax rate leads to a decline in expenditures on e-learning (see 29). This result is the sum of two effects. The first term captures the direct negative effect of lower after-tax income. The second term captures the indirect negative effect of the lower marginal product of expenditures on e-learning, which results from an improved public education sector (due to more teachers and hence smaller class size) and which therefore entices more people to study there (recall : $\frac{\partial\theta_*}{\partial\tau} > 0$).

Third, we analyze the effect of the tax rate on consumption in after-tax income, $\frac{c}{y}$ (see 30). Again there is a direct and indirect effect. The direct negative effect results from lower after-tax income. The indirect positive effect results from the negative effect of the tax rate on the expenditures in e-learning. Thus, there is more income that can be consumed.

Fourth, we analyze the effect of the tax rate on the catch-up factor, $\left(\frac{\bar{h}}{h}\right)_*$ (see 31), defined as the ratio of the frontier of knowledge to the level of current human capital. A higher tax rate translates into a larger gap between frontier of knowledge and current human capital due to smaller expenditures on e-learning and hence a lower level of catch-up (or, equivalently, a higher catch-up factor).

Proposition 5 (Effects of the Tax Rate τ) *The effects of the tax rate on the unique interior BGP equilibrium values of θ_*, $\left(\frac{e}{h}\right)_*$, $\left(\frac{c}{y}\right)_*$, and $\left(\frac{\bar{h}}{h}\right)_*$ are given by*

$$\frac{\partial\theta_*}{\partial\tau} = \left(\frac{B^p}{\Omega}\right)^{\frac{1}{\eta\mu}} > 0, \tag{28}$$

$$\frac{\partial\left(\frac{e}{h}\right)_*}{\partial\tau} = -\frac{A}{1+1/\left(\Gamma(1-\theta_*)\right)} - \frac{\left(\frac{e}{h}\right)_*}{1+1/\left(\Gamma(1-\theta_*)\right)}\frac{1}{(1-\theta_*)^2}\frac{\partial\theta_*}{\partial\tau} < 0, \tag{29}$$

$$\frac{\partial\left(\frac{c}{y}\right)_*}{\partial\tau} = -\frac{1}{(1-\tau)^2 A}\left(\frac{e}{h}\right)_* - \frac{1}{(1-\tau)A}\frac{\partial\left(\frac{e}{h}\right)_*}{\partial\tau}, \tag{30}$$

$$\frac{\partial\left(\frac{\bar{h}}{h}\right)_*}{\partial\tau} = -\frac{1-\alpha}{\alpha}\left(\frac{e}{h}\right)_*^{-\frac{1}{\alpha}}\frac{\partial\left(\frac{e}{h}\right)_*}{\partial\tau} > 0, \tag{31}$$

respectively.

Proof. The proof is straightforward. ■

Effects of Infrastructure Government Spending, g The effects of the government's spending on infrastructure, and especially the spending on e-learning related infrastructure, on the model variables are summarized in Proposition 6. First, an increase in government spending on infrastructure crowds out spending on the public education sector and thus reduces the number of teachers and increases class size in it. Both these effects increase the productivity of e-learning and thus attract more students; θ declines. See (32)

Second, we analyze the effect on e-learning of the government's spending on infrastructure, $\left(\frac{e}{h}\right)_*$. This effect is indirect and works its magic through the number of students in the e-learning sector. Specifically, we find that this effect is positive. The intuition is that higher government spending on infrastructure causes a decline in the steady state level of number of students in the public education sector to keep the BGP rate unchanged. However, this leads to smaller class size and hence also to higher productivity in the public education sector. To satisfy the MRT condition, the productivity of e-learning has to increase as well and thus spending on e-learning will increase. See (33).

Third, we analyze the effect on the share of consumption in after-tax income, $\frac{c}{y}$, of the government's spending on infrastructure. We find that higher expenditures on e-learning come at the cost of lower consumption. See (34).

Fourth, we analyze the effect of the government's spending on infrastructure on the catch-up factor, $\left(\frac{\bar{h}}{h}\right)_*$. We find that there are two negative effects. Both of these effects work through the increase in productivity in the e-learning sector since the higher productivity of this education sector allows the economy to catch up with the higher level of human capital, thus decreasing the knowlede gap. The increase in productivity is caused directly by the government's spending and indirectly by higher private spending on e-learning. See (35).

Proposition 6 (Effects of the Share of Government Infrastructure Spending g**)** *The effects of the share of government infrastructure spending on the unique interior BGP equilibrium values of* θ_*, $\left(\frac{e}{h}\right)_*$, $\left(\frac{c}{y}\right)_*$, *and* $\left(\frac{\bar{h}}{h}\right)_*$ *are given by*

$$\frac{\partial\theta_*}{\partial g} = -\left(\frac{B^p}{\Omega}\right)^{\frac{1}{\eta\mu}} < 0, \tag{32}$$

$$\frac{\partial\left(\frac{e}{h}\right)_*}{\partial g} = -\frac{\left(\frac{e}{h}\right)_*}{1+1/\left(\Gamma(1-\theta_*)\right)}\frac{1}{(1-\theta_*)^2}\frac{\partial\theta_*}{\partial g} > 0, \tag{33}$$

$$\frac{\partial\left(\frac{c}{y}\right)_*}{\partial g} = -\frac{\partial\left(\frac{e}{h}\right)_*}{\partial g} < 0, \tag{34}$$

$$\frac{\partial\left(\frac{\bar{h}}{h}\right)_*}{\partial g} = -\left(\frac{\phi}{\alpha\eta(1-\mu)}g^{-1} + \frac{1-\alpha}{\alpha}\left(\frac{e}{h}\right)_*^{-1}\right)\left(\frac{\bar{h}}{h}\right)_* < 0, \tag{35}$$

respectively.

Proof. The proof is straightforward. ■

Effects of the growth rate of the frontier of knowledge, Ω The effects on the model variables of the growth rate of the frontier of knowledge are summarized in Proposition 7. First, we analyze the effect of this growth rate on the share of time spent in the public education sector, θ_*. We find that a faster growing economy will have a larger e-learning sector and a smaller public education sector (which is, compared to its previous incarnations, more productive because of its smaller class size; see 36).

Second, we analyze the effect of this growth rate on e-learning expenditures, $\left(\frac{e}{h}\right)_*$. We find that a higher growth rate leads to an increase in e-learning expenditures (see 37). This results from the higher marginal product of e-learning expenditures which in turn results from the smaller share of time spent in the public education sector.

Third, we analyze the effect of this growth rate on the share of consumption in after-tax income, $\frac{c}{y}$. We find that the effect on the share of consumption is negative due to the positive effect of growth on e-learning expenditures (see 38).

Fourth, we analyze the effect of the growth rate of the frontier of knowledge on the catch-up factor, $\left(\frac{\bar{h}}{h}\right)_*$ - see (39). A faster growing economy has an ambiguous effect on the catch-up factor, $\frac{\partial Q^e_*}{\partial \Omega} = \frac{1}{\eta(1-\mu)} \frac{Q^e_*}{\Omega} > 0$. A positive effect is brought about by the positive effect on the relative quality of teachers. A negative effect is brought about by the positive effect of the growth rate on e-learning expenditures.

Proposition 7 (Effects of the growth rate of the frontier of knowledge Ω**)** *The effects of the growth rate of the frontier of knowledge on the unique interior BGP equilibrium values of* θ_*, $\left(\frac{e}{h}\right)_*$, $\left(\frac{c}{y}\right)_*$, *and* $\left(\frac{\bar{h}}{h}\right)_*$ *are given by*

$$\frac{\partial \theta_*}{\partial \Omega} = -\frac{1}{\eta\mu}\frac{\theta_*}{\Omega} < 0, \tag{36}$$

$$\frac{\partial \left(\frac{e}{h}\right)_*}{\partial \Omega} = -\frac{\left(\frac{e}{h}\right)_*}{1 + 1/\left(\Gamma(1-\theta_*)\right)}\frac{1}{(1-\theta_*)^2}\frac{\partial \theta_*}{\partial \Omega} > 0, \tag{37}$$

$$\frac{\partial \left(\frac{c}{y}\right)_*}{\partial \Omega} = -\frac{1}{(1-\tau)A}\frac{\partial \left(\frac{e}{h}\right)_*}{\partial \Omega} < 0, \tag{38}$$

$$\frac{\partial \left(\frac{\bar{h}}{h}\right)_*}{\partial \Omega} = \frac{1}{\alpha}\frac{\left(\frac{\bar{h}}{h}\right)_*}{Q^e_*}\frac{\partial Q^e_*}{\partial \Omega} - \frac{1-\alpha}{\alpha}\frac{\left(\frac{\bar{h}}{h}\right)_*}{\left(\frac{e}{h}\right)_*}\frac{\partial \left(\frac{e}{h}\right)_*}{\partial \Omega} <> 0, \tag{39}$$

respectively.

Proof. The proof is straightforward. ■

4 Transitional Dynamics

In this section we briefly demonstrate two typical transitions: *the catch-up transition* along 'good' equilibria with e-learning toward the interior positive-growth

BGP, and *the stagnation transition* along 'bad' equilibria toward the autarchy equilibrium with zero e-learning. The analysis shows that the interior steady state is locally semi-stable; it is stable for good equilibria and unstable for bad ones. There is also a third mode of behavior when the economy is permanently stuck in the autarchy equilibrium.

Knowing the initial level of human capital in the economy h_0 and the sequence of frontier knowledge states $\{\bar{h}_t = \bar{h}_0 e^{\Omega t}\}_{t=0}^{\infty}$, we can obtain the evolution of time spent in the public sector, $\{\theta_t\}_{t=0}^{\infty}$, e-learning expenditures, $\{\frac{e_t}{h_t}\}_{t=0}^{\infty}$, and human capital of young agents, $\{h_{t+1}\}_{t=0}^{\infty}$, by using equations (21), (22) and (18).

Catch-Up Transition Let us assume that the knowledge gap is initially larger than its steady state value, $\left(\frac{\bar{h}}{h}\right)_0 > \left(\frac{\bar{h}}{h}\right)_*$ (see Proposition 4), and the economy is initially in a good equilibrium with high e-learning expenditures and low public education attendance (point A_0 at the intersection of curves f_I and f_{II} in Figure 2a). Since $\theta_0 < \theta_*$ the current growth rate of human capital is larger than the steady state growth rate, $\omega_t > \Omega$, and thus the economy's knowledge gap declines and catches up with the frontier economy. The lower gap, $\left(\frac{\bar{h}}{h}\right)_1 < \left(\frac{\bar{h}}{h}\right)_0$, shifts the curve f_I upward and to the right, and thus in the next period the 'good' equilibrium is at A_1. Since the growth rate is still above the steady state growth rate, the economy continues to converge to the steady state A.

Stagnation Transition Let us assume that the knowledge gap is initially larger than its steady state value, $\left(\frac{\bar{h}}{h}\right)_0 > \left(\frac{\bar{h}}{h}\right)_*$ (see Proposition 4), but now the economy is initially in a 'bad' equilibrium with low e-learning and high public education attendance (point B_0 at the intersection of curves f_I and f_{II} in Figure 2b. Since $\theta_0' > \theta_*$ the current growth rate of human capital is smaller than the steady state growth rate, $\omega_t < \Omega$, and thus the economy's knowledge gap increases and the economy drifts away from the frontier of knowledge. The higher gap, $\left(\frac{\bar{h}}{h}\right)_1' > \left(\frac{\bar{h}}{h}\right)_0'$, shifts the curve f_I downward and to the left, and thus in the next period the 'bad' equilibrium is at B_1. Since the growth rate is again below the steady state growth rate, the economy continues to converge to the autarchy equilibrium B.

5 The Effect of Government Policies

In this section we analyze the effect of the tax rate, τ, and the share of government spending on infrastructure, g, on the dynamic equilibria.

Proposition 8 (Effects of the tax rate τ) *The effects ofchanges in the tax rate on the marginal-rate-of-transformation and the marginal-rate-of-substitution* *loci are given by*

$$\frac{\partial f_I}{\partial \tau} = \left(\frac{B^p}{\Psi g^\phi}\right)^{\frac{1}{\eta\mu}} \left(\frac{\bar{h}_t^\alpha e_t^{1-\alpha}}{h_t}\right)^{-\frac{1-\mu}{\mu}} > 0, \quad (40)$$

$$\frac{\partial f_{II}}{\partial \tau} = -\Gamma^{-1} \frac{A\left(\frac{e_t}{h_t}\right)^{-1}}{\left((1-\tau)A\left(\frac{e_t}{h_t}\right)^{-1} - 1\right)^2} < 0, \quad (41)$$

respectively.

Proof. The proof is straightforward by taking derivatives of (21) and (22) with respect to τ. ■

An increase in the tax rate will increase the number of teachers, and hence the productivity and number of students in the public sector. The MRT-locus, f_I, shifts up and to the right. Increased taxes mean lower after-tax income and thus lower consumption. Keeping the marginal rate of substitution between consumption and human capital equal to the marginal product of e-learning, human capital has to decline as well (via decreased spending). However, to keep the marginal product of spending unchanged the time spent in e-learning has to increase. The situation is captured in Figure 3 where it can be seen that increased taxes improve 'bad' equilibria - i.e. more resources are used in the private education sector. At the same time, increased taxes worsen 'good' equilibria - i.e. less resources are used in the private education sector.

Proposition 9 (Effects of the government expenditures g**)** *The effects of the share of government expenditures on the marginal-rate-of-transformation and the marginal-rate-of-substitution loci are given by*

$$\frac{\partial f_I}{\partial g} = -\left(\frac{B^p}{\Psi g^\phi}\right)^{\frac{1}{\eta\mu}} \left(\frac{\bar{h}_t^\alpha e_t^{1-\alpha}}{h_t}\right)^{-\frac{1-\mu}{\mu}} (1 + \frac{\phi}{\eta\mu}\frac{\tau - g}{g}) < 0, \quad (42)$$

$$\frac{\partial f_{II}}{\partial g} = 0, \quad (43)$$

respectively.

Proof. The proof is straightforward by taking derivatives of (21) and (22) with respect to g. ■

The increase in government (telecommunications) infrastructure spending primarily improves the productivity in the e-learning sector and secondarily decreases available resources to the public education sector, i.e. fewer teachers, which goes in the same direction. Thus e-learning gets cheaper and less spending is necessary to keep up with the public education sector and the MRT-locus shifts down and to the left. There is no effect of government spending on the MRS-locus. In Figure 4 it can be seen that bad equilibria are worsened - i.e. the public school sector increases. Being at 'bad' or 'passive parents' equilibrium with high current consumption and low human capital of youth, the improved private education sector allows parents to spend less on their kids to get the same amount of their human capital. However, this further increases parents' consumption; such an increase in welfare allows them to spend even less in the private education sector. Government spending improves 'good' equilibria, i.e. the private education sector increases. And vice versa for 'good' or 'active parents' equilibrium with low consumption and high human capital of youth.

6 Conclusion

We proposed a general equilibrium model of endogenous growth in which human capital investment is the engine of growth. Within that model we analyzed the potential role of e-learning in the elimination of the human capital bootstrapping problem so typical for transition economies. We find that e-learning can indeed speed up convergence to the frontier of knowledge. The intuitive idea behind this result is the ability of a transition economy to access external knowledge sources that do not require local teachers to first learn the requisite skills before they can teach them. Our results are derived in an economy with two education sectors: traditional public classroom and private e-learning which are modelled in a unified manner, with class size and relative teacher quality being the key variables. The endogenous class size creates a negative externality in the public education sector and is responsible for multiple equilibria: 'bad' or 'passive parents' equilibrium, 'good' or 'active parents' equilibrium, and corner zero e-learning trap. Specifically, our model produces three equilibrium configurations of which two depict multiple equilibria that can be ranked in terms of their growth or catch-up rates and one that has a single zero e-learning equilibrium.

We show that there are typically two transitions: 'catch-up transition' along good equilibria to balanced growth path equilibrium with sustained growth, and 'stagnation transition' along bad equilibria to autarchic zero e-learning equilibrium. Depending on whether the economy is currently at 'bad'/ 'good' equilibrium, a pro-e-learning policy can have negative/positive effect on the performance of the economy.

The intuition is straightforward: Government spending on (telecommunications) infrastructure will improve the performance of the economy through the increase in e-learning only if the economy is in a good equilibrium. If the economy is at a bad equilibrium, government spending on (telecommunications) infrastructure will worsen the economy's performance because this investment (which clearly is costly) will not generate returns that make it worth it. To put it simply, if you don't use e-learning it doesn't make sense to spend a lot on (telecommunications) infrastructure (at least within the strictures of our model; investment in such infrastructure may well have other benefits as the recent work of Roeller and Waverman,[20] suggests.)

The indeterminacy that we found suggests, within the strictures of our model another role for government: to help avoid coordination failure. While this is a necessary condition for government policy to be effective, it is by no means sufficient.

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