Microeconomics 3 **Andreas Ortmann, Ph.D.**
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Office hours: Tuesdays 17 - 19 or by appointment

Week of June 21, lecture 15:
On sequential rationality, weak perfect Bayesian and sequential equilibria.

Key readings:

MWG chapter 9.A. - 9.C.

[for Tuesday June 22, afternoon]
Problem set # 6 ~ downloadable from
http://home.cerge-ei.cz/ortmann/CourseMaterials.html.

[for Thursday June 24]
Dufwenberg (2002), Marital Investments, Time Consistency, and Emotions. Journal of Economic Behavior & Organization 48.1., 57 - 69.

[Four-page paper: Thursday June 24]

[for Tuesday June 29, morning]
MWG chapter 9.D.

Cachon & Camerer (1996), Loss Avoidance and Forward Induction in Experimental Coordination Games. Quarterly Journal of Economics 111.1, 165 - 194.

[for Tuesday June 29, afternoon]
Problem set # 7 ~ downloadable from
http://home.cerge-ei.cz/ortmann/CourseMaterials.html.

[for Thursday July 1]
MWG 12.D. and 12App (Repeated Interaction)

Kuebler et al. (2003), Job Market Signaling and Screening: An Experimental Comparison. Working paper.

Proposition* 9.B.4. (*successive play of T simultaneous-moves games):

Consider an *I*-player extensive form game Γ_E involving successive play of *T* simultaneous-move games, $\Gamma^t_N = [I, \{\Delta(S^t_i)\}, \{u^t_i(\cdot)\}]$ for t = 1, ..., t, with the players observing the pure strategies played in each game immediately after its play is concluded. Assume that each player's payoff is equal to the sum of her payoffs in the plays of the *T* games. If there is a unique Nash equilibrium in each game Γ^t_N, say $\sigma^t = (\sigma^t_1, ..., \sigma^t_I)$, then there is a unique SPNE of Γ_E and it consists of each player i playing the Nash equilibrium strategy in each game Γ^t_N regardless of what has happened previously.

[Discussion finitely repeated PDG.]

Note: As long as each of the successive T simultaneous-moves games have a unique Nash equilibrium, SPNE strategies cannot be history dependent. Well, theoretically that is, again assuming common knowledge of rationality and the structure of the game. Empirically, see for a famous example, Selten and Stoecker (1986), End Behavior in Sequences of Finite Prisoner's Dilemma Supergames. <u>Journal of Economic Behavior and Organization</u> 7, 47 - 70. This paper generated the incomplete information approach to finitely repeated games that the "gang of four" (e.g., Kreps, Milgrom, Roberts, and Wilson (1982), Rational Cooperation in the Finitely Repeated Prisoner's Dilemma. <u>Journal of Economic Theory</u> 92, 805 - 24) invented.

Where do we stand at this point (equilibrium concepts so far) ...

S Dominance
S Nash equilibrium in static games of complete information
S Subgame-perfect Nash equilibrium in dynamic games of complete information
S Bayesian Nash equilibrium in static games of incomplete information

and where we are headed:

S Weak Perfect Bayesian in dynamic games of incomplete information (akin to Nash equilibrium)
S Sequential equilibrium in dynamic games of incomplete information (akin to SPNE)

Note: Progressively richer games require stronger solution concepts that help us sort out implausible (noncredible etc.) solutions.

Definition 9.C.1. (System of beliefs):
A system of beliefs μ in extensive form game Γ_E is a specification of probability $\mu(x) \in [0,1]$ for each decision node x in Γ_E such that $\sum_{x \in H} \mu(x) = 1$ for all information sets.

Why do we need it?

[Discussion examples 9.B.1 and 9.C.1, Figure 9.C.1;
the problem is that subgame perfection fails us here. Specifically, we can not apply backward induction since the only subgame is the whole game. The key to solving this particular game is to find a strategy that is dominant and allows us to identify straightforward beliefs.
Note that
- the strategy profile σ is sequentially rational given μ
- μ is derived from the strategy profile σ
    - E looked at E's options and chose In_1
    - I ferrets out E's choice and forms belief]

Note: A system of beliefs is a probabilistic assessment by a player who moves at an information set about the relative likelihood of being at the information set's various decision nodes. Beliefs are about what has happened in the game up to that point.

Definition* 9.C.2. *(Sequential rationality given a system of beliefs):
A strategy profile $\sigma = (\sigma_1, \sigma_2, ..., \sigma_I)$ in extensive form game Γ_E is sequentially rational at information set *H* given a system of beliefs μ, if a player who moves at information set *H* has no incentive to revise her strategy given her beliefs. If strategy profile σ satisfies this condition for <u>all</u> information sets *H*, then we say that σ is sequentially rational given belief system μ.

Note: For a slightly more formal definition, see MWG p. 284.

Reminder (Bayes' rule or Bayes' theorem): Prob (x|*H*,F) = Prob (x|F)/$3_{x',H}$ Prob(x'|F)

Definition 9.C.3. (Weak perfect Bayesian equilibrium)**:**
A profile of strategies and systems of beliefs (F, :) is a weak PBE in extensive form games $'_E$ if it has the following properties:
(i) The strategy profile F is sequentially rational given belief system : .
(Ii) The system of beliefs : is derived from strategy profile F through Bayes' Rule whenever that is possible. That is, for any information set *H* such that Prob(*H*|F) > 0, we must have : (x) = Prob (x|F)/Prob(*H*|F) for all x 0 H

Note 1: This extends the principle of sequential rationality by introducing beliefs: Equilibrium strategies should specify optimal behavior from any point in game onward, given one's opponents' strategies and one's beliefs about what has happened so far in the game. Beliefs, in addition, have to be consistent with strategies. Specifically, we are looking for a fixed point at which the behavior generated by beliefs is consistent with these beliefs in the sense that these beliefs are anchored by the behavior.

Note 2: When a system of beliefs : can not be derived from strategy profile F through Bayes' Rule then everything goes (i.e., every belief is possible = "agnostic view").

Proposition 9.C.1.
(Nash equilibrium, restated in terms of systems of beliefs)**:**
A strategy profile F is a Nash equilibrium of extensive form game $'_E$ if and only if there exists a system of beliefs : such that
(i) the strategy profile F is sequentially rational given belief system : <u>at information sets H such that Prob(*H*|F) > 0.</u>
(ii) the system of beliefs : is derived from strategy profile F through Bayes' Rule whenever that is possible.

Note: The idea of consistency of beliefs is similar to the idea of Nash equilibrium. For a Nash equilibrium, however, we require sequential rationality only if on the equilibrium path. Hence, a weak perfect Bayesian equilbrium is always a Nash equilibrium but not every Nash equilibrium is a weak perfect Bayesian equilibrium.

[Discussion example 9.C.2, Figure 9.C.2;
the problem is here too that subgame perfection fails us. Again, we can not apply backward induction since the only subgame is the whole game. The key to solving this particular game is to find a strategy that is dominant and allows us to identify straightforward beliefs.]

Comparing example 9.C.1 and example 9.C.2:

In both, the key is to find a dominant strategy that acts as lever to identify straightforward beliefs.

9.C.1	9.C.2
I looks at E, puts herself into E's shoes, and reasons as E just did.	I looks at E2 and sees that Acc strictly dominates Decline
	I looks at E1 and sees that Prop strictly dominates other options given E2's choice.

E looks at I and sees that
- F never a best response
- A always a best response

E decides to play In_1

In both, the players that have to take action at the information set, form beliefs about their opponents' likely action given common knowledge of rationality and structure of the game, and then make a decision that is sequentially rational.

I then forms a belief : $(In_1) = 1$	I then forms a belief : (Accept) = 1

To check for a weak perfect Bayesian equilibrium we now have to check whether
- the strategy profile (and in particular I's strategy profile component) are sequentially rational given these beliefs.
- the beliefs have been derived from those parts of the strategy profile

leading up to the information set.

[Discussion example 9.C.3, Figure 9.C.3;
in the preceding examples trick was to find the right lever, i.e., the dominant strategy that allowed one to unravel the game. This example has been cooked up in such a way that this easy route is no longer available. Firm I’s belief of where E might end up are now represented by a probability distribution that puts weight on both :(In$_1$) and :(In$_2$).]

Note 1: Problems with the weak PBE:

- specified beliefs may not be sensible, e.g. MWG Example 9.C.4

- they may not specify a Nash equilibrium in the post-entry subgame (hence weak PBE strategies may not constitute a subgame perfect equilibrium), e.g. MWG Example 9.C.5

The weak PBE concept puts minimal consistency requirements on beliefs: they have to be derived via Bayes' Rule from the strategy profile but if Bayes' Rule doesn't apply everything goes. Specifically, in those situations that are off the equilibrium paths (i.e., information sets that are not reached with positive probability) no restrictions are placed on the beliefs.

Examples 9.C.4 and 9.C.5 illustrate why a strengthening of the weak PBE is desirable. The idea of the trick (that leads to sequential equilibrium and similar refinements) is simple. Assume (as in the case of trembling hand equilibria) that you are thrown off the equilibrium path by some perturbation and then make sure that the beliefs fulfill certain consistency requirements.

[Discussion example 9.C.4, Figure 9.C.4]

[Discussion example 9.C.5, Figure 9.C.5]

Note 2: Extra consistency requirements generate PBE and sequential equilibrium.

Definition* 9.C.4 *(Sequential equilibrium):
A profile of strategies and systems of beliefs (F, :) is a SE of extensive form game $'_E$ if it has the following properties:
(i) The strategy profile F is sequentially rational given belief system : .
(ii) There exists a sequence of completely mixed strategies $\{F^k\}_{k=1, ..., 4}$, with $\lim_{k\to 4} F^k \to F$, such that $: = \lim_{k\to 4} :^k$, where $:^k$ denotes the beliefs derived from strategic profile F^k using Bayes' rule.

Proposition* 9.C.4 *(Sequential equilibrium):
In every SE of $'_E$ the equilibrium strategy profile constitutes a SPNE of $'_E$.

Note 1: Subgame perfection often captures the principle of sequential rationality but not always. Hence, to capture the spirit of subgame perfection, the *weak Perfect Bayesian equilibrium* concept (F is sequentially rational given belief system :; belief system : is derived from F through Bayes' rule whenever possible.) Unfortunately, the weak PBE concept carries us only so far as it fails in those situations where the probability of reaching an information set is zero, making the straightforward application of Bayes' rule impossible and leading to strategies that are not subgame perfect (e.g. example 9.C.5). This fact, in turn, motivated us to impose further restrictions on beliefs (namely the consistency requirements of the *sequential equilibrium* concept.) This concept, like the weak PBE has two parts. In fact, part 1 (F is sequentially rational given belief system :) is identical to part 1 of the definition of a *weak PBE*. Part 2 circumvents the problem of the non-applicability of Bayes' rule by generating sequences of completely (strictly) mixed strategy profiles F_n and sequences of completely (strictly) mixed belief systems $:_n$ which are [and now can be] derived from the strategy profiles F_n through Bayes' rule.

Note 2: Sequential equilibria, however, may be sustained with beliefs that are not at all intuitively sensible. This fact leads to the definition of "intuitive" and "divine" criteria - further restrictions on beliefs in order to generate reasonable solutions.

Guiding questions to Dufwenberg (2002), Marital investments, time consistency and emotions, JEBO 48, 57 - 69.

(Work together with up to two other people you have not worked together with)

1. What are the basic facts of marital investments that MD asserts?
   - the major part of wealth in marriages is in human capital
   - many families concentrate on one spouse's education, still mostly the man's
   - "no-fault" divorce legislation is widespread and allows the husband to walk out of a marriage without major sanction.
   - prenuptial agreements are often not a viable option.
2. In terms of the payoffs of the investment game that MD proposes, why does such an asymmetric investment strategy make sense? (Hint: Compare the payoffs of Wife choosing the No option and the payoffs of the Wife choosing the Yes option <u>and</u> the husband choosing the Stay option!)
   - There are dramatic efficiency gains from the latter strategy. (This is essentially a variation on the old comparative advantage theme.)
3. Given the parameterization of the marital investment game as shown in Figure 1, why are there incentives for opportunistic divorce? And what do they have to do with time consistency?
   - Time consistency is obviously just another word for subgame perfection. The subgame perfect solution brings about opportunistic divorce.
4. What's wrong with the picture?
   - Well, if that's how marital investments typically unravel, we should have a hundred percent divorce rate, shouldn't we? But that's not what we typically have.
   - So, what is wrong with the picture? The marital investment game in Figure 1 doesn't take account of the emotions (e.g., **!** and, more importantly, "guilt")
5. The key emotion in Dufwenberg's model is guilt.[1] How exactly is guilt being added to the model in Figure 1? Intuitively, what is it that guilt does? (Hint: What is psychological forward induction and, in fact, what interesting problem does it induce?) And how is that intuition reflected in Figure 2? (Be specific about the changed payoff associated with the Divorce action. Before that make sure that you understand the meaning of F, J, J', J ''!!! Also discuss assumption 1)
   - Guilt is added (additively - see assumption 2), subtracting from the payoffs the husband stands to collect if he divorces his wife.)
   - J' is the wife's expectation of her husband staying with her, J.
   - J '' is the husband's expectation of J '. And guilt is his sense of violating the wife expectation, His sensitivity to his own opportunism enters through ((which is a multiplicative modifier of his belief in

[1] In a sense, the equilibrium in the marital investment game is not supported by something positive such as **!** but something negative such as the threat of feeling guilty. That's not a comforting thought. ain't it?

her trust and correspondingly his utility).

S Guilt gets triggered, or “forced” by the decision of Wife to choose Yes, thus signaling to Hubby that she trusts him. See the comment on Rabin (1993) on p. 64

6. What happens if (= 0? Intuition?
   - The psychological marital investment game in Figure 2 collapses to the marital investment game in Figure 1. The husband is an insensitive opportunistic bastard.
7. Whenever (> 0, the resultant game is not a standard game. Why?
   - Technically, rather then u_i: S - > ú we now have SxB -> ú. (See also p. 63)
8. So, how to solve the psychological marital investment game? Discuss (and understand intuitively) what (< 2 and (> 4 imply.
   - Low guilt sensitivity ((< 2) implies unravelling of the efficient marital investment equilibrium irrespective of J ’’.
   - High guilt sensitivity ((> 4) implies bliss ever after, irrespective of J ’’. Ah, ! conquers all!
9. What about the case (O (2,4)? (Make sure to discuss the definition of a marital equilibrium in Definition 1 on page 65, especially the consistency condition J, J’, and J’‘.
   - See page 65.
10. When does mixed matrimony emerge? And how is it different from the trusting twosome case?
   - See page 65.

Note that (> 0 implies that subgame perfection – as defined before – doesn’t work. Somehow we have to nail down beliefs given various parameter realizations of the guilt sensitivity parameter.

Take the ***case of (< 2***:

4 - (Ë$^0_1$ => B_H O (2,4) <= compare this to 2 from strategy *Stay*

Knowing the low guilt sensitivity of Hubby, and having common knowledge of rationality and the structure of the game, Wife will anticipate Hubby choosing the Divorce option and not pay his education..Hence, J’’ = J’ = J = 0.

Take the ***case of (> 4:***

4 - (Ë$^0_1$ => B_H O (0,4) <= compare this to 2 from strategy *Stay.*

Knowing the high guilt sensitivity of Hubby, and having common knowledge of rationality and the structure of the game, Wife will signal her trust to Hubby but she will do so only if she expects $B^{Yes/Stay}$ \$ B^{No}. Which requires J’ = J \$ 1/2. Again by common knowledge of rationality and the structure of the game, Hubby will understand the trust that has been invested in him. Specifically, J’’ = J’ = J \$ 1/2. But this then induces - by the third condition of the marital equilibrium condition - the second condition, as follows: $J'' = J' = J = 1$.

This is what Dufwenberg calls “psychological forward induction”: For a Wife to say *Yes* is a signal to Hubby that she trusts him, at least to a reasonable degree: $J \ \$ \ 1/2$. Hubby figures this out and if called upon to play must believe $J' = J \ \$ \ 1/2$. But this implies $J'' \ \$ \ 1/2$. And, hence, he *Stays.* This, in turn, Wife figures out, and hence chooses *Yes.* Love therefore conquers all. He and She will live happily ever after.

Take the ***case of (O [2,4]***
[Apply similar reasoning; and illustrate the relevant parameter space in (J'' space.]