

Microeconomics 3
Summer 2004

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Week of June 21, lecture 14:
On sequential rationality, backward induction, subgame perfection (again), beliefs and sequential rationality, and weak perfect Bayesian equilibria.

Key readings:

MWG chapter 9.A. - 9.C.

[for Tuesday June 22, afternoon]

Problem set # 6 ~ downloadable from

<http://home.cerge-ei.cz/ortmann/CourseMaterials.html>.

[for Thursday June 24]

Dufwenberg (2002), Marital Investments, Time Consistency, and Emotions. Journal of Economic Behavior & Organization 48.1., 57 - 69.

[Four-page paper: Thursday June 24]

[for Tuesday June 29, morning]

MWG chapter 9.D.

Cachon & Camerer (1996), Loss Avoidance and Forward Induction in Experimental Coordination Games. Quarterly Journal of Economics 111.1, 165 - 194.

[for Tuesday June 29, afternoon]

Problem set # 7 ~ downloadable from

<http://home.cerge-ei.cz/ortmann/CourseMaterials.html>.

[for Thursday July 1]

MWG 12.D. and 12App (Repeated Interaction)

Kuebler et al. (2003), Job Market Signaling and Screening: An Experimental Comparison. Working paper.

We are going back a bit ...

Definition 9.B.1 (subgame):

A subgame of an extensive form game Γ_E is a subset of the game having the following properties:

(i) It begins with an information set containing a single decision node, contains all the decision nodes that are successors (both immediate and later) of this node, and contains *only* these nodes.

(ii) If decision node x is in the subgame, then every $x' \in H(x)$ is also, where $H(x)$ is the information set that contains decision node x .

Note 1: The game as a whole is a subgame, as may be some strict subsets of the game.

Note 2: In a finite game of perfect information, every decision node initiates a subgame.

Reminder (Definition 7.C.1): A game is one of perfect information if each information set contains a single decision node. Otherwise, it is a game of imperfect information.

Definition 9.B.2 (subgame perfect Nash equilibrium):

A strategy profile $s = (s_1, \dots, s_I)$ in an I -player extensive form game Γ_E is a subgame perfect Nash equilibrium (SPNE) if it induces a Nash equilibrium in every subgame of Γ_E .

[Previously discussed application of these concepts to some of the sample games, e.g., entry deterrence/chain store game, alternating offer games, AoN, Centipede, etc. Also discussed how subgame perfection can select among equilibria.]

[See also MWG discussion of Matching Pennies B and C on pages 228 - 230.]

[Discussion Stackelberg follower - leader example]

Recall: A strategy is a complete contingent plan, or decision rule, that specifies how a player will act in every possible distinguishable circumstance in which she might be called upon to move. One initially counterintuitive aspect of this definition is that strategies specify actions at information sets that may never be reached during the actual play, even quite possibly by one's own intentions. But then you never know. Something (e.g., a tremble) might happen ...

[Discussion of Example 9.B.1 predation game: Why is (out, fight if firm E plays "in") not a sensible prediction for the outcome of the game?]

Note 1:

The problem with the predation/entry deterrence/chain store game is this:

- (out, fight if firm E plays "in") means that we never reach the fight/accommodation stage. So, for this equilibrium the actions of the Incumbent – including the empty threat to fight – is inconsequential.
- if, however, the Entrant would enter (say, through a tremble), then clearly a rational Incumbent should not fight. This is what the *principle of sequential rationality* is all about: Equilibrium strategies should specify optimal behavior from any point in the game onward. It guarantees that "reasonable" equilibria are selected.

Note 2: The way to guarantee sequential rationality is backward induction which is particularly easy to apply to finite games of perfect information (which allow us in principle to fold the game back to its root by substituting subgames successively with their "value" to the players.)

[Discussion Figure 9.B.1&2, Example 9.B.1]

[Discussion Figure 9.B.3, Example 9.B.2;

this example explained in class with more or less randomly filled in payoffs but of course the logic of backward induction applies to it nonetheless]

[Make sure you understand Proposition 9.B.1 - Zermelo's Theorem which says that every finite game of perfect information has a pure strategy equilibrium that can be derived via backward induction and which states that this equilibrium is indeed unique if no player has the same payoffs at any terminal node]

[Discussion Figure 9.B. 4, Example 9.B.3 (unique SPNE):

- same situation as in Example 9.B.1. but now
I and E play simultaneous-move game after entry,
each choosing “fight” or “accommodate”
- back induction still works but have to substitute simultaneous-move game
with its “value” to the players]

Proposition 9.B.3. (generalized backward induction procedure):

Consider an extensive form game Γ_E and some subgame S of Γ_E . Suppose that strategy profile F^S is an SPNE in subgame S , and let Γ_E^* be the reduced game formed by replacing subgame S by a terminal node with payoffs equal to those arising from play of F^S . Then:

- (i) In any SPNE F of Γ_E in which F^S is the play in subgame S , players' move at information sets outside subgame S must constitute an SPNE of the reduced form game Γ_E^* .
- (ii) If F^* is an SPNE of Γ_E^* , then the strategy profile F that specifies the moves in F^S at information sets in subgame S and that specifies the moves in F^* at information sets not in S is an SPNE of Γ_E .

Note 1: This is generalized backward induction procedure is explained somewhat more informally on top of page 277. The essence is that you successively fold back the outer reaches of the game tree to the root by substituting the payoffs of the Nash equilibria of the respective final subgames for them. This is even possible with multiple Nash equilibria although it gives you multiple SPNE. [The proof of the proposition is straightforward and recommended (albeit not required) reading. The key argument is that if a strategy profile F is not an SPNE in subgame S , then there must a be a profitable deviation for some player.]

Note 2: The generalized backward induction procedure applies to both games of perfect and imperfect information.

[Discussion Figures 9.B.6 and 9.B.7, example 9.B.4 (multiple equilibria)]

Note 3: If a player's strategy specify the optimal action at every point in the game tree, then that player's strategy obeys the *principle of sequential rationality*. Generalized backward induction is obviously a way to guarantee that that principle is adhered to.

Proposition 9.B.4. (successive play of T simultaneous-moves games):

Consider an I -player extensive form game Γ_E involving successive play of T simultaneous-move games, $\Gamma_N^t = [I, \{S_i^t\}, \{u_i^t(\cdot)\}]$ for $t = 1, \dots, T$, with the players observing the pure strategies played in each game immediately after its play is concluded. Assume that each player's payoff is equal to the sum of her payoffs in the plays of the T games. If there is a unique Nash equilibrium in each game Γ_N^t , say $F^t = (F_1^t, \dots, F_I^t)$, then there is a unique SPNE of Γ_E and it consists of each player i playing the Nash equilibrium strategy in each game Γ_N^t regardless of what has happened previously.

[Discussion finitely repeated PDG.]

Note: As long as each of the successive T simultaneous-moves games have a unique Nash equilibrium, SPNE strategies cannot be history dependent. Well, theoretically that is, again assuming common knowledge of rationality and the structure of the game. Empirically, see for a famous example, Selten and Stoecker (1986), End Behavior in Sequences of Finite Prisoner's Dilemma Supergames. Journal of Economic Behavior and Organization 7, 47 - 70. This paper generated the incomplete information approach to finitely repeated games that the "gang of four" (e.g., Kreps, Milgrom, Roberts, and Wilson (1982), Rational Cooperation in the Finitely Repeated Prisoner's Dilemma. Journal of Economic Theory 92, 805 - 24) invented.

Where do we stand at this point (equilibrium concepts so far) ...

- S Dominance
- S Nash equilibrium in static games of complete information
- S Subgame-perfect Nash equilibrium in dynamic games of complete information
- S Bayesian Nash equilibrium in static games of incomplete information

and where we are headed:

- S Weak Perfect Bayesian in dynamic games of incomplete information
- S Sequential equilibrium in dynamic games of incomplete information

Note: Progressively richer games require stronger solution concepts that help us sort out implausible (noncredible etc.) solutions.

Quotations of the day:

“[This chapter] is long and complex, and you should expect it to take a number of readings before it begins to make sense to you (if it ever does).”

[Kreps 1990, p. 387 on “solution concepts for non-cooperative games”]

“... the hallmark of all the literature on refinements. For normal form games, one is concerned with how strategies perform if one’s opponents take actions that have “zero” probability in equilibrium; one has to reason about the *relative* likelihood of things that one feels are not going to happen with any significant probability. In terms of extensive form games, ... most refinements that are based on extensive form considerations are keyed to what will happen “out of equilibrium” - that is, at points in the game tree

that will not be reached if the equilibrium is played. But how can we reason about what will happen conditional on things that are not meant to happen in the first place?” [Kreps 1990, p. 420]

“The notion of consistency (guaranteeing a sequential equilibrium) ... may look innocuous, if it doesn’t look completely mysterious, but it isn’t innocuous at all. (It may well be completely mysterious.) ... a lot of bodies are buried in this definition of consistency.” [Kreps 1990, p. 430]

[A good recent discussion of the empirical reality of people’s ability to play refinements is Camerer (2003), Behavioral Game Theory, Chapter 8.]

Definition 9.C.1. (System of beliefs):

A system of beliefs β in extensive form game Γ_E is a specification of probability $\beta(x) \in [0,1]$ for each decision node x in Γ_E such that $\sum_{x \in H} \beta(x) = 1$ for all information sets.

Why do we need it?

[Discussion examples 9.B.1 and 9.C.1, Figure 9.C.1; the problem is that subgame perfection fails us here. Specifically, we can not apply backward induction since the only subgame is the whole game. The key to solving this particular game is to find a strategy that is dominant and allows us to identify straightforward beliefs.

Note that

- the strategy profile F is sequentially rational given β
 - β is derived from the strategy profile F
- S E looked at E's options and chose I_{n_1}
- S I ferrets out E's choice and forms belief]

Note: A system of beliefs is a probabilistic assessment by a player who moves at an information set about the relative likelihood of being at the information set's various decision nodes. Beliefs are about what has happened in the game up to that point.

Definition 9.C.2. (Sequential rationality given a system of beliefs):

A strategy profile $F = (F_1, F_2, \dots, F_I)$ in extensive form game Γ_E is sequentially rational at information set H given a system of beliefs β , if a player who moves at information set H has no incentive to revise her strategy given her beliefs. If strategy profile F satisfies this condition for all information sets H , then we say that F is sequentially rational given belief system β .

Note: For a slightly more formal definition, see MWG p. 284.

Reminder (Bayes' rule or Bayes' theorem): $\text{Prob}(x|H, \mathbf{F}) = \text{Prob}(x|\mathbf{F}) / \sum_{x', H} \text{Prob}(x'|\mathbf{F})$

Definition 9.C.3. (Weak perfect Bayesian equilibrium):

A profile of strategies and systems of beliefs $(\mathbf{F}, :)$ is a weak PBE in extensive form games Γ_E if it has the following properties:

- (i) The strategy profile \mathbf{F} is sequentially rational given belief system $:$.
- (ii) The system of beliefs $:$ is derived from strategy profile \mathbf{F} through Bayes' Rule whenever that is possible. That is, for any information set H such that $\text{Prob}(H|\mathbf{F}) > 0$, we must have $:(x) = \text{Prob}(x|\mathbf{F}) / \text{Prob}(H|\mathbf{F})$ for all $x \in H$.

Note 1: This extends the principle of sequential rationality by introducing beliefs: Equilibrium strategies should specify optimal behavior from any point in game onward, given one's opponents' strategies and one's beliefs about what has happened so far in the game. Beliefs, in addition, have to be consistent with strategies. Specifically, we are looking for a fixed point at which the behavior generated by beliefs is consistent with these beliefs in the sense that these beliefs are anchored by the behavior.

Note 2: When a system of beliefs $:$ can not be derived from strategy profile \mathbf{F} through Bayes' Rule then everything goes (i.e., every belief is possible = "agnostic view").

Proposition 9.C.1.

(Nash equilibrium, restated in terms of systems of beliefs):

A strategy profile \mathbf{F} is a Nash equilibrium of extensive form game Γ_E if and only if there exists a system of beliefs $:$ such that

- (i) the strategy profile \mathbf{F} is sequentially rational given belief system $:$ at information sets H such that $\text{Prob}(H|\mathbf{F}) > 0$.
- (ii) the system of beliefs $:$ is derived from strategy profile \mathbf{F} through Bayes' Rule whenever that is possible.

Note: The idea of consistency of beliefs is similar to the idea of Nash equilibrium. For a Nash equilibrium, however, we require sequential rationality only if on the equilibrium path. Hence, a weak perfect Bayesian equilibrium is always a Nash equilibrium but not every Nash equilibrium is a weak perfect Bayesian equilibrium.

[Discussion example 9.C.2, Figure 9.C.2;
the problem is here too that subgame perfection fails us. Again, we can not apply backward induction since the only subgame is the whole game. The key to solving this particular game is to find a strategy that is dominant and allows us to identify straightforward beliefs.]

Comparing example 9.C.1 and example 9.C.2:

In both, the key is to find a dominant strategy that acts as lever to identify straightforward beliefs.

9.C.1

9.C.2

I looks at E, puts herself into E's shoes, and reasons as E just did.

I looks at E2 and sees that Acc strictly dominates Decline

I looks at E1 and sees that Prop strictly dominates other options given E2's choice.

E looks at I and sees that

- F never a best response
- A always a best response

E decides to play In_1

In both, the players that have to take action at the information set, form beliefs about their opponents' likely action given common knowledge of rationality and structure of the game, and then make a decision that is sequentially rational.

I then forms a belief : $(In_1) = 1$

I then forms a belief : $(Accept) = 1$

To check for a weak perfect Bayesian equilibrium we now have to check whether

- the strategy profile (and in particular I's strategy profile component) are sequentially rational given these beliefs.
- the beliefs have been derived from those parts of the strategy profile

leading up to the information set.

[Discussion example 9.C.3, Figure 9.C.3; in the preceding examples trick was to find the right lever, i.e., the dominant strategy that allowed one to unravel the game. This example has been cooked up in such a way that this easy route is no longer available. Firm I's belief of where E might end up are now represented by a probability distribution that puts weight on both : (I_n_1) and : (I_n_2) . Details in class.]