~~A~~LOCATION OF m TIMES AND n GOODS

$X = [X_1, \dots X_n]$ ~ goods — T ~ Total time available
L ~ work time — w ~ wage
$\ell = [\ell_1, \dots \ell_m]$ ~ leisure times

$$u(X_1, \dots X_n; \ell_1 \dots \ell_m) \qquad \mathcal{L} = u(\cdot) + \lambda[X_0 + wL - \sum p_i x_i]$$

$$\text{BC1: } \sum p_i x_i \equiv X_0 + wL \qquad = u(\cdot) + \lambda[X_0 + wT - w\sum \ell_i - \sum p_i x_i]$$

$$\text{BC2. } \sum \ell_i + L = T \rightarrow L = T - \sum \ell_i$$

FONC
$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_i} &= u_{x_i} - \lambda p_i \overset{!}{=} 0 \\ \frac{\partial \mathcal{L}}{\partial \ell_i} &= u_{\ell_i} - \lambda w \overset{!}{=} 0 \end{aligned}\right\} \quad \left.\begin{aligned} u_{x_i} &= \lambda p_i \\ u_{\ell_i} &= \lambda w \end{aligned}\right\} \quad \boxed{\frac{u_{\ell_i}}{u_{x_i}} = \frac{w}{p_i}}$$

w ~ marginal price of time

If ℓ_i, x_i normal goods $\Rightarrow \nearrow X_0 \rightarrow \searrow L$

$\nearrow w \rightarrow ?\, L$ income? subst?

FONC $\forall i$ & $\forall x, \ell$: $\rightarrow$ 3 sets of conditions

a) x_i vs. x_j $\longrightarrow \boxed{\frac{u_{x_i}}{u_{x_j}} \overset{!}{=} \frac{p_i}{p_j}}$ $\forall i, j$

$$dx_i \rightarrow \frac{\partial u}{\partial x_i} dx_i \qquad dx_j = \frac{dx_i\, p_i}{p_j}$$

$$dx_j \rightarrow \frac{\partial u}{\partial x_j} dx_j = \frac{\partial u}{\partial x_j} \frac{p_i}{p_j} dx_i$$

$$u_{x_i} \cancel{dx_i} \overset{!}{=} u_{x_j} \frac{p_i}{p_j} \cancel{dx_i}$$

$$\boxed{\frac{u_{x_i}}{u_{x_j}} = \frac{p_i}{p_j}}$$

b) ℓ_i vs. ℓ_j $\rightarrow \boxed{\frac{u_{\ell_i}}{u_{\ell_j}} \overset{!}{=} 1}$ $\forall i, j$

c) x_i vs. ℓ_j $\rightarrow \boxed{\frac{u_{x_i}}{u_{\ell_j}} = \frac{p_i}{w}}$

AMENDS ON CASE III

Combining FONC #2 & #3 we get several sets of conditions

a) x_i vs. x_j $\rightarrow$ $\frac{u_{x_i}}{u_{x_j}} \overset{!}{=} \frac{p_i}{p_j}$ $\forall i,j$

b) l_i vs. l_j $\rightarrow$ $\frac{u_{l_i}}{u_{l_j}} \overset{!}{=} 1$ $\forall i,j$

c) x_i vs. l_j $\rightarrow$ $\frac{u_{x_i}}{u_{l_j}} = \frac{p_i}{w}$ $\forall i,j$

You have 1 hour to use $\rightarrow$ work $\rightarrow \Delta U = \frac{\partial U}{\partial X_i} \Delta X_i$

$\rightarrow$ leisure$_i$ $\rightarrow \Delta U = \frac{\partial U}{\partial l_i} \Delta l_i$

work $dU = \frac{\partial U}{\partial x_i} dx_i = \frac{\partial U}{\partial x_i} \frac{w \cdot dl_i}{p_i}$

leisure $dU = \frac{\partial U}{\partial l_i} \cdot dl_i$

have to be equal $\Rightarrow$

$$\Rightarrow \frac{\partial U}{\partial x_i} \frac{w\, dl_i}{p_i} \overset{!}{=} \frac{\partial U}{\partial l_i} dl_i \Rightarrow \frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial l_i}} = \frac{p_i}{w}$$

FONC

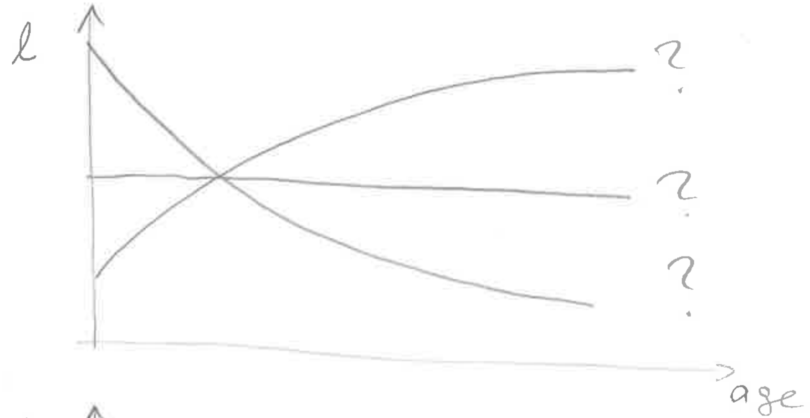
$$w = \frac{\Delta U}{\Delta l}$$

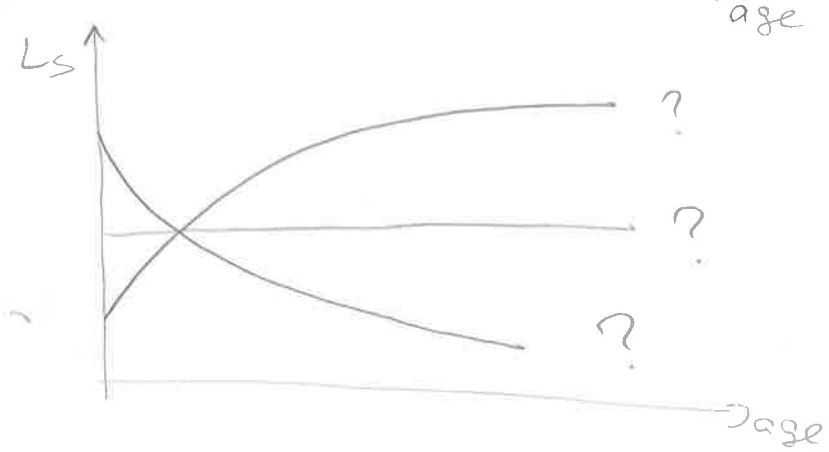
$$\lambda = \frac{\Delta U}{p_i \Delta X_i}$$

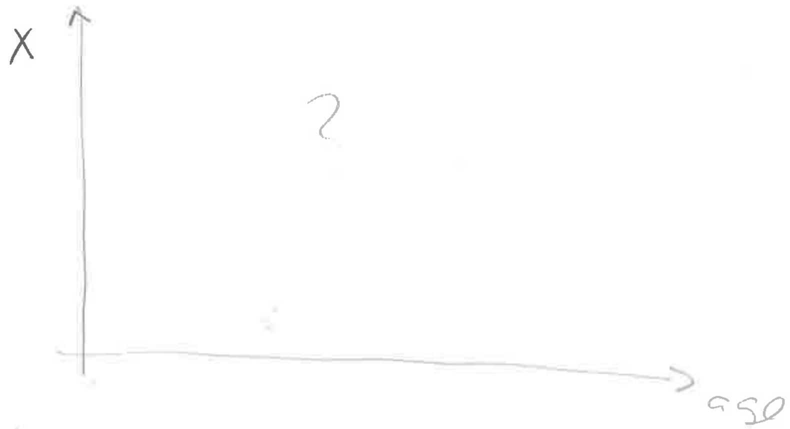
$$\frac{w}{\lambda} = \frac{\cancel{\Delta U}}{\Delta l} \frac{p_i \Delta X_i}{\cancel{\Delta U}}$$

$$= \left(\frac{p_i \Delta x_i}{\Delta l}\right) =$$

MOTIVATION FOR LIFE HORIZON LABOR SUPPLY







- what is likely to play role ?
  - preferences — risk aversion, foresight, prices
  - capital market
  - household work
  - hh members

o. PERIOD MODEL

Y_0, Y_1 income in periods $t = 0, 1$

X_0, X_1 consumption

l_0, l_1 (L_0, L_1) leisure (Labor supply)

Assume perfect capital market with interest rate r

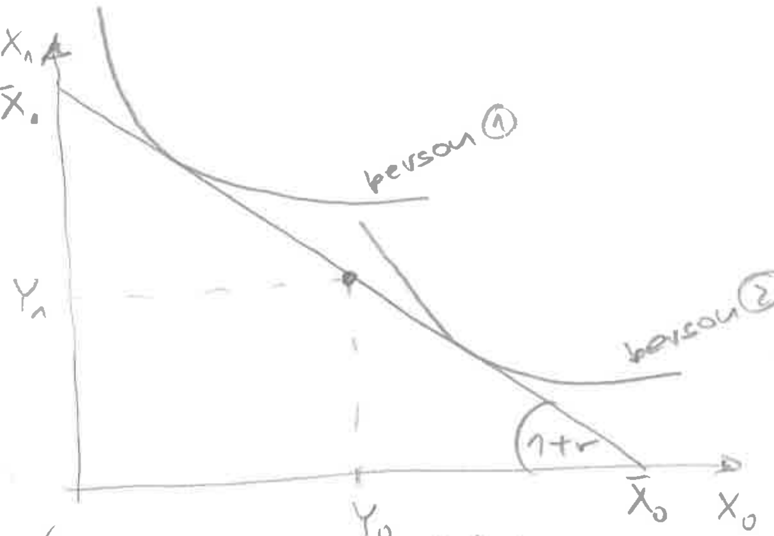
1. $\text{Max } X_0 \equiv \bar{X}_0 = Y_0 + \frac{Y_1}{1+r} \Rightarrow X_1 = 0$

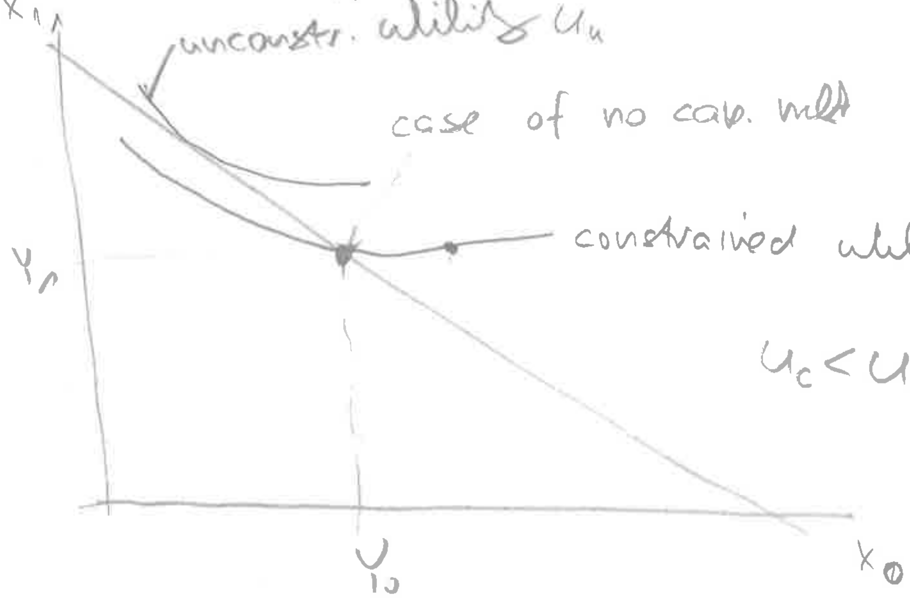
2. $\max X_1 \equiv \bar{X}_1 = (1+r)Y_0 + Y_1 \Rightarrow X_0 = 0$

3. Consuming in both periods

$$X_1 = \bar{X}_1 - (1+r)X_0 \Rightarrow \underbrace{\overbrace{X_0 + \frac{X_1}{1+r}}^{\text{choice}} = \overbrace{Y_0 + \frac{Y_1}{1+r}}^{\text{fixed}}}_{\text{Budget constraint}}$$

slope: $\frac{\bar{X}_1}{\bar{X}_0} = \frac{Y_1 + (1+r)Y_0}{\frac{Y_1}{1+r} + Y_0} = (1+r)\frac{\cancel{\frac{Y_1}{1+r} + Y_0}}{\cancel{\frac{Y_1}{1+r} + Y_0}}$





consider regimes: socialism, communism, mkt

LTIPERIOD CASE → n+1 periods

$$\text{Max } U = u[(x_0,l_0), \dots (x_n,l_n)]$$

$$z_i = f_i(x_i, l_i)$$

$$= \sum_{i=0}^{n} \beta^i z_i$$

$$\text{s.t. } \sum_{i=0}^{n} \left(\frac{1}{1+r}\right)^i x_i = x_0 + \sum_{i=0}^{n} \frac{1}{1+r} w_i \underbrace{(H_0 - l_i)}_{L_i}$$

$$\sum \alpha^i x_i + \sum \alpha^i l_i w_i = x_0 + H_0 \sum \alpha^i w_i$$

$$\mathcal{L} = \sum \beta^i \underline{\underline{z_i}} + \lambda\left[x_0 + H_0 \sum \alpha^i w_i - \sum \alpha^i \underline{\underline{x_i}} - \sum \alpha^i \underline{\underline{l_i}} w_i\right]$$

FONC: $\dfrac{\partial u}{\partial x_i} = \dfrac{\partial u}{\partial z_i}\dfrac{\partial z_i}{\partial x_i} = \beta^i \dfrac{\partial z_i}{\partial x_i} \overset{!}{=} \lambda \alpha^i$

$$\frac{\partial u}{\partial l_i} = \frac{\partial u}{\partial z_i}\frac{\partial z_i}{\partial l_i} = \beta^i \frac{\partial z_i}{\partial l_i} \overset{!}{=} \lambda \alpha^i w_i$$

assume separability

L_i does not affect L_j

assume $f_i' > 0$, $f_i'' < 0$

assuming additivity of U

β – time preference param

$= \dfrac{1}{1+\rho}$ → rate of time preference

for $\rho > 0$ → today is better

$\alpha = \dfrac{1}{1+r}$ discounting → int. rate of interest

$f_i = f \ \forall i$

w – exogenous param.

#1 $\dfrac{\partial u / \partial l_i}{\partial u / \partial x_i} = w_i$ as in 1 period model

#2 $\dfrac{\partial z_i / \partial x_i}{\partial z_j / \partial x_j} = \dfrac{\alpha^{i-j}}{\beta^{i-j}} \overset{!}{=} \left(\dfrac{1+\rho}{1+r}\right)^{i-j}$

EXPLORE: consumption over time x_i

Assume: $i > j$; $\rho < r$ $\Rightarrow$ $\underbrace{\left(\dfrac{1+\rho}{1+r}\right)}_{<1}^{\overset{>0}{i-j}} < 1 \Rightarrow \dfrac{\partial z_i}{\partial x_i} < \dfrac{\partial z_j}{\partial x_j}$

slope (i)

slope (j)

consumption increasing with t (age)

SUMMARY REPETITION OF MULTIPERIOD MODEL

$$u = \sum_{t=1}^{T} \beta^{t-1} z_t$$

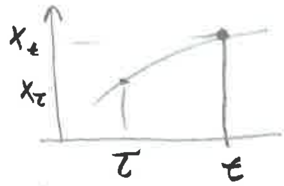
$\beta \sim$ time pref coef. $= \frac{1}{1+\rho} < 1$

$\rho > 0$ better today than tomorrow

#1 $\frac{\partial u / \partial l_t}{\partial u / \partial x_t} \overset{!}{=} w_t \quad \forall t$

$p_k = \left(\frac{1}{1+r}\right)^k$ ... discount factor

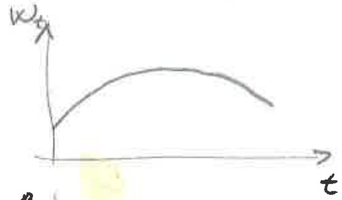
#2 $\frac{\partial z_t / \partial x_t}{\partial z_\tau / \partial x_\tau} \overset{!}{=} \frac{p^{t-\tau}}{\beta^{t-\tau}} = \left(\frac{1+\rho}{1+r}\right)^{t-\tau} \rightarrow$

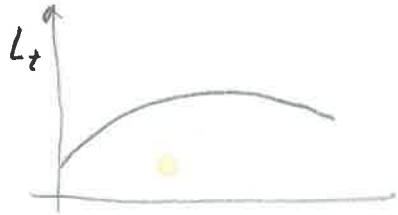


if $\rho < r$

$t > \tau$

#3 $\frac{\partial z_t / \partial l_t}{\partial z_\tau / \partial l_\tau} \overset{!}{=} \left(\frac{1+\rho}{1+r}\right)^{t-\tau} \frac{w_t}{w_\tau} \Rightarrow$





- Participation: depends on reserv. wage over time

CASES for $t > \tau$: $w_t \overset{?}{>} w_\tau^*$ non-partic: $\frac{\partial u / \partial l_t}{\partial u / \partial x_t} > w_t$

$w_t > w_t^*$ and $w_t < w_t^*$

if $w_\tau^* = w_t^* = const(?)$ → high participation when w is high

if $w_\tau^* \neq w_t^*$ – in case of women with kids time at home becomes expensive → high w^*

at older ages retirement programs introduce disincentives → steeper decline what about leisure l_i vs l_j $\quad i>j$

CASE 1

$$\boxed{\frac{\partial z_i / \partial l_i}{\partial z_j / \partial l_j} \overset{!}{=} \left(\frac{1+\rho}{1+r}\right)^{i-j} \frac{w_i}{w_j}}$$

CASE 1 assume $\rho = r \rightarrow (1)^{i-j} = 1 \quad \forall\, i>j$

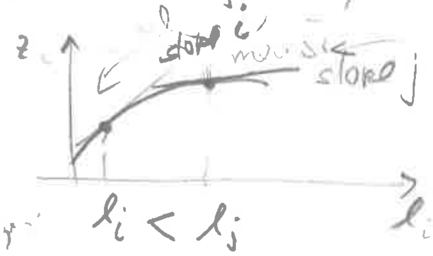
$$\frac{\partial z_i}{\partial l_i} = (1)\, \frac{w_i}{w_j}\, \frac{\partial z_j}{\partial l_j}$$

a) if $w_i = w_j = w_{const} \rightarrow l_i$ const over time

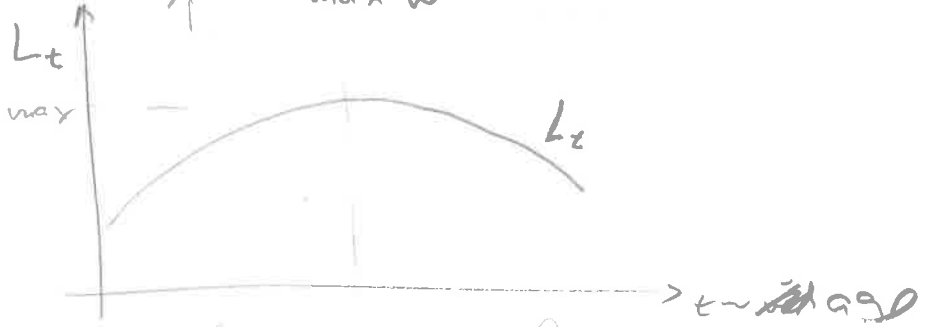
b) if w_i

$\rightarrow \frac{w_i}{w_j} > 1$

$$\Rightarrow \frac{\partial z_i}{\partial l_i} = (1) \overbrace{\left(\frac{w_i}{w_j}\right)}^{>1} \left(\frac{\partial z_j}{\partial l_j}\right)$$



growing $\Leftarrow$ growing



permanent change

c_1^*