

Figure 3.2 The sequence of human capital outputs during and after school

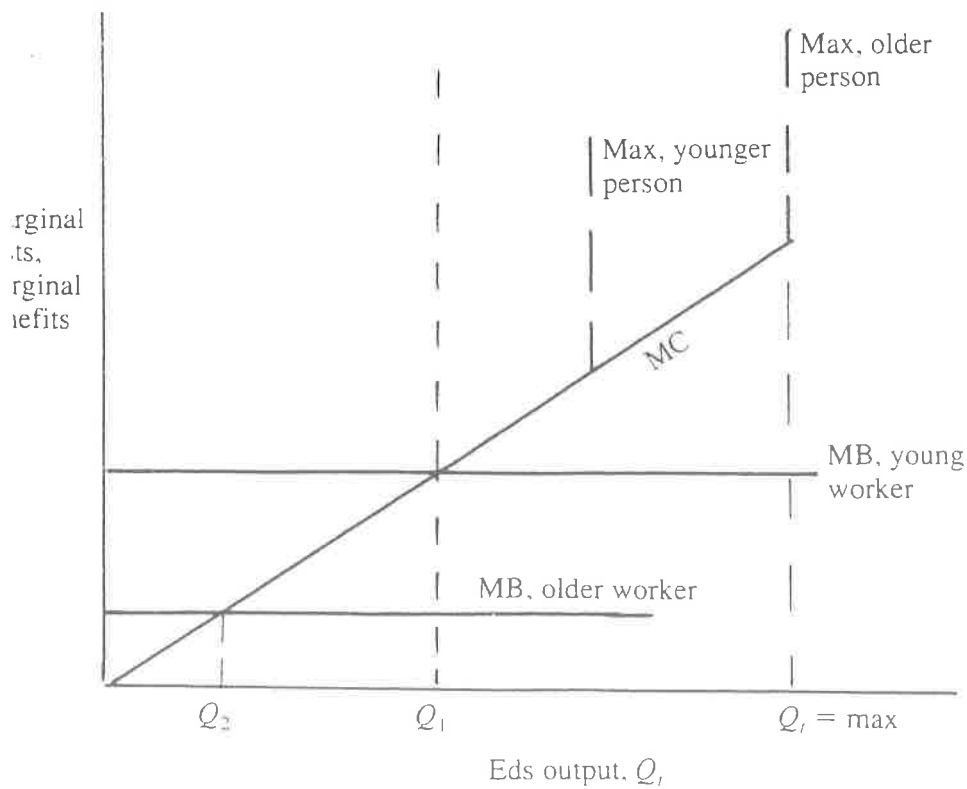


Figure 2.6 Total and marginal benefits and costs of producing eds

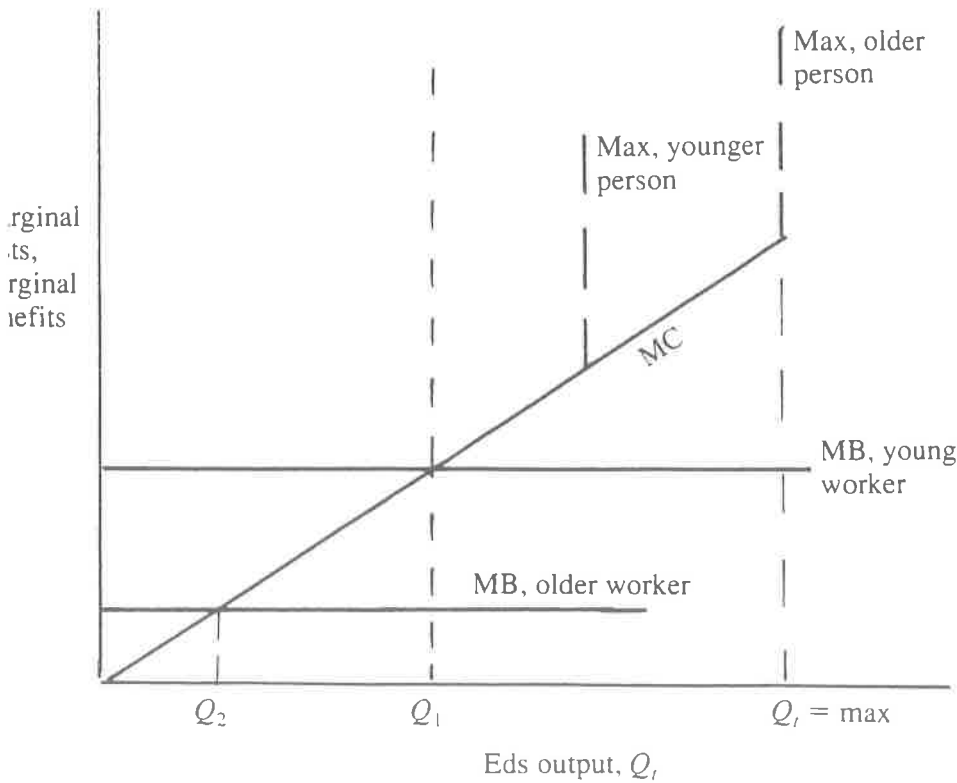
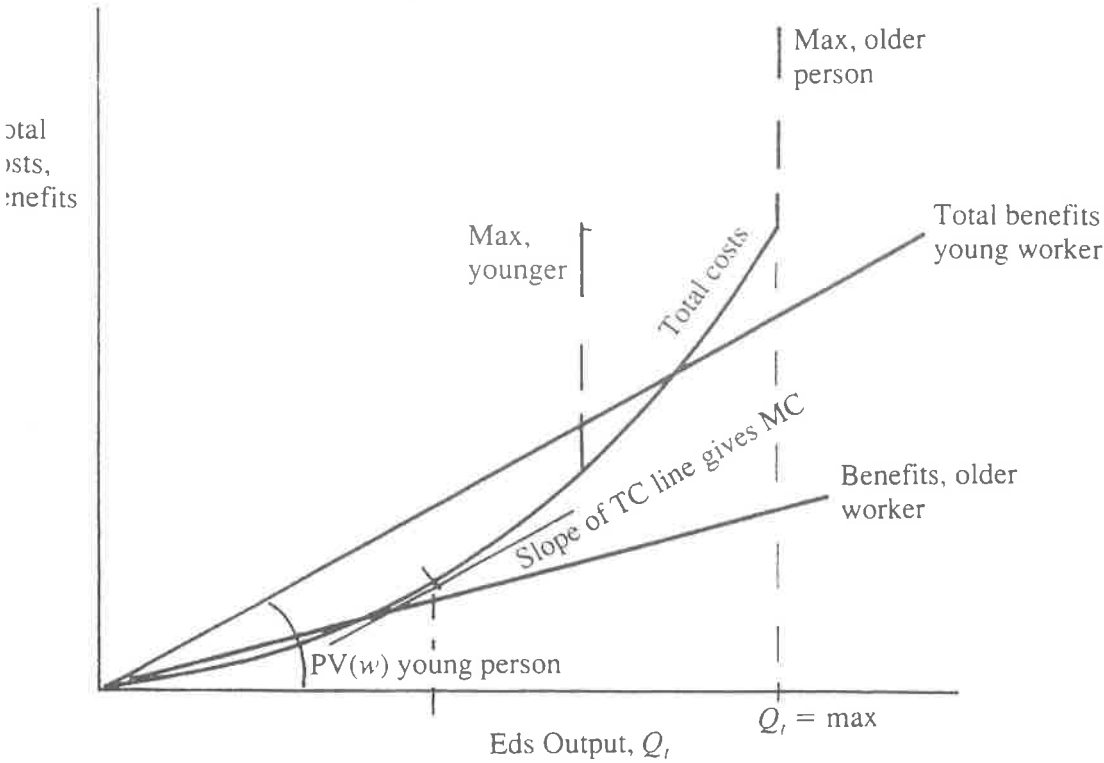


Figure 2.6 Total and marginal benefits and costs of producing eds

Human capital is a very profitable investment, but the individual cannot produce enough of it, and so chooses $s_t = 1$. Q_t then declines continuously as one gets older reaching zero at retirement. Accumulated human capital is computed by adding the annual investments. The process of adding yearly human capital investments yields a stock of human capital curve, K_t , as depicted in figure 2.7. Note that the stock of human capital increases quickly in the period when $s_t = 1$, then more slowly in middle age, and stops increasing

• SIMPLE MODEL OF OPTIMIZING INVESTMENT INTO HC

$$E_t = E_{t-1} + r C_{t-1}, \quad C_t = C_0 \left[1 - \frac{t}{T} \right] \quad \text{Ben Porath}$$

or

$$dE_t = r C_{t-1} \quad \frac{C_t}{E_t} = h_0 \left[1 - \frac{t}{T} \right]$$

$w \equiv 1$ (assume)

• OPTIMIZATION ON C_t

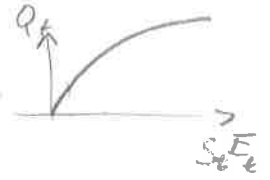
$r \dots$ discount rate

we know $\frac{\Delta \$}{r} [1 - \alpha^T] \geq \Delta \text{Costs}$

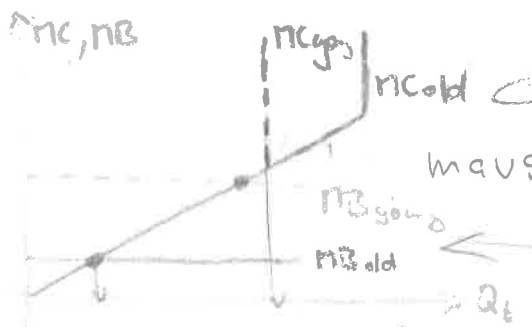
$S \dots$ as $\frac{1}{b}$ -time allocation to study

1) Assume $E_t = E_{t-1} + Q_t \equiv \frac{dE_t}{dt} = Q_t, \quad w = 1$

2) HC prod. fcn: $Q_t = (S_t E_t)^b, \quad \alpha < b < 1, \quad \alpha < S_t < 1$



3) Compare marginal costs: $\text{Cost}_t = S_t E_t \Rightarrow \text{Cost}_t = Q_t^{1/b}$



for $b = .5$

$$MC = \frac{\partial \text{Cost}_t}{\partial Q_t} = \frac{1}{b} Q_t^{1/b - 1}$$

marginal benefits: $BEN_t = \frac{1}{r} \left(1 - \frac{1}{(1+r)^{T-t}} \right) Q_t$

$$MB_t = \frac{\partial BEN_t}{\partial Q_t} = \frac{1}{r} \left(1 - \frac{1}{(1+r)^{T-t}} \right)$$

$$MC = MB \Rightarrow \frac{\partial BEN_t}{\partial Q_t} = \frac{\partial \text{Cost}_t}{\partial Q_t} \Rightarrow \frac{1}{r} \left(1 - \frac{1}{(1+r)^{T-t}} \right) = \frac{1}{b} Q_t^{1/b - 1}$$

$$Q_t^* = \left[\frac{b}{r} \left(1 - \frac{1}{(1+r)^{T-t}} \right) \right]^{b/(1-b)}$$

$$Y_t = E_t - S_t E_t = (1 - S_t) E_t^*$$

It could be that $S_t^* > 1 \rightarrow$ Full inv
 $MB > MC$

see graph

$$\int dE_t^* = \int Q_t^* dt$$

$$E_t^* = E_s + \dots \Rightarrow S_t^* = \frac{Q_t^{*1/b}}{E_t^*}$$