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An Empirical Model of Labor Supply in a Life-Cycle Setting

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This paper formulates and estimates a structural intertemporal model of labor supply. Using theoretical characterizations derived from an economic model of lifetime behavior, a two-step empirical analysis yields estimates of intertemporal and uncompensated substitution effects which provide the information needed to predict the response of hours of work to life-cycle wage growth and shifts in the lifetime wage path.

Introduction

Over the past several years there has been considerable activity in formulating life-cycle models of labor supply. Most of this work has gone unnoticed in the empirical literature. This study develops an estimable model of labor supply that fully incorporates life-cycle factors, and it devises simple econometric procedures for estimating this model.

Most empirical work on labor supply assumes decision making in a one-period context. Typically, annual hours of work are regressed on the current hourly wage rate and some measure of property income. A worker, however, determines his current labor supply in a life-cycle setting. Unless credit markets are "perfectly imperfect" and there is no human capital accumulation, the supply of labor is a function of

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current and future discounted wage rates as well as wealth and constraints in other periods. Accordingly, regressions of hours of work on current hourly wage rates yield a wage coefficient that confuses the response of labor supply to wage changes of three types: those arising from movements along a given lifetime wage profile, those arising from shifts in the wage profile, and those arising from changes in the profile slope. As a result, the wage coefficient usually reported in empirical studies has no behavioral interpretation in the context of a life-cycle framework.

The theory underlying the model of lifetime consumption and hours of work in this paper represents a natural extension of Friedman's (1957) permanent income theory to a situation in which the relative price of consumption and leisure varies over the life cycle. Theoretical characterizations of consumption and labor supply derived from this theory sharply distinguish between factors determining a consumer's dynamic behavior and factors determining differences in consumption and hours of work across consumers. This separation leads to a manageable empirical model that allows one to discriminate the responses of labor supply to wage changes attributable to movements along a lifetime wage profile from those responses attributable to parametric changes in this profile. In addition, one can estimate the effects of wealth and demographic characteristics on lifetime hours of work.

The organization of this paper is as follows. Section I outlines an economic model of life-cycle behavior. Section II develops and discusses an empirical model of labor supply. Section III interprets the parameters of this empirical model. Section IV contains the empirical analysis.

I. A Life-Cycle Model of Consumption and Labor Supply

The consumer is assumed to choose consumption and hours of leisure at each age to maximize a lifetime preference function that is strongly separable over time, subject to a wealth constraint. Let utility at age t be given by the concave function $U[C(t), L(t)]$, where $C(t)$ is the amount of market goods consumed and $L(t)$ is the number of hours spent in nonmarket activities at age t. The consumer starts life with assets $A(0)$ and operates in an environment of perfect certainty. At each age t he faces a real wage rate equal to $W(t)$ assumed to be exogenously given. The consumer can freely borrow and lend at a real rate of interest equal to $r(t)$ in period t, and his rate of time preference is ρ. A lifetime is assumed to consist of $T + 1$ periods with L^* being the total number of hours in each period.

Formally, the consumer's problem is to choose $C(t)$ and $L(t)$ at each age to maximize the lifetime preference function

$$G\left\{\sum_{t=0}^{T} \frac{1}{(1+\rho)^t} U[C(t), L(t)]\right\} \tag{1}$$

subject to the wealth constraint

$$A(0) + \sum_{t=0}^{T} R(t)N(t)W(t) = \sum_{t=0}^{T} R(t)C(t), \tag{2}$$

where $G(\cdot)$ is a monotonically increasing function, $N(t) \equiv L^* - L(t)$ is hours of work at age t, and $R(t) \equiv 1/\{[1 + r(1)][1 + r(2)] \ldots [1 + r(t)]\}$ is the discount rate which converts real income in period t into its period 0 equivalent with $R(0) = 1$.

Conditions for an optimum are satisfaction of the lifetime budget constraint and

$$U_1[C(t), L(t)] = R(t)(1 + \rho)^t \lambda, \qquad t = 0, \ldots, T, \tag{3}$$

$$U_2[C(t), L(t)] \geqslant R(t)(1 + \rho)^t \lambda W(t), \qquad t = 0, \ldots, T, \tag{4}$$

where subscripts denote partial derivatives and λ is defined by $\lambda = \lambda^*/G'$, where λ^* is the Lagrange multiplier associated with the wealth constraint (i.e., λ^* is the marginal utility of wealth in period 0) and G' is the derivative of G. According to condition (3), consumption is chosen so that the marginal utility of consumption equals the marginal utility of wealth after adjusting for a discount factor which depends on the rate of time preference and the rate of interest. Condition (4) determines the consumer's choice of leisure. If it is an equality, then a positive amount of labor is supplied to the market. If it is a strict inequality, then all time is devoted to nonmarket activities.

Using the definition of labor supply (i.e., $N[t] \equiv L^* - L[t]$) and the implicit-function theorem, it is possible to solve equations (3) and (4) for consumption and labor supply as functions of the form

$$C(t) = C[R(t)(1 + \rho)^t \lambda, W(t)], \qquad t = 0, \ldots, T, \tag{5}$$

$$N(t) = N[R(t)(1 + \rho)^t \lambda, W(t)], \qquad t = 0, \ldots, T. \tag{6}$$

The functions $C(\cdot, \cdot)$ and $N(\cdot, \cdot)$ depend only on the functional form of $U(\cdot, \cdot)$. As a consequence of concavity of U and the assumption that consumption and leisure are normal goods, they satisfy

$$C_1 < 0, N_1 \geqslant 0, N_2 \geqslant 0.[1] \tag{7}$$

[1] See Heckman (1974, 1976) for proofs of these inequalities. Heckman develops and uses functions equivalent to those given by eqq. (5) and (6) in his analysis of the behavior of consumption and labor supply over the life cycle.

These consumption and labor-supply functions allow for corner solutions for hours of work either at age t or at any other age t'. No matter what the consumer's labor-force participation pattern over his lifetime, consumption and labor-supply decisions at any age (including the decision to set hours of work equal to zero) are completely determined by the functions $C(\cdot,\cdot)$ and $N(\cdot,\cdot)$ and the values of the variables $R(t)(1 + \rho)^t\lambda$ and $W(t)$.[2]

The relationships given by (5) and (6) hereafter will be referred to as the "λ constant" consumption and labor-supply functions. They represent the marginal utility of wealth constant demand functions for consumption and leisure for a particular form of the lifetime preference function given by (1), namely, the one obtained when G is the identity transformation. For this particular choice of G, λ is the marginal utility of wealth in period 0. Given a choice of $U(\cdot,\cdot)$, it is theoretically possible to compute a unique value for λ using data on an individual's consumption, labor supply, and wage rate at a point in time. This fact receives much attention in the formulation of the empirical model which is discussed in the next section.

Substituting the λ constant consumption and labor-supply functions into the budget constraint given by (2) yields the equation

$$A(0) = \sum_{t=0}^{T} R(t)\{C[R(t)(1 + \rho)^t\lambda, W(t)]\}.$$
$$- W(t)\{N[R(t)(1 + \rho)^t\lambda, W(t)]\}. \tag{8}$$

This equation implicitly determines the optimal value of λ; λ, then, can be expressed as a function of initial assets, lifetime wages, interest rates, rates of time preference, and "consumer tastes." Concavity of preferences implies

$$\frac{\partial\lambda}{\partial A(0)} < 0 \quad \text{and} \quad \frac{\partial\lambda}{\partial W(t)} \leq 0, \qquad t = 0, \ldots, T.\text{[3]} \tag{9}$$

[2] If it is optimal for the consumer to work at age t, then condition (4) is an equality and the functions $C(\cdot,\cdot)$ and $N(\cdot,\cdot)$ represent the solutions of eqq. (3) and (4) for the variables $C(t)$ and $N(t) = L^* - L(t)$, respectively. If, on the other hand, the necessary condition given by (4) is an inequality, i.e., $U_2[C(t), L^*] > R(t)(1 + \rho)^t\lambda W(t)$, then the consumer chooses not to work. In this case, $N(\cdot,\cdot) = 0$ and the function $C(\cdot,\cdot)$ is the solution of the equation $U_1[C(t), L^*] = R(t)(1 + \rho)^t\lambda$ for $C(t)$. In either case, $C(\cdot,\cdot)$ and $N(\cdot,\cdot)$ only contain the variables $R(t)(1 + \rho)^t\lambda$ and $W(t)$ as arguments and their functional form depends only on the form of the period t utility function $U(\cdot,\cdot)$. For further discussion on this issue see Heckman and MaCurdy (1980). Introducing age dependence into the utility function does not change any of this analysis. If the utility function at age t is given by $U[C(t), L(t), X(t)]$, where $X(t)$ is a vector of time-varying determinants of "consumer tastes," then a third argument $X(t)$ enters the consumption and labor-supply functions given by (5) and (6). These functions satisfy the restrictions given by (7), and they also allow for corner solutions.
[3] See Heckman (1974, 1976) for proof of these propositions.

Inspection of the λ constant functions reveals that consumption and labor-supply decisions at a point in time are related to variables outside the decision period only through λ. Thus, except for the value of the current wage rate, λ summarizes all information about lifetime wages and property income that a consumer requires to determine his optimal current consumption and labor supply. At any age, any path of wages or property income over a consumer's lifetime that keeps λ and the current wage constant implies the same optimal current consumption and labor-supply behavior.

The λ constant functions represent an extension of Friedman's (1957) permanent income theory to a situation in which the relative price of consumption and leisure varies over the life cycle. According to these functions, current consumption and labor-supply decisions depend on a permanent component and the current wage rate. The variable λ is like permanent income in the theory of the consumption function. At each point in time it is a sufficient statistic for all historic and future information about lifetime wages and property income that is relevant to the current choice of consumption and labor supply. The usual concept of permanent income or wealth does not qualify as a sufficient statistic for this retrospective and prospective information.[4] Given knowledge of permanent income, a consumer also requires information on future wages to determine his optimal current consumption and labor supply. Only if wages are constant over the life cycle, or labor supply is exogenously determined, can λ be written as a simple function of permanent income or wealth.

The λ constant consumption and labor-supply functions fully characterize a consumer's dynamic behavior in a world of perfect certainty.[5] According to these functions, there are two reasons why a consumer might change his consumption or hours of work as he ages: (1) the real wage rate changes, or (2) the rate of interest varies and is not equal to the rate of time preference.[6]

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[4] Permanent income here is defined as that stream of income whose discounted value equals the consumer's wealth in present value terms. Formally, permanent income in period 0, Y_p, is defined by the equation $A(0) + \Sigma_{t=0}^{T} R(t)N(t)W(t) = Y_p \Sigma_{t=0}^{T} R(t)$.

[5] See MaCurdy (1978, 1980, in press) for a discussion of the uncertainty case. It is shown in these papers that with minor modifications the empirical specifications of labor supply developed in this section and their implementation in the following sections are consistent with a world in which the consumer is uncertain about his future lifetime path of wages and property income.

[6] Changes in a consumer's tastes can also be a reason for adjustments in consumption and labor supply over the life cycle. As discussed in n. 2 above, if the period utility function is age dependent, the λ constant consumption and labor-supply functions will also be age dependent. It is still true, however, that the λ constant functions fully characterize a consumer's dynamic behavior.

II. An Empirical Model

This section formulates an empirical model of labor supply that is based on the economic model described above. Specific functional forms are proposed for the λ constant labor-supply function and for the relationship between λ and such variables as lifetime wages and initial assets. The following discussion assumes the availability of panel data.

An Empirical Specification for the λ Constant Labor-Supply Function

Assume that consumer i at age t has utility given by

$$U_i[C_i(t), L_i(t)] = Y_{1i}(t)[C_i(t)]^{\omega_1} - Y_{2i}(t)[N_i(t)]^{\omega_2}, \tag{10}$$

where $0 < \omega_1 < 1$ and $\omega_2 > 1$ are time-invariant parameters common across workers, and $Y_{1i}(t)$, $Y_{2i}(t) > 0$ are age-specific modifiers of "tastes." The variables $Y_{1i}(t)$ and $Y_{2i}(t)$ depend on all of consumer i's characteristics which plausibly affect his preferences at age t; these characteristics may include such variables as the number of children present at age t, the consumer's education, and even age itself.

Assuming an interior optimum,[7] the implied λ constant labor-supply function for consumer i at age t in natural logs is

$$\ln N_i(t) = \frac{1}{\omega_2 - 1} \Big\{ \ln \lambda_i - \ln Y_{2i}(t) - \ln \omega_2$$
$$+ \ln[R(t)(1 + \rho)^t] + \ln W_i(t) \Big\}. \tag{11}$$

Assuming that "tastes" for work are randomly distributed over the population according to the equation $\ln Y_{2i}(t) = \sigma_i - u_i^*(t)$, the labor-supply function can be written as

$$\ln N_i(t) = F_i + \delta \sum_{k=0}^{t} [\rho - r(k)] + \delta \ln W_i(t) + u_i(t), \tag{12}$$

where $F_i = [1/(\omega_2 - 1)](\ln \lambda_i - \sigma_i - \ln \omega_2)$, $\delta = 1/(\omega_2 - 1)$, $u_i(t) = \delta u_i^*(t)$, $r(0) = \rho$, and we have used the approximations $\ln[1 + r(t)] \approx r(t)$ and $\ln(1 + \rho) \approx \rho$. The unobserved variables σ_i and $u_i(t)$ represent the unmeasured characteristics of consumer i; σ_i is a permanent component, and $u_i(t)$ is a time-varying error term with zero mean. If we assume that the real rate of interest, $r(t)$ for $t \geq 1$, is constant over

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[7] Since this study's empirical objective is to examine the labor-supply behavior of prime-age males, this assumption is not unreasonable.

the life cycle and equal to r, then the λ constant hours of work function reduces to

$$\ln N_i(t) = F_i + bt + \delta \ln W_i(t) + u_i(t), \tag{13}$$

where $b = \delta(\rho - r)$.[8]

The intercept term F_i in this equation represents a time-invariant component that is unique to individual i. This study treats F_i as a fixed effect. Since F_i contains $\ln \lambda_i$ as one of its components, one cannot assume that F_i is a "random factor" uncorrelated with exogenous variables of the model. Inspection of equation (8) reveals that λ depends on the values of variables and constraints in all periods. By construction, F_i is correlated with any exogenous variables used to predict a consumer's wages or wealth. Hence, treating F_i as part of the error term would result in biased parameter estimates of the labor-supply function. Treating F_i as a fixed effect, on the other hand, avoids this bias.

Estimating the parameters of equation (12) only requires variables observed within the sample period. Regressions of current hours of work on individual specific intercepts and current wage rates produce a full set of parameter estimates.[9] Because F_i captures the effect of $\ln \lambda_i$, its estimated value summarizes all of the retrospective and prospective information relevant to consumer i's current choices. As there is no need to forecast any life-cycle variables that are outside the

[8] There are other forms of the utility function that have convenient empirical specifications for the λ constant consumption and leisure demand functions. Two such functions are

$$U_i(t) = \kappa_i(t)[C_i(t) + \mu_i^*(t)]^{\omega^*}[L_i(t) + \mu_i(t)]^{\omega},$$

$$\kappa_i(t), \omega^*, \omega > 0, \omega^* + \omega < 1 \text{ or } \kappa_i(t), \omega^*, \omega < 0;$$

$$U_i(t) = \kappa_i^*(t) \frac{[C_i(t) + \mu_i^*(t)]^{\omega^*} - 1}{\omega^*} + \kappa_i(t) \frac{[L_i(t) + \mu_i(t)]^{\omega} - 1}{\omega},$$

$$\kappa_i^*(t), \kappa_i(t) > 0, \omega^*, \omega < 1,$$

where $\kappa_i^*(t)$, $\kappa_i(t)$, $\mu_i^*(t)$, $\mu_i(t)$, ω^*, and ω are all parameters. Both of these functions are concave. They include Cobb-Douglas, addilog, CES, and Stone-Geary as special cases. The λ constant functions for consumption and leisure are log linear in λ, wages, and the coefficients $\kappa_i^*(t)$ and $\kappa_i(t)$, which represent specific modifiers of tastes. This study uses the utility function given by (10) to formulate an empirical model because it implies a form for the labor-supply function which can be readily compared with labor-supply equations found in existing empirical work.

[9] If utility at age t depends on measured characteristics of the consumer that vary over the sample period, then current values of these "taste-shifter" variables would also be included as regressors. A natural way to introduce such taste-shifter variables is to model the taste coefficient $Y_i(t)$ as a function of the form $\ln Y_i(t) = \sigma_i + X_i(t)\beta - u_i^*(t)$, where $X_i(t)$ is a vector of variables influencing tastes and β is a parameter. For this case, $X_i(t)\beta\delta$ enters as an additional linear term in the λ constant labor-supply equation given by (12) or (13).

sample period, the λ constant functions afford a considerable simplification of the empirical analysis.

Use of the λ constant functions allows one to estimate parameters needed to characterize dynamic behavior without introducing any assumptions regarding a consumer's behavior outside the sample period. To appreciate this point, consider the problem of predicting the additional hours of work a consumer will supply in response to observing a higher wage rate than he observed at a younger age. To obtain an estimate of this response using a traditional model of life-cycle labor supply, one must formally incorporate the worker's future plans in the model. For example, if the worker anticipates an early retirement, future wages corresponding to the retirement years do not influence current labor supply. Thus, the researcher must not include these wages as explanatory variables. Such considerations lead to difficult data requirements and complicated estimation procedures. Using equation (12), on the other hand, it is possible to analyze this problem without knowing anything about a worker's future plans; an individual constant term for each worker accounts for a worker's future plans in a parametrically simple way.

An Empirical Specification for Individual Effects

Estimation of the λ constant labor-supply function given by (12) does not directly estimate all of the parameters required to characterize all aspects of labor supply. Differences among individuals in initial wealth or lifetime wage paths affect the level of hours of work through F_i. To explain any aspect of labor supply other than dynamic behavior (e.g., how the hours of work of two individuals differ at a point in time), one must confront the problem of predicting individual effects.

From the theoretical analysis above, we know that the value of F, or more properly λ, is uniquely determined by the implicit equation given by (8). This equation does not admit an analytical solution for λ given the specific form of the utility function given by (10), even if it is known that this function applies to all ages and the consumer works in each period. The variable λ is a complicated function of initial assets, lifetime wages, the interest rate, the rate of time preference, and parameters representing unobserved "taste" variables.[10] Using such a relationship as an empirical specification is not feasible.

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[10] The equation determining λ_i is

$$A_i(0) = \sum_{t=0}^{T} R(t) \left\{ \left[\frac{1}{\Upsilon_{1i}(t)\omega_1} R(t)(1 + \rho)^t \lambda_i \right]^{1/(\omega_1 - 1)} \right.$$
$$\left. - W_i(t) \left[\frac{1}{\Upsilon_{2i}(t)\omega_2} R(t)(1 + \rho)^t \lambda_i W_i(t) \right]^{1/(\omega_2 - 1)} \right\}.$$

This study assumes that equation (8) implies a solution for λ in which $\ln \lambda$ can be approximated as a linear function of measured characteristics, the natural log of wages at each age, initial wealth, and an unobserved random variable representing unmeasured characteristics. With this assumption the implied equation for F_i is

$$F_i = Z_i\phi + \sum_{t=0}^{T^*} \gamma(t) \ln W_i(t) + A_i(0)\theta + a_i, \qquad (14)$$

where Z_i is a vector of observed variables (e.g., family background variables), a_i is an error term, and ϕ, $\gamma(t)$, and θ are parameters assumed to be constant across consumers.[11] This structural relationship for F_i implicitly assumes that each consumer has a working life of $T^* + 1$ years. According to the theoretical restrictions given by (9), the $\gamma(t)$'s and θ should all be negative.

Unfortunately, to formulate an estimable version of an equation for F_i, we require additional assumptions concerning the forms of the lifetime wage and income paths. In contrast to the λ constant labor-supply function, estimating the parameters of equation (14) requires data which normally are not available. Most variables appearing in this equation are not directly observed, including the dependent variable F_i, wages outside the sample period, and initial wealth. While estimates of F_i are obtained as a by-product from estimating equation (12), we still require a mechanism for predicting wages outside the sample period and initial permanent income. We do this by introducing lifetime profiles for wages and income.

This study assumes that the lifetime wage path is

$$\ln W_i(t) = \pi_{0i} + t\pi_{1i} + t^2\pi_{2i} + V_i(t), \qquad (15)$$

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[11] The assumption that these parameters are constant across consumers is, of course, only an approximation. Formally, it can be shown that

$$\gamma(t) = \delta\frac{\partial \ln \lambda}{\partial \ln W(t)} = \delta\left[\frac{\partial \ln N(t)}{\partial \ln \lambda} + 1\right]\frac{\partial \ln \lambda}{\partial A(0)}E^*(t),$$

where $E^*(t) = N(t)W^*(t)$ and $W^*(t) = W(t)R(t)$ are the period 0 present value of earnings and wages in period t. This relationship for $\gamma(t)$ is derived by differentiating Roy's identity for $N(t)$ with respect to $A(0)$ to obtain the equation

$$\frac{\partial \lambda}{\partial W^*(t)} = \lambda\frac{\partial N(t)}{\partial A(0)} + N(t)\frac{\partial \lambda}{\partial A(0)} = \lambda N(t)\left[\frac{\partial \ln N(t)}{\partial \ln \lambda} + 1\right]\frac{\partial \ln \lambda}{\partial A(0)}$$

$$= \frac{\lambda}{W^*(t)}\left[\frac{\partial \ln N(t)}{\partial \ln \lambda} + 1\right]\frac{\partial \ln \lambda}{\partial A(0)}E^*(t).$$

Since the empirical specification for the λ constant hours of work function implies $[\partial \ln N(t)]/(\partial \ln \lambda) = \delta$, we see that we cannot formally have $\gamma(t) = \delta (\partial \ln \lambda)/[\partial \ln W(t)]$ and $\theta = \delta(\partial \ln \lambda)/[\partial A(0)]$ constant. Notice that the effects of the interest rate and time preference on F_i are absorbed into the coefficients of specification (14).

where π_{0i}, π_{1i}, and π_{2i} are linear functions of the form

$$\pi_{ji} = M_i g_j, \qquad j = 0, 1, 2, \tag{16}$$

M_i is a vector of exogenous determinants of wages which are constant over the consumer's lifetime (e.g., education and background variables), g_j, $j = 0$, 1, 2, are vectors of parameters, and $V_i(t)$ is an error term. This path assumes that wages follow a quadratic equation in age with an intercept and slope coefficients that depend on age-invariant characteristics of the consumer.[12]

Predicting a consumer's initial wealth is complicated by the fact that most data sets do not contain extensive measures of even the consumer's current wealth. Some measure of the consumer's property or nonwage income during the sample period, however, is usually available.[13] Let $Y_i(t)$ and $A_i(t)$ denote the property income and assets of consumer i at age t. If $Y_i(t)$ is the income flow generated by investing assets $A_i(t)$ at a rate of interest equal to r, we have the relationship $Y_i(t) = A_i(t)r$. Assume that the following quadratic equation in age approximates the lifetime path for property income:

$$Y_i(t) = \alpha_{0i} + t\alpha_{1i} + t^2\alpha_{2i} + \nu_i(t), \tag{17}$$

where α_{0i}, α_{1i}, and α_{2i} are linear functions of the form

$$\alpha_{ji} = S_i q_j, \qquad j = 0, 1, 2, \tag{18}$$

S_i is a vector of measured age-invariant characteristics of consumer i (e.g., education and background variables), q_j, $j = 0$, 1, 2, are parameter vectors, and $\nu_i(t)$ is an error term.[14] The intercept α_{0i} can be thought of as a measure of consumer i's permanent income at age 0; that is, $\alpha_{0i} = A_i(0)r$.[15]

Combining the lifetime paths for wages and income with equation (14) creates an equation for F_i that can be estimated using data

[12]In the following analysis it is assumed that this wage equation generates unbiased predictions for lifetime wages. These predictions need not be efficient nor do they need to be the same predictions used by consumers. It is possible to introduce many alternative forms for the wage equation, such as higher-order polynomials or polynomials with other functions of time (e.g., reciprocals) replacing t and t^2, with only minor modifications of empirical specifications used in the following analysis.

[13]These income measures seldom include imputed income generated by consumer durables, which is a major source of property income for most consumers.

[14] In contrast to $W(t)$, $Y(t)$ is determined endogenously in this model. Eq. (17) can be viewed as an approximation to the optimal lifetime path for $Y(t)$ expressed as a function of the exogenous variables of the model.

[15]Formally, this relationship between α_{0i} and $A_i(0)$ is correct only in a continuous time framework. When modeling the problem in discrete time, one must distinguish assets held at the beginning, at the end of the period, and exactly when asset income is earned within the period.

observed within the sample period. Substituting the wage process given by (15) and the relationship $\alpha_{0i} = rA_i(0)$ into equation (14) yields

$$F_i = Z_i\phi + \pi_{0i}\bar{\gamma}_0 + \pi_{1i}\bar{\gamma}_1 + \pi_{2i}\bar{\gamma}_2 + \alpha_{0i}\bar{\theta} + \eta_i, \qquad (19)$$

or substituting relations (16) and (18) yields

$$F_i = K_i\psi + a_i, \qquad (20)$$

where

$$\bar{\gamma}_j = \sum_{t=0}^{T^*} t^j\gamma(t), \qquad j = 0, 1, 2, \qquad \bar{\theta} = \theta/r, \qquad (21)$$

K_i is a vector including all age-invariant characteristics determining either wages, income, or λ (i.e., all the elements of M_i, S_i, and Z_i), ψ is a vector of coefficients, and η_i is a disturbance term which is randomly distributed across workers with zero mean. Equation (19) is a structural relationship between F and the characteristics of a consumer's wage and income profiles. The empirical analysis of this study focuses on estimating the parameters of this equation, ϕ, $\bar{\gamma}_0$, $\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{\theta}$. As we shall see shortly, these parameters have a sound economic interpretation. Equation (20) is essentially a reduced-form equation for (19). By estimating the parameters of this equation, it is possible to predict how F varies across consumers using only age-invariant characteristics of the consumer as explanatory variables.

III.   Interpretation of Parameters

In investigating the effect of changes in wages on labor supply, it is important to separate parametric change of the sort usually contemplated in comparative static exercises from evolutionary change due to movement along a life-cycle wage path.[16] A parametric wage change refers to shifts in a life-cycle wage profile (e.g., a shift from path II to path I in fig. 1), while an evolutionary wage change refers to movements along a given profile (i.e., along any path in fig. 1). Thus, parametric wage changes refer to differences in wages across consumers, while evolutionary wage changes refer to differences in wages across time for the same consumer.[17]

Consider the behavior of labor supply over the life cycle. As a consumer ages, he adjusts his hours of work in response to the

[16] This distinction goes back to Ghez and Becker (1975).

[17] This statement is true only in an environment of perfect certainty. If there is uncertainty about the future, a consumer can experience parametric wage changes as he acquires new information about his lifetime wage path. For a discussion of these issues see MaCurdy (1980, in press).
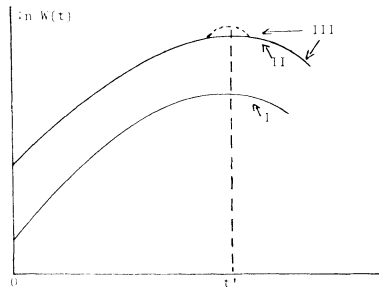
FIG. 1

different wage rates he observes at each point in his lifetime. These labor-supply adjustments represent responses to evolutionary wage changes; they reflect the consumer's desire to supply more hours in those periods with highest wages. There is no wealth effect associated with this kind of wage variation since the wage profile is known to the consumer at the beginning of his lifetime and changes in wages are due only to movement along this given profile. It is apparent from the labor-supply function given by (12) that the value of the parameter δ determines the hours of work response to evolutionary wage changes. Hereafter, I will refer to δ as the intertemporal substitution elasticity. The theoretical prediction for its sign is positive. For the particular form of the utility function given by (10), δ is also the direct elasticity of substitution for hours of work in any two periods.

Now compare the labor-supply profiles of two consumers who face wage paths II and III, respectively. As illustrated in figure 1, the wage profiles for consumers II and III are the same except at age t' when consumer III's wage rate is higher than consumer II's. Let Δ denote the absolute value of this difference in period t' wages. This wage difference represents a parametric wage change because it involves a shift in the lifetime path of wages. It causes the labor-supply profiles for consumers II and III to be different at all ages. Comparing these labor-supply profiles is the sort of problem usually considered in comparative static exercises. In terms of the empirical model outlined above, this higher wage rate has two effects on consumer III's labor supply. The first effect is on the value of F. According to equation (14), consumer III will set a value for F which is lower than the value of F for consumer II by an amount equal to $\gamma(t') \cdot \Delta$. This decline in F implies that at all ages other than t' consumer III's labor supply will be less than consumer II's by a constant fraction. At age t' there is a second effect of the wage difference. Neglecting the decline in F, consumer III's labor supply at age t' will be higher by an amount equal to $\delta \cdot \Delta$. Thus, the total impact on consumer III's hours of work

at age t' is $[\delta + \gamma(t')] \cdot \Delta$. Since $\delta > 0$ and $\gamma(t') < 0$, there is no sign prediction for $\delta + \gamma(t')$, so consumer III's hours of work at age t' may be greater than or less than consumer II's.

The parameters $\gamma(t')$ and $\delta + \gamma(t')$, then, determine the difference in consumer II's and consumer III's labor-supply profiles which is due to the discrepancy in their wage rates at age t'. The quantities $\gamma(t')$ and $\delta + \gamma(t')$ correspond to the usual concepts of cross- and own-uncompensated substitution elasticities. These elasticities describe the response of labor supply to parametric wage changes. They can be used to predict differences in labor supply across consumers. These elasticities do not directly provide information on the response of labor supply to evolutionary wage changes, so they cannot be used to predict differences in a given consumer's labor supply over time. Since the intertemporal substitution elasticity exceeds the own-uncompensated substitution elasticities (i.e., $\delta > \delta + \gamma[t']$), one expects an evolutionary wage change to induce a larger labor-supply response than a comparable parametric wage change. The wealth effect associated with a parametric wage change accounts for the smaller labor-supply response.

Comparing the labor-supply profiles for consumers I and II also involves a parametric wage change. As illustrated in figure 1, consumer II's wage profile exceeds consumer I's by a constant fraction over the entire life cycle. The parametric wage change associated with moving from consumer I to consumer II's wage profile is analogous to increasing the value of the intercept of the lifetime wage path, π_0. This has two effects on labor supply in each period. First, a consumer adjusts his value of F in response to the profile shift. According to equation (19), F declines by an amount equal to $\bar{\gamma}_0 = \Sigma_{t=0}^{T^*} \gamma(t)$ times the increase in the value of π_0. This decline in F implies a fall in hours of work at each age. Second, there is a direct impact on each period's labor supply. Holding the value of F constant, a consumer increases his hours of work by an amount equal to δ times the increase in π_0. The implied total impact on each period's labor supply, therefore, is $\bar{\gamma}_0 + \delta$ times the change in π_0. Because $\bar{\gamma}_0$ is unambiguously negative, there is no sign prediction for this total impact. Since $\bar{\gamma}_0 + \delta$ is less than $\gamma(t') + \delta$, however, the response of labor supply to a shift in π_0 should be less in algebraic value than the response to a shift in the wage profile only at age t'. The wealth effect associated with a shift in π_0 is greater. The labor-supply profile for consumer II, then, can lie above or below consumer I's labor-supply profile. It will lie above consumer I's if $\bar{\gamma}_0 + \delta$ is positive.

The empirical specification of life-cycle supply given by equations (12) and (19) provides a convenient framework for estimating the response of labor supply to the different kinds of wage changes

described above. Estimation of the λ constant labor-supply equation produces an estimate of the intertemporal substitution elasticity δ. This elasticity can be used to predict the response of labor supply to evolutionary wage changes; it provides the information one needs to describe a consumer's dynamic behavior. Estimating the equation for F given by (19) produces estimates of the parameters $\bar{\gamma}_0$, $\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{\theta}$. These estimates provide the additional information one requires to predict the response of labor supply to parametric wage and wealth changes, and they can be used to explain labor-supply differences across consumers. Combining the estimates of $\bar{\gamma}_0$, $\bar{\gamma}_1$, and $\bar{\gamma}_2$ with the estimate of δ allows one to predict how labor-supply profiles adjust to changes in the wage-path coefficients π_0, π_1, and π_2. This includes both shifts and slope changes of the wage profiles. The estimate of $\bar{\theta}$ provides the information one needs to predict the response of labor-supply profiles to changes in a consumer's initial permanent income. Estimating the empirical model proposed in this paper, then, fully characterizes a consumer's lifetime labor-supply behavior.

Three Substitution Elasticities

Nowhere in the above interpretation of parameters was there any mention of compensated substitution elasticities. In terms of the above notation, it can be shown using Slutsky's equation that the own- and the cross-compensated elasticities are $\delta + \gamma(t) - E(t)\theta$ and $\gamma(t) - E(t)\theta$, respectively, where $E(t) \equiv N(t)W(t)$ is real earnings at age t.[18] In the analysis of life-cycle behavior, it is important to distinguish sharply compensated elasticities from the intertemporal elasticity (i.e., δ) and the uncompensated elasticities (i.e., own effects $\delta + \gamma[t]$ and cross effects $\gamma[t]$) discussed above.

Some researchers incorrectly infer that intertemporal and compensated substitution elasticities are the same.[19] The fact that an indi-

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[18] According to Slutsky's equation,

$$\left.\frac{\partial N(t)}{\partial W(t)}\right|_U = \left.\frac{\partial N(t)}{\partial W(t)}\right|_{A(0)} - N(t)\frac{\partial N(t)}{\partial A(0)}.$$

So

$$\left.\frac{W(t)}{N(t)}\frac{\partial N(t)}{\partial W(t)}\right|_U = \left.\frac{W(t)}{N(t)}\frac{\partial N(t)}{\partial W(t)}\right|_{A(0)} - W(t)N(t)\frac{\partial \ln N(t)}{\partial A(0)} = \delta + \gamma(t) - E(t)\theta.$$

[19] There has been some confusion concerning the interpretation of wage coefficients estimated in labor-supply studies such as those of Ghez and Becker (1975) and Smith (1977), who use synthetic cohort data. Because these studies estimate the response of hours of work to life-cycle wage growth, the wage coefficient is an intertemporal substitution elasticity. While this elasticity constitutes an upper bound for compensated and uncompensated elasticities, it is not one of the familiar elasticities associated with

vidual is at the same level of lifetime utility at all ages in a world of perfect certainty suggests that responses in hours of work to changes in the wage rate over the life cycle represent compensated substitution effects, which in turn suggests that the intertemporal and compensated elasticities are equivalent. These elasticities, however, are used to predict labor-supply responses to different kinds of wage changes. As discussed above, intertemporal elasticities determine hours of work responses to evolutionary wage changes. Compensated elasticities, on the other hand, are like uncompensated elasticities in the sense that they determine responses to parametric wage changes (i.e., wage changes due to shifts in wage profiles rather than movements along these profiles). Compensated elasticities, then, can be used to predict differences in hours of work across consumers whose wage profiles are different and whose lifetime utility is the same.

Formally, the intertemporal substitution effect can also be interpreted as an elasticity that is associated with a particular kind of parametric wage change. In particular, it determines the response of hours of work at age t to a shift in the age t wage rate holding λ or the marginal utility of wealth constant. Thus, whereas uncompensated elasticities hold financial wealth constant and compensated elasticities hold lifetime utility constant, the intertemporal elasticity is equivalent to a parametric wage elasticity that holds λ constant. In the literature on consumer demand, this particular elasticity is known as the specific substitution effect.[20] This correspondence between intertemporal and specific substitution effects is a direct consequence of the assumption that utility is additive over periods.

While the three types of substitution elasticities are related, they are distinct and reduce to the same value only if income or wealth effects are zero. Assuming leisure is a normal good in all periods, these elasticities can be ordered as follows: $\delta > \delta + \gamma(t) - E(t)\theta > \delta + \gamma(t)$; that is, intertemporal responses are greater than compensated responses which are in turn greater than uncompensated responses.[21] It

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parametric wage changes. Thus, without imposing nontrivial restrictions on preferences, wage coefficients estimated using synthetic cohort data cannot be used for policy analysis of the sort encountered in predicting labor-supply responses to negative income tax experiments or other proposed changes in tax policies. Elsewhere, Smith (1975) uses these estimated coefficients for exactly this purpose, namely, to predict the effect of income-maintenance programs on hours of work.

[20] See Phlips (1974, pp. 47–50) for further discussion.

[21] We can relate these elasticities using solutions of what is known as the fundamental matrix equation in the literature on consumer demand (see, e.g., ibid.). For leisure demand, it can be shown that: (1) the intertemporal effect is

$$\frac{\partial L(t)}{\partial W(t)}\bigg|_{\lambda} = \lambda\mu^{22},$$

where μ^{22} is the (2, 2) element of the inverse of the hessian matrix of $U[C(t), L(t)]$; (2)

is also easy to show that intertemporal and compensated responses must be positive, while uncompensated responses may be either positive or negative. Intertemporal and uncompensated substitution effects, then, provide an upper and a lower bound for compensated substitution elasticities.

IV. Empirical Analysis

The λ constant hours of work function given by (12) and the structural equation for F given by (19) provide a manageable empirical model for analyzing labor-supply behavior in a life-cycle setting. This model naturally suggests a two-step estimation procedure. In the first step, one estimates the parameters of the λ constant labor-supply equation. This step provides an estimate of the intertemporal substitution elasticity and all the information a researcher requires to predict how the hours of work of a given consumer will differ at two points in time. In the second step, one uses estimated "individual effects" as dependent variables to estimate the structural equation for F which produces estimates of wealth effects and uncompensated substitution elasticities associated with shifts in the intercept and the slope of the lifetime wage path. This step provides the additional information one requires to predict how labor supply will differ across consumers. Using results from both steps, it is possible to compute the average own- and cross-uncompensated substitution elasticities associated with wage changes in a single year. These results also permit the calculation of upper and lower bounds for compensated elasticities. This two-step estimation procedure exploits the special characteristics of panel data to characterize life-cycle behavior with a minimal amount of computational burden.[22]

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the compensated effect is

$$\frac{\partial L(t)}{\partial W(t)}\bigg|_U = \frac{\partial L(t)}{\partial W(t)}\bigg|_\lambda + \Omega\left[\frac{\partial L(t)}{\partial A(0)}\right]^2,$$

where $\Omega = -\lambda[\partial\lambda/\partial A(0)]^{-1} > 0$; and (3) the uncompensated effect is

$$\frac{\partial L(t)}{\partial W(t)}\bigg|_A = \frac{\partial L(t)}{\partial W(t)}\bigg|_U + N(t)\frac{\partial L(t)}{\partial A(0)}.$$

So, if $L(t)$ is a normal good and U is concave, we have

$$\frac{\partial L(t)}{\partial W(t)}\bigg|_\lambda < \frac{\partial L(t)}{\partial W(t)}\bigg|_U < \frac{\partial L(t)}{\partial W(t)}\bigg|_A.$$

The proposed restrictions on labor-supply elasticities follow immediately using this result.

[22] There is an alternative strategy for estimating this structural model of lifetime labor supply. Instead of the procedure suggested above, one could use a one-step procedure. Substituting the right-hand side of eq. (19) for F directly into the λ constant

The empirical work reported here uses the randomly designed sample from the Michigan Panel Study of Income Dynamics. The sample consists of observations of 513 prime-age, white, married males for the years 1967–76. Only males continuously married to the same spouse during the period 1968–77 and who were 25–46 years old in 1967 were included in the sample.[23] The labor-supply variable used in the empirical analysis is annual hours of work. The wage variable is average hourly earnings deflated by the Consumer Price Index.

To avoid confusion in this section we must be careful to distinguish between age variables and indexes representing a sample period. As in the previous discussion, t denotes the age of a consumer; in particular, $t = 0$ when the consumer is 25 years old, $t = 1$ when he is 26, etc. The index j, on the other hand, denotes the sample period; $j = 1$ when the observation is from year 1967, $j = 2$ when it is from 1968, etc. The following analysis assumes that there are a total of τ sample periods ($\tau = 10$ for the sample used in this paper). Finally, the notation $t(j)$ denotes the consumer's age in sample period j. Thus, ln $W_i(t)$ is consumer i's wage at age t, ln $W_i(j)$ is his wage in period j, and ln $W_i(t) = $ ln $W_i[t(j)] = $ ln $W_i(j)$.

Estimates of the Intertemporal Substitution Elasticities

Estimating the parameters of the λ constant labor-supply equation is simplified by working with a first-differenced version of this equation.

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labor-supply equation given by either (12) or (13) produces a new equation containing all of the structural parameters of interest. It is possible to estimate this equation in a single step using constrained simultaneous-equation estimation procedures. The two-step procedure offers advantages over this one-step method when there exists misspecification of the equation for F. The presence of any misspecification errors in the F-equation leads to inconsistent estimators for all parameters of the labor-supply equation in the case of the one-step procedure. For the two-step estimation scheme, however, only the second-step estimators for the parameters of the F-equation are inconsistent.

Houthakker and Taylor (1970, chap. 5) and Phlips (1974, pp. 190–93, 250–60) have estimated marginal utility of wealth constant demand functions as an intermediate computational step toward estimating a system of ordinary demand functions. In contrast to their work, I directly estimate the λ constant demand functions as a means of characterizing a consumer's dynamic behavior. I treat λ as a fixed effect which summarizes the effect of historic and future information on current decisions. I thank James Heckman and Orley Ashenfelter for the Houthakker-Taylor reference.

[23] A worker had to satisfy the following criteria as well to be included in the sample: (1) He must be classified as employed or unemployed (i.e., permanently disabled and retired were deleted). (2) Wage and labor-supply data must be available for all years. (3) A worker must report less than 4,680 hours worked per year. The absolute value of the difference in his real average hourly earnings in adjacent years cannot exceed $16 or a change of 200 percent. The absolute value of the difference in the number of hours he works in adjacent years cannot exceed 3,000 hours or a change of 190 percent. The purpose of this last criterion is to minimize difficulties arising from the presence of outliers.

First differencing equation (12) yields

$$D \ln N_i(j) = \delta[\rho - r(j)] + \delta D \ln W_i(j) + \epsilon_i(j), \qquad j = 2, \ldots, \tau, \quad (22)$$

where D is the difference operator, that is, $D \ln N_i(j) = \ln N_i(j) - \ln N_i(j - 1)$, and $\epsilon_i(j) = Du_i(j)$ is a disturbance. This equation relates a consumer's labor-supply changes to changes in his wage rate. Notice that differencing eliminates individual effects, and thus it avoids the introduction of incidental parameters. Step one of the estimation procedure is to use this equation to estimate the intertemporal substitution elasticity, δ.

Combining these equations for a given worker into a single system of simultaneous equations creates a model that is particularly well suited for an empirical analysis. Stacking equation (22) for worker i according to the j index yields

$$
\begin{bmatrix}
D \ln N_i(\tau) \\
D \ln N_i(\tau - 1) \\
\cdot \\
\cdot \\
\cdot \\
D \ln N_i(3) \\
D \ln N_i(2)
\end{bmatrix}
=
\begin{bmatrix}
\beta_\tau \\
\beta_{\tau-1} \\
\cdot \\
\cdot \\
\cdot \\
\beta_3 \\
\beta_2
\end{bmatrix}
+
\begin{bmatrix}
D \ln W_i(\tau) \\
D \ln W_i(\tau - 1) \\
\cdot \\
\cdot \\
\cdot \\
D \ln N_i(3) \\
D \ln N_i(2)
\end{bmatrix}
\delta
+
\begin{bmatrix}
\epsilon_i(\tau) \\
\epsilon_i(\tau - 1) \\
\cdot \\
\cdot \\
\cdot \\
\epsilon_i(3) \\
\epsilon_i(2)
\end{bmatrix},
\quad (23)
$$

where $\beta_j = \delta[\rho - r(j)], j = 2, \ldots, \tau$. Putting this system into vector notation, we have

$$D \ln N_i = \beta + D \ln W_i \delta + \epsilon_i, \qquad i = 1, \ldots, n, \quad (24)$$

where $D \ln N_i, \beta, D \ln N_i$, and ϵ_i are $(\tau - 1) \times 1$ vectors with $D \ln N_i' = [D \ln N_i(\tau), \ldots, D \ln N_i(2)], \beta' = (\beta_\tau, \ldots, \beta_2), D \ln W_i' = [D \ln W_i(\tau), \ldots, D \ln W_i(2)], \epsilon_i' = [\epsilon_i(\tau), \ldots, \epsilon_i(2)]$, and n is the total number of workers in the sample. The following analysis assumes that the error vectors ϵ_i are independently distributed across individuals once common time effects are removed with the inclusion of year dummies in the labor-supply equations. No restrictions are imposed on the covariance matrix of ϵ_i, which permits arbitrary forms of serial correlation.

The parameters of equation (24) are estimated using standard two-stage and three-stage least-squares procedures which permit the imposition of equality constraints across equations.[24] The wage-growth variables, $D \ln W_i(j), j = 2, \ldots, \tau$, are treated as endogenous variables. The set of instruments used to predict $D \ln W_i(j)$ includes

[24] The two-stage procedures must account for the fact that the covariance matrix of the ϵ_i's is not proportional to the identity matrix when computing standard errors for the estimates.

family background variables,[25] education, age, interactions between
education and age, and dummy variables for each year of the sam-
ple. Estimation of (24) using simultaneous equation methods takes
advantage of the time-series aspect of panel data to estimate the
intertemporal substitution elasticity δ with a minimal amount of com-
putational burden. These methods avoid biases arising from pure
reporting error in earnings and hours of work, and they offer a
flexible framework for testing and estimating alternative functional
forms.

Another equation that can be used to estimate δ is one that relates
changes in hours of work to changes in earnings. Adding $\delta \cdot D \ln N_i(j)$
to both sides of (24) and solving this new equation for $D \ln N_i(j)$ yields

$$D \ln N_i(j) = \frac{\delta[\rho - r(j)]}{1 + \delta} + \frac{\delta}{1 + \delta} D \ln E_i(j) + \frac{\epsilon_i(j)}{1 + \delta},$$
$$j = 2, \ldots, \tau, \qquad (25)$$

where $E_i(j) \equiv N_i(j)W_i(j)$ is real earnings in period j. Stacking these
equations creates a model like (24), except that $D \ln W_i$, δ, and ϵ_i are
replaced by $D \ln E_i$, $\delta/(1 + \delta)$, and $[1/(1 + \delta)]\epsilon_i$, respectively, and the
elements of β become $\beta_j = \{\delta[\rho - r(j)]\}/(1 + \delta), j = 2, \ldots, \tau$. Exactly
the same procedures described above for estimating equations (24)
are applied to estimate the parameters of the stacked representation
of (25). Using the coefficient on earnings, $\delta/(1 + \delta)$, it is possible to
construct an estimate of δ.

Table 1 presents estimates of the intertemporal substitution elastic-
ity. Two specifications of the labor-supply equation are considered.
One assumes that the interest rate, $r(t)$, is constant over time, and it
constrains intercepts in the labor-supply equations (i.e., the elements
of β) to be equal over the sample. The other allows $r(t)$ to be different
in each period by including dummy variables for each year without
any constraints on their coefficients, which permits intercepts to be
different each period. These alternative empirical specifications yield
similar results. All of the implied estimates of the intertemporal sub-
stitution elasticity are positive. According to the estimates of the wage
coefficients, δ lies in the range .10–.23. The earnings coefficients
indicate a range of .25–.45 for δ. The earnings coefficients indicate a

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[25] Family background variables include the education (in years) of both the father
and the mother of the consumer and dummy variables indicating parents' economic
status at the time the consumer was growing up. Both education and education squared
and interactions between these two education variables and age are included as instru-
ments. Coefficients in the "prediction equation" for wage growth are constrained to be
equal across time periods. Formal hypothesis tests accept this restriction. In the appli-
cation of constrained three-stage least squares, the wage-growth equations are not
treated as part of the simultaneous-equation system.

TABLE 1

SIMULTANEOUS-EQUATION ESTIMATION OF FIRST-DIFFERENCED LABOR-SUPPLY-EQUATION
ESTIMATES OF THE INTERTEMPORAL SUBSTITUTION ELASTICITY

Estimation Procedure	D(Log Wage)	D(Log Earning)*	Intercept	Average of Year Dummies
2SLS	.23	...	−.009	...
	(2.42)		(4.02)	
3SLS	.14	...	−.008	...
	(1.97)		(4.26)	
2SLS	...	.35	−.006	...
		(2.22)	(4.26)	
3SLS	...	.25	−.006	...
		(2.63)	(5.18)	
2SLS	.15	...	...	−.008
	(.98)			
3SLS	.10	...	...	−.008
	(.80)			
2SLS	...	.45	...	−.007
		(1.54)		
3SLS	...	.30	...	−.007
		(1.67)		

NOTE.—Absolute values of t-statistics are in parentheses.
* The estimates and t-statistics reported in the "Log Earning" column are for δ; they are computed using the coefficient on earnings, denoted ψ, and its t-statistics. We have $\hat{\delta} = \hat{\psi}/(1 - \hat{\psi})$. To convert the t-statistics reported for ψ to those for δ requires division by the quantity $(d\delta/\partial\psi) = (1 + \delta)^2$ evaluated at $\delta = \hat{\delta}$.

higher estimate for δ in all cases, but these differences are small relative to their standard errors. The estimates of the intercepts indicate that the real rate of interest exceeds the rate of time preference on average by about 2–4 percentage points.

A comparison of these results with others in the literature is difficult since most studies use cross-section data for their empirical analysis where differences in lifetime wage paths are the primary source of wage variation across observations. As a consequence, they do not estimate the intertemporal substitution elasticity. Estimating equation (13) using cross-section data is complicated by the presence of individual effects, F_i. As discussed above, economic theory implies that F_i is correlated with a consumer's wages and all of his other characteristics. Therefore, it is not reasonable to assume that F_i is a "random factor" uncorrelated with explanatory variables. One, then, cannot directly use observations on individuals from a cross section to estimate the parameters of (13) even if one uses simultaneous equation estimation procedures. Such procedures implicitly treat individual effects as random variables, and this leads to inconsistent parameter estimates. To estimate the intertemporal elasticity using cross-section data, one requires a specification of the labor-supply equation where variation in wages reflects evolutionary wage change.

One approach constructs synthetic cohorts to estimate equation (13) using cross-section data as implemented by Ghez and Becker (1975) and Smith (1977). A synthetic cohort is constructed by computing geometric means of wage rates and hours worked for each age group, and it is assumed to represent the life cycle of a typical individual. The basic assumption underlying this approach is that there are no cohort effects, so that group individual effects (i.e., the average of the F_i's for each age group) are the same for all age groups after adjusting for "smooth" vintage effects. In this case, least-squares estimation of equation (13) using synthetic cohort data produces consistent parameter estimates.[26] This approach allows for measured "taste-shifter" variables such as family size and age. In principle, this approach accounts for the endogeneity of wages and measured characteristics by using group averages as instruments. The problem of treating group effects as random variables does not arise since it is assumed that they are the same for all age groups.

The estimates of the intertemporal substitution elasticity obtained by Ghez and Becker and Smith are comparable to those estimates presented in table 1 of this paper. Becker forms synthetic cohorts using the 1960 U.S. Census, and he obtains estimates for δ ranging from $-.068$ to $.44$.[27] Smith, on the other hand, treats the family as the relevant decision unit and uses the 1967 Survey of Economic Opportunity to form his synthetic cohorts. He estimates δ to be about $.32$.[28] Comparing these estimates of δ and those reported in table 1 suggests that cohort effects do not seriously bias estimates based on synthetic cohort data.

Estimates of Responses to Parametric Wage Changes

Estimating the structural equation for F given by (19) provides the additional information we require to predict a consumer's labor-supply response to parametric wage changes. Estimating this equation is not as difficult as it may first appear. It is true that all of the variables appearing in this equation (F_i, π_{0i}, π_{1i}, π_{2i}, and α_{0i}) are not directly observable. But it is possible to construct observable quantities that have expected values equal to these variables. If one replaces the unobserved variables by their observed counterparts, one can employ standard two-stage least-squares procedures to estimate the structural parameters of interest.

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[26] Ghez and Becker and Smith do not interpret their parameter estimates as those of a λ constant labor-supply function. Given their log-linear specifications, however, ln λ is absorbed into the intercept of their regression specifications.

[27] Estimates obtained from Ghez and Becker (1975, pp. 112, 114).

[28] Estimates obtained from Smith (1977, p. 244).

First consider the coefficients of the lifetime path for wages. Define the difference operator D_k as $D_k \ln W_i(j) = \ln W_i(j) - \ln W_i(j - k)$. Applying this operator to the wage equation given by (15) and dividing the result by k yields

$$\frac{D_k \ln W_i(j)}{k} = \pi_{1i} + \pi_{2i}[2t(j) - k] + \frac{D_k V_i(j)}{k}.$$

Subtracting $D_1 \ln W_i(2)$ from this equation and dividing the result by $1/(2j - k - 3)$ creates a new equation:

$$\frac{1}{2j - k - 3}\left[\frac{D_k \ln W_i(j)}{k} - D_1 \ln W_i(2)\right] = \pi_{2i} +$$
$$\frac{D_k V_i(j)}{(2j - k - 3)k} - \frac{D_1 V_i(2)}{2j - k - 3}.$$

Notice that the dependent variable of this last equation has mean equal to π_{2i}. Replacing π_{2i} in the previous equation by this dependent variable allows one to create another observable variable whose mean is π_{1i}. Further substituting these two observable variables for π_{2i} and π_{1i} in the original wage equation allows one to create a third measurable variable whose expected value is π_{0i}. Following this strategy and taking averages to use all the available data, consider the following definitions:

$$\tilde{\pi}_{2i} = \frac{1}{\tau - 2}\sum_{j=1}^{\tau-2}\frac{1}{j}\left[\frac{D_{j+1}\ln W_i(j + 2)}{j + 1} - D_1 \ln W_i(2)\right] \qquad (26)$$

$$\tilde{\pi}_{1i} = \frac{1}{\tau - 1}\sum_{j=1}^{\tau-1}\left\{\frac{D_j \ln W_i(j + 1)}{j} - \tilde{\pi}_{2i}[2t(j + 1) - j]\right\} \qquad (27)$$

$$\tilde{\pi}_{0i} = \frac{1}{\tau}\sum_{j=1}^{\tau}\{\ln W_i(j) - \tilde{\pi}_{1i}t(j) - \tilde{\pi}_{2i}[t(j)]^2\}, \qquad (28)$$

where τ is the total number of sample periods. It can be shown that $E(\tilde{\pi}_{hi}) = \pi_{hi}$, $h = 0, 1, 2$. This is an important result because the $\tilde{\pi}_{hi}$'s are observable variables, and it is possible to use the $\tilde{\pi}_{hi}$'s as dependent variables in a simultaneous-equation analysis to estimate the π_{hi}'s consistently.

Similarly, given observations on consumer i's income over the sample period, one can construct the variable $\tilde{\alpha}_{0i}$ using definition (28) with $Y_i(j)$ replacing $\ln W_i(j)$; $\tilde{\alpha}_{0i}$, then, can be used in a simultaneous-equation analysis to predict the intercept of the lifetime income path, α_{0i}, which is a measure of consumer i's initial permanent income.

An analogous strategy can be used to construct a measurable variable to serve as a proxy for F_i. From equation (13) we see that an

average of the quantities $\ln N_i(j) - bt(j) - \delta \ln W_i(j)$ has an expected value equal to F_i, but unfortunately, this average cannot be directly observed since it depends on unknown parameters b and δ. We have estimates of these parameters, however, from the first step of the empirical analysis. A logical alternative to the above average, then, is to form the variable

$$\tilde{F}_i = \frac{1}{\tau} \sum_{j=1}^{\tau} [\ln N_i(j) - \hat{b}t(j) - \hat{\delta} \ln W_i(j)].$$

Asymptotically, $\tilde{F}_i$ has an expectation equal to F_i.

Collecting the above results, we have a complete simultaneous-equations model given by

$$\tilde{\pi}_{hi} = M_i g_h + \eta_h, \qquad h = 0, 1, 2, \tag{29}$$

$$\tilde{\alpha}_{0i} = S_i q_0 + \eta_3, \tag{30}$$

$$\tilde{F}_i = \phi + \tilde{\pi}_{0i} \bar{\gamma}_0 + \tilde{\pi}_{1i} \bar{\gamma}_1 + \tilde{\pi}_{2i} \bar{\gamma}_2 + \tilde{\alpha}_{0i} \bar{\theta} + \eta_4, \tag{31}$$

where the vectors of exogenous variables M_i and S_i and the coefficient vectors g_0, g_1, g_2, and q_0 are defined by (16) and (18), and the η_h's are disturbances. We have one set of equations for each consumer i. The endogenous variables in this model are $\tilde{F}_i$, $\tilde{\alpha}_{0i}$, and the $\tilde{\pi}_{hi}$'s; the exogenous variables are the elements of M_i and S_i; and the structural parameters of interest are $\bar{\gamma}_0$, $\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{\theta}$. The vectors M_i and S_i contain variables determining the coefficients of the lifetime wage and income paths. In the following empirical analysis, they include the consumer's education, his education squared, and family background variables.[29]

To estimate the parameters of the structural equation for F given by (31) consistently, one can employ a standard two-stage least-squares procedure. The standard errors reported by this procedure are valid if the number of time-series observations for each consumer is sufficiently large.[30]

Table 2 presents estimates for the structural parameters of the individual effects equation given by (31), where $\tilde{F}_i$ is computed using three different sets of estimates for b and δ. The estimates of $\bar{\gamma}_0$, $\bar{\gamma}_1$,

[29] Family background variables include the education of both the father and the mother of the consumer and dummy variables indicating parents' economic status at the time the consumer was growing up.

[30] If one does not have a sufficiently large number of these time-series observations, however, the usual standard errors are invalid. The problem lies in the fact that we use estimated values for b and δ to form the dependent variable F_i. In cases where the number of time-series observations is small, one must adjust the usual standard errors to account for errors in estimating b and δ. The precise form of this adjustment can be obtained from the author upon request. While this adjustment is not complicated, it does require the use of matrix operations. This adjustment was very minor in every instance for which it was used in this study, which suggests that even 10 observations per person is large enough to neglect making any adjustments in standard errors.

TABLE 2

SIMULTANEOUS-EQUATION ESTIMATION OF FIXED-EFFECTS EQUATIONS

$\hat{b}$, $\hat{\delta}$	$\bar{\gamma}_0$	$\bar{\gamma}_1$	$\bar{\gamma}_2$	$\bar{\theta}$	Intercept
$-.009$, $.1$	$-.05$	$-.83$	-10.47	$-.00026$	7.81
	(1.5)	(1.4)	(1.2)	(.48)	(157)
$-.009$, $.15$	$-.07$	-1.08	-13.03	$-.0001$	7.77
	(1.78)	(1.65)	(1.31)	(.16)	(137)
$-.009$, $.23$	$-.10$	-1.46	-16.86	$.0001$	7.68
	(2.13)	(1.9)	(1.4)	(.2)	(119)

Note.—Absolute values of t-statistics are in parentheses. The dependent variable $\tilde{F}_i$ (i.e., the proxy variable for F_i) is computed using the estimates b and δ reported in the stub column. Income is measured in thousands of dollars.

and $\bar{\gamma}_2$ are all negative as theory predicts. These estimates decrease monotonically as one uses a higher estimated value of δ to compute the proxy variable for individual effects, $\tilde{F}_i$. Since uncompensated substitution elasticities associated with permanent wage changes are calculated by adding the estimate of δ to the estimate of $\bar{\gamma}_0$, an inverse relationship between the estimates of δ and $\bar{\gamma}_0$ is required if uncompensated permanent wage elasticities are to remain constant for different choices of $\hat{\delta}$ in computing $\tilde{F}_i$.

All of the estimates of the initial permanent income coefficient, $\bar{\theta}$, are statistically insignificant and very small. The measure of property income (i.e., $Y_i[t]$) used in this empirical analysis is total family income minus husband's earnings in thousands of 1967 dollars. While the estimates of $\bar{\theta}$ are negative as theory predicts when low estimated values of δ are used to construct $\tilde{F}_i$, we see that the estimated effects of a change in income on hours of work are minute; a \$1,000 increase in initial permanent income leads to at most a .026 percent decrease in hours of work. Several other measures of property income were tried in the empirical analysis;[31] in every case the estimates obtained for $\bar{\theta}$ were of the order of magnitude reported in table 2.

Combining estimates of $\bar{\gamma}_0$ and δ allows one to form estimates of cross- and own-uncompensated substitution elasticities and bounds for compensated elasticities associated with wage changes in a single year. Dividing the estimate of $\bar{\gamma}_0$ by the length of the working life produces an estimate of the average cross-uncompensated elasticity. Using results from the second row of table 2 and assuming a working life of 40 years implies a cross elasticity equal to $-.0018$. Adding the estimate for δ (which is .15 for the second row of table 2) to this cross elasticity implies a value of .15 for the average own-uncompensated

[31] One measure of income excluded wife's earnings. Others included imputed income for house ownership. Due to data limitations, it was not possible to include imputed income from other forms of consumer durables, which certainly constitute a major component of a consumer's nonwage income.

elasticity. Since the average own-compensated elasticity lies between the intertemporal and the average own-uncompensated elasticity, we conclude that this compensated elasticity is also approximately equal to .15. Increasing a consumer's wage rate in period t by 10 percent, then, leads to about a 1.5 percent increase in his hours of work in period t and approximately no change in his hours of work at other ages.

Combining estimates of $\bar{\gamma}_0$, $\bar{\gamma}_1$, and $\bar{\gamma}_2$ with estimates of δ provides the information needed to predict a consumer's labor-supply response to shifts in his wage profile. In response to a uniform 10 percent increase in wages at all ages (i.e., a parallel shift in the log wage profile), the estimates of the second row of table 2 predict that a consumer will adjust his hours of work by an amount equal to $(\hat{\bar{\gamma}}_0 + \hat{\delta})10\% = (-.07 + .15)10\% = .8$ percent at all ages. There is, then, a small positive response in a consumer's labor supply to parallel shifts in his log wage profile. If the slope of a consumer's wage profile is altered by changing the coefficient on the linear term (i.e., π_{1i}) by Δ percent, the estimates for $\bar{\gamma}_1$ and δ indicate that his hours of work at age t change by $(\hat{\bar{\gamma}}_1 + \hat{\delta}t)\Delta\% = (-1.08 + .15t)\Delta\%$. Recall that t here measures a consumer's age and takes a value of 0 when the consumer is 25 years old. Hence, hours of work decline at early ages (i.e., prior to age 32) and they increase at later ages in response to this sort of increase in the slope of the wage profile. The same is true when the slope of the wage profile is altered by changing the coefficient on the quadratic term (i.e., π_{2i}) by Δ percent. Hours of work at age t adjust by an amount equal to $(-13.03 + .15t^2)\Delta\%$, which also implies a decline in hours of work at younger ages (i.e., prior to age 34) and an increase at older ages.

The estimates above of uncompensated elasticities are generally consistent with results found in cross-section studies and the popular notion that the lifetime labor-supply curve of prime-age males is not very responsive to permanent wage changes. If, in a cross-section analysis, one purges wages and income of their transitory components using a simultaneous-equation method and interprets the estimated coefficients as some sort of "lifetime average" relationship, then one finds small positive estimates for wage coefficients and negative or zero estimates for income coefficients for prime-age males,[32] which agrees with the results above. In contrast to previous work, however, the estimates above also indicate that to predict the response of labor

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[32] Accounting for the endogeneity of ln $W_i(t)$ and $Y_i(t)$ has been shown to have a significant effect on cross-section estimates. DaVanzo, Detray, and Greenberg (1976) find that treating both wages and income as endogenous variables leads to positive estimates for wage coefficients and negative or zero estimates for income coefficients for prime-age men, which are consistent with the empirical results reported above. Neglecting this endogeneity produces estimated coefficients with opposite signs.

supply to shifts in the lifetime wage path one must be careful to specify the particular shift involved and the time of the life cycle relevant for evaluating the response. The lifetime labor-supply curve for prime-age males is backward bending for some types of wage changes over part of the life cycle, and it is positively sloped for other age ranges and wage changes.

V.   Conclusion

This study formulates a manageable empirical model of labor supply that fully incorporates life-cycle considerations. This model naturally divides the analysis into two steps. In the first step, the analysis concentrates on measuring parameters relevant for describing a consumer's dynamic behavior. Here one estimates the response of hours of work to evolutionary wage changes. In step two, the analysis focuses on measuring parameters relevant for explaining differences in labor supply across consumers. This step produces estimates of the impact of parametric changes in wealth and in wages on hours of work over the life cycle. This two-step analysis offers a very tractable estimation procedure with minimal data requirements.

We have seen that there are three types of substitution elasticities relevant for predicting the response of hours of work to changes in the wage rate. The existence of these three elasticities reflects the fact that the effect of a wage change depends on its source. The intertemporal elasticity determines the labor-supply response to wage changes resulting from life-cycle wage growth and movements over a perfectly foreseen business cycle. Uncompensated and compensated elasticities, on the other hand, determine the hours of work response to shifts in wage profiles. When specifying these latter elasticities one must identify not only the particular wage profile shift involved but also that part of the profile that is being held constant. While the three types of substitution elasticities are distinct, they are related with intertemporal and uncompensated elasticities providing an upper and lower bound for compensated elasticities.

This paper presents a full set of estimates required to describe the lifetime labor-supply behavior of prime-age males. Estimates of the intertemporal substitution elasticity indicate that a 10 percent increase in the real wage rate which is due to life-cycle wage growth induces a 1–5 percent increase in hours worked. The estimates of own-period uncompensated and compensated substitution elasticities range between .1 and .5, and cross-uncompensated elasticities associated with a 1-year wage change are approximately zero. Estimates of uncompensated elasticities associated with shifts in the entire wage profile indicate that a uniform 10 percent increase in wages at all ages leads to a

0.5–1.3 percent increase in hours of work, and an increase in the slope of the profile leads to a decline in hours of work at early ages and an increase at later ages. The interpretation of these empirical results is, of course, dependent upon strong theoretical assumptions. The analysis in this paper neglects the presence of taxes, and it assumes that capital markets are perfect. There is an implicit assumption that hours of work are perfectly flexible and chosen freely by the worker. This paper also ignores the role of human capital investment in measuring both the supply of labor and the returns to work.

The important point for an analyst to extract from this study is the following: Recognizing that individuals make their decisions in a life-cycle setting is crucial if one's objective is to estimate economically meaningful parameters. Creating an empirical model that accounts for such a setting need not complicate the analysis, and it generally leads to a more complete understanding of consumer behavior.

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