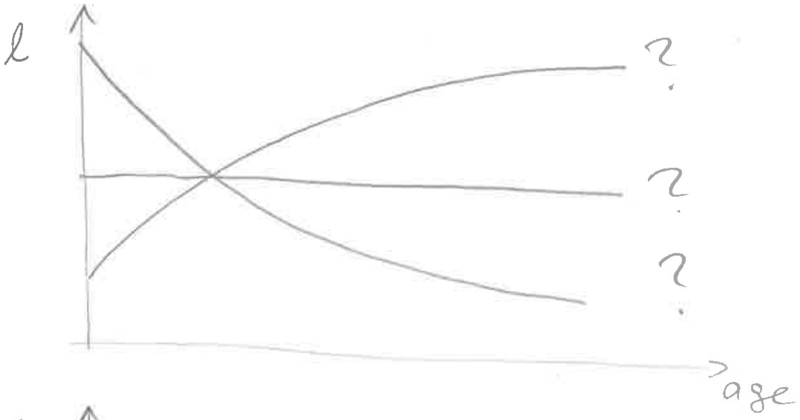
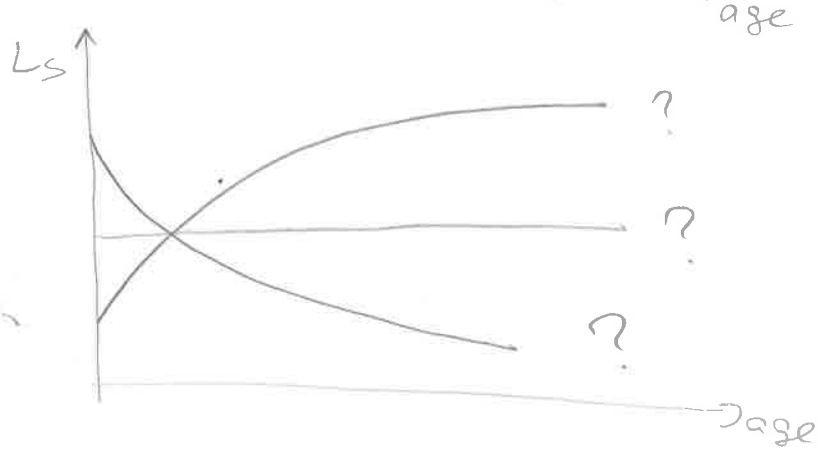
MOTIVATION FOR LIFE HORIZON LABOR SUPPLY





- What is likely to play role?
  - preferences
    - risk aversion
    - foresight
    - prices
  - capital market
  - household work
  - hh members

2 PERIOD MODEL

Y_0, Y_1 income in periods $t = 0, 1$

X_0, X_1 consumption

l_0, l_1 (L_0, L_1) leisure (Labor supply)

Assume perfect capital market with interest rate r

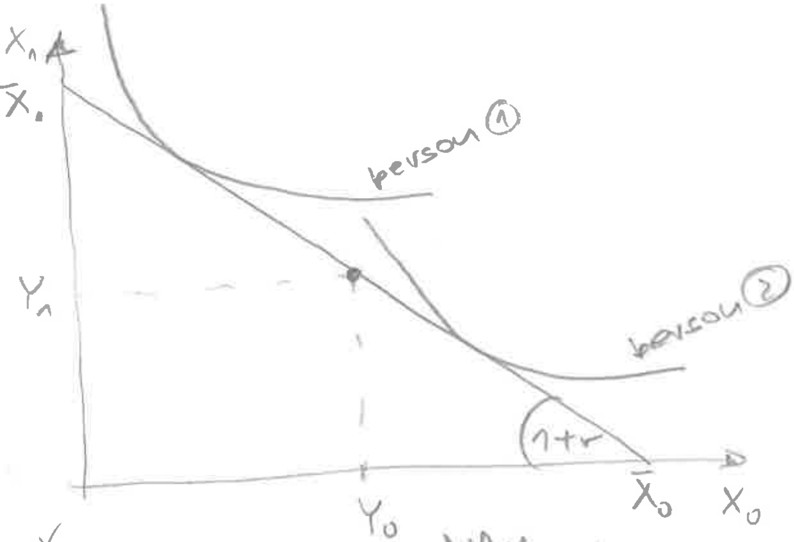
1. $\max X_0 \equiv \bar{X}_0 = Y_0 + \frac{Y_1}{1+r} \Rightarrow X_1 = 0$

2. $\max X_1 \equiv \bar{X}_1 = (1+r)Y_0 + Y_1 \Rightarrow X_0 = 0$

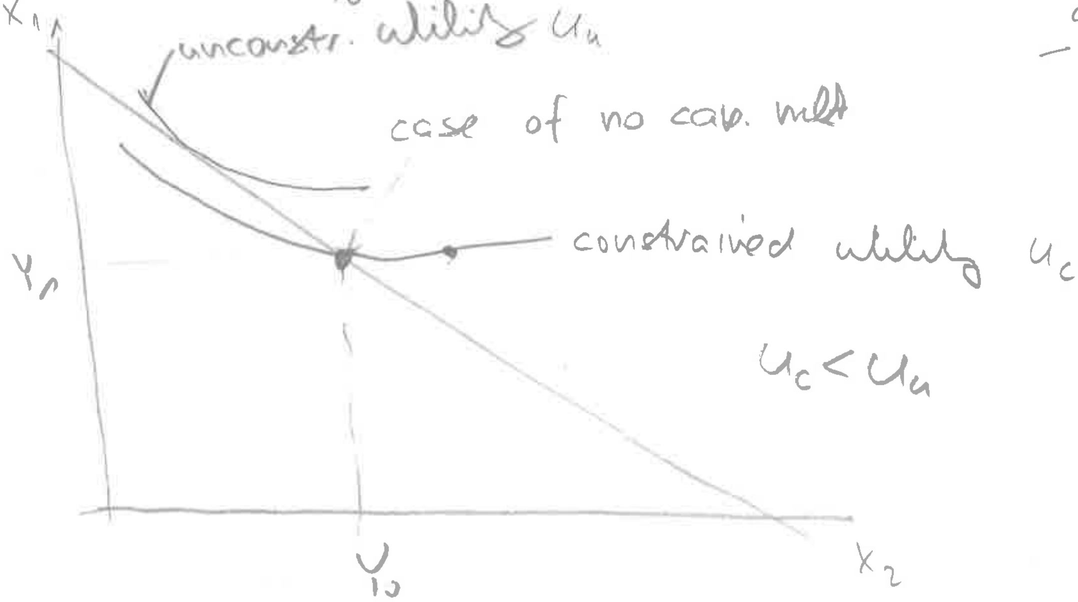
3. Consuming in both periods

$$X_1 = \bar{X}_1 - (1+r)X_0 \Rightarrow \underbrace{\overset{\text{choice}}{X_0 + \frac{X_1}{1+r}} = \overbrace{Y_0 + \frac{Y_1}{1+r}}^{\text{fixed}}}_{\text{Budget constraint}}$$

slope: $\frac{\bar{X}_1}{\bar{X}_0} = \frac{Y_1 + (1+r)Y_0}{\frac{Y_1}{1+r} + Y_0} = \underline{\underline{(1+r)}} \frac{\cancel{\frac{Y_1}{1+r} + Y_0}}{\cancel{\frac{Y_1}{1+r} + Y_0}}$



consider regimes: socialism, communism, mkt



LTIPERIOD CASE → n+1 periods

$$\text{Max } U = u[(x_0,l_0), \ldots (x_n,l_n)]$$

$$z_i = f_i(x_i, l_i)$$

$$= \sum_{i=0}^{n} \beta^i z_i$$

$$\text{s.t. } \sum_{i=0}^{n}\left(\frac{1}{1+r}\right)^i x_i = x_0 + \sum_{i=0}^{n}\frac{1}{1+r} w_i (\underbrace{H_0 - l_i}_{L_i})$$

assume separability

L_i does not affect L_j

assume $f_i' > 0$, $f_i'' < 0$

assuming additivity of U

β - time preference param

$= \frac{1}{1+\rho}$ → rate of time preference

for $\rho > 0$ z today is better

$\alpha = \frac{1}{1+r}$ discounting → int. rate of interest

$f_i = f \ \forall \ i$

w - exogenous param.

$$\sum \alpha^i x_i + \sum \alpha^i l_i w_i = x_0 + H_0 \sum \alpha^i w_i$$

$$\mathcal{L} = \sum \beta^i z_i + \lambda\left[x_0 + H_0 \sum \alpha^i w_i - \sum \alpha^i x_i - \sum \alpha^i l_i w_i\right]$$

$$\text{FONC: } \frac{\partial u}{\partial x_i} \equiv \frac{\partial u}{\partial z_i}\frac{\partial z_i}{\partial x_i} = \beta^i \frac{\partial z_i}{\partial x_i} \overset{!}{=} \lambda \alpha^i$$

$$\frac{\partial u}{\partial l_i} \equiv \frac{\partial u}{\partial z_i}\frac{\partial z_i}{\partial l_i} = \beta^i \frac{\partial z_i}{\partial l_i} \overset{!}{=} \lambda \alpha^i w_i$$

#1 $\frac{\partial u/\partial l_i}{\partial u/\partial x_i} = w_i$ as in 1 period model

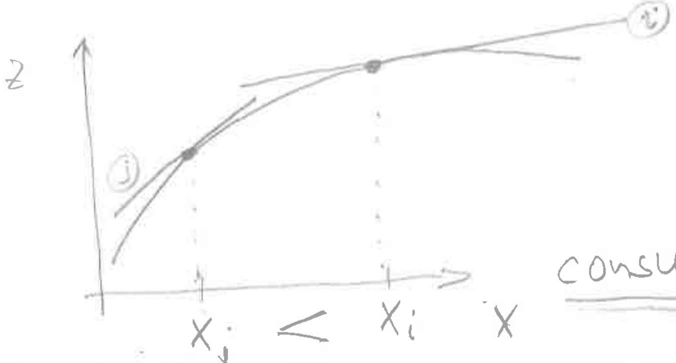
#2 $\frac{\partial z_i/\partial x_i}{\partial z_j/\partial x_j} \equiv \frac{\alpha^{i-j}}{\beta^{i-j}} \overset{!}{=} \left(\frac{1+\rho}{1+r}\right)^{i-j}$

EXPLORE: consumption over time x_i

Assume: $i > j$; $\rho < r$ ⇒ $\underbrace{\left(\frac{1+\rho}{1+r}\right)}_{<1}^{\overbrace{i-j}^{>0}} < 1$ ⇒ $\frac{\partial z_i}{\partial x_i} < \frac{\partial z_j}{\partial x_j}$

slope (i)

slope (j)



consumption increasing with t (age)

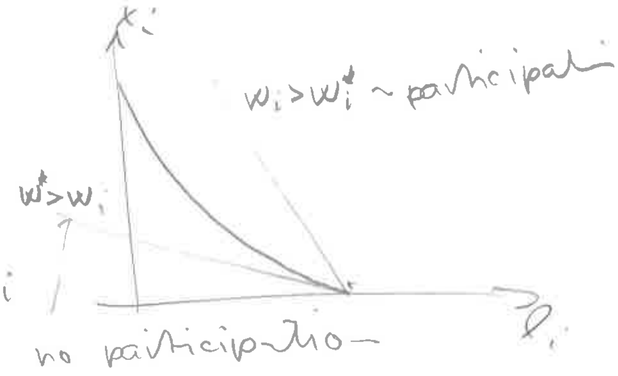
articipation over life-cycle

↳ depends on reservation wage over time w_i^*

we know $\frac{\partial u/\partial l_i}{\partial u/\partial x_i} = w_i$

$\Downarrow$

$\frac{\partial z_i/\partial l_i}{\partial z_i/\partial x_i} = w_i^* > w_i$



=> Participation depends on shape of $z_i(x_i, l_i)$ over time

=> w^*

CASE if w_i^* = const $\forall i$ & w_i growing over time ->

-> participation increases over life-cycle

CASE if w_i^* ~~declines~~ increases $\begin{cases} \text{maternity} \\ \text{retirement} \end{cases}$ -> lower participation when old or with kids

SUMMARY REPETITION OF MULTIPERIOD MODEL

$$U = \sum_{t=1}^{T} \beta^{t-1} z_t$$

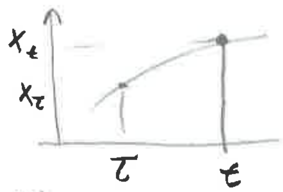
$\beta \sim$ time pref coef. $= \frac{1}{1+\rho} < 1$

$\rho > 0$ better today than tomorrow

$P_k = \left(\frac{1}{1+r}\right)^k$ ... discount factor

#1 $\frac{\partial u / \partial l_t}{\partial u / \partial x_t} \overset{!}{=} w_t \quad \forall t$

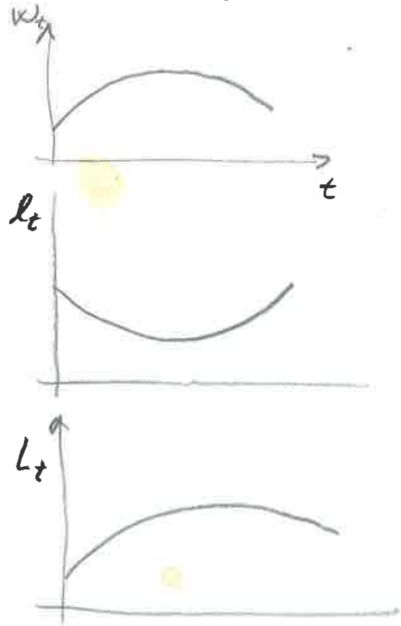
#2 $\frac{\partial z_t / \partial x_t}{\partial z_\tau / \partial x_\tau} \overset{!}{=} \frac{p^{t-\tau}}{\beta^{t-\tau}} = \left(\frac{1+\rho}{1+r}\right)^{t-\tau} \Rightarrow$



if $\rho < r$

$t > \tau$

#3 $\frac{\partial z_t / \partial l_t}{\partial z_\tau / \partial l_\tau} \overset{!}{=} \left(\frac{1+\rho}{1+r}\right)^{t-\tau} \frac{w_t}{w_\tau} \Rightarrow$



• Participation: depends on reserv. wage over time

CASES for $t > \tau$: $w_t \overset{?}{>} w^*_\tau$ non-partic: $\frac{\partial u / \partial l_t}{\partial u / \partial x_t} > w_t$

$w_t > w^*_t$ aneb $w_t < w^*_t$

if $w^*_\tau = w^*_t = const(t)$ --> high participation when w is high

if $w^*_\tau \neq w^*_t$ - in case of women with kids time t at home becomes expensive -> high w^*

at older ages retirement programs introduce disincentive -> steeper decline what about leisure l_i vs l_j $i > j$

CASE 1

$$\boxed{\frac{\partial z_i / \partial l_i}{\partial z_j / \partial l_j} \overset{!}{=} \left(\frac{1+\rho}{1+r}\right)^{i-j} \frac{w_i}{w_j}}$$

CASE 1 assume $\rho = r \rightarrow (1)^{i-j} = 1 \quad \forall\, i > j$

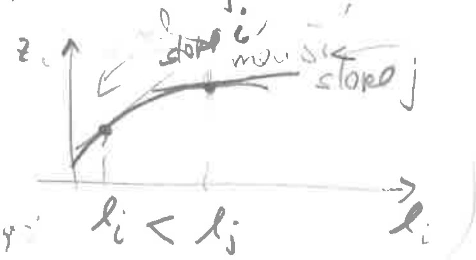
$$\frac{\partial z_i}{\partial l_i} = (1)\, \frac{w_i}{w_j}\, \frac{\partial z_j}{\partial l_j}$$

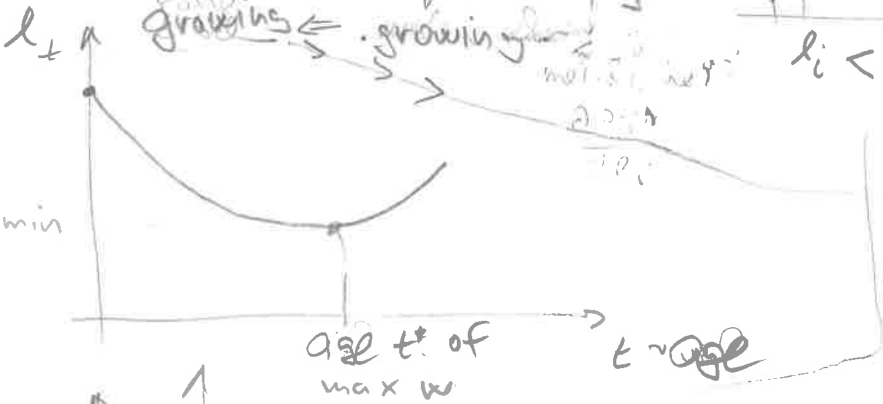
a) if $w_i = w_j = w_{const} \rightarrow l_i$ const over time

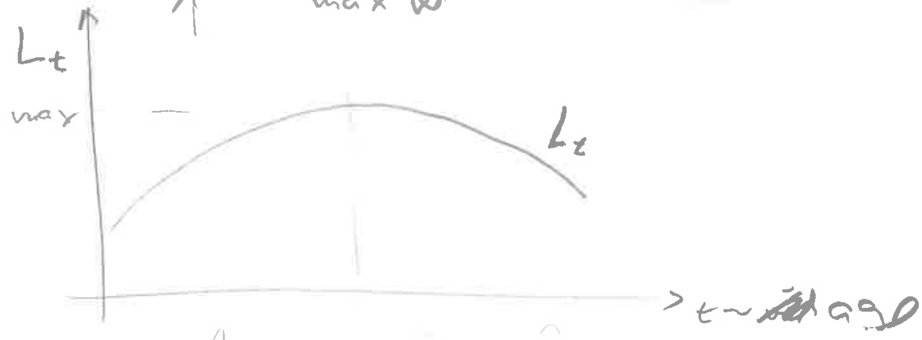
b) if w_i

$\rightarrow \frac{w_i}{w_j} > 1$

$$\Rightarrow \frac{\partial z_i}{\partial l_i} = (1) \underbrace{\left(\frac{w_i}{w_j}\right)}_{>1} \left(\frac{\partial z_j}{\partial l_j}\right)$$







permanent change