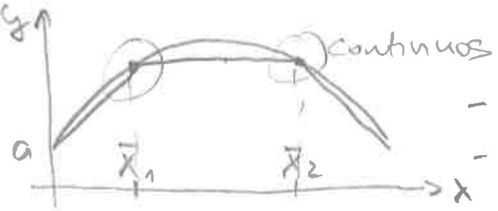
Notes on estimation

Splines $\rightarrow$ $y = a + b_1 x + b_2(x - \bar{x}_1) D[x > \bar{x}_1] + b_3(x - \bar{x}_2) D[x > \bar{x}_2]$



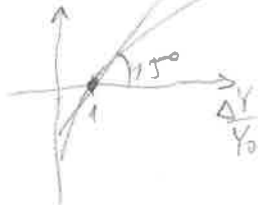
D ... indicator fcio = dummy

- polynomial versions possible
- possible for more x's
- regions of x by definition or optional

$\ln y = \beta x + \delta D \; : \; D = 1, 0$

$$\left.\begin{aligned} \ln y_0 &= \beta x + 0 \\ \ln y_1 &= \beta x + \delta \end{aligned}\right\} \ln y_1 - \ln y_0 = \ln \frac{y_1}{y_0} = \ln\left(\frac{Y_0 + \Delta Y}{Y_0}\right) = \ln\left(1 + \frac{\Delta Y}{Y_0}\right) = \delta$$

$$\% \Delta Y = \frac{\Delta Y}{Y_0} = e^{\delta} - 1 \Rightarrow \delta \sim \% \text{ for small } \delta$$

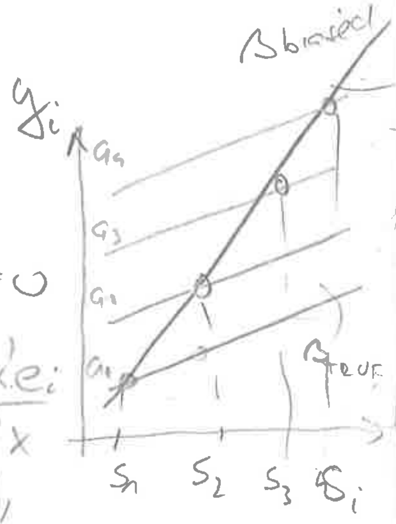


Ability bias

$y_i = \beta x_i + \underbrace{a_i + e_i}_{\varepsilon_i}$, $\operatorname{cov}(a_i, x_i) > 0$

$\operatorname{cov}(a_i, e_i) = \operatorname{cov}(x_i, e_i) = 0$

$$\hat{\beta}_{OLS} = \frac{x'y}{x'x} = \frac{x'(\beta x + a + e)}{x'x} = \beta + \underbrace{\frac{x'a}{x'x}}_{\downarrow \, > 0 \, \downarrow \, \text{bias}} + \underbrace{\frac{x'e_i}{x'x}}_{\downarrow \, 0}$$



- error in variables

RHS : EDU $\left\langle \begin{array}{l} \text{recall error} \\ \text{imputation error} \\ \text{drop outs} \end{array} \right\} \Rightarrow$ bias $\nearrow \searrow$

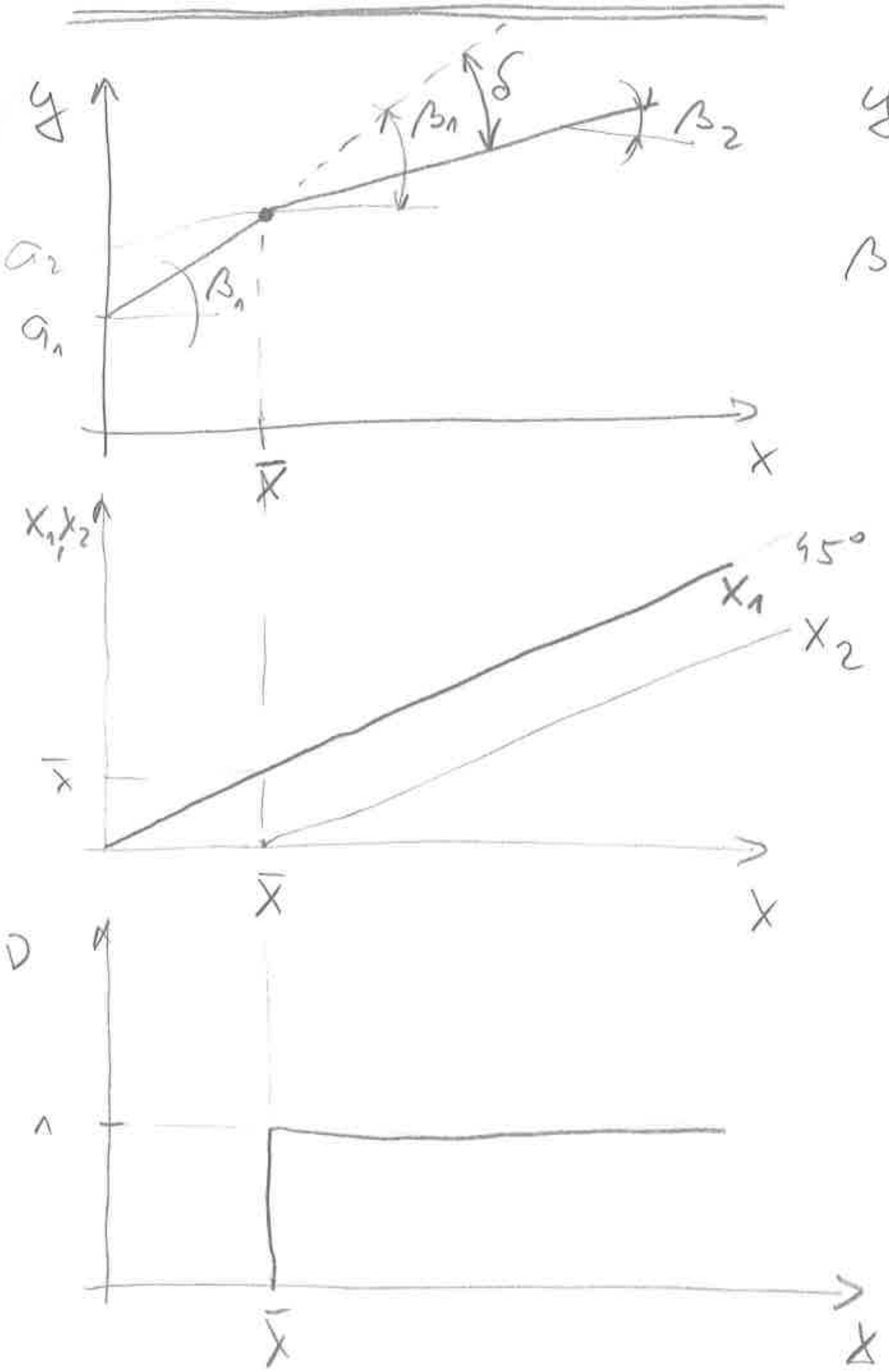
LHS : Y ... recall $\rightarrow$ no bias

taxes $\rightarrow$ bias

$EXP = AGE - EDU - 6 - \underset{\text{missing}}{h}$

- specification EDU $\left\langle \begin{array}{l} \text{years ?} \\ \text{levels ?} \end{array} \right.$

tuition on SPLINES



$$y = a_1 + \beta_1 \underbrace{X}_{X_1} + \underbrace{(X-\bar X)\, D(x>\bar x)}_{X_2}\, \delta$$

$$\beta_2 = \beta_1 + \delta$$

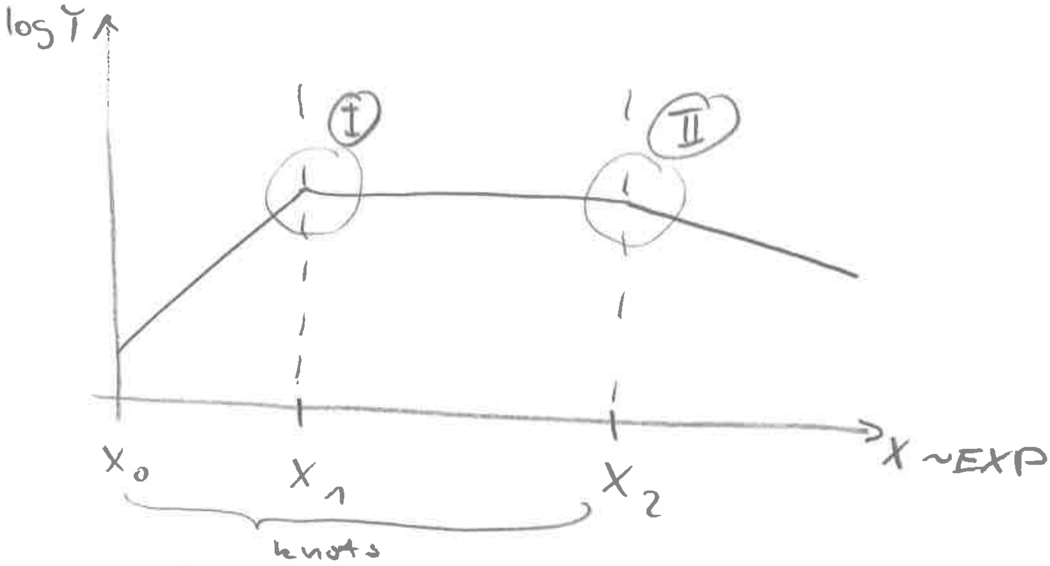
or

$$\cancel{y = a_1 + \beta_1 X\, D(X \le \bar X) + a_2 + (X-\bar X)\, D(X > \bar X)\, \beta_2}$$

SPLINES

- use if theory does not predict fc. form
- use if continuity $f(x)$ needed

- CASE of EXPerience ~ EXP profile



Choice of knots:

- arbitrary
- other rules

$\bar{x}_0, \bar{x}_1, \bar{x}_2$ ~ knots = const

$D_1 = 1$ if $\bar{x}_0 \leq x < \bar{x}_1$

$D_2 = 1$ if $\bar{x}_1 \leq x < \bar{x}_2$

$D_3 = 1$ if $\bar{x}_2 < x$

$$Y = [a_1 + b_1(x - \bar{x}_0)] D_1 + [a_2 + b_2(x - \bar{x}_1)] D_2 + [a_3 + b_3(x - \bar{x}_2)] D_3 + u$$

Impose continuity at I & II

$$a_2 = a_1 + b_1(\bar{x}_1 - \bar{x}_0)$$

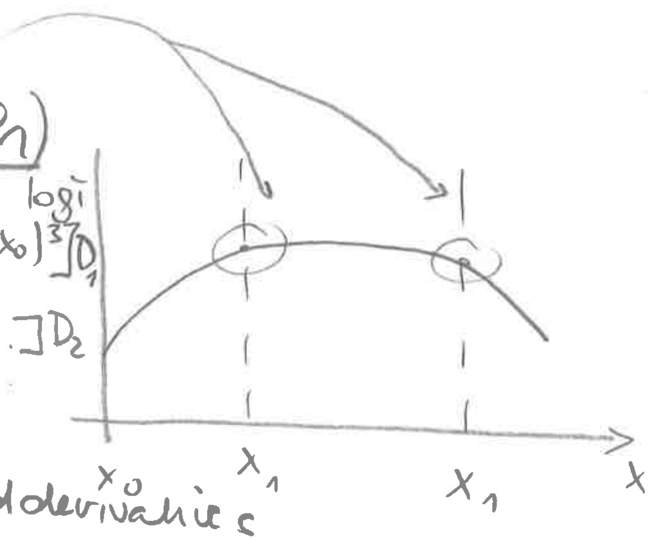
$$a_3 = a_2 + b_2(\bar{x}_2 - \bar{x}_1)$$

- POLYNOMIAL SPLINE (smooth)

$$Y = [a_1 + b_1(x - x_0) + c_1(x - x_0)^2 + d_1(x - x_0)^3] D_1$$

$$+ [a_2 \ldots \ldots] D_2$$

$\vdots$



Impose continuity & continuity of 1st & 2nd derivatives

$\to a_2 = a_1 + b_1(x_1 - x_0) + c_1(x_1 - x_0)^2 + d_1(x_1 - x_0)^3$

$\to b_2 = \ldots$

Simple formula (for general specification)

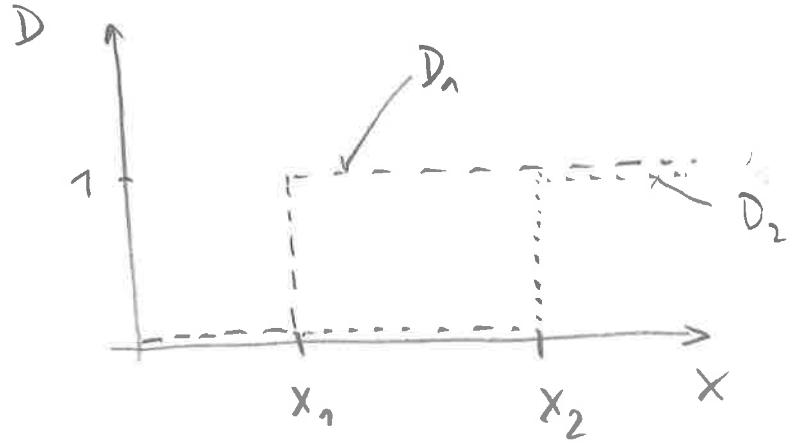
$$\boxed{Y = a_1 + b_1(X - X_0) + c_1(X - X_0)^2 + d_1(X - X_0)^3 + \sum_{i=1}^{k} (d_{i+1} - d_i)(X - X_i)^3 D_i^*}$$

$$\boxed{\begin{aligned} D_i^* &= 1 \quad \text{if} \quad X \geq X_i \\ &= 0 \quad \text{otherwise} \end{aligned}}$$

marginal definitn

$k+1 = 3 \sim$ # intervals

$\underbrace{a_1, b_1, c_1, d_1, d_2, \dots d_k}_{\text{parameters}}$



- more X's can be defined by splines

CASE of 1st order:

$D_1 = 1(X \geq X_1)$ $\quad$ $D_2 = 1(X \geq X_2)$

$$Y = a + b_1(X - X_0) + b_2(X - X_1)D_1 + b_3(X - X_2)D_2$$

CASE of 2nd order:

$$Y = a + b_1(X - X_0) + c_1(X - X_0)^2 + C_2(X - X_1)^2 D_1 + C_3(X - X_2)^2 D_2$$

CASE of 3rd order: