ALLOCATION OF m TIMES AND n GOODS

$x = [x_1, \dots x_n]$ ~ goods
L ~ work time
$\ell = [\ell_1, \dots \ell_m]$ ~ leisure times
T ~ Total time available
w ~ wage

$$u(x_1, \dots x_n; \ell_1 \dots \ell_m)$$

$$\mathcal{L} = u(\cdot) + \lambda[X_0 + wL - \sum p_i x_i]$$
$$= u(\cdot) + \lambda[X_0 + wT - w\sum \ell_i - \sum p_i x_i]$$

BC1: $\sum p_i x_i \equiv X_0 + wL$

BC2: $\sum \ell_i + L = T \rightarrow L = T - \sum \ell_i$

FONC
$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_i} &= u_{x_i} - \lambda p_i \overset{!}{=} 0 \\ \frac{\partial \mathcal{L}}{\partial \ell_i} &= u_{\ell_i} - \lambda w \overset{!}{=} 0 \end{aligned}\right\} \quad \left.\begin{aligned} u_{x_i} &= \lambda p_i \\ u_{\ell_i} &= \lambda w \end{aligned}\right] \quad \boxed{\frac{u_{\ell_i}}{u_{x_i}} = \frac{w}{p_i}}$$

w ~ marginal price of time

If ℓ_i, x_i normal goods $\Rightarrow \nearrow X_0 \rightarrow \searrow L$

$\nearrow w \rightarrow ?\, L$ income? subst?

FONC $\forall i$ & $\forall x, \ell$: $\rightarrow$ 3 sets of conditions

a) x_i vs. x_j $\rightarrow \boxed{\frac{u_{x_i}}{u_{x_j}} \overset{!}{=} \frac{p_i}{p_j}}$ $\forall i, j$

$$dx_i \rightarrow \frac{\partial u}{\partial x_i} dx_i \qquad dx_j = \frac{dx_i\, p_i}{p_j}$$

$$dx_j \rightarrow \frac{\partial u}{\partial x_j} dx_j = \frac{\partial u}{\partial x_j} \frac{p_i}{p_j} dx_i$$

$$u_{x_i} \cancel{dx_i} \overset{!}{=} u_{x_j} \frac{p_i}{p_j} \cancel{dx_i}$$

$$\boxed{\frac{u_{x_i}}{u_{x_j}} = \frac{p_i}{p_j}}$$

b) ℓ_i vs. ℓ_j $\rightarrow \boxed{\frac{u_{\ell_i}}{u_{\ell_j}} \overset{!}{=} 1}$ $\forall i, j$

c) x_i vs. ℓ_j $\rightarrow \boxed{\frac{u_{x_i}}{u_{\ell_j}} = \frac{p_i}{w}}$

MMENTS ON CASE III

Combining FONC #2 & #3 we get several sets of conditions

a) x_i vs. x_j -> $\frac{u_{x_i}}{u_{x_j}} \overset{!}{=} \frac{p_i}{p_j} \quad \forall i,j$

b) l_i vs. l_j -> $\frac{u_{l_i}}{u_{l_j}} \overset{!}{=} 1 \quad \forall i,j$

c) x_i vs. l_j -> $\frac{u_{x_i}}{u_{l_j}} = \frac{p_i}{w} \quad \forall i,j$

You have 1 hour to use -> work -> $\Delta U = \frac{\partial U}{\partial X_i} \Delta X_i$; leisure$_i$ -> $\Delta U = \frac{\partial U}{\partial l_i} \Delta l_i$

work $dU = \frac{\partial U}{\partial x_i} dx_i = \frac{\partial U}{\partial x_i} \frac{w \cdot dl_i}{p_i}$

leisure $dU = \frac{\partial U}{\partial l_i} \cdot dl_i$

have to be equal =>

$$\Rightarrow \frac{\partial U}{\partial x_i} \frac{w\, dl_i}{p_i} \overset{!}{=} \frac{\partial U}{\partial l_i} \cancel{dl_i} \Rightarrow \frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial l_i}} = \frac{p_i}{w}$$

FOC

$$\omega = \frac{\Delta U}{\Delta l}$$

$$\lambda = \frac{\Delta U}{p_i \Delta X_i}$$

$$\frac{\omega}{\lambda} = \frac{\cancel{\Delta U}}{\Delta l} \frac{p_i \Delta X_i}{\cancel{\Delta U}} = \frac{p_i \Delta X_i}{\Delta l} =$$

CASE III G. Becker

$z_1, \dots, z_n$ = activities

$z_i = f_i(x_i, l_i)$

x_i ... goods

l_i ... ~~leisure~~ time used

T_0 ... total time available

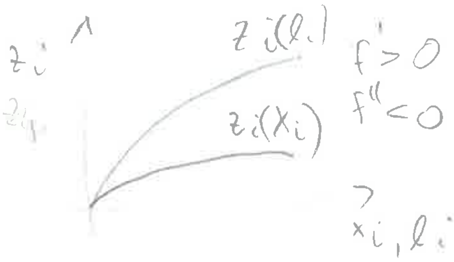
L_0 ... Labor time

$u(z_1, \dots, z_n)$

BC: $z_i = f(x_i, l_i)$

$$\sum_i p_i x_i = X_0 + W(L_0)$$

$$\sum l_i = H_0 - L_0$$



Assume separability:

$$u = u[(x_1, l_1), (x_2, l_2), \dots, (x_n, l_n)]$$

FONC: $u_{x_i} = \dfrac{\partial u}{\partial f_i} \dfrac{\partial f_i}{\partial x_i} = \lambda p_i$

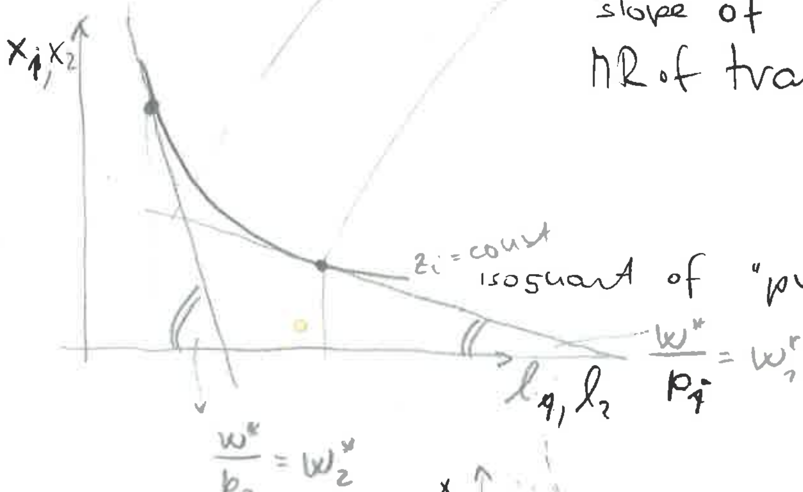
$$\mathcal{L} = u + \lambda[\sum p_i x_i - X_0 - W(L_0)] - \mu[\sum l_i - H_0 + L_0]$$

$u_{l_i} = \dfrac{\partial u}{\partial f_i} \dfrac{\partial f_i}{\partial l_i} = \mu$

$\lambda = \dfrac{dU}{dX_0}$, $\mu = \dfrac{dU}{dH_0} \Rightarrow \dfrac{\mu}{\lambda} = \dfrac{dU}{dH_0} \cdot \dfrac{dX_0}{dU} = \dfrac{dX_0}{dH_0}$ shadow price of time

$\forall i,j$: $\dfrac{u_{l_i}}{u_{x_i}} \overset{!}{=} \dfrac{\mu/\lambda}{p_i} = \dfrac{w^*}{p_i} = \dfrac{\frac{\partial f_i}{\partial l_i}}{\frac{\partial f_i}{\partial x_i}}$ w^* .. shadow price of time

slope of f isoquant
MR of transformation



DESCRIPTION:

- if $w \nearrow$ → move along isoproduction line → → $l_1 \searrow$, $x_1 \nearrow$
- if technology changes → → $l_1 \searrow$, $x_1 \nearrow$