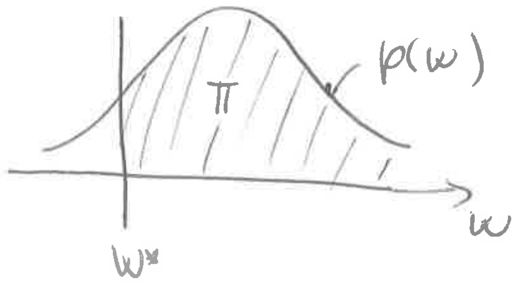
# ~~D~~URATION MODELS

- Not the incidence, but the duration of unemployment is more related to welfare
- Duration plays role in ~~duration~~ search models

OTHER AREAS:

- duration of marriages
- spacing of births
- adoption of new technologies
- time between trades on financial markets
- product durability
- life-times of firms
- duration of wars
- time in office
- length of PhD

Simple model of search:




wage distribution $p(w)$

reservation rate $w^*$

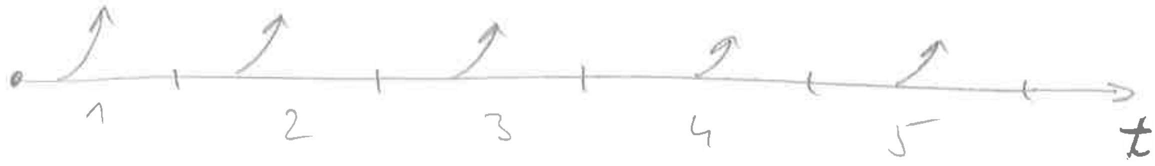
$$\text{Prob(acceptable offer)} = \pi = \int_{w^*}^{\infty} p(w)\,dw$$

$$\text{Prob(job offer arrival)} = \lambda$$

$$\text{Prob(job match)} = \lambda \pi + f(t) \longrightarrow \text{exp distribution of } t$$

$$f(t) = \lambda \exp(-\lambda t)$$

## Discrete time framework




Prob( match ) = $\lambda$ (= const) ~ cond. prob

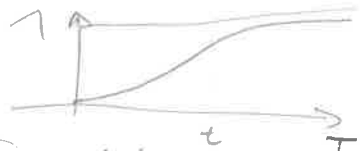
Prob( match at $T=t$ ) = $\lambda(1-\lambda)^{t-1}$ ~ unconditional prob

Prob( survive till $T=t$ ) = $(1-\lambda)^t$ ~ survival fce

Prob( match before $T=t$ ) = $\cancel{\lambda}\ 1-(1-\lambda)^t$ − distribution of spells

## Continuous time framework

$F(t) = Pr(T<t)$ ~ distribution f. of spells




$f(t) = \frac{dF}{dt}$ ~ density function

$T$ ~ random variable

$S(t) = 1 - F(t) = Prob(T \geq t)$

$\lambda(t) = \frac{f(t)}{S(t)}$ ~ hazard function $= \frac{f(t)}{1-F(t)}$ ~ mills ratio

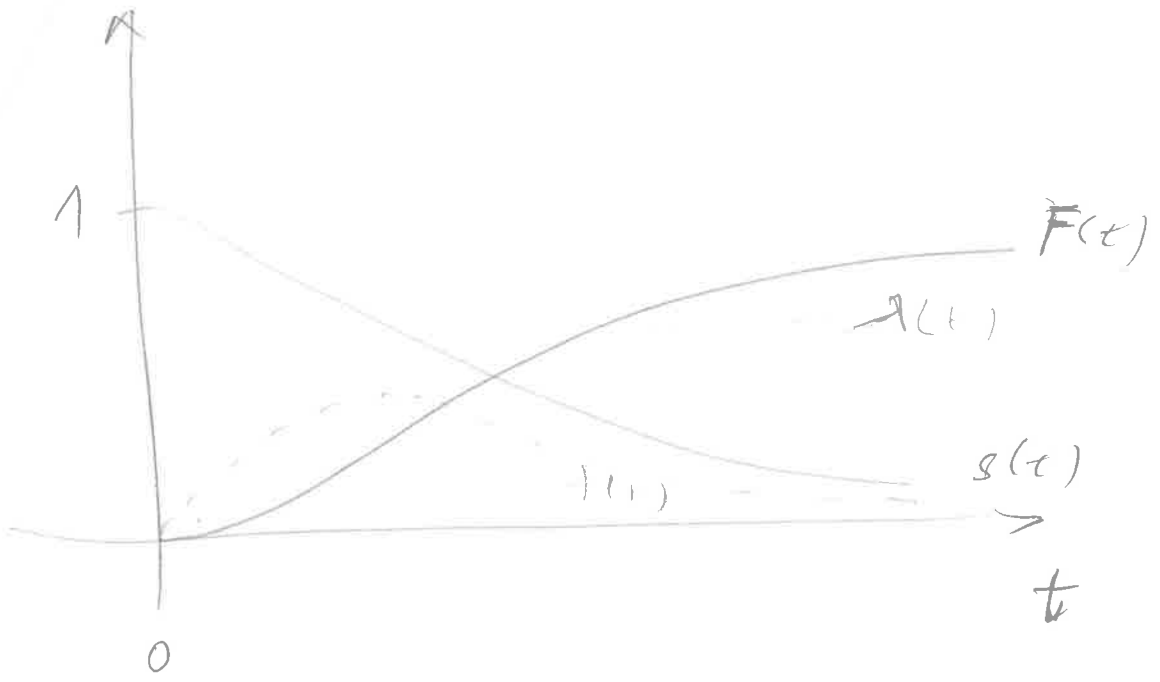
~ continuous version of conditional prob.

$= \frac{dF/dt}{S(t)} = \frac{-dS/dt}{S} = -d\ln S(t)/dt$

$F(t)$, $S(t)$, $\lambda(t)$ describe the same but $\lambda(t)$ is simpler element to work with and model than $F(t)$, $S(t)$

Frequent assumption $\lambda(t) = \text{const}$ → simple distr. function

~~M~~ Normal and lognormal $F(t)$ do not have simple hazard functions.




## Duration dependence

No: $\frac{\partial \lambda(t)}{\partial t} = 0$ at $t = t^*$

Possitive: $\frac{\partial \lambda(t)}{\partial t} > 0$ at $t = t^*$ (search effort increasing)

Negative: $\frac{\partial \lambda(t)}{\partial t} < 0$ — I — (increasing reservation wage or decreasing wage)

Some distributions:

EXPONENTIAL - memory-less

$\lambda(t) = \gamma$ →

$S(t) = \exp(-\gamma t)$

$F(t) = 1 - \exp(\cdot)$

$f(t) = \gamma \exp(-\gamma t)$

$$\underbrace{\lambda = -\frac{ds/dt}{S}}_{\text{we know}} \Rightarrow \gamma dt = -\frac{ds}{S}$$

$$\int \gamma dt = \int -\frac{ds}{S}$$

$$\gamma t = -\ln S$$

$$S = \exp(-\gamma t)$$

$$F = 1 - \exp(-\gamma t)$$

$$f = \frac{dF}{dt} = \gamma \exp(-\gamma t)$$

model $\lambda_1(x, \beta) = \exp(x_1' \beta_1)$

Why is <u>duration</u> of unemployment important?

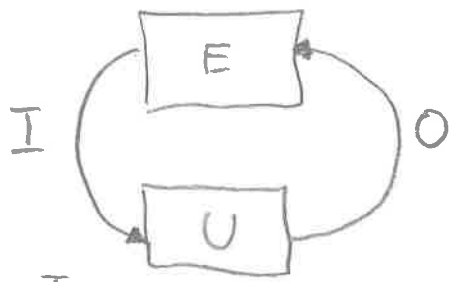
↳ it is important determinant of unemployment rate

unemployment rate (in equilibrium) $= u_r$

$$\boxed{u_r = \frac{i_r}{i_r + o_r}} \qquad \text{where } u_r = \frac{U}{LF}$$

probability of outflow $\quad i_r = \frac{I}{E}$ ~ infl. rate

How to get this formula? $\quad o_r = \frac{O}{U}$ ~ outfl. rate



in equilibrium: $I = O$

$$E\, i_r = o_r U \rightarrow U = \frac{E\, i_r}{o_r} \rightarrow$$

$$U_r = \frac{U}{LF} = \frac{E\, i_r}{o_r\, LF} = \frac{E\, i_r}{o_r\,(E+U)} = \frac{i_r}{o_r + \frac{O}{\cancel{U}}\frac{\cancel{U}}{E}}$$

$$= \frac{i_r}{o_r + \frac{O}{E}} = \left| \begin{array}{c} \text{we know that} \\ I = O \end{array} \right| = \underline{\underline{\frac{i_r}{o_r + i_r}}}$$

Duration of unemployment vs. heterogeneity