HUMAN

CAPITAL

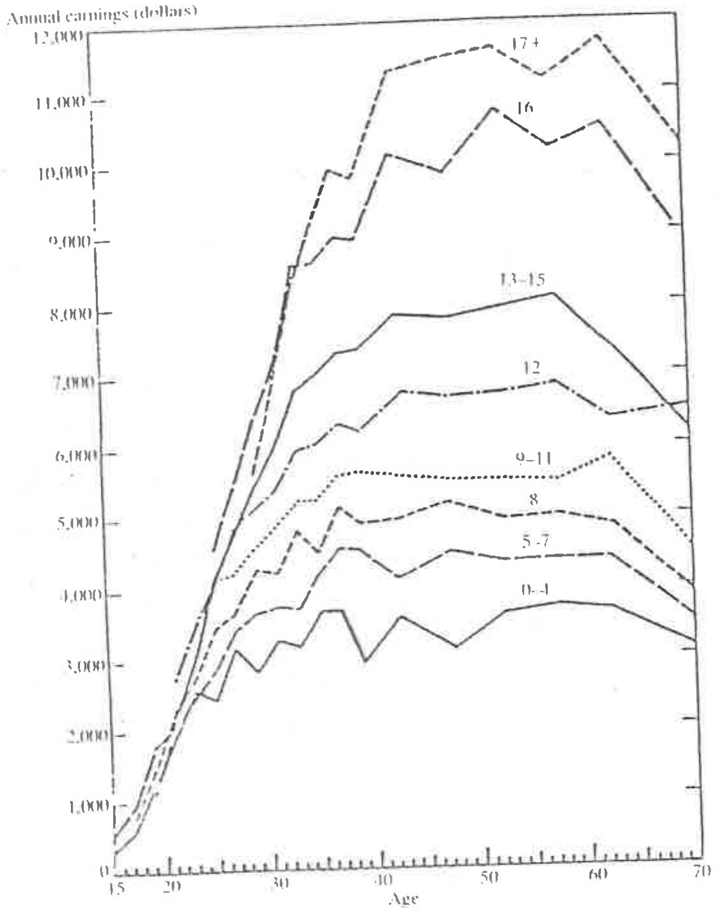


Figure 2.2 Age–earnings profiles of white non-farm men by schooling level, 1959. *Source:* Mincer, 1974, 66

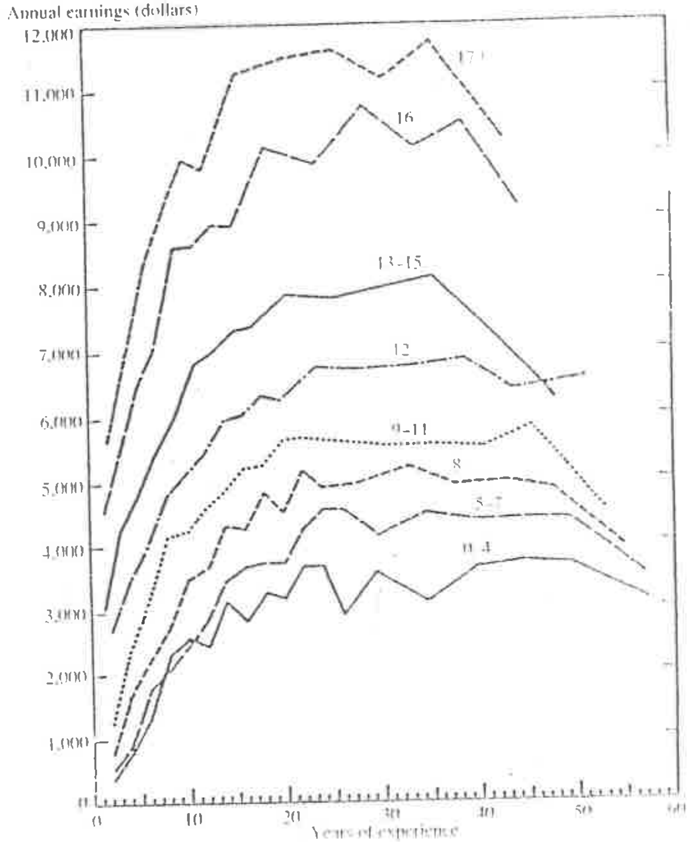


Figure 2.3 Experience–earnings profiles for white non-farm men by schooling level, 1959. *Source:* Mincer, 1974, 67

In figure 2.3 the two earnings patterns alluded to earlier are eminently clear: (1) earnings rise with age at a diminishing rate and (2) earnings profiles are

INVESTMENT IN HUMAN CAPITAL

- Assumptions: - investment into onself one can increase own productivity
  - investment is also giving up of current earning

- CONTENT: simple examples → simple, asym..., screening, education, estimation

CASE

$$-C_0 + \sum_{i=1}^{n} \alpha^i Y_i > 0$$

C_0 .... current investment cost

$Y_1 \dots Y_n$ .. generated income

$\alpha_i = \frac{1}{1+r_i}$

if $r_i = r$; $Y_i = Y$

$$-C_0 + Y \sum \alpha^i = -C_0 + \frac{Y}{r}(1-\alpha^n)$$

→ Finite income stream
→ see next page →

$$\boxed{\frac{Y}{r}(1-\alpha^n) > C_0}$$ criterium for investment

using IRR - Internal Rate of Return approach

$$\frac{Y}{\rho}\left(1 - \frac{1}{(1+\rho)^n}\right) \stackrel{!}{=} C_0$$

→ find ρ^* and see if $\rho > r$
→ advantageous investment

CASE

$Y_i = \overline{w} E_i \overline{H}$; $\Delta Y = Y_1 - Y_0 = \overline{w}\overline{H}\Delta E$

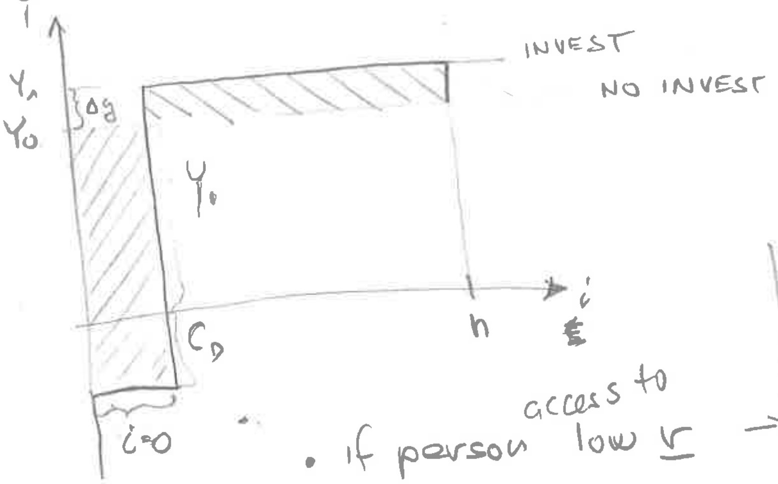
$C_0 = C_D + Y_0$

$H = \overline{H}$ ... hours worked

E – human capital

$w = \overline{w}$ rental value of k = wage

C_0 - cost < direct, forgone income



$$\boxed{\frac{\Delta Y}{r}(1-\alpha^n) > Y_0 + C_D}$$

- if person access to low $\underline{r}$ → $\frac{\Delta Y}{r}\uparrow$ → invest more
- the longer n → invest more
- invest early in carier
- C_D - case of taxes
- returns to society from individual investment
- child labor

A note on geometric series

NOTE

#2

$$S_n = 1 + q + \dots q^n$$

$$\left(S_{n+1} = 1 + q + \dots q^n + q^{n+1} \right)$$

$$q S_n = \underbrace{q + q^2 \dots q^n}_{} + q^{n+1}$$

$$= S_n - 1 + q^{n+1}$$

$$S_n = \frac{1-q^{n+1}}{1-q}$$

$$q = \frac{1}{1+r} \qquad S_n = \frac{1 - \left(\frac{1}{1+r}\right)^{n+1}}{1 - \frac{1}{1+r}} = \frac{1 - (\quad)}{\frac{\cancel{1}+r-\cancel{1}}{1+r}} = \frac{1}{r}\left[(1+r) - \frac{1}{(1+r)^n}\right]$$

#1

$$S_n = q + q^2 \dots q^n = ? \qquad q = \frac{1}{1+r}$$

$$\left(S_{n+1} = \dots \dots q^n + q^{n+1} \right)$$

$$q S_n = q^2 + \dots q^n + q^{n+1}$$

$$= S_n - q + q^{n+1}$$

$$S_n = \frac{q - q^{n+1}}{1-q} = \left| q = \frac{1}{1+r} \right| = \frac{\frac{1}{1+r} - \left(\frac{1}{1+r}\right)^{n+1}}{\frac{r}{1+r}} = \frac{1}{r}\left[1 - \left(\frac{1}{1+r}\right)^n\right]$$

$$\equiv \frac{1}{r}\left[1 - q^n\right]$$

Or; using #1 & #2:

$$S_n = 1 + q + \dots q^n = \frac{1}{r}\left[(1+r) - \frac{1}{(1+r)^n}\right] \Rightarrow$$

$$\Rightarrow S_n - 1 = \frac{1}{r}\left[(1+r) - \frac{1}{(1+r)^n}\right] - 1 = \dots = \frac{1}{r}\left[1 - \left(\frac{1}{1+r}\right)^n\right]$$

Original idea of HC model - Mincer

$\Downarrow E_t$

- At any period of life, t, decission on how to alocalo a unit of time (day, year)

$0<\Phi_t<1$; if $1-k_t$ ~ share devoted to work $\Rightarrow Y_t = \overset{*w=1}{E_t}(1-k_t)$

k_t ' — " — to studies:

forgone earnings (costs)

$C_t = k_t E_t$

increase in HC in the future

$E_{t+1} = E_t + r C_t$

$r > 0$ ~ return

$\Delta E_t = r C_t$

$\Delta E_t / C_t = r$

- Questions:
  - shape of life-cycle profile of Y_t, E_t, C_t?
  - decissions driving k_t?
  - shape of profile during formal schooling?
  - decission driving optimal schooling?
  - testable predictions?

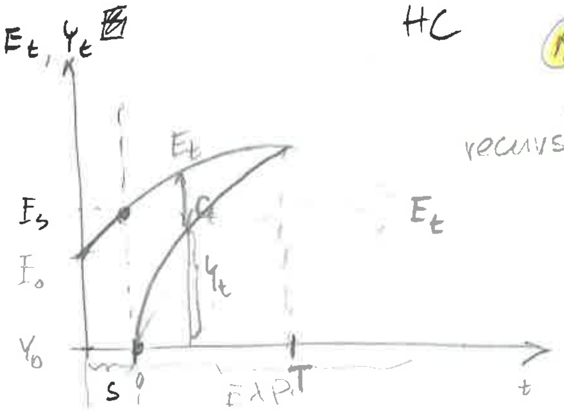
MODEL Jacob Mincer & Becker - general investment – assume 1 HC → DISCRETE INVestment = forgone earnings

- investment & HC not observable, only $Y_t = w E_t - C_t$ (1)

E... Earning capacity = HC

C... investment in HC

Y... earnings

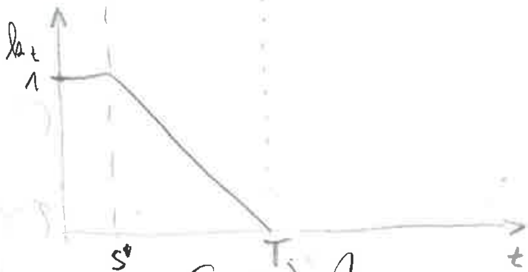


(1) $E_t = E_{t-1} + r\,C_{t-1}$

recursively → $= (E_{t-2} + rC_{t-2}) + rC_{t-1}$

$= E_S + r \sum_{\tau=1}^{t-1} C_\tau$ → not observed

(5) $Y_t = E_t - C_t = E_S + r \sum_{\tau=1}^{t-1} C_\tau - C_t$



(3) $\dfrac{\Delta E_t}{\Delta t} = r\,C_{t-1}$ → if $\delta = 0$ → E increasing

(4) $\dfrac{\Delta Y_t}{\Delta t} = \dfrac{\Delta E_t}{\Delta t} - \dfrac{\Delta C_t}{\Delta t}$ → steeper than E_t ($\dfrac{\Delta C_t}{\Delta t} < 0$)

$C_t = f(?)$!

ASSUMPTION I: $C_t = C_0\left(1 - \dfrac{t}{T}\right) \Rightarrow \dfrac{dE_t}{dt} = r\,C_t \Rightarrow dE_t = r\,C_t\,dt$

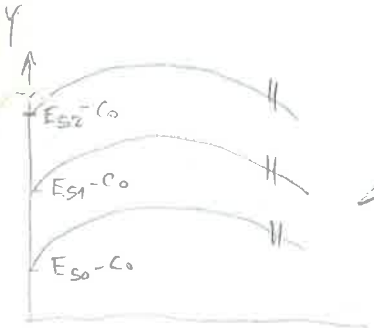
$E_t = E_S + r\int C_0\left(1 - \dfrac{\tau}{T}\right) d\tau =$ | $\int dE_t = \int^t r\,C_0\left(1-\frac{t}{T}\right) + E_S$

$= E_S + rC_0\left[\tau - \dfrac{\tau^2}{2T}\right] =$

$= E_S + rC_0 t - rC_0 \dfrac{t^2}{2T}$ we do not know E_t

$Y_t = E_t - C_t = E_S + rC_0 t - rC_0\dfrac{t^2}{2T} - C_0\left(1 - \dfrac{t}{T}\right)$

$= (E_S - C_0) + \underbrace{C_0\left(r + \frac{1}{T}\right)t - rC_0\frac{t^2}{2T}}_{\text{no } S}$



slope at $t = 0$:

$\left.\dfrac{\partial Y_t}{\partial t}\right|_{t=0} = C_0\left(r + \dfrac{1}{T}\right) + \theta$

but we observe in dat

→ modify our model → %

OTHER WAY - (CONTINUOUS CASE)

assume rate of investment declining over time

$$\frac{C_t}{E_t} = k_0\left(1-\frac{t}{T}\right) = k_t$$

instead of $E_t = E_{t-1} + rC_{t-1}$

we write

$$\frac{dE_t}{dt} = rC_t = rE_t k_0\left(1-\frac{t}{T}\right)$$

$$Y_t = E_t - C_t =$$

$$Y_t = E_t\left[1-k_0\left(1-\frac{t}{T}\right)\right]$$

$$\frac{dE_t}{E_t} = r k_t \, dt \qquad //$$

$$\ln Y_t = \ln E_t - k_0\left(1-\frac{t}{T}\right)$$

$$\int_0^t \frac{dE_t}{E_t} = \int_0^t r k_t \, dt + \text{cons}_0$$

$$\ln E_t = r k_0\left[t - \frac{t^2}{2T}\right] + \text{cons}_0 \qquad \Big|\ t=0:\ \ln E_s = \text{cons}_0$$

$$\ln Y_t = r k_0\left[t - \boxed{\frac{t^2}{2T}}\right] - k_0\left(1-\frac{t}{T}\right) + \ln E_s$$

during schooling: the same investment equation but simple

$$E_t = E_{t-1} + rE_{t-1}$$

$$\frac{dE_t}{dt} = rE_{t-1}$$

$$\int \frac{dE_t}{E_t} = r\int 1\, d\tau + \text{const.} \qquad Y_s = 0$$

$$\ln E_t = tr + \text{cons}_{00} \quad E_0$$

$$\ln E_s = rS + \ln E_{00} \qquad / \text{ for } t = S \sim \text{school years}$$

Altogether $\ln Y_t = (\ln E_0 - k_0) + (rk_0 + k_0)\, t - rk_0 \boxed{\frac{t^2}{2T}} + rS$

$$\ln Y_t = \alpha + \beta_1 EXP + \beta_2 EXP^2 + \gamma\, EDU$$

see pictures

ASSUMPTION II

$$k_t \equiv \frac{C_t}{E_t} = k_0\left(1-\frac{t}{T}\right)$$ → rate of investment declines over time

discrete world solution

$$E_t = E_{t-1} + rC_{t-1} = E_s + r\sum_{\tau=1}^{t-1} C_\tau$$

$$Y_t = E_t - C_t = E_s + r\sum_{\tau=1}^{t-1} C_\tau - C_t$$

$$E_t = E_{t-1}(1+rk_{t-1}) = E_s \prod_{\tau=1}^{t-1}(1+rk_\tau)$$

$$\ln E_t = \ln E_s + \sum \ln(1+rk_\tau)$$

$$\approx \ln E_s + r\sum k_\tau = \ln E_s + r\int_0^t k_0\left(1-\frac{\tau}{T}\right)d\tau$$

$$\approx \ln E_s + rk_0\left[t-\frac{t^2}{2T}\right]$$ concave shape

we do not know E_t!

$$Y_t = E_t(1-k_t) \leftarrow Y_t = E_t - C_t$$

$$\ln Y_t = \ln E_t + \ln(1-k_t) \doteq \ln E_t - k_t$$

$$\ln Y_t = \ln E_s + rk_0\left[t-\frac{t^2}{2T}\right] - k_0\left(1-\frac{t}{T}\right)$$

$$\ln E_s = \ln E_0 + r\sum_{\tau=1}^{s} k_\tau = \ln E_0 + rs$$

$$\ln Y_t = \underbrace{(\ln E_s - k_0)}_{const} + \left(r+\frac{1}{T}\right)k_0 t - rk_0\frac{t^2}{2T}$$

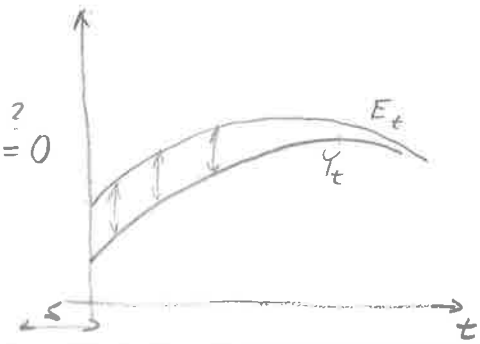
$$\boxed{\ln Y_t = \ln E_0 + rs + (\beta_1)t + (\beta_2)t^2}$$

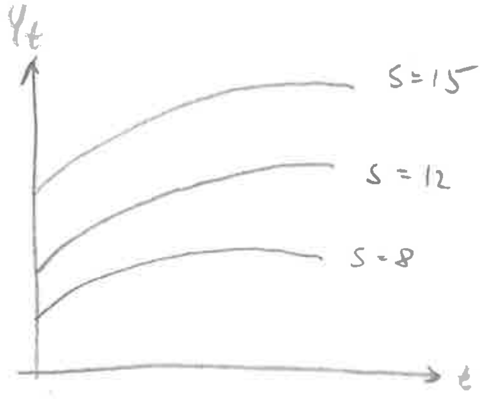
JACOB MINCER

$$\frac{\partial \ln Y_t}{\partial s} = r$$

$$\frac{\partial \ln Y_t}{\partial t} = \beta_1 + 2\beta_2 t \overset{?}{=} 0$$

$$\left.\frac{\partial \ln Y_t}{\partial t}\right|_{t=0} = \beta_1$$



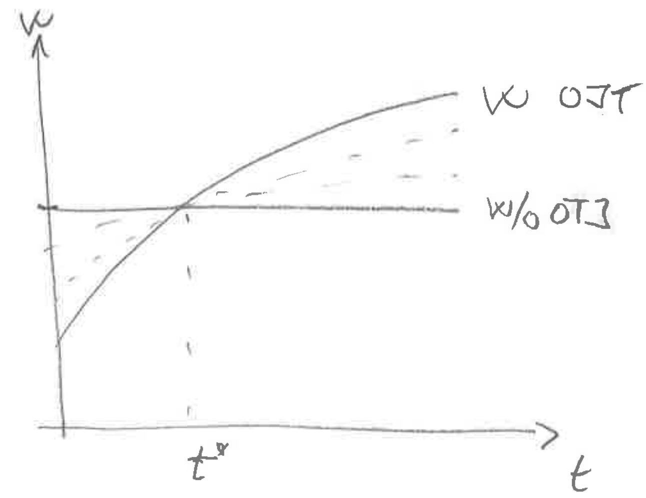


Sample of Mincer regression coefficients

	Coefficient on EXP	100*Coefficient on EXP2	Years of maximum returns` from experience	Average years of schooling 1995	Coefficient on EDU	Period
Argentina	0.052	-0.07	37.1	0	0.11	1989
Austria	0.039	-0.067	29.1	11.9	0.04	1987
Bolivia	0.046	-0.06	38.3	0	0.07	1989
Brazil	0.073	-0.1	36.5	5.3	0.15	1989
Britain	0.091	-0.15	30.3	12.1	0.10	1972
Canada	0.025	-0.046	27.2	13.2	0.04	1981
Chile	0.048	-0.05	48.0	0	0.12	1989
China	0.019	0	n.a.	0	0.05	1985
Colombia	0.059	-0.06	49.2	0	0.15	1989
Czech R.	0.021	-0.04	26.3	12.5	0.03	Men, 1989
Czech R.	0.021	-0.04	26.3	12.5	0.07	Men, 1996
Czech R.	0.028	-0.059	23.7	0	0.04	1988
Czech R.	0.032	-0.063	25.4	0	0.09	1996
Denmark	0.033	-0.057	28.9	12.4	0.05	1990
Ecuador	0.054	-0.08	33.8	0	0.10	1987
Greece	0.039	-0.088	22.2	10.9	0.03	1985
Guatemala	0.044	-0.06	36.7	0	0.14	1989
Hungary	0.034	-0.059	28.8	11.3	0.04	1987
India	0.041	-0.05	41.0	0	0.06	1981
Indonesia	0.094	-0.1	47.0	0	0.17	1981
Ireland	0.061	-0.1	30.5	10.8	0.08	1987
Israel	0.029	-0.046	31.5	0	0.06	1979
Italy	0.01	-0.027	18.5	10	0.03	1987
Kenya	0.044	-0.2	11.0	0	0.09	1980
South Korea	0.082	-0.14	29.3	0	0.11	1986
Malaysia	0.013	-0.004	162.5	0	0.09	1979
Mexico	0.084	-0.1	42.0	0	0.14	1984
Netherlands	0.035	-0.049	35.7	12.7	0.07	1983
Pakistan	0.106	-0.06	88.3	0	0.10	1979
Poland	0.021	-0.036	29.2	11.1	0.02	1986
Portugal	0.025	-0.04	31.3	10	0.09	1985
Singapore	0.062	-0.1	31.0	0	0.11	1974
Spain	0.049	-0.06	40.8	11.2	0.13	1990
Sweden	0.049	0	n.a.	12.1	0.03	1981
Switzerland	0.056	-0.069	40.6	12.6	0.07	1987
Thailand	0.071	-0.088	40.3	0	0.09	1971
USA	0.032	-0.048	33.3	13.5	0.09	1989
West Germany	0.045	-0.077	29.2	13.4	0.08	1988
AVERAGE	0.049	-0.069	42.9	11.6	0.09	-

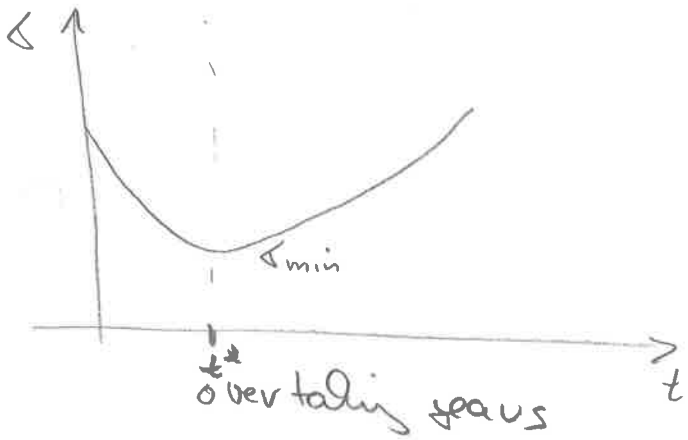
Mark Bils and Peter J. Klenow, 1997, Does Schooling Cause Growth or the Other Way Around?, pp.36.

OVERTAKING YEARS



max correl. of (w, s) at t^*

OJT on-the-job training



$$\ln Y = s * r \Rightarrow Y = e^{s*r} \rightarrow \frac{dY}{ds} = re^{sr} \leftarrow \text{growing with } \underline{\underline{s}}$$

$$\rightarrow \underline{\underline{\%\Delta Y}} = \frac{\frac{dY}{s}}{Y} = \frac{re^{sr}}{e^{sr}} = \underline{\underline{r}}$$

$$\frac{d \ln Y}{ds} = r$$

$$\frac{dY}{ds} \frac{1}{Y} = r$$

$$\boxed{\frac{\Delta \% Y}{\Delta s} = r}$$

• DEPRECIATION

$$\frac{dE_t}{dt} = r C_t - \rho E_t$$

$$= r k_0 \left(1 - \frac{t}{T}\right) E_t - \rho E_t = \left[r k_0 \left(1 - \frac{t}{T}\right) - \rho\right] E_t$$

$$\frac{dE_t}{E_t} = \dots$$

$$\ln E_t = \ln E_s + r k_0 \left(t - \frac{t^2}{2T}\right) - \rho t$$

$$= \ln E_s + t[r k_0 - \rho] - t^2 \frac{r k_0}{2T}$$

$$Y_t = E_t (1 - k_t)$$

$$\ln Y_t = \ln E_t + \ln(1 - k_t) \approx \ln E_t - k_t$$

$$= \ln E_s + t[r k_0 - \rho] - t^2 \frac{r k_0}{2T} - k_0 \left(1 - \frac{t}{T}\right)$$

$$= \underbrace{[\ln E_s - k_0]}_{\text{const}} + t \underbrace{k_0 \left[r + \frac{1}{T} - \frac{\rho}{k_0}\right]}_{\beta_1} - t^2 \underbrace{\frac{r k_0}{2T}}_{\beta_2}$$

new term smaller than if $\rho = 0$

$\Rightarrow t^*(E = E_{max})$ is smaller

$$\frac{\partial \ln Y}{\partial t} = \beta_1 + 2 * \beta_2 t^* = 0$$

$$\boxed{t^* = \frac{\beta_1}{2\beta_2}}$$ smaller $\Rightarrow t^*$ smaller

$$\boxed{\text{Maximum of } E_t \text{ \& } Y_t}$$

$$\ln E_t = r k_0 \left[t - \frac{t^2}{2T} \right] + \ldots \text{ const}$$

$$\frac{\partial \ln E_t}{\partial t} = r k_0 \left[1 - \frac{2t^*}{2T} \right] \overset{!}{=} 0 \rightarrow \boxed{t^* = T}$$

$$\ln Y_t = r k_0 \left[t - \frac{t^2}{2T} \right] - k_0 \left(1 - \frac{t}{T} \right)$$

$$\frac{\partial \ln Y_t}{\partial t} = r \cancel{k_0} \left[1 - \frac{2t^{**}}{2T} \right] + \cancel{k_0}/T \overset{!}{=} 0 \quad / * T$$

$$\boxed{\begin{aligned} t^{**} &= \frac{1}{r} + T \quad ; \text{ if } r \approx .07 \\ &= 14.2 + T > T \end{aligned}}$$

Gender wage gap formation