• SIMPLE MODEL OF OPTIMIZING INVESTMENT INTO HC

Ben Porath

$E_t = E_{t-1} + r C_{t-1}$, $C_t = C_0 \left[1 - \frac{t}{T}\right]$

or

$dE_t = r C_{t-1}$ $\qquad \frac{C_t}{E_t} = k_0 \left[1 - \frac{t}{T}\right]$

$w \equiv 1$ (assume)

r ... discount rate

s ... as k - time allocation to study

• OPTIMIZATION ON C_t

we know $\frac{\Delta \$}{r}\left[1 - \mathcal{L}^T\right] \geq \Delta \text{Costs}$

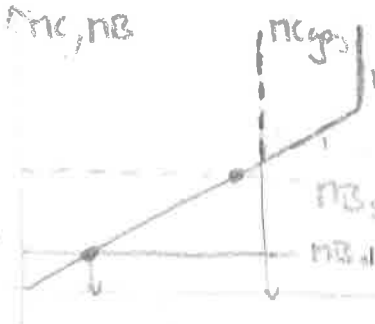
1) Assume $E_t = E_{t-1} + Q_t \equiv \frac{dE_t}{dt} = Q_t$, $w = 1$

2) HC prod. fcion: $Q_t = (s_t E_t)^b$, $0 < b < 1$, $0 \leq s_t \leq 1$; (ability, costs)

3) Compare marginal costs $\text{Cost}_t = s_t E_t \Rightarrow \text{Cost}_t = Q_t^{1/b}$

$MC = \frac{\partial C_t}{\partial Q_t} = \frac{1}{b} Q_t^{(1-b)/b}$

for $b = .5$



marginal benefits: $BEN_t = \frac{1}{r}\left(1 - \frac{1}{(1+r)^{T-t}}\right) Q_t$

$MB_t = \frac{\partial BEN_t}{\partial Q_t} = \frac{1}{r}\left(1 - \frac{1}{(1+r)^{T-t}}\right)$

$MC = MB \Rightarrow \frac{\partial BEN_t}{\partial Q_t} \overset{!}{=} \frac{\partial \text{Cost}_t}{\partial Q_t} \Rightarrow \frac{1}{r}\left(1 - \frac{1}{(1+r)^{T-t}}\right) = \frac{1}{b} Q_t^{(1-b)/b}$

MB : MC

Q_t^*

r	$\gg$	→ lower growth of earnings E_t
b	$\nearrow$	→ higher growth of E_t, Y_t
$\frac{t}{T}$	$\searrow$	

$Q_t^* = \left[\frac{b}{r}\left(1 - \frac{1}{(1+r)^{T-t}}\right)\right]^{b/1-b}$

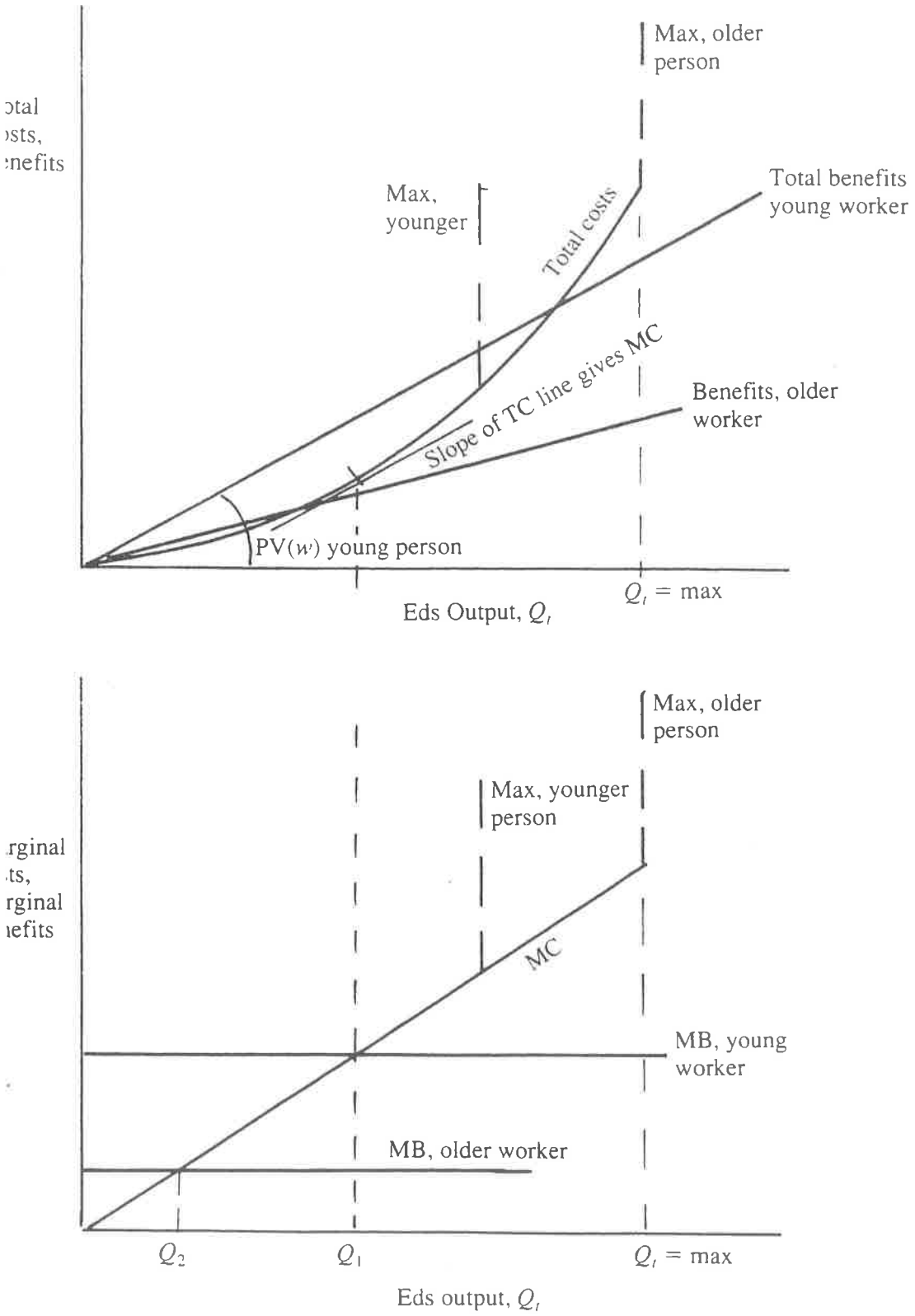
$Y_t = E_t - s_t E_t = (1 - s_t^*) E_t^*$

It could be that $s_t^* \geq 1$ → "Full time" MB > MC

↓ see graph

$\int dE_t^* = \int Q_t^* \, dt$

$E_t^* = E_s + \ldots \Rightarrow s_t^* = \frac{Q_t^{*1/b}}{E_t^*}$



ure 2.6 Total and marginal benefits and costs of producing eds

ially human capital is a very profitable investment, but the individual ınot produce enough of it, and so chooses $s_t = 1$. Q_t then declines con-ıously as one gets older reaching zero at retirement. Accumulated human ıital is computed by adding the annual investments. The process of adding ırly human capital investments yields a stock of human capital curve, K_t, as ıicted in figure 2.7. Note that the stock of human capital increases quickly he period when $s_t = 1$, then more slowly in middle age, and stops increas-

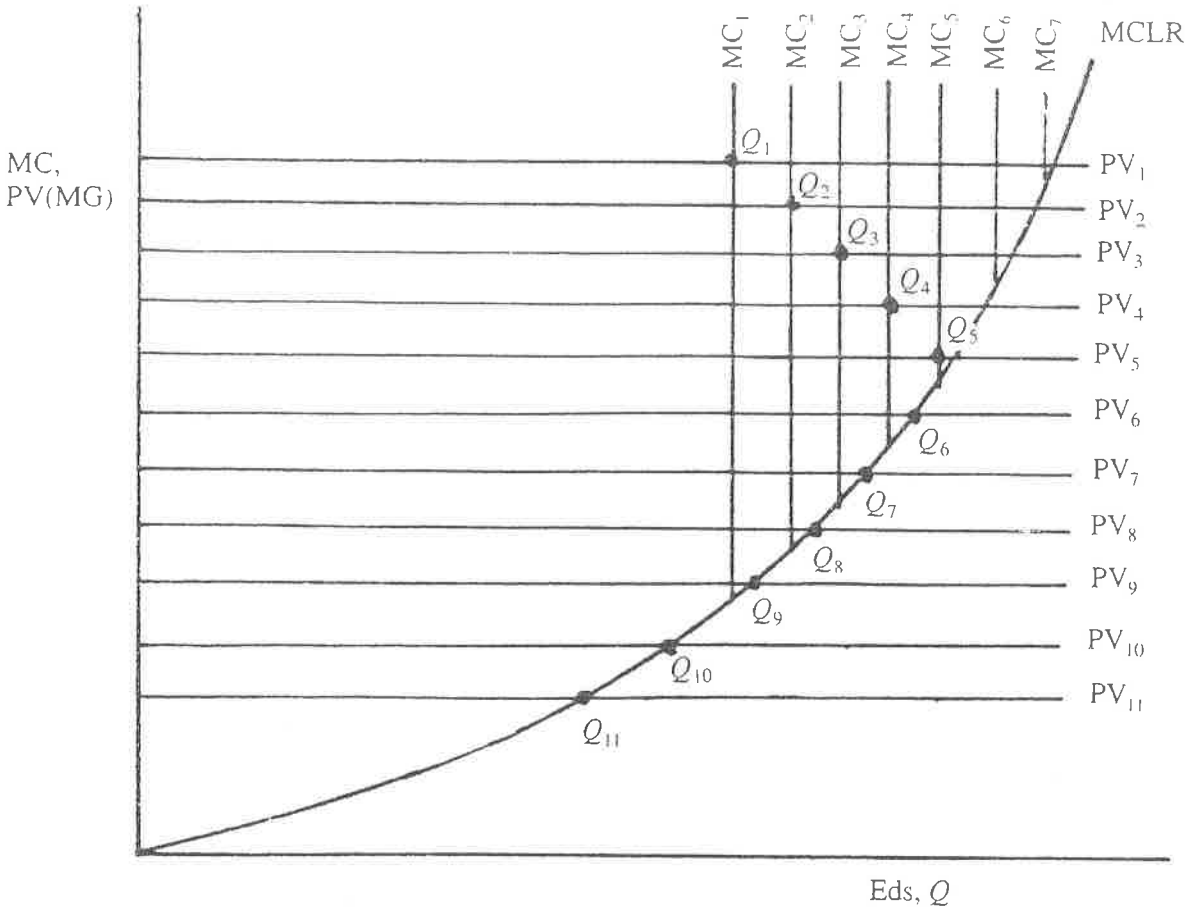
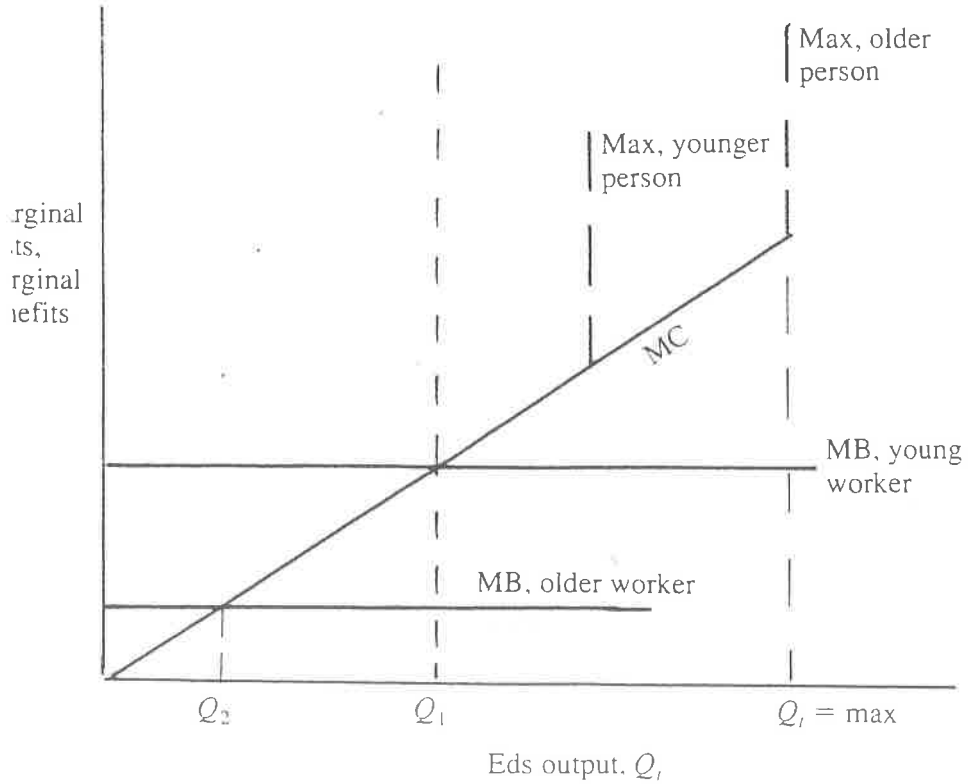


Figure 3.2 The sequence of human capital outputs during and after school

Eds Output, Q_t

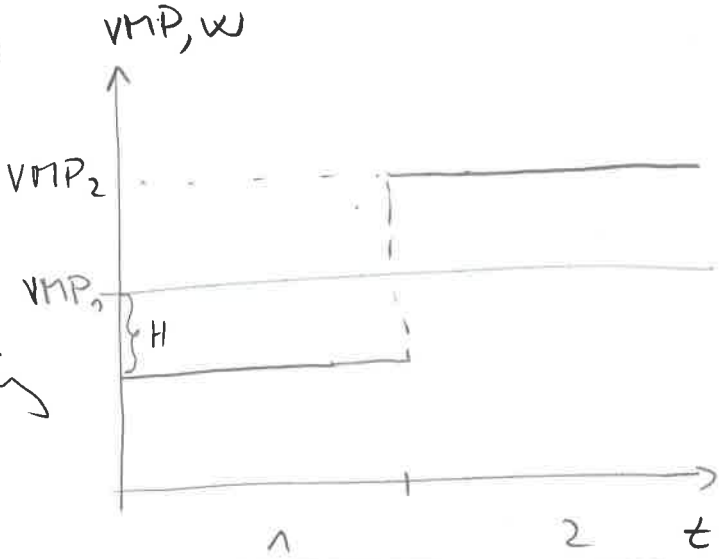


ure 2.6 Total and marginal benefits and costs of producing eds

2 period model of OJT

If perfect competition

$$TC_1 + \frac{TC_2}{1+r} \leq VMP_1 + \frac{VMP_2}{1+r}$$



OJT in t=1 : H costs of training

$$w_1 + H + \frac{w_2}{1+r} \leq VMP_1 + \frac{VMP_2^*}{1+r}$$

$$w_1 + H \leq VMP_1 + \underbrace{\frac{VMP_2^* - w_2^*}{1+r}}_{G}$$

NOTE:

$$VMP = MP \cdot P = \frac{\partial Q}{\partial L} P$$

If competition

$$VMP = w$$

WHO PAYS FOR H?

General HC: $G \overset{!}{=} 0$ ← forced by mkt to prevent leave

↳ $w_1 + H \leq VMP_1$

worker gets $[VMP_1 - H] + \frac{VMP_2^*}{1+r}$

- medical interns
- out of firm courses π, by $w_1 \leq VMP_1 - H$
- monopsonies

Specific HC: consider $G = 0$ {costs covered by (w)}

alternative w not affected by H

↳ $w_1 = VMP_1 - H$ → save costs

$w_2 = VMP_2^*$ → uncertain gains for (w)

Prob layoff

consider $w_1 = VMP_1$ {costs covered by (F)

$$H \leq \underbrace{\frac{VMP_2^* - w_2}{1+r}}_{>0}$$

↳ save costs H

↳ uncertain gains for (F)

Alternative w not affected by H

ECIFIC HC.

• alternative wage not affected by training

WHO PAYS & WHO GETS RETURN?

① Firm PAYS: $H_E > 0$, $G > 0$
but if worker quit → loss

② Worker PAYS: $H_w > 0$, danger the laid-off → loss

G. Becker: $\bar{w} < w_2 < VMP_2$ sharing returns
↳ sharing cost H

IMPLICATIONS

IF SP. TRAINING
• at T_1 ... worker's get $w_1 < VMP_1$
↳ share on H
• at T_2 ... worker's get $w_2 < VMP_2^*$
↳ firm gets part of return

$W < MP * p$!!

• formal & informal contracts

• if negative demand shock MP↓ since $W < MP * P$ → buffer of senior workers → less lay-offs
→ firing low tenure

inv into spec. HC →
→ ↓ job termination
→ lower turnover with Tenure

last hired - first laid off
↳ skilled workers are less unemployed
↳ temporary lay-off...