

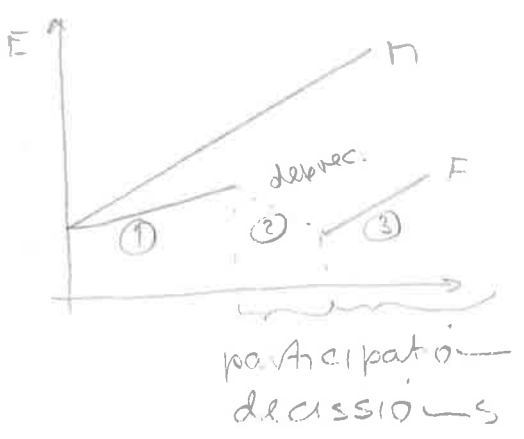
About r

- r ... different for schooling levels
- r ... stable during 1940-1970 in the USA
- r ... decline in 70's growing later
- r ... high in developing countries
- lower in developed
- very low in centrally planned & Scandinavian

Gender wage differentials



- discrimination
- job segregation
- different Xs: $EXP = Age - S - G - h$



- ① Low investment due to lower $PV(Y)$
- ② No \bar{i} + deprec $\sim 3\%$ / year
- ③

Mincer & Polachok: $\ln Y_t = a + b_s s + b_1 x + b_2 x^2 - b_3 h - b_4 h^2$

- h ... employers expecting h do not offer career track jobs
- LH is cheaper than household cleaner / nanny
 - earlier retirement against women
 - legislation: is not very effective (unemployment)

COEFFICIENT ON EDUCATION

- in Mincerian regression - private rate of return

↓
valid only if ability does not exist

- productivity effect

↳ abilities that are unobserved must be $\text{cov}(\text{EDU}, A) = 0$

- private and social marg. returns differ \Rightarrow inefficient outcome

1) $r_p > r_s \rightarrow$ over investment

2) $r_p < r_s \rightarrow$ underinvestment

↳ improved matching (Stiglitz '75)

CASE unpleasant schooling people - skilled having comparative advantage in low skilled jobs

If employers cannot distinguish \rightarrow

\rightarrow signaling helps to raise both wages

- Ability and education

$\uparrow A_i \rightarrow$ more productive at every level

\rightarrow higher opportunity costs of schooling
↳ $\downarrow \text{EDU}$

\rightarrow value future more \rightarrow discount less
↳ $\uparrow \text{EDU}$

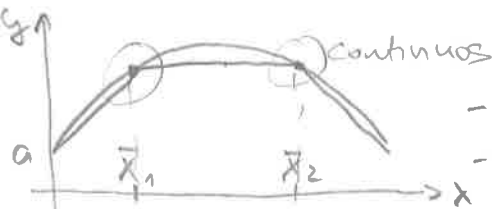
\rightarrow learn faster

↳ $\uparrow \text{EDE}$

\rightarrow easier
OJT \rightarrow
 $\rightarrow \uparrow W$

Notes on estimation

Spline $\rightarrow y = a + b_1x + b_2(x - \bar{x}_1)D[x > \bar{x}_1] + b_3(x - \bar{x}_2)D[x > \bar{x}_2]$



- polynomial versions possible
 - possible for more x 's
 - regions of x by definition or options
- D ... indicator func = dummy

log $= \beta x + \delta D$; $D = 1, 0$

$$\left. \begin{aligned} \ln y_0 &= \beta x + 0 \\ \ln y_1 &= \beta x + \delta \end{aligned} \right\} \ln y_1 - \ln y_0 = \ln \frac{y_1}{y_0} = \ln \left(\frac{y_0 + \Delta y}{y_0} \right) = \ln \left(1 + \frac{\Delta y}{y_0} \right) = \delta$$

$$\% \Delta Y \equiv \frac{\Delta y}{y_0} = e^\delta - 1 \Rightarrow \delta \sim \% \text{ for small } \delta$$

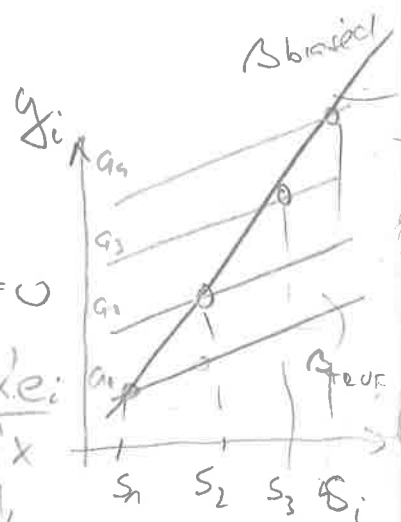
Ability bias

$$y_i = \beta x_i + \underbrace{a_i}_{\varepsilon_i} + e_i, \quad \text{cov}(a_i, x_i) > 0$$

$$\text{cov}(a_i, e_i) = \text{cov}(x_i, e_i) = 0$$

$$\hat{\beta}_{OLS} = \frac{X'Y}{X'X} = \frac{X'(\beta X + a + e)}{X'X} = \beta + \frac{X'a}{X'X} + \frac{X'e}{X'X}$$

\downarrow \downarrow
 > 0 0
 \downarrow \downarrow
 bias 0



- error in variables

RHS: EDU $\left\{ \begin{array}{l} \text{recall error} \\ \text{imputation error} \\ \text{drop outs} \end{array} \right\} \Rightarrow \text{bias} \nearrow \searrow$

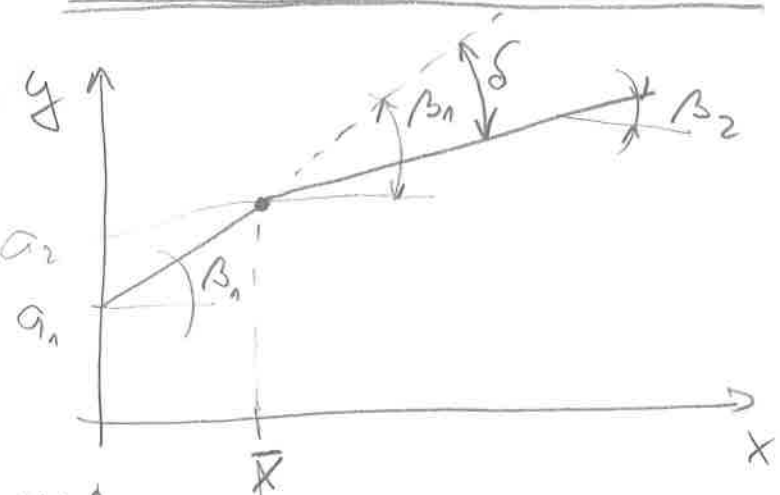
LHS: Y ... recall \rightarrow no bias
taxes \rightarrow bias

missing

$$\text{EXP} = \text{AGE} - \text{EDU} - 6 - h$$

- specification EDU $\left\{ \begin{array}{l} \text{years?} \\ \text{levels?} \end{array} \right.$

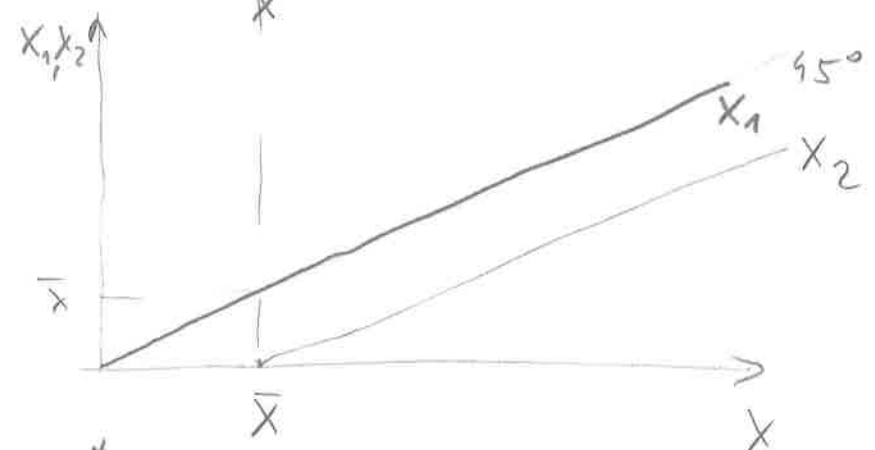
Tuition on SPLINES



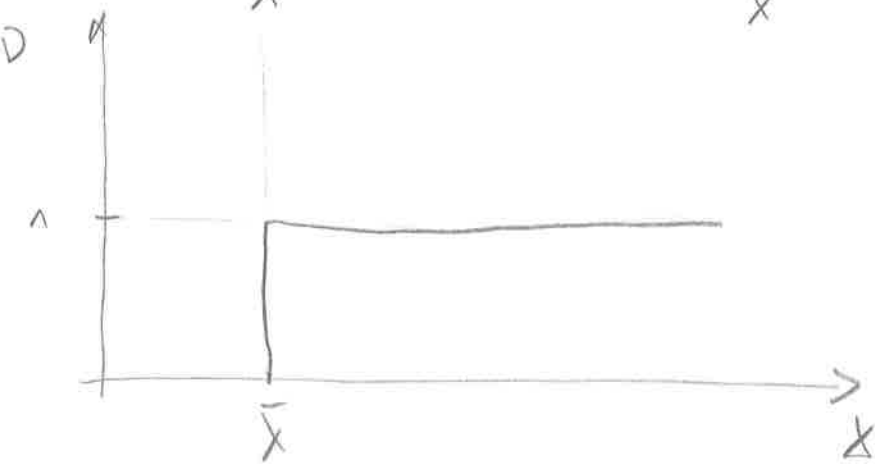
$$y = a_1 + \beta_1 x + \underbrace{(x - \bar{x}) D(x > \bar{x})}_{x_2} \delta$$

$$\beta_2 = \beta_1 + \delta$$

or



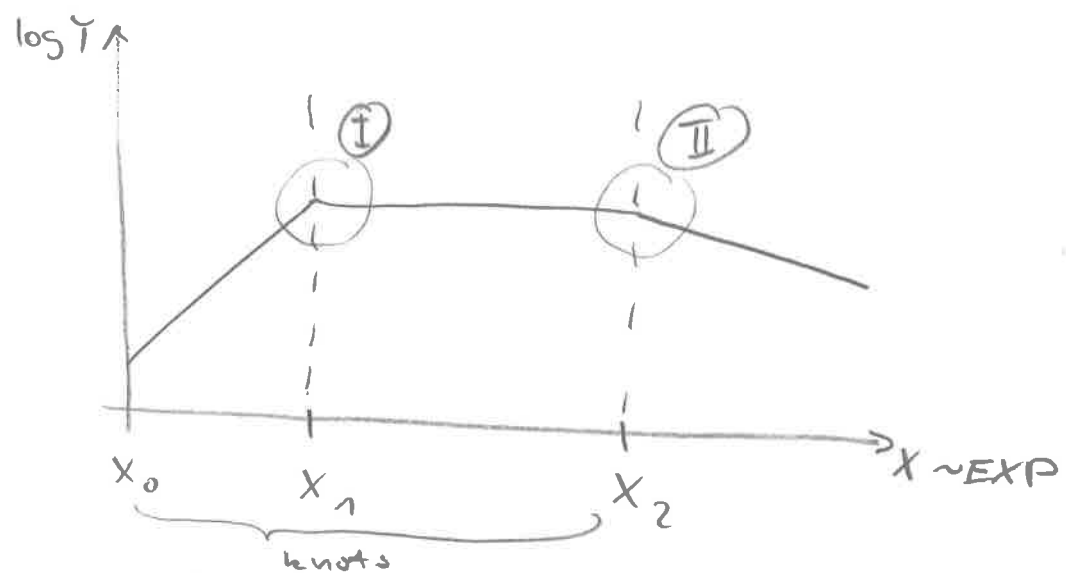
$$y = a_1 + \beta_1 x D(x \leq \bar{x}) + a_2 + (x - \bar{x}) D(x > \bar{x}) \beta_2$$



SPLINES

- use if theory does not predict fr. form
 - use if continuity $f(x)$ needed

• CASE OF EXPERIENCE \sim EXP profile



choice of knots:

- arbitrary
- other rules

$\overline{X_0, X_1, X_2} \sim$ knots
 " const

$$Y = [a_1 + b_1(x - \bar{x}_0)] D_1 + [a_2 + b_2(x - \bar{x}_1)] D_2 + [a_3 + b_3(x - \bar{x}_2)] D_3 + c$$

$$D_1 = 1 \text{ if } \bar{x}_0 \leq x < \bar{x}_1$$

$$D_2 = 1 \text{ if } \bar{x}_1 \leq x < \bar{x}_2$$

$$D_3 = 1 \text{ if } \bar{x}_2 < x$$

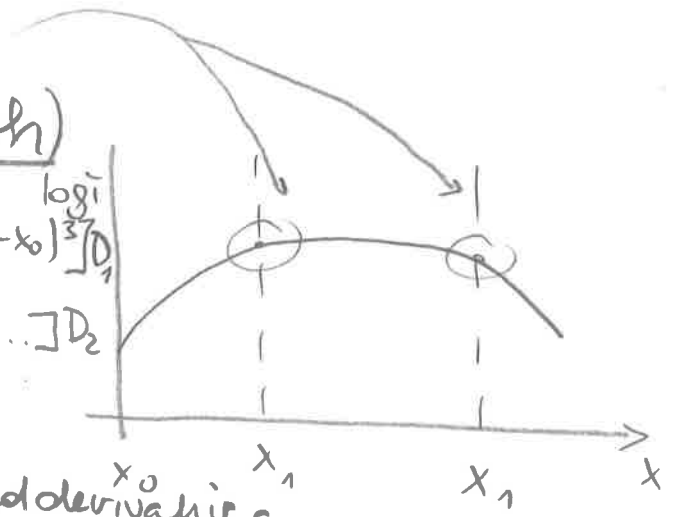
Impose continuity at ① & ②

$$a_2 = a_1 + b_1(\bar{x}_1 - \bar{x}_0)$$

$$a_3 = a_2 + b_2(\bar{x}_2 - \bar{x}_1)$$

POLYNOMIAL SPLINE (smooth)

$$Y = [a_1 + b_1(x - x_0) + c_1(x - x_0)^2 + d_1(x - x_0)^3] D_1 + [a_2 + \dots] D_2 + \dots$$



Impose continuity & continuity of 1st & 2nd derivative

$$\rightarrow a_2 = a_1 + b_1(x_1 - x_0) + c_1(x_1 - x_0)^2 + d_1(x_1 - x_0)^3$$

$$\rightarrow b_2 = \dots$$

simple formula (for general specification)

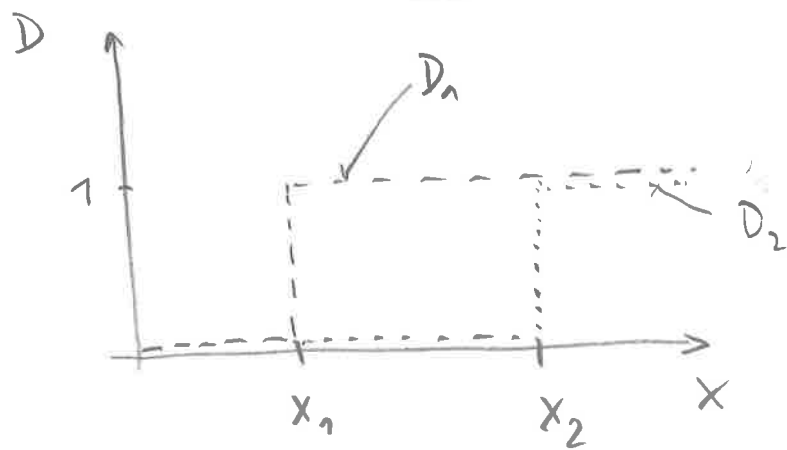
$$Y = a_1 + b_1(x - x_0) + c_1(x - x_0)^2 + d_1(x - x_0)^3 + \sum_{i=1}^k (d_{i+1} - d_i)(x - x_i)^3 D_i^*$$

$$D_i^* = \begin{cases} 1 & \text{if } x \geq x_i \\ 0 & \text{otherwise} \end{cases}$$

marginal
definit

$k+1 = 3 \sim \# \text{ intervals}$

$a_1, b_1, c_1, d_1, d_2, \dots, d_k$
parameters



• more x_i 's can be defined by splines

CASE of 1st order:

$$D_1 = 1(x \geq x_1)$$

$$D_2 = 1(x \geq x_2)$$

$$Y = a + b_1(x - x_0) + b_2(x - x_1) D_1 + b_3(x - x_2) D_2$$

CASE of 2nd order:

$$Y = a + b_1(x - x_0) + c_1(x - x_0)^2 + c_2(x - x_1)^2 D_1 + c_3(x - x_2)^2 D_2$$

CASE of 3rd order: