ROY (1951) model

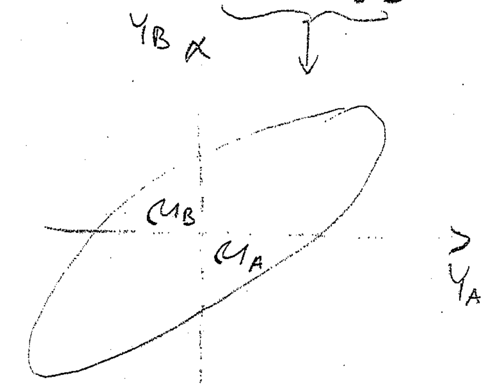
- self-selection
- choosing professions
- observed distr of earnings determined by choic

EXAMPLE:

$$\left.\begin{array}{l} Y_{iA} \ldots\ldots \text{hunter} \\ Y_{iB} \ldots\ldots \text{fisher} \end{array}\right\} \quad \text{if} \quad \begin{array}{l} Y_{iA} > Y_{iB} \to \text{hunter} \\ Y_{iB} < Y_{iB} \to \text{fisher} \end{array}$$

Assume: joint normal distr. of $(Y_A, Y_B) \sim$ means (μ_A, μ_B)

$$\sim \text{COV} \underbrace{\begin{bmatrix} \sigma_A^2 & \sigma_{AB} \\ \sigma_{AB} & \sigma_B^2 \end{bmatrix}}$$



$$\boxed{\begin{array}{ll} \text{Question:} & E(Y_A \mid \text{hunter}) = ? \\ & E(Y_B \mid \text{fisher}) = ? \\ & E(Y_A \mid \text{fisher}) = ? \\ & E(Y_B \mid \text{hunter}) = ? \end{array}}$$

Define: $u_A = Y_A - \overset{\text{will become } X\beta_A}{\mu_A}$, $u_B = Y_B - \overset{X\beta_B}{\mu_B}$, $\sigma^2 = VAR(u_A - u_B)$

$$Z = \frac{\mu_A - \mu_B}{\sigma}, \quad u = \frac{u_B - u_A}{\sigma}, \quad \begin{array}{l} \sigma_{AU} = COV(U_A, U) \\ \sigma_{BU} = COV(U_B, U) \end{array}$$

$$E(Y_A \mid \text{hunter}) = E(Y_A \mid Y_A - Y_B > 0) = E\left(Y_A \,\middle|\, \frac{\mu_A - \mu_B}{\sigma} > \frac{u_B - u_A}{\sigma}\right)$$

$$= E(Y_A \mid Z > u) \overset{\text{see NOTE}}{=} \underline{\underline{\mu_A - \sigma_{AU} \frac{\phi(Z)}{\Phi(Z)}}}$$

$$E(Y_B \mid \text{fisher}) = E(Y_B \mid Y_A - Y_B < 0) = \ldots =$$

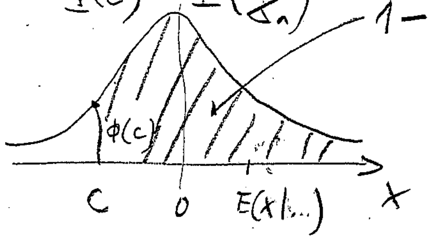
$$= \underline{\underline{\mu_B + \sigma_{BU} \frac{\phi(Z)}{1 - \Phi(Z)}}}$$

Mean of truncated normal

if $u_1 \sim N(0, \sigma_1)$

$$\phi(c) = \frac{1}{\sigma_1}\phi\left(\frac{c}{\sigma_1}\right)$$

$$\Phi(c) = \Phi\left(\frac{c}{\sigma_1}\right)$$



SIMPLE CASE

Let $u_1 \sim N(0,1)$

$$E(u_1) = 0$$

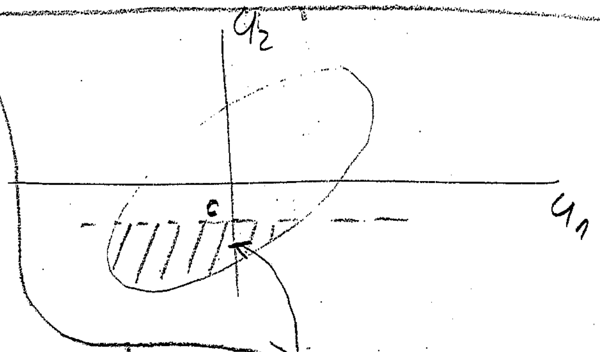
$$E(u_1 | u_1 \geq c) = \frac{\phi(c)}{1-\Phi(c)}$$

$$E(u_1 | u_1 \leq c) = -\frac{\phi(c)}{\Phi(c)}$$

COMPLICATED CASE

Let $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \sim \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$

σ_2^2: assume 1



$$E(u_1 | u_2 < c) = ?$$

$$E(u_1 | u_2 < c) = \left| E(u_1|u_2) = \sigma_{12} u_2 \right| = \sigma_{12} E(u_2 | u_2 < c) =$$

$$= -\sigma_{12} \frac{\phi(c)}{\Phi(c)}$$

SPECIAL COMPLICATED CASE

Let $\begin{bmatrix} u_A \\ \underbrace{u_B - u_A}_{u} \end{bmatrix} \sim \begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{bmatrix} \sigma_A^2 & \sigma_{AU} \\ \sigma_{AU} & \sigma_u^2 \end{bmatrix}$ assume $= 1$

$$E(u_A | u_B - u_A < c) = -\sigma_{AU} \frac{\phi(c)}{\Phi(c)} =$$

$$\sigma_{AU} = cov(u_A, u_B - u_A) = \frac{1}{N}\sum(u_A - 0)(u_B - u_A - 0) = \frac{1}{N}\sum u_A u_B - \frac{1}{N}\sum u_A^2 = \sigma_{AB} - \sigma_A^2$$

$$= -(\sigma_{AB} - \sigma_A^2)\frac{\phi(c)}{\Phi(c)}$$

CONSIDER POSSIBLE CASES (if $\sigma \neq 1$)

NOTE that $\underbrace{\sigma_{AU} = \frac{\sigma_{AB} - \sigma_A^2}{\sigma} \cancel{\gtrless 0} \; ; \; \sigma_{BU} = \frac{\sigma_B^2 - \sigma_{AB}}{\sigma} \cancel{\gtrless 0}}_{\boxed{\sigma_{BU} - \sigma_{AU} > 0}}$ $\quad \rho = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$

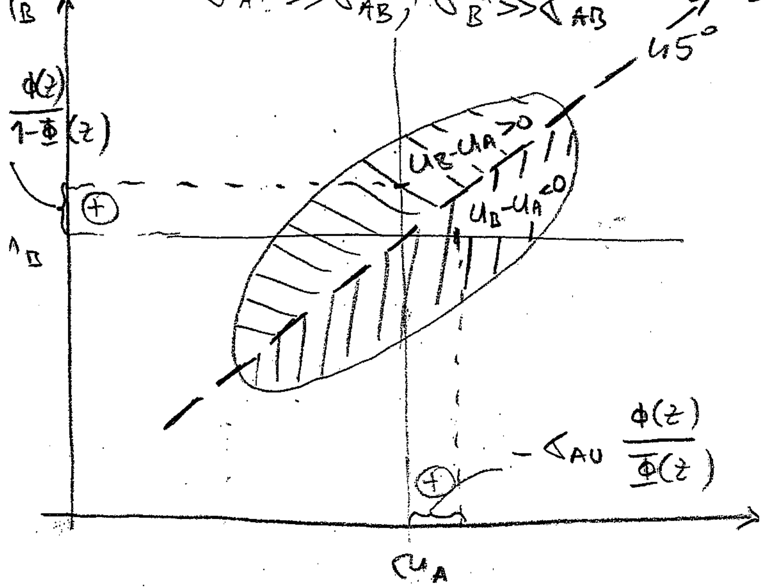
$\sigma_B^2 - \sigma_{AB} - \sigma_{AB} + \sigma_A^2 > 0$

$\sigma_B^2 + \sigma_A^2 > 2\sigma_{AB}$

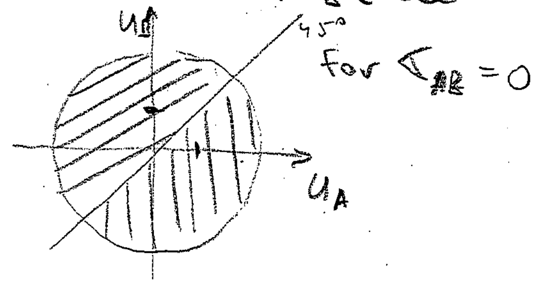
$\sigma_B^2 + \sigma_A^2 > 2\rho\sigma_A\sigma_B$

$\boxed{\text{CASE I: } \sigma_{AU} < 0 \;,\; \sigma_{BU} > 0}$

$\sigma_A^2 >> \sigma_{AB}$; $\sigma_B^2 >> \sigma_{AB}$



- Mean income of hunters is above μ_A (all)
- Mean income of fishers is above μ_B (all
-

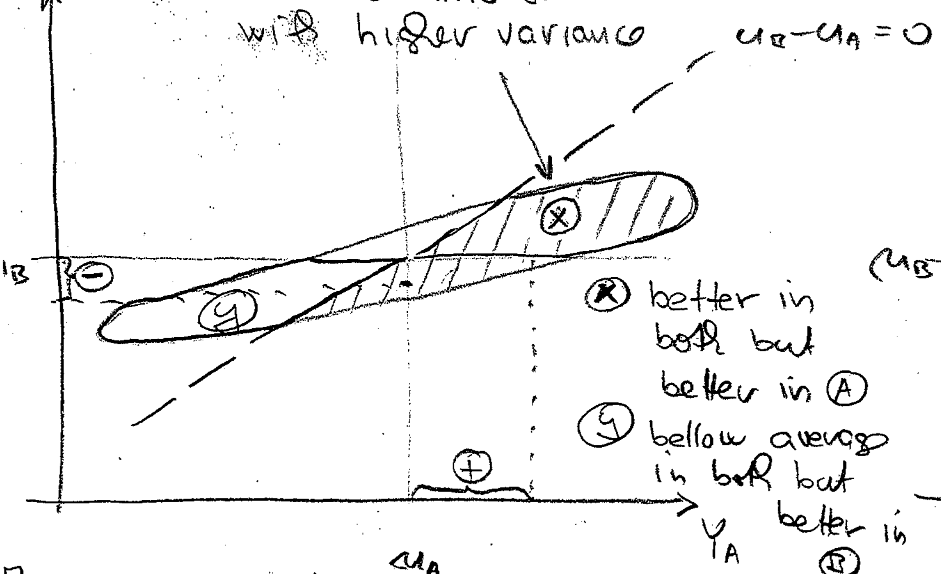


$\boxed{\text{CASE II}_a\text{: } \sigma_{AU} < 0 \;;\; \sigma_{BU} < 0}$

$\Downarrow$

$\sigma_A^2 >> \sigma_{AB}$, $\sigma_{AB} > \sigma_B^2 \Rightarrow \sigma_A^2 > \sigma_B^2$

more skilled into jobs with higher variance



(x) better in both but better in (A)

(y) bellow average in both but better in (B)

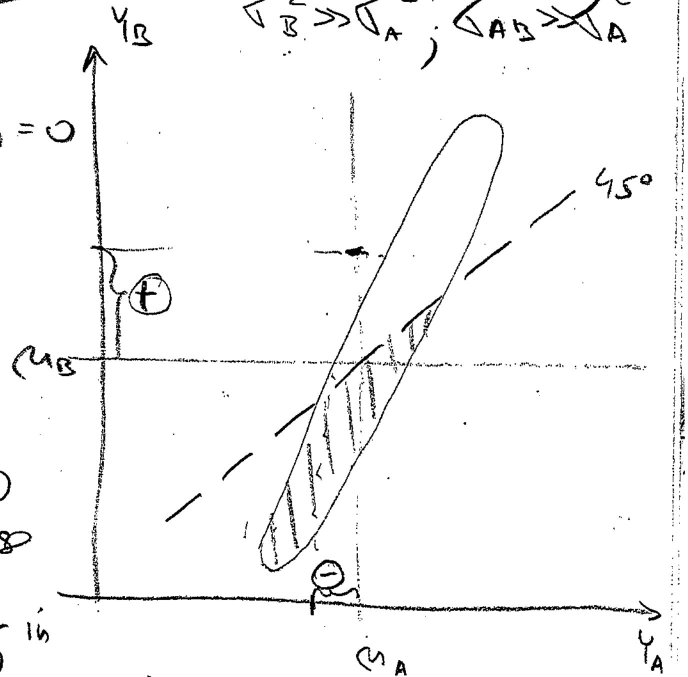
Mean income hunters $> \mu_A$

— " — fisherman $< \mu_B$

$\boxed{\text{CASE II}_b\text{: } \sigma_{AU} > 0 \;,\; \sigma_{BU} > 0}$

$\Downarrow$

$\sigma_B^2 >> \sigma_A^2$; $\sigma_{AB} > \sigma_A^2$



$\boxed{\cancel{\text{CASE: } \sigma_{AU} > 0 \;,\; \sigma_{BU} < 0}}$

ALTERNATIVE CHOICES - continued

sign matters

$$E(Y_B \mid Y_A > Y_B) = E(Y_B \mid u > z) = \underline{\underline{\mu_B - \sigma_{BU}\frac{\phi(z)}{\Phi(z)}}}$$

↓

would hunters choose fishing

sign matters

$$E(Y_A \mid Y_B > Y_A) = E(Y_A \mid z < u) = \underline{\underline{\mu_A + \sigma_{AU}\frac{\phi(z)}{1-\Phi(z)}}}$$

↓

would fishermen choose hunting

Observed difference in mean incomes:

$$E(\bar{Y}_A - \bar{Y}_B) = [\mu_A - \mu_B] \underbrace{- \sigma_{AU}\frac{\phi(z)}{\Phi(z)} - \sigma_{BU}\frac{\phi(z)}{1-\Phi(z)}}_{\text{bias} = f(\sigma_{AU},\, \sigma_{BU})}$$

$\sigma_{AU} < 0,\ \sigma_{BU} > 0$	CASE I: ambiguous bias
$\sigma_{AU} < 0,\ \sigma_{BU} < 0$	CASE IIa: upward bias
$\sigma_{AU} > 0;\ \sigma_{BU} > 0$	CASE IIb: downward bias