CONCEPTUAL FRAMEWORK

Gross Unadjusted wage differential

$$G_{WB} = W_W / W_B - 1 \qquad [\%]$$

Pure Productivity difference (w/o discrim)

$$Q_{WB} = W^0_W / W^0_B - 1$$

Market Discrimination Coefficient

$$D_{WB} = \frac{(G_{WB} + 1) \rightarrow \text{due to discr.}}{Q_{WB} + 1 \rightarrow \text{due to product. diff}} - 1$$

$$\Rightarrow \log\underbrace{(G_{WB} + 1)}_{\frac{W_W}{W_B}} = \underbrace{\log(D_{WB} + 1)}_{\text{discr}} + \underbrace{\log(Q_{WB} + 1)}_{\text{product.}} =$$

Decompose D_{WB}: $\quad D_{WB}+1 = \dfrac{\frac{W_W}{W_B}}{\frac{W^0_W}{W^0_B}} = \dfrac{W_W}{W^0_W} \cdot \dfrac{W^0_B}{W_B}$

define $\begin{cases} \text{let } \delta_{W0} = \dfrac{W_W}{W^0_W} - 1 & [\% \text{ difference}] \\ \text{let } \delta_{0B} = \dfrac{W^0_B}{W_B} - 1 & [\text{— " —}] \end{cases}$

$$D_{WB} = (\delta_{W0} + 1)(\delta_{0B} + 1)$$

$$\log(G_{WB}+1) = \underline{\log(\delta_{W0}+1) + \log(\delta_{0B}+1)} + \log(Q_{WB}+1)$$

Assume semilogarithmic wage equation & OLS estimati:

$\tilde{W}$ ~ geom. mean wage

$\bar{X}$ ~ vector of mean values

$\hat{\beta}$ ~ est. coefs

$$w = \log W = X\beta$$

$$\tilde{W} = \sqrt[n]{\prod W_i} \Rightarrow \log \tilde{W} = \frac{1}{n}\sum^n \log W_i$$

$$\bar{w} = \bar{X}\hat{\beta} \qquad = \frac{1}{n}\sum^n X\beta = \bar{X}\beta$$

$$\log(G_{WB}+1) = \log(\delta_{wo}+1) + \log(\delta_{ob}+1) + \ln(Q_{WB}+1)$$

$$\log(G_{WB}+1) = \underbrace{\bar{X}_w'(\hat{\beta}_w - \beta^*) + \bar{X}_b'(\beta^* - \hat{\beta}_b)}_{\text{Discriminat.}} + (\bar{X}_w - \bar{X}_B)'\beta^*$$

$$\delta_{wo}+1 = \frac{\overline{W}_w}{\overline{W}_o} - 1 + 1 = \frac{\overline{W}_w}{\overline{W}_o} \Rightarrow \log(\delta_{wo}+1) = \overline{\log W_w} - \overline{\log W_o} = \bar{X}_w'\hat{\beta}_w - \bar{X}_w'\beta^*$$

$$= \bar{X}_w'(\beta_w - \beta^*)$$

$$\delta_{ob}+1 = \frac{\overline{W}_B^o}{\overline{W}_B} - 1 + 1 = \frac{\overline{W}_B^o}{\overline{W}_B} \Rightarrow \log(\delta_{ob}+1) = \overline{\log W_B^o} - \overline{\log W_B} = \bar{X}_B'\hat{\beta}^* - \bar{X}_B'\hat{\beta}_B =$$

$$= \bar{X}_B'(\hat{\beta}^* - \hat{\beta}_B)$$

$$Q_{WB}+1 = \frac{\overline{W}_w^o}{\overline{W}_B^o} - 1 + 1 = \frac{\overline{W}_w^o}{\overline{W}_B^o} \Rightarrow \log\left(\frac{W_w^o}{W_B^o}\right) = \overline{\log W_w^o} - \overline{\log W_B^o} = \bar{X}_w'\beta^* - \bar{X}_B'\beta^* =$$

$$G_{WB}+1 = \frac{\overline{W}_W}{\overline{W}_B} - 1 + 1 = \frac{\overline{W}_w}{\overline{W}_B} \Rightarrow \log\frac{\tilde{W}_w}{\tilde{W}_B} =$$

$$= \beta^*(\bar{X}_w' - \bar{X}_B')$$

$$= \log\tilde{W}_w - \log\tilde{W}_B = \underbrace{\bar{X}_w'\hat{\beta}_w - X_B'\hat{\beta}_B}$$

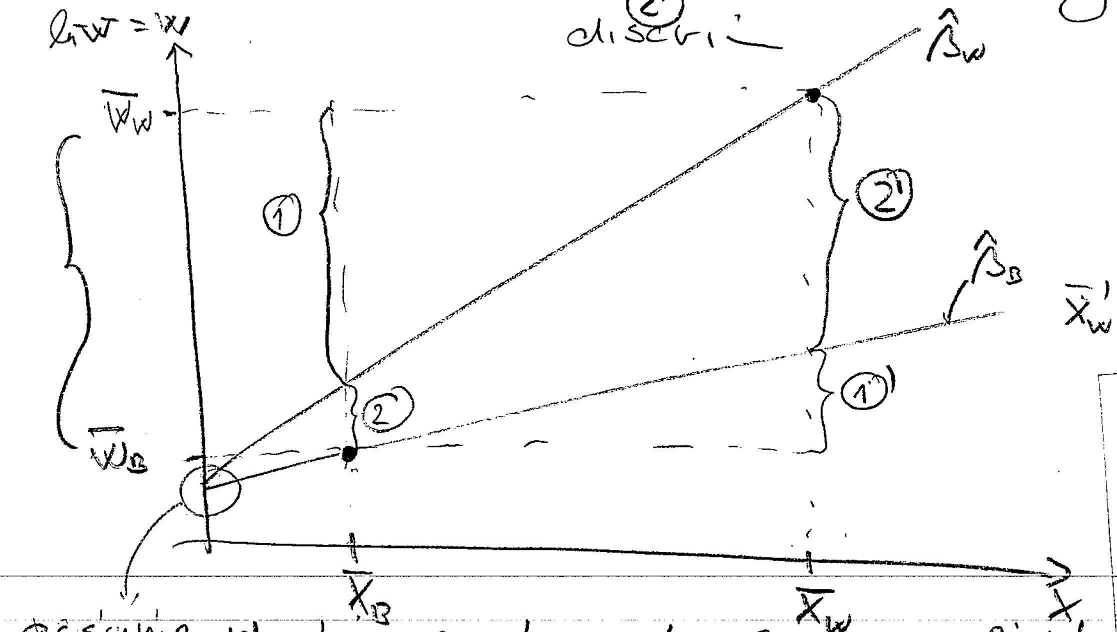
β^* not known !!!

$$\boxed{\text{Assume } \beta^* = \hat{\beta}_W:}$$

$$\underbrace{\bar{X}_w'\hat{\beta}_W - \bar{X}_B'\hat{\beta}_B}_{\overline{W}_w - \overline{W}_B} = \underbrace{\bar{X}_w'(\cancel{\hat{\beta}_W - \hat{\beta}_W})}_{0} + \underbrace{\bar{X}_B'(\hat{\beta}_W - \hat{\beta}_B)}_{\substack{>0 \\ \text{discrim} \\ 2}} + \underbrace{(\bar{X}_w - \bar{X}_B)\beta_w}_{\substack{\text{non-discr} \\ 1}}$$

$$\boxed{\text{Assume } \beta^* = \hat{\beta}_B}:$$

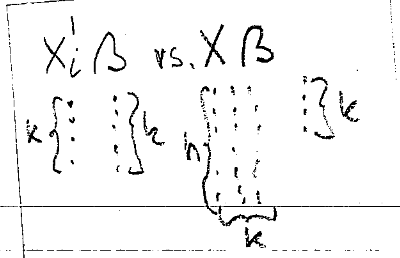
$$\bar{X}_W'\hat{\beta}_W - \bar{X}_B'\hat{\beta}_B = \underbrace{X_W'(\hat{\beta}_W - \hat{\beta}_B)}_{\substack{2' \\ \text{discri}}} + \underbrace{\bar{X}_B'(\cancel{\hat{\beta}_B - \hat{\beta}_B})}_{0} + \underbrace{(\bar{X}_W - \bar{X}_B)\hat{\beta}_B}_{\substack{\text{non-discr} \\ 1'}}$$



NOTE: $\hat{\alpha}_W - \hat{\alpha}_B$ in $\hat{\beta}_W - \hat{\beta}_B$

$2' > 2$

$$\bar{X}_W'(\hat{\beta}_W - \hat{\beta}_B) > \bar{X}_B'(\hat{\beta}_W - \hat{\beta}_B)$$



assume identical intercepts for simplicity

What is β^*?

- $\hat{\beta}_B$ or $\hat{\beta}_W$
- between $\langle \hat{\beta}_B, \hat{\beta}_W \rangle$ => $\beta^* = \Omega \hat{\beta}_W + (1-\Omega)\hat{\beta}_B$

in matrix case

$$\beta^* = \Omega \hat{\beta}_W + (I - \Omega)\hat{\beta}_B$$

$$I = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$

None is satisfactory because arbitrary

Oaxaca '73: For $\Omega = 1 \Rightarrow \beta^* = \hat{\beta}_W$

$\Omega = 0 \Rightarrow \beta^* = \hat{\beta}_B$

Reimers '83 ← $\Omega = .5$ | $\Omega = I * .5$

Cotton '88 ← $\Omega = P_W$ | $\Omega = I * P_W$

proportion of white