In this equality we have set $a_j = z\alpha_j$, $j = 0, 1, 2$, where z is a vector of observable characteristics unchanging over time, α_j is a vector of parameters, and u_t is a random element. The term t^2 is introduced to account for a possible nonlinearity in the relation between wages and experience, which is generally confirmed by empirical work on this subject (see below, chapter 6, section 4.3). The simultaneous estimation of equations (17) and (18) allows us to obtain the parameters needed to assess the impact of an overall change in wages on labor supply. Parameter α_w measures the impact of a change in the current wage w_t, while parameters γ_i measure the consequences of changes in the overall wage profile (see Blundell and MaCurdy, 1999, pp. 1600–1603, for more details).

To sum up, it is necessary to define precisely the set of variables that explain labor supply—in particular, the indicators of non-wage income—in order to see what type of elasticity the model utilized allows us to estimate. Having thus set out the ingredients that go to make up an empirical labor supply equation of type (16), we can now present the principles that guide this estimation.

2.1.2 A Short Guide to Estimating Labor Supply

Estimating the basic equation by ordinary least squares leads to biased results, since it neglects to take into account participation decisions. If we want to obtain unbiased estimators of the elasticity of labor supply, we have to estimate jointly decisions to participate and decisions about the number of hours worked. These estimates oblige us to attribute a fictitious wage to those who do not participate in the labor market.

What We Must Not Do

The first idea that comes to mind is to apply the method of ordinary least squares to equation (16) alone. Until the 1970s most studies proceeded in this way. But it is not a correct method, for it fails to distinguish decisions about *participation* in the labor market from those about the number of hours an agent is prepared to offer. The question that faces the econometrician is, given a sample of individuals, how to take into account persons who do not work (or episodes during which an agent has not worked if the data are equally temporal)? Certain studies subsequent to the 1970s simply set $h_i = 0$ for these persons. In other words, these studies took the view that certain workers choose exactly $h_i = 0$, just like any other value of h_i, which entails that equation (16) holds for any wage value of h_i and w_i. It is precisely this last hypothesis that is false. Equation (16) is only valid for wages *above* the reservation wage, and for *all other* wages, labor supply is null. Making do with equation (16) and setting $h_i = 0$ for episodes of nonwork thus leads to specification errors. An alternative solution was simply to exclude the unemployed, and nonparticipants in the labor market, from the sample. But in this case the econometrician commits a selection bias, forgetting that not to supply any hours of work is a decision in the same way that supplying them is. The fact that this type of decision is not described by equation (16) does not authorize us to set it aside purely and simply. The solution is to employ an empirical

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What We Must Do

The approach most often utilized today is "structural." It combines an explicit functional form for the direct utility function of consumers, dependent in parametric fashion on the different observable characteristics of an individual, and a random term representing the nonobserved heterogeneity among individuals. We then write the budget constraint, from which we deduce, by the usual maximization procedure, labor supply and the reservation wage. The participation condition is then arrived at using the probability distribution of the random term, by positing that the wage offered must be superior to the reservation wage. We estimate the model at which we arrive using cross-sectional data that specify, for each individual, the values of every variable we are interested in, and his or her decisions to participate or not in the labor market. Let us illustrate this approach using an example, for purely pedagogic purposes, based on the static model of section 1.1.1, with a utility function of the Cobb-Douglas type.

The utility of a consumer will then take the form $C^{1-\beta}L^{\beta}$, $1 > \beta > 0$, and the budget constraint continues to be written $C + wL = wL_0 + R$. We assume that the explanatory variables and the random term intervene via the coefficient β according to the linear form $\beta = x\theta + \varepsilon$. Following the static model of section 1.1.1, we know that the reservation wage w_A is equal to the marginal rate of substitution U_L/U_C taken at point (R, L_0) and that the maximization of utility subject to the budget constraint gives the optimal value of leisure. After several simple calculations, we find that:

$$w_A = \frac{\beta}{1-\beta}\frac{R}{L_0} \quad \text{and} \quad L = \begin{cases} \beta\left(L_0 + \dfrac{R}{w}\right) & \text{if } w \geq w_A \\ L_0 & \text{if } w \leq w_A \end{cases}$$

Since the coefficient β is a function of the random term ε, the inequality $w \geq w_A$ is equivalent to an inequality on the values of ε, which is written:

$$w \geq w_A \Leftrightarrow \varepsilon \leq \frac{wL_0}{R + wL_0} - x\theta$$

In conclusion, the decisions concerning labor supply $h = L_0 - L$ and participation may be summed up in this fashion:

$$h = \begin{cases} L_0 - (x\theta + \varepsilon)\left(L_0 + \dfrac{R}{w}\right) & \text{if } \varepsilon \leq \dfrac{wL_0}{R + wL_0} - x\theta \\ 0 & \text{if } \varepsilon \geq \dfrac{wL_0}{R + wL_0} - x\theta \end{cases} \tag{19}$$

This expression of labor supply is related, as regards the interior solution, to the basic equation (16). But we see that taking account of participation decisions constrains the variations of the random term, making them depend on explanatory variables. In these circumstances, the use of ordinary least squares is seen to be inadequate.

Joint Estimations of Hours Worked and Participation Decisions

Let us suppose that the econometrician has at his or her disposal a sample of individuals, N in size, specifying that individuals $i = 1, \ldots, J$ have worked h_i hours and that individuals $i = J+1, \ldots, N$ have not worked. Let us denote by $F(.)$ and $f(.)$ respectively the cumulative distribution function and the probability density of the random term ε (the random term is most often assumed to follow a normal distribution). It is then possible to write the likelihood of the sample. Following rule (19) giving the optimal decisions of an agent, when an individual i has worked h_i hours, that means that the random term has taken the value $\varepsilon_i = w_i(L_0 - h_i)/(R_i + w_i L_0) - x_i\theta$. In this case its contribution to the likelihood of the sample is equal to $f(\varepsilon_i)$. If agent i has not worked, that means that the random term is bounded above by the value $\tilde{\varepsilon}_i = [w_i L_0/(R_i + w_i L_0)] - x_i\theta$. In this case, its contribution to the likelihood of the sample is given by $\Pr\{h_i = 0\} = 1 - F(\tilde{\varepsilon}_i)$. Setting $\bar{F} = 1 - F$, the likelihood function of the sample is written in logarithmic form:

$$\mathscr{L} = \sum_{i=1}^{i=J} \ln f\left[\frac{w_i(L_0 - h_i)}{R_i + w_i L_0} - x_i\theta\right] + \sum_{i=J+1}^{i=N} \ln \bar{F}\left[\frac{w_i L_0}{R_i + w_i L_0} - x_i\theta\right] \tag{20}$$

The maximization of the likelihood function by appropriate techniques (in this case of the *probit* type, since there is a mixture of continuous and discrete variables) furnishes estimates of the parameters in which we are interested. The expression of the likelihood function also permits us to understand clearly the mistakes made in failing to formalize participation decisions completely. If we set $h_i = 0$ for persons who do not work, that amounts to believing that their contribution to the likelihood is equal to $f[(w_i L_0/(R_i + w_i L_0)) - x_i\theta]$, which comes down to substituting function f for function $\bar{F}$ in the second term of the right-hand side of relation (20). If we exclude persons who do not work from the sample, then we are neglecting to take account of the second term on the right-hand side of relation (20). These two solutions result in biased estimators.

A Nonparticipant's Wage

The expression (20) of the likelihood function also highlights a delicate problem. By definition, the econometrician does not observe the wages of individuals $i = J+1, \ldots, N$ who do not work. However, relation (20) shows that it is necessary to attribute a fictitious *wage* to these individuals if we want to maximize the likelihood function. We thus have to be able to assign a quantity to the (unobserved) wage notionally offered to an individual, which he or she has refused. The most common solution at present consists of deducing the wage of a nonparticipant using the wage received by participants with similar characteristics in terms of educational qualification, experience, age, and so on. In practice we can explain the wages of individuals participating in the labor market by a regression of the type $w_i = y_i\theta_p + u_i$ in which variables. In these circumstances, the use of ordinary least squares is seen to be inadequate.

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the vector y_i represents the characteristics of an individual i participating in the labor market, and θ_p designates the vector of the parameters to be estimated. Let us use $\hat{\theta}_p$ to denote the vector of the estimates of θ; we can then use this vector $\hat{\theta}_p$ to calculate the wage w_k of a nonparticipant k, using the vector y_k of his or her characteristics and setting $w_k = y_k\hat{\theta}_p$. This simple technique unfortunately presents a *selection bias*, since it assumes that the regression equation $w_i = y_i\theta_p + u_i$ also applies to the notional wages of nonparticipants. This hypothesis is highly likely to be mistaken, inasmuch as participants in the labor market must on average have nonobserved characteristics that allow them to demand wages higher than those that nonparticipants can demand. Formally this means that the distribution of the random disturbance u_i should not be the same for participants and nonparticipants. The distribution that applies to participants ought to weight the high values of the random factor more strongly than the one that applies to nonparticipants, and consequently the estimation procedure described previously will *overestimate* the notional wage attributable to a nonparticipant. One way to correct this bias consists of making simultaneous estimations of equations explaining wages and decisions to supply labor (see Heckman, 1974, for an application).

2.1.3 Nonlinear Budget Constraint

In practice, the budget constraint of an agent does not come down to a simple segment of a line, as in the basic model of section 1.1.1. Mandatory contributions and transfers make this constraint (at best) piecewise linear. The estimation of labor supply then runs into a new problem, that of the endogeneity of the choice of the "piece" on which an agent will settle. The method of virtual incomes and the construction of a differentiable approximation of the budget constraint make such an estimation possible, however.

The Method of Virtual Incomes

In all countries, the systems of tax and subsidy that agents come under present important differences according to income, so that, from the point of view of empirical estimations, it is not possible to assume that the budget constraint of an agent is represented by a single segment of a line, as in the basic model of section 1.1.1. In practice, the different schedules of marginal rates according to income brackets, and the different deductions to which certain contributors are entitled, imply that the budget constraint of an agent is piecewise linear. By way of illustration, let us consider the example of a tax system in which an agent whose income does not exceed an exogenous threshold $R_{\max}$ is not taxed, whereas if his or her income crosses this threshold, his or her wage will be taxed at rate τ. Let us use w and R to denote respectively the wage and the non-wage income of this agent. Our example of a fiscal system starts to tax the consumer from the point at which his or her working time surpasses the value $h_{\max}$ defined by $wh_{\max} + R = R_{\max}$. To this maximal value of working time there corresponds a value for leisure of $L_{\min} = L_0 - h_{\max}$. Figure 1.6 represents the budget constraint associated with this rudimentary fiscal system in the plane (L, C). In reality, the