

### System of Equations

This problem set should give you experience with estimation of parameters of systems of equations. The logarithmic transcendental flexible form of firm production function is the underlying specification. The Chapter 17 in Greene's Econometrics (2<sup>nd</sup> edition) illustrates how to estimate a cost function of this form. You should apply the described techniques **without** imposing the assumption of constant returns to scale as is assumed in the textbook (section 17.4). The data is described and listed in the textbook on page 502.

- 1) As usual, start by familiarizing with the data. Compute basic characteristics and report them. You will need them later anyway.
- 2) Write down the system of equations implied by a translog cost function form electric power **without imposing the assumption of constant returns to scale**. Hint: Remember that the second partial cross derivatives are symmetric and that your model should also include second order terms that involve  $\ln Y$ .
- 3) Since the factor shares must sum up to one, what restriction you have to impose on your system of equations? Impose the restrictions and rewrite the system of equations.
- 4) Compare the number of parameters in specifications 2) and 3). How you will get the estimates of parameters that are in specification 2) but not in specification 3)?
- 5) Estimate system in 2) as well as in 3) using OLS, i.e., each equation separately. Do the estimated parameters approximately satisfy the cross equation and symmetry restrictions given by the theoretical specification?
- 6) Now estimate your equations as a full system (seemingly unrelated regression), imposing all of the theoretical restrictions (i.e., system in 3)). Re-estimate the parameters using maximum likelihood estimation. Construct table to compare both these sets of estimates and the OLS estimates in 5).
- 7) Using your parameter estimates and descriptive statistics, compute estimates of the elasticities of substitution and own and cross price elasticities for the three factor inputs at the point of means.
- 8) Test the following two hypotheses (use both likelihood ratio and Wald test statistics):
  - a) Returns to scale are constant.
  - b) Returns to scale may not be constant, but they are price invariant.