How Much Intergenerational Risk Sharing Does the Social Security System Really Provide?

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Abstract

Pay-as-you-go pension systems redistribute income across cohorts and hence allow sharing of risk across generations. This paper quantitatively assesses the degree of intergenerational risksharing provided by the Social Security system in the U.S. throughout the 20th century. We analyze the outcomes of the Social Security as actually implemented through the combination of default rules and various pension reforms. We propose simple measures of risk sharing that can be computed from data on the histories of wages, contributions, benefits, and stock market returns. They relate net transfers from Social Security received by each cohort to the wages and returns on savings earned by that cohort as well as other cohorts. We calculate the risk sharing measures for cohorts born between 1900-1985 and estimate policy functions that characterize net transfers as a function of shocks to wages and returns on savings. The Social Security system did not provide systematic risk sharing to the shocks to wages but appears to have provided complete risk sharing to the shocks to returns on savings. There are large disparities between cohorts in the mean and variance in our measures of risk sharing. Risk sharing to a given cohorts is provided by cohorts born 20 or more years later but not by more adjacent cohorts. Changes in expected transfers over time cannot be explained by risk-sharing considerations.

Keywords: social security, intergenerational risk sharing, policy risk, pension reform

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1 Introduction

The ability to share risk across generations is a desirable feature of public pay-as-you-go pension systems. People generally want to insure against major shocks to their income, such as a reduction in their wages or in the value of their savings. Shocks that affect members of a given generation equally (such as a stock market crash just before the generation’s retirement or a major recession during the generation’s working years) can only be insured vis-à-vis another generation, requiring a transfer of income between generations. While the pay-as-you-go pension systems vary in numerous details, they always transfer income from the young to the old generation and typically link the benefits received by the old to the wage growth. By providing the old with a source of income that is not explicitly linked to the stock market fluctuations, they mitigate the impact of the shocks to the value of savings. By linking benefits to the wage growth, they effectively pass some of the wage growth enjoyed by the young to the old and compensate a generation that suffered a negative wage shock during its working years.

Substantial theoretical literature has compared the risk sharing properties of prototypical pension systems and has shown that a properly structured scheme can implement intergenerational transfers that are optimal from a risk sharing perspective (Rengel and Zeckhauser (2001), Bohn (2001), Wagener (2004), Ball and Mankiw (2007), Auerbach and Lee (2009), among others). That, by itself, does not guarantee that the pay-as-you-go systems that are actually implemented by individual countries provide the optimal level of risk sharing. The political considerations that determine the benefits and contributions do not coincide with those of a benevolent social planner. Several political economy models show that pay-as-you-go systems provide transfers to the old that are too stable (D’Amato and Galasso (2008)) or that transfers from the young to the old are much more likely to constitute a political equilibrium than reverse transfers even though reverse transfers are often times necessary for optimal risk sharing (Rengel and Zeckhauser (2001)).

Looking at the issue from a different perspective, it has been well documented that the pension systems redistribute income (in a lifetime sense) across cohorts and sometimes explicitly treat different cohorts differently.\textsuperscript{1} Whether such implicit intergenerational transfers can be rationalized as optimal risk sharing remains an open question.

There are two broad mechanisms through which a pension system as implemented may fail to

\textsuperscript{1}Leimer (2007) computes the net lifetime transfers from social security for all cohorts and shows that they vary widely across cohorts. McHale (2001) and Dusek and Kopcsni (2008) compute the changes in the social security wealth induced by pension reforms in various OECD countries and show that the change in the social security wealth differed substantially between cohorts for some reforms.
achieve optimal intergenerational risk sharing:

(1) The pension systems typically operate under default rules that are supposed to last for decades. They specify formulas and parameters that translate economic outcomes (e.g. wages, inflation rates) into benefits and may also automatically adjust parameters to economic and demographic shocks. Poorly designed default rules would fail to achieve optimal intergenerational transfers in response to shocks such as a stock market crash.

(2) The default rules may prescribe an allocation that is deemed undesirable upon a realization of a shock, or that is financially unsustainable. The government then takes a discretionary action, which may take the form of a one-time adjustment in the system’s parameters or a systemic reform changing the default rules for the future. The discretionary response may not necessarily be optimal from the risk-sharing perspective.

The risk sharing properties of default rules can be studied analytically by simulations\(^2\) which assume that any given default rule will be in place forever. Such analysis is useful for the optimal design of the default rules. In reality, however, the rules change over time and each cohort lives through a pension system that combines stable default rules for some time with occasional ad-hoc adjustments and radical reforms. Little is known about the degree of risk sharing that has been actually provided by such real-world, "as implemented" pension systems. Knowing it is potentially useful for making long-term projections of the pay-as-you-go system’s finances or for evaluating the optimal balance between the funded and the pay-as-you-go pillar. In such exercises one should take into account that the pay-as-you-go default rules will not be in place forever and should evaluate the pay-as-you-go system "as implemented".

\section{Contribution}

The objective of the paper is twofold: First, to assess quantitatively the degree of intergenerational risk-sharing historically provided by the Social Security system in the United States during the 20th century. We propose and compute simple measures of intergenerational risk sharing which relate differences in lifetime net transfer from the Social Security between cohorts to differences in their lifetime wages and returns on savings. Our measures are based on outcomes (contributions actually paid and benefits actually received) and hence summarize the myriad of Social Security provisions as well as the changes in those provision as they affected the actual transfers between cohorts.

\(^2\)Wagener (2004) and Auerbach and Lee (2009) are examples.
The second objective is to interpret the observed patterns of risk sharing in terms of the optimal risk sharing literature. While the existing normative models\(^3\) vary in their assumptions, they broadly agree on qualitative prescriptions. For example, if a particular generation suffers a negative income shock, all current and future generations should provide a transfer to the affected generation (if such transfers are technologically feasible); the compensating transfers should be spread equally across the other generations; if the young suffer a negative shock, the old should also provide a transfer (i.e., the transfers need not necessarily flow from the young to the old). We evaluate whether the observed patterns of risk sharing are consistent with such qualitative prescriptions.

We use data on the histories of wages, returns on savings, contributions, and benefits that a representative agent in every was expected to realize in each year from 1939 till 2005, based on a given forecasting rule for the future and on the social security legislation valid in that year. From those we construct the real present value of lifetime wages, excess savings (savings accumulated at the end on the agent’s working history minus the savings that the agent would have accumulated if the returns on savings were constant), and net transfers (Social Security benefits received minus contributions paid). The dataset covers cohorts born in 1900, 1905, 1910, etc. till 1980. We provide two sets of results for our measures of risk sharing. One set is constructed from the "terminal" outcomes for each cohort, i.e., the lifetime wages, excess savings, and benefits that the cohort realized by the end of its life, or for younger cohorts, that it realized by the last year of the dataset and expected to realize for the remainder of its life. The "terminal" outcomes hence allow a cross-sectional comparison between cohorts. The second set of results is constructed from lifetime outcomes that the cohorts expected every five years, i.e., 1940, 1945, etc. till 2005. The expected outcomes allow to make the cross-sectional comparison between cohorts at many points in time and allow to analyze whether expected net transfers to each cohort adjust with shocks to wages and returns to savings that had differential impact on individual cohorts.

The cross-sectional comparison of terminal outcomes shows that differences between cohorts in their lifetime wages are not negatively related to differences in lifetime net transfers. The Social Security system as implemented in the United States thus did not provide systematic risk sharing of wage shocks. On the other hand, differences in lifetime excess savings translate almost one-for-one into differences in lifetime net transfers for the pre-1960 cohorts, and more than one-for-one for the post-1960 cohorts. Social Security system thus appears to have provided complete (possibly more than complete) risk sharing of the stock market risk. This finding is

\(^3\)For example, Gordon and Varian (1988), Ball and Mankiw (2007), D’Amato and Galasso (2008), Bohn (2009).
particularly striking given the fact that the Social Security rules do not contain any explicit provision that links stock market returns, contributions, and benefits.

The panel analysis of expected lifetime outcomes again does not find any evidence of risk sharing with respect to wages and finds some evidence of risk sharing with respect to excess savings although much smaller in magnitude than in the cross-section. The latter result is driven by comparing cohorts within a year; when comparing pairs of cohorts over time we do not find evidence of risk sharing even with respect to excess savings. Outcomes and transfers of the most nearby cohorts are positively related while they are negatively related for cohorts with larger age difference (20 years and more).

The observed patterns of risk sharing are different for cohorts for which we observe complete (or near-complete) lifetime outcomes and for cohorts for which the lifetime outcomes are based mostly on expectations about the future. The Social Security system “as implemented” with numerous reforms accumulating during the cohorts lifetime provides different pattern of risk sharing than if system’s default rules were held unchanged forever.

Our research contributes to an emerging empirical literature on the policy risk in pay-as-you-go schemes and the determinants and outcomes of pension reforms. Borgmann and Heidler (2003) compute the changes in relative generosity of the German pension system and show that they are largely influenced by changes in the population projections. Dusek and Kopcsni (2008) compute the changes in the social security wealth for different cohorts, genders, and education levels for all major pension reforms in Hungary, Czech Republic and Slovakia. While some reforms were adopted in response to economic or demographic shocks, the authors argue that relative outcomes of individual cohorts under these reforms are frequently inconsistent with optimal intergenerational risk sharing.

Leimer (1994, 2007) investigated intergenerational transfers throughout the Social Security’s history by calculating the lifetime net transfers from Social Security for all cohorts affected by the program. The reported transfers vary substantially by cohort. Shoven and Slavov (2006) compute how the lifetime net transfer varied over time at the cohort level. Specifically, they calculate the internal rate of return on contributions in the U.S. Social Security system for cohorts born in 1900, 1905, 1910, etc till 1985 in each year based on the legislation valid in that year. The IRRs also varied substantially within a cohort over time; measures taken to restore the financial solvency of the system were the major direct cause of that variation.

While the Leimer and Shoven and Slavov papers present descriptive evidence on the variation in lifetime net transfers, the contribution of this paper is in investigating whether, and to what
extent, can the relative differences between cohorts in the lifetime net transfers be explained by relative differences in the cohort’s economic outcomes, namely lifetime wages and returns on savings. The auxiliary contribution is in evaluating whether the observed relationship between transfers and economic outcomes is consistent with optimal intergenerational risk sharing. The fact that some cohorts obtain positive and other cohorts negative lifetime transfer need not imply that the system generates an ad-hoc redistribution of income or is in some sense inefficient. If the cohorts receiving large positive transfers are also those who experienced relatively worse economic outcomes and vice versa, the variation in net transfers across cohorts may be consistent with optimal intergenerational risk sharing. We in fact find evidence that the inter-cohort differences in net transfers can be partly rationalized by differences in outcomes.

3 Empirical implementation

3.1 Relative differences in lifetime outcomes

We propose several simple measures to quantify the degree of intergenerational risk sharing. The first measure relates the difference between lifetime wages and savings of two cohorts to a difference between their lifetime net transfers, i.e., benefits net of the contributions paid into the pension system. Intuitively, consider three cohorts, \(i\), \(j\), and \(k\) which earned different incomes due to unexpected shocks such that the cohort \(i\) is the poorest and cohort \(k\) is the richest. The pension system provides each cohort with a net transfer (positive or negative). The political process that determines the net transfers may take into consideration two factors: the political influence of each cohort and the desire to optimally share risk across generations. If only risk sharing considerations determine the transfers, then cohort \(i\) should receive the highest and cohort \(k\) the lowest transfer. If both political influence and risk sharing considerations determine the transfers and cohort \(k\) happens to be the most influential (such that it receives the highest net transfer despite being the richest), the risk sharing considerations vis-a-vis the remaining cohorts still imply that the gap in net transfers between cohorts \(i\) and \(k\) should be smaller than between \(j\) and \(k\). This is captured by parameters \(\gamma_{ij}^W\) and \(\gamma_{ij}^S\):

\[
(B_i - C_i) - (B_k - C_k) = \gamma_{ik}^W (W_i - W_k), \quad \forall i, k \tag{1}
\]

\[
(B_i - C_i) - (B_k - C_k) = \gamma_{ik}^S (S_i - S_k) \tag{2}
\]
where $B, W$ and $C$ denote per capita present value of benefits and wages received and contributions paid through the lifetime of a cohort, $S$ denotes the return on savings accumulated during the lifetime of the cohort, and the subscripts $k,i$ indicate a "base" cohort and a "comparison" cohort. All variables are measured in real dollars. There are many cohorts with whom income can be shared with, hence we compute separate lifetime risk sharing parameters $\gamma_{ik}$ for all combinations of cohorts. Optimal risk sharing prescribes that $\gamma_{ik}$ should be negative (higher wages imply lower net transfer) and between minus one and zero.\footnote{An illustrative benchmark is a pure defined contribution system in which the savings of each cohort are invested in an asset that yields a constant return - each cohorts simply "eats what it gets" - and $\gamma^S_{ik}$ is therefore zero for $i \neq k$.}

### 3.2 Differences in net transfers as a function of the differences in lifetime outcomes

An alternative way of quantifying the degree of risk sharing is by estimating a policy function

\[
(B_i - C_i) - (B_k - C_k) = \beta^W (W_i - W_k) + \beta^S (S_i - S_k) + \beta^X X_{ik} + \lambda_k + \epsilon_{ik}
\]

which explains the differences in lifetime net transfers between cohorts as a function of differences in wages and returns on savings. The units of observation for the estimating equation 3 are pairwise combinations of cohorts. The coefficients $\beta^W$ and $\beta^S$ can be thought of as the regression counterparts of the parameters $\gamma^W_{ik}$ and $\gamma^S_{ik}$ from equations 1 and 2. Variable $X_{ik}$ includes other variables that could potentially affect the difference in transfers between cohort, such as the difference in lifetime accumulated inflation between the two cohorts or the projected gap between Social Security’s revenues and expenditures. $\lambda_k$ denotes a cohort dummy which captures the fact each cohort may have received relatively larger or smaller net transfer for reasons unrelated to its economic outcomes vis-a-vis other cohorts. The estimates are then identified out of deviations from the mean difference in net transfers, wages, and savings for a cohort.

We consider two alternative specifications of the policy function. One estimates separate $\beta'$s for each distance between cohorts in order to capture a "waning effect" that the degree of risk sharing is likely to get weaker the more distant two cohorts are in time. The other also attempts to quantify which cohorts provide the risk sharing by estimating...
Equation 4 models the net lifetime transfer of cohort $i$ as a function of the cohort $i$’s own outcomes as well as outcomes of other cohorts that were born before and after. While very intuitive, the equation can be identified only for a small number of other cohorts (in the actual estimation we include cohorts born 5 years before and 5, 10, and 15 years after).

Equations 1-4 define measures based on “terminal” outcomes, observed after the history of contributions and benefits has been realized. They all can be defined analogously for outcomes that were expected at various points during the cohort’s life. Expected lifetime outcome (e.g., expected lifetime benefits) as of year $s$, $B_i^s$, is defined as

$$B_i^s = B_i^{t \leq s} + E_s[B_i^{t > s}]$$

where $B_i^{t \leq s}$ denotes the present value of benefits received by cohort $i$ until year $s$, and $E_s[B_i^{t > s}]$ denotes the present value of expected benefits received by cohort $i$ after year $s$, the expectation being taken as of year $s$. The expected future contributions and benefits are computed under the assumption that the pension legislation valid in year $s$ will remain valid through the cohort’s lifetime.\(^5\) Observing the cohorts over time allows exploiting the panel structure of the data; for example, fixed cohorts effects remove any permanent differences in net transfers between cohorts that are due to permanent differences in their political influence. If cohort $k$ has higher political influence than other cohorts, it expects in every year to receive a higher lifetime net transfer. However, if between years $s - l$ and $s$ the wages of cohort $i$ were rising faster than wages of cohort $k$, the risk sharing consideration should reduce the gap in lifetime net transfer between cohorts $i$ and $k$, despite the fact that cohort $k$ may still expect both higher wages and higher transfer as of year $s$. Such adjustments of net transfers to differential shocks to outcomes can only be estimated with the panel data.

\(^5\)The assumption need not imply that the parameters of the Social Security system will not change through the cohort’s lifetime since the Social Security legislation sometimes stipulates changes in parameters in the future. Such changes are reflected in the expectation as of year $s$. 

$$B_i - C_i = \beta^W W_i + \beta^W_{-1} W_{i-1} + \beta^W_{-2} W_{i-2} + \ldots + \beta^W_{i+1} W_i + \beta^W_{i+2} W_{i+2} + \ldots$$

$$+ \beta^W S_i + \beta^W_{-1} S_{i-1} + \beta^W_{-2} S_{i-2} + \ldots + \beta^W_{i+1} S_{i+1} + \beta^W_{i+2} S_{i+2} + \ldots + \epsilon_i$$ (4)
3.3 Transfers and variance of lifetime incomes

The very fact that economic shocks affect different cohorts differently is manifested in the variance of lifetime incomes between cohorts. Should social security at least partially provide inter-generational risksharing, the variance of net lifetime incomes (incomes plus net transfers) should be lower than the variance of incomes only.

\[
V(W + S + T) - (V(W) + V(S)) = V(T) + 2\text{cov}(W + S, T)
\]

As long as there is variance in transfers between cohorts, the presence of transfers mechanically adds to the overall variance in lifetime net incomes. In order to reduce the overall variance, it is not sufficient that the covariance between transfers and incomes is negative, but it must be sufficiently negative. Our additional measure of risk-sharing is thus the the right-hand side of equation 5, the contribution of social security transfers to the overall variance.

The same notion can be applied to changes in expected lifetime incomes and transfers over time. If there is variance between cohorts in the change in expected lifetime wages and returns on savings between years \(s - l\) and \(s\), the changes in net transfers should produce a lower variance in the change in expected net lifetime incomes.

3.4 Data

We use the dataset created by Shoven and Slavov (2006). They compute the internal rate of return on contributions in the U.S. Social Security system for each cohort born in 5-year intervals after 1900 (1900, 1905, 1910, till 1985) in each year between 1939-2005 under the social security legislation valid in that year. To do so, they first estimate the age-wage profile for an average worker and scale it by the average wage in each year to obtain the age-wage profile for each year. The worker’s wage then evolves as he moves along the profile and the profile also shifts up with the average wage growth. They assume that the worker works from age 20 till the official retirement age and has a deterministic length of life of 80 years. For each year, they compute the contributions paid and benefits received historically based on the past social security rules. For each year, they construct the projections of future wages and inflation rates, based on adaptive expectations (the future growth rate of a variable will be equal to the average growth rate from the previous five years). Finally, they construct the projections of future contributions and

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6 We are extremely grateful to Sita Slavov for the willingness to share the data.

7 All cohorts therefore experience the same shock to the level of their wages and expected wage growth rate. Still, the common shock has a differential impact on the lifetime wages of each cohort, since a younger cohort will
benefits from the wage and inflation projections by applying the social security legislation that was on the books in that year.

Our key variables are expected lifetime wages, excess savings, and net transfers in each year of expectation $s$, or precisely, the present value of the realized past and expected future flows of real wages, excess savings, and net transfers. We construct them by first converting the realized and projected wages etc. into real (2005) dollars. Next, we compute their present values based on a 4% discount factor. All variables are discounted to the year when the cohort is born, which implies that comparisons across cohorts are not affected by differential discounting. Finally we construct the variable "real lifetime net transfer" as the difference between the (present value of) real lifetime benefits and real lifetime contributions.

Since we also measure the risk sharing with respect to the returns on savings we generate evolution of savings for each cohort. Although the shocks to the stock markets returns are common to all cohorts in a given year, the timing of the shocks during the lifecycle of the cohort generates variation in the amount of savings that each cohorts accumulates upon retirement. We assume that each cohort saves 9% of its annual wages towards retirement every working year. The savings are invested in a portfolio consisting of 60% stocks that yield a real return equal to the inflation-adjusted return on the S&P500 index, and 40% in bonds that yield a safe 1% real return. We construct the histories of accumulated and expected savings for each cohort and expectation year by assigning the portfolio returns to the cohorts’ accumulated savings. The stock returns expected in the future are assumed to be equal to the average S&P return during the past 20 years. We then compute the present value of (realized or expected) lifetime savings in each expectation year $s$, which are equal the to savings accumulated at the year of retirement (if the cohort has already retired by year $s$) or the savings that the cohort expects it will have accumulated at the year of retirement (if the cohort is still working). From that we subtract the savings that the cohort would have achieved if the return on savings were constant in all years and equal to the average return from 1901 till 2005. The difference gives the variable of interest which denoted as "lifetime excess savings". The excess savings are used instead of the simple level of savings because savings are automatically higher for cohorts with higher wages, and thus

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8. For example, wages received in year 1940 by a 1900 cohort are discounted by the same discount factor as wages received in 1965 by a 1925 cohort.

9. First, one cohort may experience higher average return during its working years than another cohort. Second, a large shock (such as a stock market crash) has a larger affect if it hits a cohort shortly before retirement, since it changes the value of most their accumulated savings, while it may have little effect on the lifetime savings if it hits the cohort in its early working years.

10. This assumption is taken from Poterba et al. (1998) who report that the average 401(k) contribution represents 9% of the contributing household’s income.

11. This division of assets corresponds to calculations in Feldstein and Rangelova (2001).
higher savings for a particular cohort capture both a good stock market history or a good wage history. By removing the part of the savings that a cohort accumulated through higher wages, we isolate the former from the latter.

4 Results

4.1 Terminal outcomes

The first set of results evaluates the "terminal" outcomes, i.e. the present values of lifetime wages, savings, and net transfers observed in the final year of the cohort’s life if the cohort has died by 2005, or, if the cohort is alive as of 2005, the outcomes expected observed in 2005. It is essentially a cross-sectional comparison between cohorts. Figure 1 shows the lifetime wages for each cohort, which were rising sharply between 1900 and 1940 cohorts, declined somewhat between 1940-1960 cohorts, and then were rising for all the subsequent cohorts, Figure 2 shows lifetime excess savings. The variation between cohorts is substantial, with the 1900, 1935, and all post-1960 cohorts experiencing high positive returns while the cohorts from 1910 to 1930 all experiencing substantially negative returns. Last, Figure 3 depicts the lifetime net transfers from Social Security. The profile of the transfers shows positive net transfers for the early cohorts, particularly high net transfers for the 1920-1940 cohorts, and increasingly negative net transfers for all cohorts 1955 and later.\(^\text{12}\)

Figure 4 plots the differences in the lifetime net transfers between cohorts against the differences in lifetime wages. Each subpanel represent a cohort \((k)\), and the axis measure the outcome of cohort \(i\) (denoted by the label on each datapoint) minus the outcome of cohort \(k\) (such that positive values imply that cohort \(i\) had a higher wages or transfers than cohort \(k\)).\(^\text{13}\) The figure shows two rather striking patterns. First, optimal intergenerational risk sharing implies that all datapoints should be located either in the upper-left or lower-right quarters of the graph. This is true for some pairwise combinations of cohorts (namely, whenever any cohort 1925 or older is compared with any cohort 1955 or younger) but is not true for a large number of combinations of cohorts (namely, when the oldest (1900-10) cohorts are compared with somewhat younger (1915-35) cohorts that had both higher wages and higher net transfer, and when the 1930-45 cohorts are compared with somewhat younger (1950-65) cohorts that had both lower wages and lower net transfer).

Second, the relationship between net transfers and wages should be negative. In fact, there

\(^{12}\)This pattern of net transfers has been well documented (Leimer 2007)

\(^{13}\)Differences in both wages and transfers are normalized by the lifetime wages of cohort \(k\).
are three distinct ranges of data: For cohorts 1900-35 the relationship between wages and net transfers is positive, for cohorts 1940-60 it is also positive but with a steeper slope, and only when comparing cohorts 1960-85, the relationship is negative. But there is a substantial difference between these three cohort groups: By the time of observing the outcomes (2005), the first group has already realized its full history of wages, savings, and Social Security contributions and benefits\(^\text{14}\); the second group has realized most of its working years and is nearing retirement while the third groups has a fairly short working history and its lifetime outcomes are largely based on future expectations as opposed to actual realizations. As a consequence, the observations for the first and second group reflect (fully or to large extent) the Social Security system "as implemented", with various default rules and reforms driving the resulting net transfers. The observations for the third group reflect (almost fully) the rules of Social Security system that were in force in 2005 and are assumed to be valid in the future. The negative relationship between net transfers and benefits is visible only for the third group, and it reflects the redistribution that is inherent in the current Social Security rules\(^\text{15}\). But for the cohorts for whom the data largely reflects the Social Security system "as implemented" the negative relationship does not exist and is even reversed. This difference shows that distinguishing the default and "as-implemented" pension systems is empirically relevant.

Figure 5 plots the analogous relationship for lifetime net transfers and excess savings.\(^\text{16}\) It is radically different. Most datapoints do fall into either the upper-left or the lower-right quadrant\(^\text{17}\), and the difference in net transfers declines almost linearly (and with minimal variance) with the difference in excess savings. There are again two distinct ranges of data: when comparing within the group of 1900-1955 cohorts, the net transfer "compensates" decreases nearly dollar-for-dollar with an increase in excess savings, suggesting that the Social Security provided essentially complete intergenerational risk sharing against shocks to the returns on savings. This is particularly surprising given the fact that the Social Security rules do not contain any explicit link between the stock market returns and the level of benefits. But "as implemented", the Social Security system appears to function as if it was perfectly compensating for between-cohort differences in returns on savings. When comparing within the 1960-85 cohorts, the net transfer declines much faster with an increase in excess savings, suggesting overinsurance. But again, these are cohorts

\(^{14}\)The 1930 and 35 cohorts are still alive but way into their retirement. Since benefits are indexed for the cost of living and they are rarely being changed for people who already retired, it is near-certain that the real present values of lifetime outcomes of these cohorts would not change.

\(^{15}\)Including, for example, the phased-in postponement of the retirement age.

\(^{16}\)Differences in both excess savings and transfers are normalized by the lifetime "safe-equivalent" savings of cohort \(k\), i.e., the savings that the cohort would have had if the stock market returns were constant in all years and equal to the average return over the 1901-2005 period.

\(^{17}\)One exception is the 1935 cohort, which experienced a relatively better stock market history yet received the highest net transfer of all cohorts.
for which the lifetime outcomes are based largely on future expectations.

We computed the parameters $\gamma^W$ and $\gamma^S$ for all pairwise combinations of cohorts. The summary statistics are shown in Table 1. The overall average $\gamma^W$ for wages is $-0.19$ while the average $\gamma^S$ for excess savings is $-1.69$. Hence while differences in lifetime net transfers are, on average, negatively related to differences in both lifetime wages and excess savings, the relationship is rather weak for wages and rather strong (greater than one in absolute value) for savings. The table also shows summary statistics separately for each cohort $k$ with which the outcomes of the remaining cohorts are compared. The average cohort-level $\gamma^W$ varies substantially among cohorts, from $-1.19$ (cohort 1985) to $+0.47$ (cohorts 1935 and 1940). The average $\gamma^S$ is negative for all cohorts but one (1935); it varies from $-3.55$ (cohort 1985) to $-0.62$ (cohort 1900). Individual cohorts experienced very different patterns of risk sharing with other cohorts with respect to wages while they experienced more similar patterns of risk sharing with respect to excess savings.

The distribution of $\gamma'$s across cohorts is depicted graphically in Figures 6 (for wages) and 7 (for excess savings). Figure 5 illustrates that not only does average $\gamma^W$ vary between cohorts, but so does its variance. Several cohorts have very similar values of $\gamma^W$ for all their subsequent comparison cohorts (e.g., cohorts 1900-1920, 1965-) while for other cohorts $\gamma^W$ is typically positive for one cluster of comparison cohorts and negative for another cluster. For savings, the variance is much smaller. Also, for most cohorts, $\sigma^S$ tends to be more negative for the subsequent (younger) cohorts than for the preceding (older cohorts), suggesting that shocks to the returns on savings for any given cohort are "insured" more by the subsequent rather than preceding cohorts.

Alternative estimates of the policy function 3 are presented in Tables 3 and 5. Each regression is a panel regression where the unit of observation is a pairwise combination of a "base" cohort $k$ and all subsequent "comparison" cohorts $i$, as if each $ki$ observation was an observation of a subject $k$ in different points in time in a conventional panel set-up. Standard errors are clustered by cohort, and observations are weighted by the absolute value of the percentage difference between the cohorts' lifetime wages. Alternative specifications are run without cohort dummies, with "base" cohort dummies, and with "comparison" cohort dummies. The dummy variables capture the fact that some cohorts (base or comparison) may have received relatively larger or smaller net transfers due to relative differences in political influence. The estimates are then identified out of deviations from the mean difference in net transfers, wages, and savings for a cohort.

The basic regression without the cohort dummies (column 1 of Table 3) explains 45% percent of

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18The coefficients on the cohort dummies can be interpreted as measures of such relative differences in influence.
the variation in the differences in net transfers. Differences in both wages and excess savings are negatively and statistically significantly related to differences in net transfers, but the coefficient on excess savings is about 10 times greater in magnitude. When cohort dummies are included (column 2), the coefficient on wages switches sign while the coefficient on excess savings becomes more negative. When comparison cohort dummies are included, the coefficient on both wages and excess savings are again negative, except smaller in magnitude. A 10-percent difference between the lifetime wages of two cohorts implies an 0.7-percent difference in net transfers (as a share of wages), and a 10-percent difference in excess savings implies a 6-percent difference in transfers. The specification with comparison cohort dummies has much higher $R^2$ than the specification with base cohort dummies.

Regression in Table 5 attempt to capture the notion that the degree of risk sharing is likely to depend on the age difference ("distance") between cohorts. In column 4, we do so by interacting the differences in wages and excess savings with the distance between cohorts and also including the level of the distance among explanatory variables. The level of the distance is negative, capturing the fact that Social Security systematically provides increasingly worse "deal" for every younger cohort. The both interaction terms are also negative, implying that if wages of cohorts $k + 10$ and $k + 20$ are higher than wages of cohort $k$ by 10%, the difference in net transfer between cohorts $k + 20$ and $k$ would be 0.5 percent of wages higher than between cohorts $k + 10$ and $k$. (For excess savings, the corresponding number is 1.23 percent of wages). The regression in column 5 interacts the differences in wages and excess savings with dummies for each distance, thus not requiring that the interaction effects be linear. Still, the interaction effect is negative and increasing (in absolute value) as the comparison cohort is younger. Regressions in columns 6 and 7 include comparison cohort dummies in lieu of base cohort dummies, but the coefficients are not fundamentally different. In all regressions except column 7, the coefficients on the cohort (or comparison cohort) dummies are jointly statistically significant.

Regressions in Table 7 takes a different approach to estimating the dependence of the degree of risk sharing on the age difference between cohorts. We estimate the net transfer to cohort $i$ as a function of outcomes of cohort $i$ as well as outcomes of other cohorts. Since we have only 18 cohorts in the dataset, we can include only a limited number of other cohorts to preserve degrees of freedom, and we therefore include cohorts $i - 5, i + 5, i + 10$, and $i + 15$. Four specifications are estimated: the dependent variable is always the lifetime net transfer to cohort $i$, while in

---

19 It is cheaper to execute a given transfer of lifetime income from cohort $k + l$ to cohort $k$ if $l$ is small (for example, if the government borrows money to provide extra income to cohort $k$ and the debt plus interest is to be repaid by cohort $k + l$, the burden borne by $k + l$ is smaller the smaller is $l$; a lower (potentially distortionary) tax needs to be imposed on cohort $k + l$.

20 We experimented with alternative combinations (with 5 being the highest number of other cohorts that is technically estimable, and results were similar for such specifications.
column (1) the right-hand side variables are levels of wages and excess savings for cohort $i$ and other cohorts; in column (2) the wages and excess savings are detrended from the linear trend to capture the idea the net transfers might reflect only the “surprises” to cohort outcomes but not the differences in outcomes based on a (predictable) growth trend; the right-hand side variables in column (3) are the differences in outcomes of the other cohorts from cohort $i$; and column (4) include those differences as well as the level of the outcome of cohort $i$. We should expect the own outcomes to have a negative effect on the net transfer and the outcomes of other cohorts to have a positive effect. The results do not follow this pattern. The own effect of wages is always positive and large. The effect of wages of the other cohorts is indeed positive, but the effect for the most distant cohort included (born 15 years later) is negative. For excess savings, the own effect is negative (and significant in two out of three specifications) while the outcomes of all other cohorts have also negative effect, contradicting the theoretical predictions. That is, the effects of outcomes of cohort $i$ and the other cohorts have the same sign for either wages or excess savings; suggesting that the cohorts bear rather than share the same risk. This finding may be entirely driven by our inability to include more cohorts (particularly more distant cohorts, for which the relationship may be different.)

Last, we look at the contribution of Social Security to the variance in net lifetime incomes. The standard deviation of gross lifetime income (wages plus returns to savings) across all 18 cohorts is 48,157. The standard deviation in net lifetime income (gross income plus net transfer) is 43,011. The net transfers from Social Security hence reduced the standard deviation in incomes by 11% (the simple correlation between net transfers and gross incomes is -0.45).

4.2 Summary and interpretation of results - terminal outcomes

The analysis of the cohorts’ terminal outcomes gives four broad results: One, the Social Security system as implemented in the United States did not provide systematic risk sharing to wage shocks. For a large subgroup of cohorts there even appears to be a perverse relationship where positive differences between cohorts in lifetime wages translate into positive differences in lifetime net transfers.

Second, the Social Security system provided essentially complete risk sharing of the stock market risk. Differences in lifetime excess savings are translate one-for-one into differences in lifetime net transfers for the pre-1960 cohorts, and more than one-for-one for the post-1960 cohorts (which appears to be over-compensating). This finding is particularly striking given the fact that the Social Security rules do not contain any explicit provision that links stock market

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returns, contributions, and benefits.

Third, the differences in net transfers across cohorts are to a large extent also driven by factors other than economic outcomes of the cohorts, as evidenced by the irregular patterns in the relationship between net transfers and wages (Figure 4), high variance in $\gamma_{ki}$ across and within cohorts, and statistically significant cohort dummies. Simply stated, some cohorts were more "lucky" than others. Should such unexplained differences indeed be unrelated to the cohorts' incomes; they can be interpreted as evidence that the policy risk embedded in the pay-as-you-go pension scheme also creates additional intergenerational risk.

Fourth, the observed patterns of risk sharing are different for cohorts for which we observe complete (or near-complete) lifetime outcome and for cohorts for which the lifetime outcomes are based mostly on expectations about the future. The Social Security system "as implemented" with numerous reforms accumulating during the cohorts lifetime provides different pattern of risk sharing than if system’s default rules were held unchanged forever.

### 4.3 Expected lifetime outcomes over time

The second set of results evaluates the expected lifetime transfers, wages, and excess savings, as they evolved from 1939 to 2005. The idea is simple - assume that as of year $t$ people had some expectations about their lifetime wages and savings, and the Social Security rules generated certain expected lifetime net transfer for each cohort. Economic shocks that were realized between years $t$ and $t+s$ affected individual cohorts differently, say by increasing the expected lifetime income of cohort $k$ relative to cohort $j$. Also as of year $t+s$, people form a new expectation about the lifetime net transfer, the difference from year $t$ being a product of the default rules automatically adjusting the contributions and benefits to the shocks, and of possible reforms that may have been enacted between years $t$ and $t+s$. If the Social Security system “as implemented” between years $t$ and $t+s$ provides intergenerational risksharing, the net transfer to cohort $k$ should fall relative to cohort $j$.\(^{21}\)

We therefore test for a negative relationship between differences in changes in wages and excess savings among cohorts and differences in changes in lifetime net transfers. We have the expectation of these variables for each cohort and each year from 1939 to 2005. Since pension reforms are relatively infrequent and one would not expect economic shocks to be reflected in Social Security rules on a year-to-year basis, we reduce the frequency to five-year intervals from 1940, 1945, etc to 2005. The expectation for each of these years actually used in the analysis is

\(^{21}\)Cohort $k$ still may expect higher net transfer as of year $t+s$ than cohort $j$, either due to its higher political influence that carries through its lifetime or due to experiencing relatively worse economic shocks in the past.
equal to a 5-year average of the current year and the preceding four years, e.g. the expectation for 1945 is the average over 1941-1945.\textsuperscript{22}

Figures 8-9 provide a descriptive snapshot by plotting the change in expected lifetime wages, excess savings, and net transfers over the five-year intervals for each cohort. The changes are expressed in a fraction of expected lifetime gross income. The large negative change in 1945 for the older cohorts stems from the end of the rapid wage growth during the WWI, which, by construction of the expectations of future wages in Shoven and Slavov (2006) fed into very high expected lifetime wages in 1940. There is visible heterogeneity between cohorts in the evolution of expected excess savings. As for the expected cohorts, a distinct group of cohorts (1915-1935) experienced a series of positive changes during the 1970’s and 1980’s, while negative changes occur occasionally for the post 1950 cohorts and in later periods. The magnitude of changes in expected transfers is always below 5% of lifetime gross income.

Figures 11 and 12 show directly whether differences between cohorts in the change in expected lifetime wages or excess savings translate into differences in the change in the expected lifetime net transfers. Each observation in the figure is a pair-wise combination of cohorts $i$ and $j$ that were alive and working or retired in each year of expectation, the horizontal axis measures the difference in the change in lifetime wages between cohorts $i$ and $j$, normalized to cohort $j$’s gross income, and the vertical axis measures the corresponding difference in net transfers.\textsuperscript{23} There is no visible evidence of a negative relationship in almost any year of expectation. In fact, for most years, the cohorts that experienced a relatively better change in either wages or excess savings also experienced either no relative change or a positive change in net transfers. (The only exception are years 1995 and 1970, when relative positive change in excess savings is accompanied with a relatively negative change in net transfer.)

The relationship between expected transfers, wages, and excess savings over time is quantified in regressions reported in Table 9, which are panel equivalents to the cross-sectional regressions performed at the terminal outcomes in Tables 3 and 5. The unit of observation is a combination of cohorts $i$ and $j$ such that $i > j$ and a year of expectation.\textsuperscript{24} The dependent variable being explained is thus the difference in lifetime net transfers between all cohorts $i$ (that are younger than cohort $j$) and the cohort $j$. The kind of relationship being identified depends on the dummy variables included.

Regressions in column (1) includes fixed effects for cohorts $j$ (which captures the average net

\textsuperscript{22}Robustness check will re-run the analysis based on the current observation for each 5th year.

\textsuperscript{23}Specifically, the horizontal axis measures $[(W_{t+5}^{i} - W_{t}^{i}) - (W_{t+5}^{j} - W_{t}^{j})]/(W_{t}^{j} + S_{t}^{j})$.

\textsuperscript{24}Excluding cohort pairs with $i < j$ avoids duplicity of observations which would only have the opposite sign from the pairs of the same cohorts with $i > j$. 

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transfer from cohort \( j \) vis-a-vis all younger cohorts across all years) and fixed effects for each year of expectation (which captures the average difference in net transfer for all younger cohorts \( i \) relative to cohorts \( j \)). The estimator is based on deviations from fixed effects both over cohorts \( j \) and over years. The coefficient on wages is negligible and statistically insignificant; the coefficient on excess savings is negative \((-0.087)\) and statistically significant at 1%. Column (2) contains fixed effects for each cohort \( j \) - year combination. It implicitly assumes a fixed “treatment” of a cohort \( j \) in a given year, and then the estimator is based on deviations of the cohorts \( i \) from the cohort \( j \) - year mean. The results show again an insignificant coefficient on wages and negative and significant \((-0.132)\) coefficient on excess savings. As such, these results, where the identifying variation is based in part or entirely on a comparisons between cohorts, produce results qualitatively similar to the results based on terminal outcomes, except that the estimated relationship between excess savings and transfers is smaller by the order of magnitude.

Very different results emerge from column (3) which includes dummies for each combination of cohorts \( i \) and \( j \). It implicitly assumes that each pair of cohorts has a fixed difference in net transfers “given” throughout their lives (reflecting, for example, the relative political power of the two cohorts), and then compares whether relative changes in wages or excess savings over time within this pair of cohorts yield to relative changes in transfers. They do not; the coefficient on wages is again insignificant while the coefficient on excess savings is actually positive \((0.057)\) and significant. The results indicate that within a pair of cohorts, changes in transfers and wages actually go in the “wrong” direction.

The remaining columns in Table 9 attempt to estimate how the degree of risk-sharing between cohorts depends on the “distance” (difference in the year of birth) between cohorts. Each regression includes interaction of wages (excess savings) with the dummy variable for age difference between cohorts being 10, 15, 20, 25, and 30 and more years (5 is the omitted category).\(^{25}\) In columns (4) and (5) that include cohort \( j \) dummies or dummies for interactions between cohort \( j \) and year, the direct effects of both wages and excess savings are positive (with one exception), and larger for excess savings. The positive direct effects most likely reflect the fact that the nearby cohorts experience similar economic shocks and also receive the most similar treatment from Social Security. The interactions are negative and become more negative for more distant cohort, implying that eventually the overall effect (direct plus interaction) becomes negative for cohorts that are younger by 20-25 years or more. Cohorts that are distant by a generation or more thus appear to share some of the shocks to their incomes. Results based on regression with fixed effects for each pairs of cohorts \( i \) and \( j \) do not show a significant relationship between

\(^{25}\)Because observations include cohort pairs with \( i > j \), the age differences are only positive.
wages, excess savings, and net transfers.

Finally, Table 11 estimates the expected lifetime net transfer to cohort $i$ as a function of the expected lifetime outcomes of cohort $i$ and outcomes of the other (nearby) cohorts (hence is equivalent to Table 7 except that we follow each cohort over time). The results for expected lifetime wages do not show a systematic pattern of risk sharing; some coefficients are implausibly large in magnitude and they have different signs for nearby cohorts. For excess savings, the own effect and the effects of the cohorts $i - 5, i + 5$ and $i + 10$ are insignificant, and only for the cohort $i + 15$ we observe a positive (as expected) and statistically significant relationship between that cohort’s excess savings and cohort $i$’s net transfer.

4.4 Summary and interpretation of results - expected outcomes over time

The analysis of expected lifetime outcomes in different years yields the following broad results: We find essentially no evidence of risk sharing with respect to wages, and some evidence of risk sharing with respect to excess savings. The latter result is driven by differences between pairs of cohorts in any given year; in fact, changes in relative outcomes within a given pair of cohorts over time do not translate into changes in net transfers in a way that is consistent with optimal risk sharing. Outcomes and transfers of the most nearby cohorts are positively related while they are negatively related for cohorts with larger age difference (20 years and more). Certain patterns of redistribution (e.g. Table 11) appear to be driven by selective treatment of certain cohorts rather than risk-sharing considerations.

References


5 Annex - sketch of a formal model

Our risk sharing measures are admittedly based on the intuitive interpretations of the normative intergenerational risk sharing literature, and the same is true about our qualitative normative
evaluations of our results in section 4. A model providing a normative benchmark and a measure of welfare would allow to better motivate the choice of the risk sharing measures and to carry out quantitative evaluations. We construct an OLG model which can be calibrated with our available data, formulate a welfare function, and compare the welfare under the observed pattern of risk sharing with the optimal allocation. It follows D’Amato and Galasso (2008) but extends it such that the variables in the model correspond to variables that are observable in our data, i.e., wages and stock market returns received by overlapping cohorts. (By this stage of the project (August 2010) we have only set-up the structure and assumptions of the model. Solving and calibrating it is the next step.)

The economy

There are three overlapping generations: young, middle-aged, and old. The young and middle-aged work, earn a wage, save a certain fraction of their wage and invest it. When old, people do not work and consume their savings plus a return on savings plus any social security benefit they may receive. When young or middle aged, people consume their wage earnings minus savings minus any social security tax they may be required to pay. The wages are pure endowments received by the particular cohorts; likewise, returns to savings are endowments that are added to (or subtracted from in the case of negative returns) to the wage endowments that the cohort put aside.

Let superscripts denote the cohort (birth period of the cohort) and subscripts the time period. Hence

\[ w_t^t \ldots \text{ wage of cohort } t \text{ when young} \]
\[ w_{t+1}^t \text{ wage of cohort } t \text{ when middle aged.} \]
\[ A_{t+2}^t \text{ financial assets accumulated by cohort } t \text{ upon retirement} \]

The wages of the young and middle aged in period \( t \) differ by a constant fraction \( a \), that is, people move along a (highly simplified) "wage profile" \( w_{t-1}^t = (1 + a)w_{t+1}^t \)

Each period the wages are subject to an aggregate (common) shock \( g_t \):

\[ w_t^t = (1 + g_t)w_{t-1}^{t-1} \]
\[ w_{t-1}^{t-1} = (1 + g_t)w_{t-1}^t \]

that is, wages of both the young and the middle aged increase by the same rate. Although the
shock is not cohort-specific, its impact on the expected lifetime earnings is cohort-specific. For the current young, it affects their earnings in two periods (when young and when middle aged); for the current middle-aged, it affects their earnings only in one period (when middle aged); the old are unaffected by the wage shock.

The wage growth is stochastic with an unconditional mean $g$. At this level of generality, we allow the wage shocks to be persistent such that the entire past history of wage shocks may affect the expected future wage shocks, e.g. $E[g_{t+1}|g_t, g_{t-1}...](specific data-generating processes will be discussed later).

When working, people save a constant fraction $s$ of their before-tax wage. The savings yield a return $r_t$ ($r_t$ is a return on assets the cohort saved up to period $t-1$ and is received at the beginning of period $t$. Upon retirement in period $t+2$, the assets of the cohort $t$ are thus

$$A_{t+2}^t = sw_t^t(1+r_{t+1})(1+r_{t+2}) + sw_{t+1}^t(1+r_{t+2})$$

The returns are stochastic with an unconditional mean $r$. We allow the expected future returns to depend on the past realizations of returns. $^{26}$ Again, while the shocks to the returns are aggregate shocks, their impact on the expected lifetime incomes are cohort-specific. A negative shock reduces the entire lifetime savings of the old; for the middle aged, it reduces the value of the savings they accumulated while young plus it reduces the expected return when they get old; for the young, it only reduces the expected return when they get middle aged and old. $^{27}$

**Consumers**

Consumers have concave utility defined over the present value of their lifetime consumption, discounted to the period of birth

$$U = U(C^t) = U(C^t + \delta C_{t+1}^t + \delta^2 C_{t+2}^t)$$

$^{26}$On one hand, there is hard evidence that the past returns do not predict future returns. On the other hand there is evidence from behavioral economics that people believe that the current returns do predict future returns.

$^{27}$In the limiting case when the expected future returns do not depend on the past returns ($E[r_{t+j}|r_t, r_{t-1}...] = r \forall j > 0$) then a realization of the shock does not affect the lifetime earnings of the young, and the change in the expected lifetime earnings of the middle aged is approximately one-half of the change for the old.
where

\[
\begin{align*}
C_t^t &= (1 - s)w_t^t - \tau_t w_t^t \\
C_{t+1}^t &= (1 - s)w_{t+1}^t - \tau_{t+1} w_{t+1}^t \\
C_{t+2}^t &= sw_t^t(1 + r_t)^{(1 + r_{t+2})} + sw_{t+1}^t(1 + r_{t+2}) + b_t^t
\end{align*}
\]

In each period, the consumption can be decomposed into income from the endowments (wages or savings) and income from social security (taxes paid or benefits received). People do not make any economic decision as savings are fixed. As they progress from young to middle aged, they build up the stock of lifetime consumption. As they become old, the ultimate level of lifetime consumption is determined by the return to savings and the government’s choice of social security benefits. While restrictive, this specification greatly simplifies the social planner’s objective function. Even though the planner maximizes the (weighted) sum of utilities of all current and future cohorts, the recursive problem can be reformulated as maximizing the lifetime utility of the current old only; higher benefits to the old however increase the taxes and therefore reduce the stock of lifetime consumption for the middle-aged and the young when these cohorts become old.

**Policy space**

At the beginning of each period, the realizations of \(g_t\) and \(r_t\) are revealed. The planner then decides on the tax rate \(\tau_t\) on the young and the middle-aged and the benefits \(b_t^{t-2}\) for the old. We restrict the tax rate to be the same for the young and for the middle-aged, emulating the actual social security system.

The objective function - ex ante optimality, no commitment, policy decided each period

\[
\max_{\{b_{t+i}\}_{i=0}^{\infty}} U(C_t^{t-2}) + \beta E_t U(C_t^{t-1}) + \beta^2 E_t U(C_t) + \ldots
\]

\(\beta\) ... social discount factor (between cohort).

Constraint:

\[
b_t = \tau_t(w_t^t + w_{t-1}^{t-1})
\]

State variables:

\(g_t\), plus past history up to \(t - k\) (past shocks up to \(k\) periods back affect expected \(E[g_{t+i}]\))
$r_t$, plus past history up to $t - k$ (past shocks up to $k$ periods back affect expected $E[r_{t+i}]$)

$\tau_{t-1}$ and $\tau_{t-2}$... past taxes on the current old and middle-aged affect the “stock” of lifetime consumption they carry to the current period

Recursive formulation:

$$V(g_{t-i}, r_{t-i}, \tau_{t-1}, \tau_{t-2}) = \max_{b_t} \{ U(C_{t-2}) + \beta E_t V(g_{t+1-i}, r_{t+1-i}, \tau_t, \tau_{t-1}) \}, \ i = 0, 1, ..., k$$

To be done:

- solve
- characterize the optimal policy
- calibrate to plausible utility function, data-generating process (as much as possible with the data at hand - the data-generating process for the expectations is under our control - both in the data and in the model) - what the optimal benefits, taxes etc should have been given the actual data on wages, returns, and expectations
- compare with actual benefits and taxes
- derive the implications for variance between cohorts
Figure 1: Lifetime wages
Figure 2: Lifetime excess savings

Accumulated savings net of equivalent risk-free savings

Figure 3: Lifetime net transfer
Figure 4: Differences in lifetime net transfers and wages between cohorts

Differences in net transfers and wages between cohorts

Graphs by cohort

Figure 5: Differences in lifetime net transfers and excess savings between cohorts

Differences in net transfers and savings deviations, relative to safe equivalent savings

Difference in real accumulated savings net of equivalent risk–free savings, rela

Graphs by cohort
Figure 6: Risk-sharing measure gamma for lifetime wages

Risk–sharing measure gamma for lifetime wages
terminal outcomes, by cohorts

Gamma wages

gamma_savings_dev

1900 1905 1910 1915 1920
1925 1930 1935 1940 1945
1975 1980 1985

Graphs by cohort

Figure 7: Risk-sharing measure gamma for lifetime excess savings

Risk–sharing measure gamma for lifetime excess savings
terminal outcomes

Gamma savings_dev

1900 1905 1910 1915 1920
1925 1930 1935 1940 1945
1975 1980 1985

Graphs by cohort
Figure 8: Changes in expected lifetime wages over time

![Change in expected lifetime wages](image)

Graphs by cohort

Figure 9: Changes in expected lifetime excess savings over time

![Change in expected lifetime excess savings](image)

Graphs by cohort
Figure 10: Changes in expected lifetime net transfer over time

![Change in expected lifetime net transfer](image)

Graphs by cohort

Figure 11: Inter-cohort differences in changes in lifetime transfers and wages

![Inter-cohort differences in changes in lifetime transfers and wages](image)

Graphs by year

30
Figure 12: Inter-cohort differences in changes in lifetime transfers and excess savings

Graphs by year

Inter–cohort differences in changes in lifetime transfers and excess savings
Table 1: Gamma parameter for terminal outcomes

<table>
<thead>
<tr>
<th>cohort</th>
<th>gamma - wages</th>
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<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std. deviation</td>
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<tr>
<td>All</td>
<td>-0.187</td>
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<tr>
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<td>0.179</td>
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<td>1905</td>
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<td>1910</td>
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<td>-0.316</td>
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<tr>
<td>1920</td>
<td>-0.508</td>
<td>0.823</td>
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<tr>
<td>1925</td>
<td>-0.554</td>
<td>1.346</td>
</tr>
<tr>
<td>1930</td>
<td>-0.102</td>
<td>2.264</td>
</tr>
<tr>
<td>1935</td>
<td>0.469</td>
<td>3.615</td>
</tr>
<tr>
<td>1940</td>
<td>0.469</td>
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<tr>
<td>1945</td>
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<td>1950</td>
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<td>1955</td>
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<td>1960</td>
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<td>1985</td>
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Table 3: Determinants of the differences in lifetime net transfers between cohorts

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<th>3</th>
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<tr>
<td>Difference in lifetime transfer</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Difference in lifetime wages</td>
<td>-0.126</td>
<td>0.134</td>
<td>-0.071</td>
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<tr>
<td>excess savings</td>
<td>-1.328</td>
<td>-2.023</td>
<td>-0.600</td>
</tr>
<tr>
<td>Constant</td>
<td>0.005</td>
<td>-0.016</td>
<td>-0.014</td>
</tr>
<tr>
<td>Own cohort dummies</td>
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<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Comparison cohort dummies</td>
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<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Observations</td>
<td>153</td>
<td>153</td>
<td>153</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.453</td>
<td>0.605</td>
<td>0.961</td>
</tr>
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Robust standard errors in parentheses.
All differences normalized by own cohort’s lifetime wages
* significant at 5%; ** significant at 1%
Unit of observation: pairwise combination of cohorts i-j, where i>j, terminal outcomes
Table 5: Determinants of the differences in lifetime net transfers between cohorts - cont.

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<th>5</th>
<th>6</th>
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<td>Difference in lifetime transfer</td>
<td>0.371</td>
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<td>0.386</td>
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<tr>
<td>(0.023)**</td>
<td>(0.588)</td>
<td>(0.025)**</td>
<td>(0.387)</td>
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<tr>
<td>Difference in lifetime wage</td>
<td>0.865</td>
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<td>0.088</td>
<td>-1.657</td>
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<tr>
<td>(0.044)**</td>
<td>(0.568)</td>
<td>(0.070)</td>
<td>(0.775)**</td>
<td></td>
</tr>
<tr>
<td>Difference in lifetime excess savings x cohort distance</td>
<td>-0.005</td>
<td>-0.003</td>
<td>-0.001</td>
<td>-0.003</td>
</tr>
<tr>
<td>(0.0003)**</td>
<td>(0.0001)**</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Diff. in lifetime excess savings x cohort distance</td>
<td>-0.012</td>
<td>-0.001</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>(0.001)**</td>
<td>(0.001)</td>
<td></td>
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<td></td>
</tr>
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<td>Cohort distance</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.003</td>
</tr>
<tr>
<td>(0.0002)**</td>
<td>(0.0002)**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.013</td>
<td>-0.032</td>
<td>-0.099</td>
<td>-0.105</td>
</tr>
<tr>
<td>(0.003)**</td>
<td>(0.003)**</td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff. in lifetime wages x 10-years cohort distance dummy</td>
<td>0.498</td>
<td>-0.284</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.303)</td>
<td>(0.257)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff. in lifetime wages x 15-years cohort distance dummy</td>
<td>0.644</td>
<td>-0.339</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.362)</td>
<td>(0.341)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Diff. in lifetime wages x 20-years cohort distance dummy</td>
<td>0.740</td>
<td>-0.362</td>
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<td></td>
</tr>
<tr>
<td>(0.383)</td>
<td>(0.378)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff. in lifetime wages x 25-years cohort distance dummy</td>
<td>0.811</td>
<td>-0.354</td>
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<tr>
<td>(0.424)</td>
<td>(0.396)</td>
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<td></td>
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<tr>
<td>Diff. in lifetime wages x 30-years cohort distance dummy</td>
<td>0.725</td>
<td>-0.352</td>
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<tr>
<td>(0.449)</td>
<td>(0.413)</td>
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<tr>
<td>Diff. in lifetime excess savings x 10-years cohort distance dummy</td>
<td>-0.393</td>
<td>0.178</td>
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<tr>
<td>(0.433)</td>
<td>(0.281)</td>
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<td></td>
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<td>Diff. in lifetime excess savings x 15-years cohort distance dummy</td>
<td>-0.795</td>
<td>0.472</td>
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<td>(0.597)</td>
<td>(0.390)</td>
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<td></td>
<td></td>
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<tr>
<td>Diff. in lifetime excess savings x 20-years cohort distance dummy</td>
<td>-0.818</td>
<td>0.602</td>
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<tr>
<td>(0.553)</td>
<td>(0.490)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Diff. in lifetime excess savings x 25-years cohort distance dummy</td>
<td>-0.601</td>
<td>0.886</td>
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<td>(0.564)</td>
<td>(0.564)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff. in lifetime excess savings x 30-years cohort distance dummy</td>
<td>-0.012</td>
<td>1.058</td>
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<tr>
<td>(0.004)*</td>
<td>(0.620)</td>
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<td></td>
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<tr>
<td>Own cohort dummies</td>
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<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Comparison cohort dummies</td>
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<td>no</td>
<td>yes</td>
<td>yes</td>
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<td>Observations</td>
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<td>153</td>
<td>153</td>
<td>153</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.990</td>
<td>0.681</td>
<td>0.998</td>
<td>0.964</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses
All differences normalized by own cohort’s lifetime wages
* significant at 5%; ** significant at 1%
Unit of observation: pairwise combination of cohorts i-j, where i>j, terminal outcomes
Table 7: Estimates of the net transfer to cohort $i$ as a function of outcomes of adjacent cohorts

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net transfer to cohort $i$ to cohort $i$</td>
<td>Levels</td>
<td>Levels detrended</td>
<td>Difference from cohort $i$</td>
<td>Difference from cohort $i$</td>
</tr>
<tr>
<td>Lifetime wage of cohort $i$</td>
<td>1.251</td>
<td>2.902</td>
<td>0.384</td>
<td>0.384</td>
</tr>
<tr>
<td></td>
<td>(0.104)**</td>
<td>(2.399)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lifetime wage of cohort $i-5$</td>
<td>0.145</td>
<td>-0.137</td>
<td>-2.753</td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>(0.107)</td>
<td>(0.660)</td>
<td>(1.412)</td>
<td></td>
</tr>
<tr>
<td>Lifetime wage of cohort $i+5$</td>
<td>1.627</td>
<td>-3.975</td>
<td>-3.939</td>
<td>-1.627</td>
</tr>
<tr>
<td></td>
<td>(0.080)**</td>
<td>(3.562)</td>
<td>(2.136)</td>
<td>(0.080)**</td>
</tr>
<tr>
<td>Lifetime wage of cohort $i+10$</td>
<td>0.723</td>
<td>2.088</td>
<td>2.820</td>
<td>0.723</td>
</tr>
<tr>
<td></td>
<td>(0.113)**</td>
<td>(2.819)</td>
<td>(1.471)</td>
<td>(0.113)**</td>
</tr>
<tr>
<td>Lifetime wage of cohort $i+15$</td>
<td>-0.108</td>
<td>-0.315</td>
<td>-1.530</td>
<td>-0.108</td>
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<tr>
<td></td>
<td>(0.148)</td>
<td>(0.787)</td>
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<td>(0.148)</td>
</tr>
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<td>Lifetime excess savings of cohort $i$</td>
<td>-0.210</td>
<td>-0.536</td>
<td>-3.545</td>
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</tr>
<tr>
<td></td>
<td>(0.020)**</td>
<td>(0.583)</td>
<td></td>
<td>(0.180)**</td>
</tr>
<tr>
<td>Lifetime excess savings of cohort $i-5$</td>
<td>-0.162</td>
<td>-0.092</td>
<td>0.281</td>
<td>-0.162</td>
</tr>
<tr>
<td></td>
<td>(0.049)*</td>
<td>(0.279)</td>
<td>(0.272)**</td>
<td>(0.049)*</td>
</tr>
<tr>
<td>Lifetime excess savings of cohort $i+5$</td>
<td>-1.072</td>
<td>-1.664</td>
<td>0.193</td>
<td>-1.072</td>
</tr>
<tr>
<td></td>
<td>(0.067)**</td>
<td>(0.588)</td>
<td>(0.324)</td>
<td>(0.067)**</td>
</tr>
<tr>
<td>Lifetime excess savings of cohort $i+10$</td>
<td>-1.299</td>
<td>-2.104</td>
<td>0.011</td>
<td>-1.299</td>
</tr>
<tr>
<td></td>
<td>(0.091)**</td>
<td>(0.747)</td>
<td>(0.241)</td>
<td>(0.091)**</td>
</tr>
<tr>
<td>Lifetime excess savings of cohort $i+15$</td>
<td>-0.799</td>
<td>-1.724</td>
<td>-0.928</td>
<td>-0.799</td>
</tr>
<tr>
<td></td>
<td>(0.045)**</td>
<td>(0.966)</td>
<td>(0.742)</td>
<td>(0.045)**</td>
</tr>
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<td>Constant</td>
<td>-100476.2</td>
<td>-10805.38</td>
<td>-683.930</td>
<td>-100476.2</td>
</tr>
<tr>
<td></td>
<td>(24822.38)*</td>
<td>(2631.633)*</td>
<td>(3463.81)**</td>
<td>(24822.38)*</td>
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<tr>
<td>Observations</td>
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<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.999</td>
<td>0.991</td>
<td>0.724</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Newey-West robust standard errors in parentheses
* significant at 5%; ** significant at 1%
Unit of observation: cohort, terminal outcomes
Table 9: Determinants of the differences in lifetime net transfers between cohorts

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>Difference in lifetime transfer</td>
<td>Difference in lifetime wage</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.0004</td>
<td>0.021</td>
<td>0.046</td>
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<td></td>
<td>Difference in lifetime excess savings</td>
<td>-0.087</td>
<td>-0.132</td>
<td>0.057</td>
<td>0.397</td>
<td>0.205</td>
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<td>Constant</td>
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<td>-0.009</td>
<td>-0.020</td>
<td>-0.024</td>
<td>-0.024</td>
<td>-0.025</td>
</tr>
<tr>
<td>Diff. in lifetime wages x 10-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 15-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 20-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 25-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 30-years cohort distance dummy</td>
<td>Diff. in lifetime excess savings x 10-years cohort distance dummy</td>
<td>Diff. in lifetime excess savings x 15-years cohort distance dummy</td>
</tr>
<tr>
<td></td>
<td>0.014</td>
<td>-0.012</td>
<td>-0.030</td>
<td>-0.019</td>
<td>-0.020</td>
<td>-0.026</td>
</tr>
<tr>
<td></td>
<td>(0.001)*</td>
<td>(0.010)*</td>
<td>(0.027)***</td>
<td>(0.010)*</td>
<td>(0.022)*</td>
<td>(0.010)*</td>
</tr>
<tr>
<td>Diff. in lifetime wages x 10-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 15-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 20-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 25-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 30-years cohort distance dummy</td>
<td>Diff. in lifetime excess savings x 10-years cohort distance dummy</td>
<td>Diff. in lifetime excess savings x 15-years cohort distance dummy</td>
</tr>
<tr>
<td></td>
<td>0.014</td>
<td>-0.012</td>
<td>-0.030</td>
<td>-0.019</td>
<td>-0.020</td>
<td>-0.026</td>
</tr>
<tr>
<td></td>
<td>(0.001)*</td>
<td>(0.010)*</td>
<td>(0.027)***</td>
<td>(0.010)*</td>
<td>(0.022)*</td>
<td>(0.010)*</td>
</tr>
<tr>
<td>Diff. in lifetime wages x 10-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 15-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 20-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 25-years cohort distance dummy</td>
<td>Diff. in lifetime wages x 30-years cohort distance dummy</td>
<td>Diff. in lifetime excess savings x 10-years cohort distance dummy</td>
<td>Diff. in lifetime excess savings x 15-years cohort distance dummy</td>
</tr>
<tr>
<td>own cohort dummies</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Combination of cohorts dummies</td>
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<td>no</td>
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<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Expectation year dummies</td>
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<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
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<tr>
<td>cohort and expectation year interaction</td>
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<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
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<td>Observations</td>
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<td>672</td>
<td>672</td>
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</tr>
<tr>
<td>R-squared</td>
<td>0.407</td>
<td>0.456</td>
<td>0.863</td>
<td>0.049</td>
<td>0.046</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses
All differences normalized by own cohort’s lifetime gross income
* significant at 10%; ** significant at 5%; *** significant at 1%
Unit of observation: pairwise combinations of cohorts i-j and year of expectation, with i>j, expected lifetime outcomes
Table 11: Estimates of the net transfer to cohort $i$ as a function of outcomes of adjacent cohorts

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net transfer to cohort $i$ to cohort $i$</td>
<td>Levels</td>
<td>Difference from cohort $i$</td>
<td>Difference from cohort $i$</td>
</tr>
<tr>
<td>Lifetime wage of cohort $i$</td>
<td>18.414 (12.582)</td>
<td>-0.009</td>
<td></td>
</tr>
<tr>
<td>Lifetime wage of cohort $i-5$</td>
<td>-8.105 (7.354)</td>
<td>-7.048 (3.182)**</td>
<td>-8.105 (7.354)</td>
</tr>
<tr>
<td>Lifetime wage of cohort $i+10$</td>
<td>3.113 (1.516)*</td>
<td>2.632 (1.096)**</td>
<td>3.113 (1.516)*</td>
</tr>
<tr>
<td>Lifetime wage of cohort $i+15$</td>
<td>0.001 (0.001)</td>
<td>0.001 (0.001)</td>
<td>0.001 (0.001)</td>
</tr>
<tr>
<td>Lifetime excess savings of cohort $i$</td>
<td>-0.296 (0.396)</td>
<td>0.100</td>
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</tr>
<tr>
<td>Lifetime excess savings of cohort $i-5$</td>
<td>-0.128 (0.408)</td>
<td>-0.257 (0.351)</td>
<td>-0.128 (0.408)</td>
</tr>
<tr>
<td>Lifetime excess savings of cohort $i+5$</td>
<td>-0.018 (0.898)</td>
<td>0.001 (0.893)</td>
<td>-0.018 (0.898)</td>
</tr>
<tr>
<td>Lifetime excess savings of cohort $i+10$</td>
<td>-0.277 (0.585)</td>
<td>-0.192 (0.667)</td>
<td>-0.277 (0.585)</td>
</tr>
<tr>
<td>Lifetime excess savings of cohort $i+15$</td>
<td>0.228 (0.092)**</td>
<td>0.192 (0.092)</td>
<td>0.228 (0.092)**</td>
</tr>
<tr>
<td>Constant</td>
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<td>-11769.2 (8055.1)</td>
<td>-10011.0 (197902.0)*</td>
</tr>
<tr>
<td>Observations</td>
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<td>84</td>
<td>84</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.155</td>
<td>0.171</td>
<td>0.155</td>
</tr>
<tr>
<td>cohort dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%

Unit of observation: cohort and year of expectation, expected lifetime outcomes