Introduction to Game Theory - MIDTERM EXAM A, November 3, 2009

This is 35-minutes closed-book exam worth 100 points in total. This exam will count for 40% of your final grade from the course. The exam has 3 pages and there are 4 problems in total. You are not allowed to use any books, notes or calculator. You are not allowed to talk between each other. Please write your name on the exam booklet. Most of you will be able to compute in the 35 minutes just Problems 1-3 so I suggest to start with Problem 4 after you have already solved Problems 1-3. If you have a question, please raise your hand. Good luck!

Name: ……………………………….

Problem 1 (20 points)

Consider the following static game, where player 1 chooses rows and player 2 chooses columns. The first number in every cell is payoff for player 1, the second payoff for player 2.

Player 2

Player 1

	A	B	C	D	E
A	1,3	5,1	5,4	2,2	8,1
B	0,5	4,6	6,2	1,5	4,1
C	1,3	3,2	6,3	2,7	1,0
D	2,6	4,2	3,2	5,6	4,1
E	2,2	5,5	7,2	3,6	2,4

a) Does either player have any strictly dominated actions in pure strategies? If so, what are they?

Strictly dominated action is such action that leads to strictly lower payoffs compared to some other action, no matter what is the other player doing. Therefore, if some action is strictly dominated, there has to exist other action for which all numbers (payoffs) are higher in every column for player 1 (the first number), or in every row for player 2 (the second number). Further the action cannot be strictly dominated if it is best response to some action of the other player.

For Player 1: A cannot be strictly dominated (best response to B and E), for the same reasons D (best response to A and D) and E (best response to A,B and C) cannot be strictly dominated. So it is enough to check B and C.

We can see that only **C is strictly dominated** by E (1<2; 3<5; 6<7; 2<3; 1<2) and we cannot find any such action for B.

For Player 2: A cannot be strictly dominated (best response to D), for the same reasons B (best response to B), C(best response to A) and D(best response to C,D and E) cannot be strictly dominated. So it is enough to check E.

We can see that **E is strictly dominated** by D (1<2; 1<5; 0<7; 1<6; 4<6).

Correct answer: For player 1: C is strictly dominated by E, for player 2 E is strictly dominated by D.

b) What actions of each player remain after iterated elimination of strictly dominated strategies (use just pure strategies)?

Now we assume that both players are rational and are iteratively eliminating strictly dominated actions.
In the first step we eliminate for player 1: C and for player 2: E.
In the second step we see that after eliminating (C for P1 and E for P2) now B is strictly dominated by E for player 1. So we will eliminate B for player 1.
In the third step we see that now B is strictly dominated by D for player 2. So we will eliminate B for player 2.
In the fourth step we see that A is strictly dominated by E for player 1. So we will eliminate A for player 1.
In the fifth step we see that C is strictly dominated by D for player 2. So we will eliminate C for player 2.
In this way of thinking we continue till the final stage.
As we see we cannot eliminate any other action for any of the two players so we end up with actions D and E for player 1 and A and D for player 2.

Player 2

		A	~~B~~	~~C~~	D	~~E~~
	~~A~~	~~1,3~~	~~5,1~~	~~5,4~~	~~2,2~~	~~8,1~~
	~~B~~	~~0,5~~	~~4,6~~	~~6,2~~	~~1,5~~	~~4,1~~
Player 1	~~C~~	~~1,3~~	~~3,2~~	~~6,3~~	~~2,7~~	~~1,0~~
	D	2,6	~~4,2~~	~~3,2~~	5,6	~~4,1~~
	E	2,2	~~5,5~~	~~7,2~~	3,6	~~2,4~~

Correct answer: For player 1: D and E remain, for player 2 A and D remain.

Problem 2 (20 points)

Consider the static game, where player 1 chooses rows and player 2 chooses columns. The first number in every cell is payoff for player 1, the second payoff for player 2.

Player 2

Player 1

	A	B	C	D
A	1,3	3,1	5,$\underline{4}$	2,0
B	0,5	4,$\underline{6}$	$\underline{7}$,2	1,5
C	1,3	$\underline{5}$,2	$\underline{7}$,3	$\boxed{\underline{3},\underline{7}}$
D	$\boxed{\underline{2},\underline{6}}$	4,2	3,2	1,$\underline{6}$

a) Find all pure strategy Nash Equilibria of the game. (you may mark them in the table above)

Using best responses (underlined highest numbers in every column for player 1 and in every row for player 2) we see that only for (C,D) and (D,A) it holds that both actions are best responses to the other player's action.

Correct answer: (C,D) and (D,A)

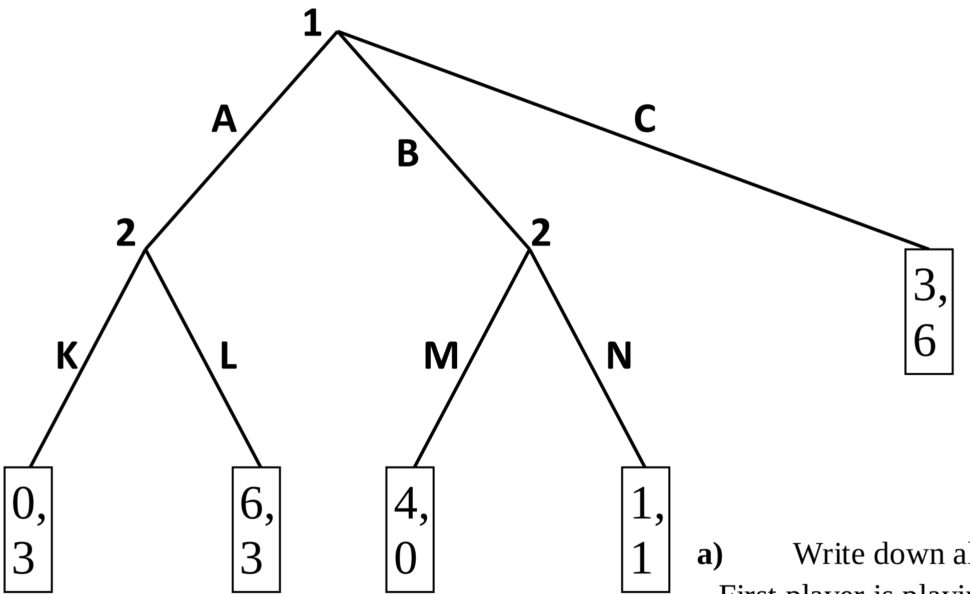
b) Which of the NE are strict?

Only for (C, D) it holds for both players that if any of them would deviate from equilibrium action he or she will be strictly worse off.

Correct answer: (C,D)

Problem 3 (25 points)

Consider the dynamic game, depicted on the tree diagram below. The first number in every end node represents payoff for player 1, the second payoff for player 2.



a) Write down all strategies for player 1

First player is playing only after history ø and he has 3 choices. Therefore he has 3 strategies A, B and C

Correct answer: A,B and C

b) Write down all strategies for player 2

Player 2 is playing after history A and B. Therefore every strategy of player 2 has to specify action after A and action after B. In every state (after A and B) he has two choices. Therefore his strategies are: (K after A, M after B), (K after A, N after B), (L after A, M after B), (L after A, N after B)

Correct answer: KM, KN, LM, LN

c) Find all Subgame Perfect Nash Equilibria of this dynamic game

In subgame of length 1 player 2 has optimal actions: after A – K and L, after B – N.

Therefore we have 2 combinations of these: after A – K, after B – N : KN
after A – L, after B – N : LN

When player 2 is playing KN, it is best for player 1 in the subgame of length 2 (whole game) to play C.

When player 2 is playing LN, it is best for player 1 in the subgame of length 2 (whole game) to play A.

Correct answer: two SBNE: (C, KN) , (A, LN)

Problem 4 (35 points)

Consider the static game, where player 1 chooses rows and player 2 chooses columns. The first number in every cell is payoff for player 1, the second payoff for player 2.

Player 2

Player 1

	A	B	C
A	1,1	3,4	5,5
B	0,7	4,3	6,2

a) Find mixed strategy of player 2 that strictly dominates one of his pure strategies.

If some action is a best response to other player's action, it cannot be strictly dominated. Therefore action A (best response to B) and action C (best response to A) cannot be strictly dominated. Therefore, we have to check if there exist some mixed strategy that dominates action B. Such strategy would exist, only if we can find such probability p that expected utility from playing A with probability p and C with probability (1-p) is always higher than playing B. Therefore we have:

If player 1 is playing A:

$$1p + 5(1 - p) > 4$$
$$1p + 5 - 5p > 4$$
$$1 > 4p$$
$$1/4 > p$$

If player 1 is playing B:

$$7\text{p} + 2(1 - \text{p}) > 3$$
$$7\text{p} + 2 - 2\text{p} > 3$$
$$5\text{p} > 1$$
$$\text{p} > 1/5$$

Correct answer: any mixed strategy for which the expected utility is always higher than playing B. One such example: player 2 is playing A with probability 0.22 and C with probability 0.78.

b) Find all Mixed Strategy Nash Equilibria of the game.

If any player has strictly dominated action, such action will be never played with positive probability in any MSNE. So we have just simple 2x2 game:

Player 2

Player 1

	A(q)	C(1-q)
A(p)	1,1	5,5
B(1-p)	0,7	6,2

The game has no MSNE in pure strategies, so the only possibility is that both of the players are mixing. Assume that player 1 is playing A with probability p and B with 1-p. Further assume that player 2 is playing A with probability q and C with 1-q.

As every player has to be indifferent in MSNE between playing A or B, or A or C, we have following:

For player 1:

$$EU(A) = EU(B)$$
$$1q + 5(1-q) = 0q + 6(1-q)$$
$$1q + 5 - 5q = 0q + 6 - 6q$$
$$5 - 4q = 6 - 6q$$
$$2q = 1$$
$$q = 1/2$$

For player 2:

$$EU(A) = EU(C)$$
$$1p + 7(1-p) = 5p + 2(1-p)$$
$$1p + 7 - 7p = 5p + 2 - 2p$$
$$7 - 6p = 2 + 3p$$
$$9p = 5$$
$$p = 5/9$$

Correct answer: The only MSNE of the game is when player 1 is playing A with probability 5/9 and B with probability 4/9 and player 2 is playing A with probability ½ and C with probability ½.