Nash Equilibrium

Lecture 2

Revision

- Static game of complete information
  - Set of players (firms, political candidates, bidders, etc.)
  - Set of actions (possible choices of player in the game)
  - Set of preferences over the set of action profiles
    - action profile – set of chosen actions by players
- Examples - Prisoner's dilemma, Bach or Stravinsky, Matching pennies, Stag hunt
- Construction of normal form of game
  - According to preferences assign payoffs for every player to each action profile
  - MxN matrix in case of 2 players – M number of actions of player 1, N number of actions of player 2

Revision

- Strictly Dominated actions
  - *Action* ***b_i*** *of i^{th} player is strictly dominated if there exists action* ***a_i*** *such that for every combination of others players' actions payoff when playing* ***a_i*** *is strictly higher than when playing* ***b_i***
- Iterative elimination of strictly dominated actions
  - Rational players do not play strictly dominated actions
  - Common knowledge: all players are rational
  - all the players know that all the players know that all the players are rational etc. – thinking infinitely steps ahead

Notation

- **a** - action profile = set of actions of all players

  $a=(a_1 ,a_2 ,a_3 ,a_4 , \dots ,a_{i-1} ,a_i ,a_{i+1} ,\dots , a_{N-2} ,a_{N-1} ,a_N)$

- $\mathbf{a_i}$ - particular action of i^{th} player
- $\mathbf{a_{-i}}$ - action profile of actions of all players except i^{th} player

  $a_{-i} =(a_1 ,a_2 ,a_3 ,a_4 , \dots , a_{i -1}, a_{i +1} ,\dots , a_{N -2} , a_{N -1} ,a_N)$

  $(a_i ,a_{-i}) = a$

- But if $b_i \neq a_i$

  $(b_i ,a_{-i}) = (a_1 , \dots , a_{i -1}, b_i ,a_{i +1} ,\dots , a_N) \neq a$

Strictly Dominated strategies

- *Strict domination in a static game with ordinal preferences* - action if it is superior no matter what the other players do
- Definition: player *i's action* ***a_i strictly dominates her action b_i if*** $u_i(a_i, a_{-i}) > u_i(b_i, a_{-i})$ *for every list* a_{-i} *of the other players' actions,* where u_i *is a payoff function that represents player i's preferences*
  - *For every combination of others players' actions payoff when playing a is strictly higher than when playing b*
    - $a_{-I} = \{a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_N\}$ *- actions of others players*
- Definition: If any action strictly dominates the action b_i, *we say that* b_i *is* ***strictly dominated***

Weakly Dominated strategies

- Definition: player *i's action* ***a_i weakly dominates her action b_i if*** $u_i(a_i, a_{-i}) \geq u_i(b_i, a_{-i})$ *for every list* a_{-i} *of the other players' actions,* where u_i *is a payoff function that represents player i's preferences*
  - *For every combination of others players' actions payoff when playing a is equal or higher than when playing b*
    - $a_{-I} = \{a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N\}$ *- actions of others players*
- Definition: If any action weakly dominates the action b_i, *we say that* b_i is ***weakly dominated***

Nash Equilibrium

- Definition:

  A *Nash equilibrium (NE) is an action profile* a^* *with the property that no* player *i can do better by choosing an action different from* a^*_i *, given* that every other player *j adheres to* a^*_j

Nash Equilibrium - assumptions

- Each player chooses best available action
  - Best action depends on other players' actions
- Each player has belief about other players' actions
  - Derived from past experience playing the game
  - Experience sufficient to know how opponents will behave
  - Does not know action of her particular opponents
- Idealized circumstances:
  - For each player - population of many such players; players are selected randomly from each population. Players gain experience about "typical" opponents, but not any specific set of opponents

Nash Equilibrium

- Definition:

  The action profile a^* *is a* Nash equilibrium if, for every player *i and every action* b_i *of player i,* a^* *is at least as good according* to player *i's preferences as the action profile* (b_i , a^*_{-i}) *in which player i chooses* b_i while every other player *chooses* a^*_{-i} *.*

  Equivalently, for every player i,

  $u_i(a^*) \geq u_i(b_i , a^*_{-i})$ *for every action* b_i *of player i,* where u_i *is a payoff function that represents player i's preferences.*

Example 1: Prisoner's Dilemma

Suspect 2

Suspect 1

	Quiet	Fink
Quiet	**2, 2**	**0, 3**
Fink	**3, 0**	**1, 1**

Example 1: Prisoner's Dilemma

Suspect 2

Suspect 1

	Quiet	Fink
Quiet	**2, 2**	**0, 3**
Fink	**3, 0**	**1, 1**

Strict Nash equilibrium

- NE is strict if each player's equilibrium action is *better than all her other actions, given the other players' actions*

- Definition: action profile a^* *is a* ***strict NE*** *if for every player i we have* $u_i(a^*) > u_i(b_i, a^*_{-i})$ *for every action* $b_i \neq a^*_i$ *of every player i*

- every non-equilibrium action for a player yields her a payoff less than does her equilibrium action, and hence does not weakly dominate the equilibrium action

Example 2: Bach or Stravinsky?

Friend 2

Friend 1

	Bach	Stravinsky
Bach	**2, 1**	**0, 0**
Stravinsky	**0, 0**	**1, 2**

Example 2: Bach or Stravinsky?

Friend 2

Friend 1

	Bach	Stravinsky
Bach	**2, 1**	**0, 0**
Stravinsky	**0, 0**	**1, 2**

Strict NE

Example 3: Matching Pennies

Person 2

Person 1

	Head	Tail
Head	**1, -1**	**-1, 1**
Tail	**-1, 1**	**1, -1**

Example 3: Matching Pennies

Person 2

Person 1

	Head	Tail
Head	**1, -1**	**-1, 1**
Tail	**-1, 1**	**1, -1**

No NE in pure strategies, only in mixed strategies – will cover next lecture

Example 4: Stag Hunt 2 hunters

Hunter 2

Hunter 1

	Stag	Hare
Stag	**2**, **2**	**0**, **1**
Hare	**1**, **0**	**1**, **1**

Example 4: Stag Hunt 2 hunters

Strict NE

Hunter 2

Hunter 1

	Stag	Hare
Stag	**2, 2**	**0, 1**
Hare	**1, 0**	**1, 1**

Strict NE

Example 4: Stag Hunt N players

(Stag, Stag, Stag, Stag , Stag) ….. NE

- all players will have part of Stag

(Hare, …   , Stag   , ... , …) ... not NE – others players playing Stag can increase their utility from changing their action to Hare as well

(Stag, …   , Hare, …., …..) … not NE – player 1 can increase their utility from changing their action to Hare as well

(Hare, Hare, Hare, Hare , Hare)…. NE

Best Response Function

$B_i(a_{-i}) =$

$\{a_i \text{ in } A_i : u_i(a_i, a_{-i}) \geq u_i(a_i, a_{-i}) \text{ for all } a_i \text{ in } A_i\}$

Every member of the set $B_i(a_{-i})$ *is a* ***best response of*** player *i to* a_{-i}*: if each of the other players adheres to* a_{-i} *then player i can do* no better than choose a member of $B_i(a_{-i})$

Best Response Function - NE

The action profile a^* is a Nash equilibrium if and only if every player's action is a best response to the other players' actions.

P 2

P 1

	L	C	R
T	1 , 2	2 , 1	1 , 0
M	2 , 1	0 , 1	0 , 0
B	0 , 1	0 , 0	1 , 2

Best Response Function - NE

The action profile a^* is a Nash equilibrium if and only if every player's action is a best response to the other players' actions.

P 2

P 1

	L	C	R
T	1 , 2	2*, 1	1*, 0
M	2*, 1	0 , 1	0 , 0
B	0 , 1	0 , 0	1*, 2

Best Response Function - NE

The action profile a is a Nash equilibrium if and only if every player's action is a best response to the other players' actions.*

P 2

P 1

	L	C	R
T	1 , 2*	2*, 1	1*, 0
M	2*, 1*	0 , 1*	0 , 0
B	0 , 1	0 , 0	1*, 2*

Best Response Function - NE

The action profile a is a Nash equilibrium if and only if every player's action is a best response to the other players' actions.*

Non-Strict NE

P 2

P 1

	L	C	R
T	1 , 2*	2*, 1	1*, 0
M	2*, 1*	0 , 1*	0 , 0
B	0 , 1	0 , 0	1*, 2*

NE – strict and non-strict

P 2

P 1

P3:A	A	B	C
A	2, 4, 1	3, 0, 3	5, 8, 8
B	1, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 1, 7	4, 3, 5

P 2

P3:B	A	B	C
A	1, 7, 8	5, 0, 1	2, 2, 5
B	4, 0, 3	6, 4, 3	7, 3, 5
C	1, 5, 3	3, 5, 6	1, 6, 1

P 2

P 1

P3:C	A	B	C
A	7, 4, 7	3, 6, 8	3, 1, 2
B	2, 5, 3	4, 6, 1	4, 1, 6
C	1, 5, 4	3, 4, 2	1, 4, 2

NE – strict and non-strict

P 2

P3:A	A	B	C
A	2, 4, 1	3, 0, 3	5, 8, 8
B	1, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 1, 7	4, 3, 5

P 1

P 2

P3:B	A	B	C
A	1, 7, 8	5, 0, 1	2, 2, 5
B	4, 0, 3	6, 4, 3	7, 3, 5
C	1, 5, 3	3, 5, 6	1, 6, 1

P 2

P3:C	A	B	C
A	7, 4, 7	3, 6, 8	3, 1, 2
B	2, 5, 3	4, 6, 1	4, 1, 6
C	1, 5, 4	3, 4, 2	1, 4, 2

P 1

Player's 1 action A weakly dominates action C

NE – strict and non-strict

P 2

P3:A	A	B	C
A	2, 4, 1	3, 0, 3	5, 8, 8
B	1, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 1, 7	4, 3, 5

P 1

P 2

P3:B	A	B	C
A	1, 7, 8	5, 0, 1	2, 2, 5
B	4, 0, 3	6, 4, 3	7, 3, 5
C	1, 5, 3	3, 5, 6	1, 6, 1

P 2

P3:C	A	B	C
A	7, 4, 7	3, 6, 8	3, 1, 2
B	2, 5, 3	4, 6, 1	4, 1, 6
C	1, 5, 4	3, 4, 2	1, 4, 2

P 1

Player's 1 action A weakly dominates action C

NE – strict and non-strict

P 2

P 1

P3:A	A	B	C
A	2, 4, 1	3, 0, 3	5, 8, 8
B	1, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 1, 7	4, 3, 5

P 2

P3:B	A	B	C
A	1, 7, 8	5, 0, 1	2, 2, 5
B	4, 0, 3	6, 4, 3	7, 3, 5
C	1, 5, 3	3, 5, 6	1, 6, 1

P 2

P 1

P3:C	A	B	C
A	7, 4, 7	3, 6, 8	3, 1, 2
B	2, 5, 3	4, 6, 1	4, 1, 6
C	1, 5, 4	3, 4, 2	1, 4, 2

Player's 1 action A weakly dominates action C

NE – strict and non-strict

P 2

P3:A	A	B	C
A	2, 4, 1	3, 0, 3	5, 8, 8
B	1, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 1, 7	4, 3, 5

P 1

P 2

P3:B	A	B	C
A	1, 7, 8	5, 0, 1	2, 2, 5
B	4, 0, 3	6, 4, 3	7, 3, 5
C	1, 5, 3	3, 5, 6	1, 6, 1

P 2

P3:C	A	B	C
A	7, 4, 7	3, 6, 8	3, 1, 2
B	2, 5, 3	4, 6, 1	4, 1, 6
C	1, 5, 4	3, 4, 2	1, 4, 2

P 1

Player's 1 action A weakly dominates action C

NE – strict and non-strict

Strict NE

P 2

P3:A	A	B	C
P 1 A	2, 4, 1	3, 0, 3	5, 8, 8
B	1, 5, 4	4, 2, 3	2, 3, 7
C	2, 5, 4	1, 1, 7	4, 3, 5

P 2

P3:B	A	B	C
A	1, 7, 8	5, 0, 1	2, 2, 5
B	4, 0, 3	6, 4, 3	7, 3, 5
C	1, 5, 3	3, 5, 6	1, 6, 1

Non-Strict NE

P 2

P3:C	A	B	C
P 1 A	7, 4, 7	3, 6, 8	3, 1, 2
B	2, 5, 3	4, 6, 1	4, 1, 6
C	1, 5, 4	3, 4, 2	1, 4, 2

Non-Strict NE

Player's 1 action A weakly dominates action C

Best Response Function - NE

Two individuals are involved in a synergistic relationship. If both individuals devote more effort to the relationship, they are both better off. For any given effort of individual *j, the return to individual i's* effort first increases, then decreases.

$$u_i = a_i(c + a_j - a_i)$$

Best Response Function - NE

$b(a_1) = \frac{1}{2}(c + a_2)$

$b(a_2) = \frac{1}{2}(c + a_1)$

NE: $a_1 = b_1(a_2)$ *and* $a_2 = b_2(a_1)$

$a_1 = \frac{1}{2}(c + \frac{1}{2}(c + a_1)) = \frac{3}{4}c + \frac{1}{4}a_1$

$a_1 = c$

unique Nash equilibrium $(a_1, a_2) = (c, c)$

Nash Equilibrium - properties

- Any finite game (finite number of players and actions) has at least one Nash equilibrium (include mixed strategies)
- All Nash equilibria survive iterative elimination of strictly dominant strategies
  - If iterative elimination give us solution to the game it is Nash equilibrium
- Nash equilibrium corresponds to a steady state
  - embodies a stable “social norm”: if everyone else adheres to it, no individual wishes to deviate from it
  - whether appropriate model of given situation – matter of judgment

Nash Equilibrium - properties

- **Is Nash Equilibrium a Good Prediction?**
  - Is the equilibrium a "good" outcome? Could the players jointly do better at another profile? (communication, repetition, sanctions) strict, non-strict NE
  - Is this game to be played just once … or a few times … or a great many times, repeatedly? - may converge to NE
  - how much experience do players have?
  - How much do the players know about one another? Have they communicated beforehand?
  - How many firms (players) are there?
    - just 2 players - easier to coordinate or punish
    - With many players, it might be difficult to detect and punish "cheaters"

Experiment

You may participate in a microeconomic experiment. You are free to leave, those of you who would like to participate please sit into the front rows of the class.

There will be several rounds. The winner of each round will receive a prize of 0.5 extra point valid toward the midterm exam.

A person so lucky as to guess the right number in all rounds might get 2 points (if we will have enough time to play 4 rounds)!

Experiment

In this game you will have to make decisions repeatedly in several rounds.

In every round you should choose a number in the interval [0–100]. You are allowed to choose 0 and 100. The winning number is the number closest to 2/3 of the average of all the numbers chosen by your group:

numbers A,B,C,D,E

(A+B+C+D+E)/5=average

winning number – the closest to the 2/3*average

Experiment

The rules of the game in each period are as follows:

Each row is separate group for the experiment. I will pass each row a paper and every person will write his or her number and name on it, and fold the paper in such a way that the written numbers are not visible. Then he or she passes it to the next person and the process continues. Please make sure that the number is not visible and that the next person is not unfolding the paper as you act as competitors in the experiment and if anybody knows your strategy he can profit from it. Also try to be honest and do not look at what are your colleagues writing.

Summary

- Experiment – results at my webpage
- Nash equilibrium
- Best response function
- Gibbons 1.1.C-1.2.D; Osborne 2.6-3

NEXT WEEK:

Examples of NE, mixed strategies, mixed strategy Nash equilibrium