STATIC GAMES with incomplete information – Bayesian Games

Lecture 10

Final exam, make up midterms

- Make-up midterms:
  - 16.12.2009 9:00 NB350 – capacity 20 (priority given to exchange students)
  - 05.01.2010 16:00 RB211 – capacity 30
  - 11.01.2010 9:00 RB210 – capacity 30
- Final exam:
  - 16.12.2009 10:30 NB350 – capacity 15 (priority given to exchange students)
  - 05.01.2010 17:45 Vencovského aula – capacity 100
  - 11.01.2010 11:00 NB D – capacity 50
  - 18.01.2010 10:15 NB D – capacity 50

Revision - Bayesian Games

A strategic game with imperfect information is called a "Bayesian game" and consists of:

- Set of players
- Set of states

And for each player:

- Set of actions
- Set of signals that she may receive and a signal function that associates a signal with each state
- for each signal that she may receive, a **belief about the states consistent with** the signal (a probability distribution over the set of states with which the signal is associated)
- vNM preferences over pairs ***(a, ω), where a is an action profile and*** *ω is a state*

How to find Nash Equilibrium

- Given the BAYESIAN GAME
  1) Find all types of players – what signal may each player receive?

     Example 1 – 2 types of RADKA → 3 players

     Example 2 – 2 types of ADAM and RADKA → 4 players
  2) What are the beliefs of each player type after receiving the signal?

     Example 1 – ADAM→ ½ and ½ . 1st Radka – 1 and 0

     2nd Radka – 0 and 1
  3) Given the beliefs, compute EU for each possible action of each type, given the action profile of the other players' types

How to find Nash Equilibrium

- Given the BAYESIAN GAME
  3) Given the beliefs, compute EU for each possible action of each type, given the action profile of the other players' types

  Example 2: If 2nd RADKA believes that 1st type ADAM plays – NOHAV., 2nd type ADAM – D.BILL: EU (D.BILL) = 2/3 * 1 + 1/3 * 0 = 2/3

  4) Given computed all the EU for all types of all players and all possible action profiles, find NE of this game → such that no type of player A have any incentive to deviate given the actions of all other players' types

  Example 1 and 2 – large table jointly for all 3 players (Ex1) or for all 4 players (Ex2)

Example 1: More information may hurt

A decision-maker in a single-person decision problem cannot be worse off if she has more information: if she wishes, she can ignore the information.

In a game the same is not true: if a player has more information and the other players know that she has more information then she may be worse off.

PLAYER 2

PLAYER 1

½ ½

1st state	L	M	R
T	**1**, **4**	**1**, **0**	**1**, **6**
B	**2**, **16**	**0**, **0**	**0**, **24**

½ ½

2nd state	L	M	R
T	**1**, **4**	**1**, **6**	**1**, **0**
B	**2**, **16**	**0**, **24**	**0**, **0**

Example 1: More information may hurt

Consider, for example, the two-player Bayesian game below. In this game there are two states, and neither player knows the state. And both players believes that the 1st state occurs with probability ½ and the 2nd state occurs with probability ½ .
As both players know that the other player cannot distinguish the state, they believe that there is just one uninformed type of the other player.

PLAYER 2

½ ½

1st state	L	M	R
T	1, 4	1, 0	1, 6
B	2, 16	0, 0	0, 24

½ ½

2nd state	L	M	R
T	1, 4	1, 6	1, 0
B	2, 16	0, 24	0, 0

P1

Example 1: More information may hurt

Signal and signal function define the amount of information players have. Both players receives uninformative signal. The quality of signal depends on the number of distinct values it may have.

As both receives for both states same signal, they cannot distinguish the state. The signal also define the types of the players. Same signal → just one type of every player.

PLAYER 2

P1

½ ½ 1st state	L	M	R
T	1, 4	1, 0	1, 6
B	2, 16	0, 0	0, 24

½ ½ 2nd state	L	M	R
T	1, 4	1, 6	1, 0
B	2, 16	0, 24	0, 0

Example 1: More information may hurt

If P1 believes that P2 will choose L:

EU (T) = ½ * 1 + ½ * 1 = 1 EU (B) = ½ * 2 + ½ * 2 = 2

If P1 believes that P2 will choose M:

EU (T) = ½ * 1 + ½ * 1 = 1 EU (B) = ½ * 0 + ½ * 0 = 0

If P1 believes that P2 will choose R:

EU (T) = ½ * 1 + ½ * 1 = 1 EU (B) = ½ * 0 + ½ * 0 = 0

PLAYER 2

PLAYER 1

½ ½

1st state	L	M	R
T	1, 4	1, 0	1, 6
B	2, 16	0, 0	0, 24

½ ½

2nd state	L	M	R
T	1, 4	1, 6	1, 0
B	2, 16	0, 24	0, 0

Example 1: More information may hurt

If P2 believes that P1 will choose T:

EU (L) = ½ * 4 + ½ * 4 = 4

EU (M) = ½ * 0 + ½ * 6 = 3

EU (R) = ½ * 6+ ½ * 0 = 3

If P2 believes that P1 will choose B:

EU (L) = ½ * 16 + ½ * 16 = 16

EU (M) = ½ * 0 + ½ * 24 = 12

EU (R) = ½ * 24 + ½ * 0 = 12

PLAYER 2

PLAYER 1

½ ½

1st state	L	M	R
T	**1, 4**	**1, 0**	**1, 6**
B	**2, 16**	**0, 0**	**0, 24**

½ ½

2nd state	L	M	R
T	**1, 4**	**1, 6**	**1, 0**
B	**2, 16**	**0, 24**	**0, 0**

Example 1: More information may hurt

As both players know that the other player cannot distinguish the state, they believe that there is just one uninformed type of the other player.

PLAYER 2

PLAYER 1

EU	L	M	R
T	**1, 4**	**1, 3**	**1, 3**
B	**2, 16**	**0, 12**	**0, 12**

Example 1: More information may hurt

When we plug the computed expected utilities, we can see, that player 2 has dominant strategy to play L, no matter what is the player 1's action.
Thus (B, L) is the unique Nash equilibrium of the game, yielding player 1 a payoff of 2 and player 2 payoff 16.

PLAYER 2

PLAYER 1

EU	L	M	R
T	**1**, **4**	**1**, **3**	**1**, **3**
B	**2**, **16**	**0**, **12**	**0**, **12**

Example 1: More information may hurt

Now consider the variant of this game in which player 2 is informed of the state: player 2's signal function τ_2 satisfies $\tau_2(\omega_1) = \tau_2(\omega_2)$. In other words, the player 2 is able, after receiving the signal, distinguish between two states. As player 1 knows that player 2 is receiving such signal, he now believes that there are two types of player 2.

PLAYER 1

½ **1st PLAYER 2**

1st state	L	M	R
T	**1, 4**	**1, 0**	**1, 6**
B	**2, 16**	**0, 0**	**0, 24**

½ **2nd PLAYER 2**

2nd state	L	M	R
T	**1, 4**	**1, 6**	**1, 0**
B	**2, 16**	**0, 24**	**0, 0**

Example 1: More information may hurt

Further, player 1 can see, that both types of player 2 has dominant action.
For first type of player 2 it is best to play R, no matter of what is P1's action
For second type of player 2 it is best to play M, no matter what is P1's action.

PLAYER 1

½ 1st PLAYER 2

1st state	L	M	R
T	1, 4	1, 0	1, 6
B	2, 16	0, 0	0, 24

½ 2nd PLAYER 2

2nd state	L	M	R
T	1, 4	1, 6	1, 0
B	2, 16	0, 24	0, 0

Example 1: More information may hurt

If P1 believes that 1st P2 will choose R and 2nd P2 - M:

EU (T) = ½ * 1 + ½ * 1 = 1   EU (B) = ½ * 0 + ½ * 0 = 0

PLAYER 1

½ **1st PLAYER 2**

1st state	L	M	R
T	**1, 4**	**1, 0**	**1, 6**
B	**2, 16**	**0, 0**	**0, 24**

½ **2nd PLAYER 2**

2nd state	L	M	R
T	**1, 4**	**1, 6**	**1, 0**
B	**2, 16**	**0, 24**	**0, 0**

Example 1: More information may hurt

When we plug the computed expected utilities, we can see, that it is better for player 1 to play T.

Thus (T, (R,M)) is the unique Nash equilibrium of the game, yielding player 1 a payoff of 1 and player 2 payoff 6.

Player 2's payoff in the unique Nash equilibrium of the original game is 16,

whereas her payoff in the unique Nash equilibrium of the game in which she knows the state is 6 in each state. Thus she is worse off when she knows the state than when she does not.

PLAYER 2

PLAYER 1

EU	R,M
T	1, 6, 6
B	0, 24, 24

Example 1: More information may hurt

Player 2's action R is good only in state ω_1 whereas her action M is good only in state ω_2. When she does not know the state she optimally chooses L, which is better than the average of R and M whatever player 1 does. Her choice induces player 1 to choose B.

When player 2 is fully informed she optimally tailors her action to the state, which induces player 1 to choose T. There is no steady state in which she ignores her information and chooses L because this action leads player 1 to choose B, making R better for player 2 in state ω_1 and M better in state ω_2.

PLAYER 2

PLAYER 1 EU	R,M
T	**1**, **6**, **6**
B	**0**, **24**, **24**

Example 2: Infection

The notion of a Bayesian game may be used to model not only situations in which players are uncertain about each others' preferences, but also situations in which they are uncertain about each others' knowledge. Consider, for example, the following Bayesian game: Player 1 receives such signal (information), that he can distinguish between states (1) and (2 or 3).

1st P 1

1	L	R
L	**2**, 2	**0**, 0
R	**3**, 0	**1**, 1

2nd P 1

¾

2	L	R
L	**2**, 2	**0**, 0
R	**0**, 0	**1**, 1

¼

3	L	R
L	**2**, 2	**0**, 0
R	**0**, 0	**1**, 1

Example 2: Infection

The notion of a Bayesian game may be used to model not only situations in which players are uncertain about each others' preferences, but also situations in which they are uncertain about each others' knowledge. Consider, for example, the following Bayesian game: Player 1 receives such signal (information), that he can distinguish between states (1 or 2) and (3).

1st P2

¾

1	L	R
L	2, 2	0, 0
R	3, 0	1, 1

¼

2	L	R
L	2, 2	0, 0
R	0, 0	1, 1

2nd P2

3	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Example 2: Infection

P2's preferences are same in all three states, and P1's preferences are the same in states *2 and 3.* In particular, in state 3, each player knows the other player's preferences, and player 2 knows that player 1 knows her preferences.

¾ 1st P2 ¼ | **2nd P2**

1st P 1

1	L	R
L	2, 2	0, 0
R	3, 0	1, 1

2nd P 1 ¾ ¼

2	L	R
L	2, 2	0, 0
R	0, 0	1, 1

3	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Example 2: Infection

The shortcoming in the players' information in state 3 is that P1 does not know that P2 knows her preferences: P1 knows only that the state is either 2 or 3, and in state 2 P2 does not know whether the state is 1 or 2, and hence does not know P1's preferences (because player 1's preferences in these two states differ).

1st P2: ¾ (states 1 and 2), ¼ (state 3)
2nd P2: state 3

1st P 1 (state 1)

1	L	R
L	2, 2	0, 0
R	3, 0	1, 1

2nd P 1 (states 2 and 3): ¾ (state 2), ¼ (state 3)

2	L	R
L	2, 2	0, 0
R	0, 0	1, 1

3	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Example 2: Infection

1st Player 1 knows exactly the state and for him it is always better to play R than L, no matter what is 1st P2 playing
(2nd P2 never affects the payoff of 1st P1)

¾ **1st P2** ¼ | **2nd P2**

1st P 1

1	L	R
~~L~~	~~2, 2~~	~~0, 0~~
R	3, 0	1, 1

2nd P 1 ¾

2	L	R
L	2, 2	0, 0
R	0, 0	1, 1

¼

3	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Example 2: Infection

If 2nd P1 believes that 1st P2 will choose L and 2nd P2 - L:

EU (L) = ¾ * 2 + ¼ * 2 = 2 EU (R) = ¾ * 0 + ¼ * 0 = 0

If 2nd P1 believes that 1st P2 will choose L and 2nd P2 - R:

EU (L) = ¾ * 2 + ¼ * 0 = 6/4=3/2 EU (R) = ¾ * 0 + ¼ * 1 = ¼

1st P2: ¾ / ¼ — 2nd P2

1st P 1

1	L	R
~~L~~	~~2, 2~~	~~0, 0~~
R	3, 0	1, 1

2nd P 1 ¾

2	L	R
L	2, 2	0, 0
R	0, 0	1, 1

¼

3	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Example 2: Infection

If 2nd P1 believes that 1st P2 will choose R and 2nd P2 - L:

EU (L) = ¾ * 0 + ¼ * 2 = ½ EU (R) = ¾ * 1 + ¼ * 0 = ¾

If 2nd P1 believes that 1st P2 will choose R and 2nd P2 - R:

EU (L) = ¾ * 0 + ¼ * 0 = 0 EU (R) = ¾ * 1 + ¼ * 1 = 1

¾ **1st P2** ¼ | **2nd P2**

1st P 1

1	L	R
~~L~~	~~2, 2~~	~~0, 0~~
R	3, 0	1, 1

2nd P 1 ¾

2	L	R
L	2, 2	0, 0
R	0, 0	1, 1

¼

3	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Example 2: Infection

If 1st P2 believes that 1st P1 will choose R and 2nd P1 - L:

EU (L) = ¾ * 0 + ¼ * 2 = ½ EU (R) = ¾ * 1 + ¼ * 0 = ¾

If 1st P2 believes that 1st P1 will choose R and 2nd P1 - R:

EU (L) = ¾ * 0 + ¼ * 0 = 0 EU (R) = ¾ * 1 + ¼ * 1 = 1

1st P2: ¾ (1st P 1), ¼ (2nd P 1). 2nd P2.

1st P 1

1	L	R
~~L~~	~~2, 2~~	~~0, 0~~
R	3, 0	1, 1

2nd P 1 — ¾

2	L	R
L	2, 2	0, 0
R	0, 0	1, 1

¼

3	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Example 2: Infection

For 2nd P2 – the EUs are exactly the payoffs in state 3

1st P2

¾ / **¼**

2nd P2

1st P 1

1	L	R
~~L~~	~~2, 2~~	~~0, 0~~
R	3, 0	1, 1

2nd P 1 **¾** / **¼**

2	L	R
L	2, 2	0, 0
R	0, 0	1, 1

3	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Example 2: Infection

We can represent the game in one joint table. Each column and row of the table is a pair of actions for the two types of players, the first action of each pair refers to the action of the 1st type the second to the action of the 2nd type. We have just two rows as 1st type of player 1 have strictly dominant action R.

First number in each cell represents EU of 1st type of P1, second number is EU of 2nd P1, third one is EU of 1st type of P2 and fourth one is EU of 2nd type of P2

		L , L	L , R	R, L	R, R
P1	R, L	**3, 2, ½ , 2**	**3,3/2, ½, 0**	**1,½, ¾ , 2**	**1, 0, ¾, 0**
	R, R	**3, 0, 0, 0**	**3, ¼ , 0, 1**	**1, ¾ , 1, 0**	**1, 1, 1, 1**

Example 2: Infection

The imperfection in player 1's knowledge of player 2's information significantly affects the equilibria of the game. If information were perfect in state 3, then both (L, L) and (R, R) would be Nash equilibria. However, the whole game has a unique Nash equilibrium ((R,R),(R,R)), in which the outcome in state 3 is (R, R).The argument shows that the incentives faced by player 1 in state 1 "infect" the remainder of the game.

		L , L	L , R	R, L	R, R
P1	R, L	**3**, **2**, **½** , **2**	**3**,**3/2**, **½**, **0**	**1**,**½**, **¾** , **2**	**1**, **0**, **¾**, **0**
	R, R	**3**, **0**, **0**, **0**	**3**, **¼** , **0**, **1**	**1**, **¾** , **1**, **0**	**1**, **1**, **1**, **1**

Example 3: Cournot's duopoly

Consider a Cournot duopoly model (two firms compete in selling a good) with inverse demand given by $P(Q) = a – Q$, where $Q=q_1+q_2$ is the aggregate quantity of the good on the market. Firm 1's cost function is $C_1(q_1) = cq_1$. Firm 2's cost function, however, is $C_2(q_2)=c_Hq_2$ with probability θ and $C_2(q_2)=c_Lq_2$ with probability $1-\theta$, where $c_L<c_H$.

Furthermore, information is asymmetric: firm 2 knows its cost function and firm 1's, but firm 1 knows its cost function and only that firm 2's marginal cost is c_H with probability θ and c_L with probability $1-\theta$. (Firm 2 could be a new entrant to the industry, or could have just invented a new technology → receives signal about its cost). All of this is common knowledge: firm 1 knows that firm 2 has superior information, firm 2 knows that firm 1 knows this, and so on.

Example 3: Cournot's duopoly

- Set of players: firm 1 and firm 2
- Set of states: { L, H }

And for each player:

- Set of actions: Each firm's set of actions is the set of its possible outputs (nonnegative numbers).
- Set of signals: Firm 1's signal function τ_1 satisfies $\tau_1(H) = \tau_2(L)$ (its signal is the same in both states); firm 2's signal function τ_2 satisfies $\tau_2(H) \neq \tau_2(L)$ (its signal is perfectly informative of the state).
- beliefs: The single type of firm 1 assigns probability *θ to state L and probability* $1 - \theta$ *to state H. Each type of firm 2 assigns probability 1 to the single* state consistent with its signal.

Example 3: Cournot's duopoly

Preferences: The firms' payoffs are their profits; if the actions chosen are (q_1, q_2) and the state is I (either L or H) then firm 1's profit is $q_1(P(q_1 + q_2) - c)$ and firm 2's profit is $q_2(P(q_1 + q_2) - c_I)$, where $P(q_1 + q_2)$ is the market price when the firms' outputs are q_1 and q_2.

Firm 1

θ

Firm 2 – type L

STATE L:

Firm 1: C

Firm 2: C_L

1-θ

Firm 2 – type H

STATE H:

Firm 1: C

Firm 2: C_H

How to find Nash Equilibrium

- Given the BAYESIAN GAME
  1) Find all types of players – what signal may each player receive?

     1 type of firm 1, 2 types of firm 2 → 3 players
  2) What are the beliefs of each player type after receiving the signal?

     The single type of firm 1 assigns probability θ to state L and probability 1 − θ to state H. Each type of firm 2 assigns probability 1 to the single state consistent with its signal.

How to find Nash Equilibrium

A Nash equilibrium of this game is a triple (q^*_1, q^*_L, q^*_H), where q^*_1 is the output of firm 1, q^*_L is the output of type L of firm 2 (i.e. firm 2 when it receives the signal $\tau_2(L)$), and q^*_H is the output of type H of firm 2 (i.e. firm 2 when it receives the signal $\tau_2(H)$), such that:

- q^*_1 maximizes firm 1's profit given the output q^*_L of type L of firm 2 and the output q^*_H of type H of firm 2
- q^*_L maximizes the profit of type L of firm 2 given the output q^*_1 of firm 1
- q^*_H maximizes the profit of type H of firm 2 given the output q^*_1 of firm 1.

Example 3: Cournot's duopoly

To find an equilibrium, we first find the firms' best response functions.

We will start with best response of type L of firm 2:

The firm is maximizing its profit π given the production q_1 of the firm 1, type L firm 2 assigns probability 1 to state L:

$$\max_{(q_L)} [a - (q_1+q_L) - c_L]q_L = (a - q_1 - c_L)q_L - q_L^2$$

taking derivative with respect to q_L

$$a - q_1 - c_L - 2\,q_L = 0$$

$$q_L = (a - q_1 - c_L)/2$$

Example 3: Cournot's duopoly

To find an equilibrium, we first find the firms' best response functions.

We will continue with best response of type H of firm 2:

The firm is maximizing its profit π given the production q_1 of the firm 1, type H firm 2 assigns probability 1 to state H:

$$\max (q_H)\ [a - (q_1+q_H) - c_H]q_H = (a - q_1 - c_H)q_H - q_H^2$$

taking derivative with respect to q_H

$$a - q_1 - c_H - 2\, q_H = 0$$

$$q_H = (a - q_1 - c_H)/2$$

Example 3: Cournot's duopoly

To find an equilibrium, we first find the firms' best response functions.

At last will compute best response of firm 1:

The firm is maximizing its profit π given the production q_L and q_H of both types of the firm 2, firm 1 assigns probability θ to state L and probability $1 - \theta$ to state H :

$$\max (q_1)\ \theta[a - (q_1+q_L) - c]q_1 + (1-\theta)[a - (q_1+q_H) - c]q_1 =$$

$$= \theta[(a - q_L - c)q_1 - q_1^2] + (1-\theta)\ [(a - q_H - c)q_1 - q_1^2]$$

taking derivative with respect to q_1

$$\theta[(a - q_L - c)] + (1-\theta)\ [(a - q_H - c)q_1] - 2q_1 = 0$$

$$q_1 = [a - c - \theta q_L - (1-\theta)\ q_H]/2$$

Example 3: Cournot's duopoly

We have best responses of firm 1 and both types of firm 2. To find NE of: chosen actions of both types of firm 2 have to be best response to firm 1 and vice versa.

Firm 1: $q_1 = [a - c - \theta q_L - (1-\theta)\, q_H]/2$

Firm 2 L: $q_L = (a - q_1 - c_L)/2$

Firm 2 H: $q_H = (a - q_1 - c_H)/2$

By plugging q_L and q_H to q_1 equation we get:

$$q_1 = [a - c - \theta\,(a - q_1 - c_L)/2 - (1-\theta)\,(a - q_1 - c_H)/2]/2$$

$$= [a - c - a/2 + q_1/2 + \theta c_L/2 + (1-\theta)c_H/2]/2$$

$$q_1 = (a/4 - c/2 + \theta c_L/4 + (1-\theta)c_H/4)*4/3$$

$$q_1 = [a - 2c + \theta c_L + (1-\theta)c_H]/3$$

Example 3: Cournot's duopoly

We have best responses of firm 1 and both types of firm 2. To find NE: chosen actions of both types of firm 2 have to be best response to firm 1 and vice versa.

Firm 1: $q_1 = [a – 2c + \Theta c_L + (1-\Theta)c_H]/3$

Firm 2 L: $q_L = (a – q_1 – c_L)/2$

Firm 2 H: $q_H = (a – q_1 – c_H)/2$

By plugging q_1 to q_L and q_H equation we get:

$q_L = (a – a/3 + 2c/3 - \Theta c_L/3 - (1-\Theta)c_H/3 – c_L)/2$

$q_L = (2a/3 + 2c/3 - 4c_L/3 + (1-\Theta)c_L/3 - (1-\Theta)c_H/3)/2$

$q_L = (a + c - 2c_L)/3 + (1-\Theta)(c_L-c_H)/6$

$q_H = (a + c - 2c_H)/3 - \Theta(c_L-c_H)/6$

Example 3: Cournot's duopoly

NE of the game:

Firm 1: $q_1 = [a - 2c + \theta c_L + (1-\theta)c_H]/3$

Firm 2 L: $q_L = (a + c - 2c_L)/3 - (1-\theta)(c_H-c_L)/6$

Firm 2 H: $q_H = (a + c - 2c_H)/3 + \theta(c_H-c_L)/6$

If firm 2's cost is high, for example, it produces less ($c_H>c_L$) because its cost is high, but also produces more ($\theta(c_H-c_L)$) because it knows that firm 1 will produce a quantity that maximizes its expected profit, which is in this game smaller than firm 1 would produce if it knew firm's cost to be high.

($\theta c_L + (1-\theta)c_H<c_H$)

Example 4: Reporting a crime

Consider the variant of the model of reporting crime, where we have just two players.
A crime is observed by 2 people. Each person would like the police to be informed, but prefers that someone else make the phone call. Specifically, suppose that each person attaches the value v to the police being informed and bears the cost c if she makes the phone call, where $v > c > 0$.
The difference is that now each of two players does not know whether she is the only witness, or whether there is another witness. Denote by π the probability each player assigns to being the sole witness. Model this situation as a Bayesian game with three states: one in which player 1 is the only witness, one in which player 2 is the only witness, and one in which both players are witnesses.

Example 4: Reporting a crime

Find a condition on π under which the game has a pure Nash equilibrium in which each player chooses Call (given the signal that she is a witness).

When the condition is violated, find the symmetric mixed strategy Nash equilibrium of the game, and check that when $\pi = 0$ this equilibrium coincides with the perfect information when $n = 2$.

Example 4: Reporting a crime

- Set of players: person 1 and person 2
- Set of states: {1: person 1 the only witness, 2: person 2 the only witness, 3: both players are witnesses }

And for each player:

- Set of actions: { Call, Nothing }
- Set of signals: observing a crime - this signal is same for states 1 and 2 for person 1 and for states 2 and 3 for person 2
- beliefs**:** each person after observing a crime assigns probability 1-θ *to state 3 and probability* θ *to state 1 in case of person 1 and state 2 in case of person 2.*

  Each person assigns probability 1 to the single state 1 or 2 when she does not observe a crime.

Example 4: Reporting a crime

In state 1, only player 1 observes the crime (2nd type of player 1), player 2 does not observe the crime.

But player 1 does not know if player 2 observes the crime as well or not (states 1 or 3). He assigns probability 1-θ to state 3 and probability θ to state 1.

θ **2nd P1** **1-θ**

1	N
C	v-c, 0
N	0, 0

3	C	N
C	v-c, v-c	v-c, v
N	v, v-c	0, 0

1st P1

2	C	N
N	0, v-c	0, 0

Example 4: Reporting a crime

In state 2, only player 2 observes the crime (2nd type of player 2), player 1 does not observe the crime.
But player 2 does not know if player 1 observes the crime as well or not (states 2 or 3). He assigns probability 1-*θ to state 3 and probability θ to state 2.*

1st P 2

1	N
C	**v-c, 0**
N	**0, 0**

2nd P 2 **1-θ**

3	C	N
C	**v-c, v-c**	**v-c, v**
N	**v, v-c**	**0, 0**

θ

2	C	N
N	**0, v-c**	**0, 0**

Example 4: Reporting a crime

If the player does not observe the crime, he will do nothing.

θ — 2nd P1 — 1-θ

1st P1

1st P 2

1	N
C	v-c, 0
N	0, 0

2nd P 2 — 1-θ

3	C	N
C	v-c, v-c	v-c, v
N	v, v-c	0, 0

θ

2	C	N
N	0, v-c	0, 0

Example 4: Reporting a crime

If the player observes the crime, his EU is:

If he believes that other player in state 3 will Call:

EU(C) = θ (v-c) + (1-θ) (v-c) =v-c

EU(N) = θ.0+ (1-θ).v = (1-θ)v

θ **2nd P1** **1-θ** **1st P1**

1st P 2

1	N
C	**v-c, 0**
N	**0, 0**

2nd P 2 **1-θ**

3	C	N
C	**v-c, v-c**	**v-c, v**
N	**v, v-c**	**0, 0**

θ

2	C	N
N	**0, v-c**	**0, 0**

Example 4: Reporting a crime

If the player observes the crime, his EU is:

If he believes that other player in state 3 will not call:

EU(C) = θ (v-c) + (1-θ) (v-c) =v-c

EU(N) = θ.0+ (1-θ).0 = 0

θ 2nd P1 1-θ

1st P2

1	N
C	v-c, 0
N	0, 0

2nd P2 1-θ

3	C	N
C	v-c, v-c	v-c, v
N	v, v-c	0, 0

1st P1

θ

2	C	N
N	0, v-c	0, 0

Example 4: Reporting a crime

We can represent the game in one joint table. Each column and row of the table is a pair of actions for the two types of players, the first action of each pair refers to the action of the 1st type the second to the action of the 2nd type. We have just two rows as 1st type of player 1 is always playing N.

First number in each cell represents EU of 1st type of P1, second number is EU of 2nd P1, third one is EU of 1st type of P2 and fourth one is EU of 2nd type of P2

P 2

		N , C	N , N
P1	N, C	**0, v-c , 0, v-c**	**0, v-c, 0, (1-θ)v**
	N, N	**0, (1-θ)v, 0, v-c**	**0, 0, 0, 0**

Example 4: Reporting a crime

We can get rid of the 1st type of players, as they are playing always N.

First number in each cell represents EU of 2nd type of P1, second number is EU of 2nd P2.

As we can see from the table below, SYMMETRIC pure strategy NE exist only when **$v-c>(1-\theta)v \rightarrow \theta>c/v$**

2nd P 2

2nd P1	C	N
C	**v-c, v-c**	**v-c, (1-θ)v**
N	**(1-θ)v, v-c**	**0, 0**

Example 4: Reporting a crime

We can get rid of the 1st type of players, as they are playing always N.

First number in each cell represents EU of 2nd type of P1, second number is EU of 2nd P2.

IF **$v-c<(1-\theta)v \rightarrow \theta<c/v$, there are two NE in pure strategies, but not symmetric**

2nd P 2

2nd P1		C	N
	C	**v-c, v-c**	**<u>v-c</u>, <u>(1-θ)v</u>**
	N	**<u>(1-θ)v</u>, <u>v-c</u>**	**0, 0**

Example 4: Reporting a crime

$\Theta < c/v$: find symmetric mixed strategy NE

If 2nd P1 is playing C with probability p and N with probability 1-p:

It has to hold for player 2:

$$p\,(v-c) + (1-p)\,(v-c) = p\,(1-\Theta)\,v + (1-p).0$$

$$v-c = p\,(1-\Theta)v$$

$$p = (v-c)/(1-\Theta)v$$

As $\Theta < c/v \rightarrow p < 1$

2nd P 2

2nd P1		C	N
	C	**v-c, v-c**	**v-c, (1-ϴ)v**
	N	**(1-ϴ)v, v-c**	**0, 0**

Example 4: Reporting a crime

When ϴ=0 : which is perfect competition

$$p = (v-c)/(1-\theta)v = (v-c)/v$$

$$p = 1 - c/v$$

So the mixed strategy NE is same as computed in lecture 4 when n=2

2nd P 2

2nd P1		C	N
	C	**v-c, v-c**	**v-c, (1-θ)v**
	N	**(1-θ)v, v-c**	**0, 0**

Summary

- Illustrations of static games with incomplete information – Bayesian games
- Gibbons 3-3.2; Osborne 9-9.6

NEXT WEEK:

Extensive games with imperfect information